Data605-Week3-HomeWork3-kamath

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09/13/2020

Problem set 1

(1) What is the rank of the matrix A?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix} \tag{1}$$

Figure 1: .

```
#defining the sample matrix:
A <- matrix(c(1, -1, 0, 5, 2, 0, 1, 4, 3 ,1, -2, -2, 4, 3, 1, -3), 4, 4)
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```
#run the function qr()
qr(A)$rank
```

```
## [1] 4
```

```
#Alternative: load the Matrix package...
require(Matrix)
```

```
## Loading required package: Matrix
```

```
#and run the function rankMatrix()
rankMatrix(A)[1]
```

```
## [1] 4
```

(2) Given an mxn matrix where $m > n$, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

Figure 2: .

==> *For an mxn matrix (assuming that the matrix is non-zero) where $m > n$; the maximum rank can be n.*

(3) What is the rank of matrix B?

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix} \tag{2}$$

Figure 3: .

```
#defining the square  matrix:
A <- matrix(c(1, 3, 2, 2, 6, 4, 1, 3, 2), 3, 3)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    3    6    3
## [3,]    2    4    2
```

```
#run the function qr()
qr(A)$rank
```

```
## [1] 1
```

```
#Alternative: load the Matrix package...
require(Matrix)
```

```
#and run the function rankMatrix()
rankMatrix(A)[1]
```

```
## [1] 1
```

Problem set 2

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \tag{3}$$

Figure 4: .

```
#defining the square  matrix:
A <- matrix(c(1, 0, 0, 2, 4, 0, 3, 5, 6), 3, 3)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    0    4    5
## [3,]    0    0    6
```

```
#eigen fucntion:
e <- eigen(A)

#eigenvalues:
e$values
```

```
## [1] 6 4 1
```

```
#eigenvectors:
e$vectors
```

```
##            [,1]      [,2] [,3]
## [1,] 0.5108407 0.5547002    1
## [2,] 0.7981886 0.8320503    0
## [3,] 0.3192754 0.0000000    0
```

4

Figure 5: .