NPTEL ONLINE CERTIFICATION COURSES

DIGITAL CONTROL IN SMPCs AND FPGA-BASED PROTOTYPING

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Module 04: Modeling Techniques and Mode Validation using MATLAB

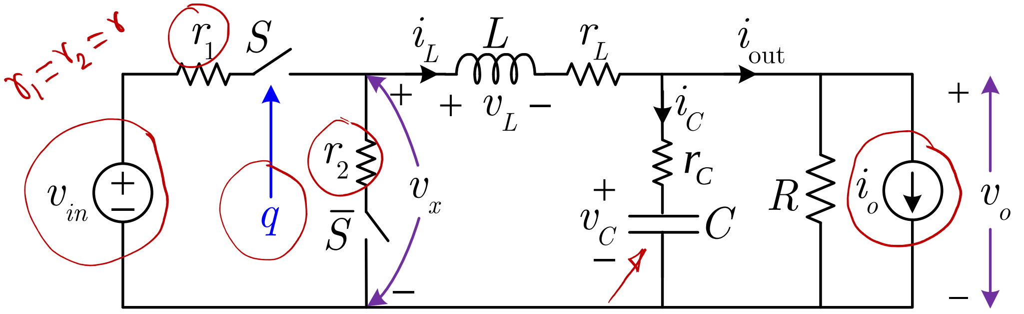
Lecture 33: State-Space Modeling and Steps For Deriving Discrete-Time Models

CONCEPTS COVERED

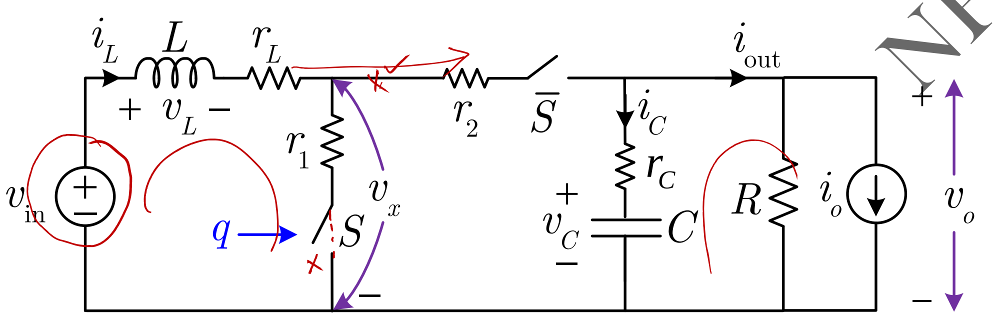
- State space modeling of buck and boost converters
- State space solution vectors
- State space solutions for individual switch configurations
- Guidelines for deriving discrete-time modeling of SMPCs

State-Space Modeling of DC-DC Converters

$r_1 = r_2 = r$



Synchronous buck converter



Synchronous boost converter

State variables:

$x_1 = i_L$ inductor current

$x_2 = v_C$ capacitor voltage

$$x = \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

Input variables:

$u_1 = v_{in}$ input voltage

$u_2 = i_o$ sink current

$$u = \begin{bmatrix} v_{in} \\ i_o \end{bmatrix}$$

State-Space Modeling of DC-DC Converters (contd…)

- State-space model

CCM

$$q = \begin{cases} 1 & \text{if controllable MOSFET is ON} \\ 0 & \text{if " " " OFF} \end{cases}$$

$$\dot{x} = A_q x + B_q u$$

Buck converter matrices

$$A_q = \begin{bmatrix} -\dfrac{(r_e + \alpha r_C)}{L} & -\dfrac{\alpha}{L} \\ \dfrac{\alpha}{C} & -\dfrac{\alpha}{RC} \end{bmatrix} \qquad B_q = \begin{bmatrix} \dfrac{q}{L} & \dfrac{\alpha r_C}{L} \\ 0 & -\dfrac{\alpha}{C} \end{bmatrix}$$

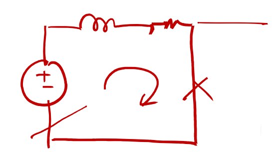
where $r_e = r_1 + r_L$ and $\alpha = \dfrac{R}{R + r_C}$ ← load resistance

$\boxed{r_1 = r_2 = r_1}$

↑ ESR of the capacitor

[For details, refer to Lecture~26, NPTEL "Control and Tuning Methods …" course (Link)]

State-Space Modeling of DC-DC Converters (contd…)



- State-space model

$$\dot{x} = A_q x + B_q u$$

$\frac{di_L}{dt}$

Boost converter matrices

$$A_q = \begin{bmatrix} -\dfrac{(r_e + (1-q)\alpha r_C)}{L} & -\dfrac{(1-q)\alpha}{L} \\ \dfrac{(1-q)\alpha}{C} & -\dfrac{\alpha}{RC} \end{bmatrix} \qquad B_q = \begin{bmatrix} \dfrac{1}{L} & \dfrac{(1-q)\alpha r_C}{L} \\ 0 & -\dfrac{\alpha}{C} \end{bmatrix}$$

where $\quad r_e = r_1 + r_L \quad$ and $\quad \alpha = \dfrac{R}{R + r_C}$

[For details, refer to Lecture~26, NPTEL "Control and Tuning Methods …" course (Link)

Considerations in Discrete-Time Modeling

- solution of state vector for each switch configuration is obtained

$$x(t) = \underbrace{\boxed{e^{A_q(t-t_o)}x(t_o)}}_{\text{Zero-input response}} + \underbrace{\boxed{\int_{t_o}^{t} e^{A_q(t-\tau)}B_q u(\tau)\,d\tau}}_{\text{Zero-state response}}$$

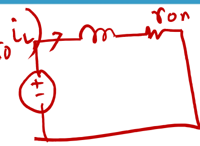
$$u = \begin{bmatrix} v_{in} \\ i_o \end{bmatrix}$$

- input voltage v_{in} and current sink i_o are considered constant
- above assumption is perfectly valid within a switching cycle

$$u(\tau) \triangleq u$$

Zero-State Response

$$\frac{di_L}{dt} = \frac{1}{L}\left(v_{in} - r_o i_L\right)$$



$$A_{on} = \begin{bmatrix} -\frac{r_{on}}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}$$

$$\int_{t_o}^{t} e^{A_q(t-\tau)} B_q u(\tau)\, d\tau = \left\{ \int_{t_o}^{t} e^{A_q(t-\tau)}\, d\tau \right\} B_q u = \left\{ \int_{0}^{t-t_o} e^{A_q\{(t-t_o)-m\}}\, dm \right\} B_q u \quad \textbf{where} \quad m = \tau - t_o$$

$$= e^{A_q(t-t_o)} \left\{ \int_{0}^{t-t_o} e^{-A_q m}\, dm \right\} B_q u = (e^{A_q(t-t_o)} - I) A_q^{-1} B_q u \quad \textbf{assume } A_q \textbf{ to be invertible}$$

Overall state-space solution

$$\boxed{x(t) = e^{A_q(t-t_o)} x(t_o) + (e^{A_q(t-t_o)} - I) A_q^{-1} B_q u}$$

State-Space Matrices under Different Switch Configurations

- Mode M_1 when the control switch S is ON

$$A_q = A_1 \quad \text{and} \quad B_q = B_1$$

- Mode M_2 when the control switch S is OFF

$$A_q = A_2 \quad \text{and} \quad B_q = B_2$$

State-Space Matrices of a Synchronous Buck Converter

$$A_1 = \begin{bmatrix} -\frac{(r_e + \alpha r_C)}{L} & -\frac{\alpha}{L} \\ \frac{\alpha}{C} & -\frac{\alpha}{RC} \end{bmatrix} \qquad B_1 = \begin{bmatrix} \frac{1}{L} & \frac{\alpha r_C}{L} \\ 0 & -\frac{\alpha}{C} \end{bmatrix} \qquad B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{(r_e + \alpha r_C)}{L} & -\frac{\alpha}{L} \\ \frac{\alpha}{C} & -\frac{\alpha}{RC} \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 & \frac{\alpha r_C}{L} \\ 0 & -\frac{\alpha}{C} \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A_1 = A_2$$

in absence of current sink

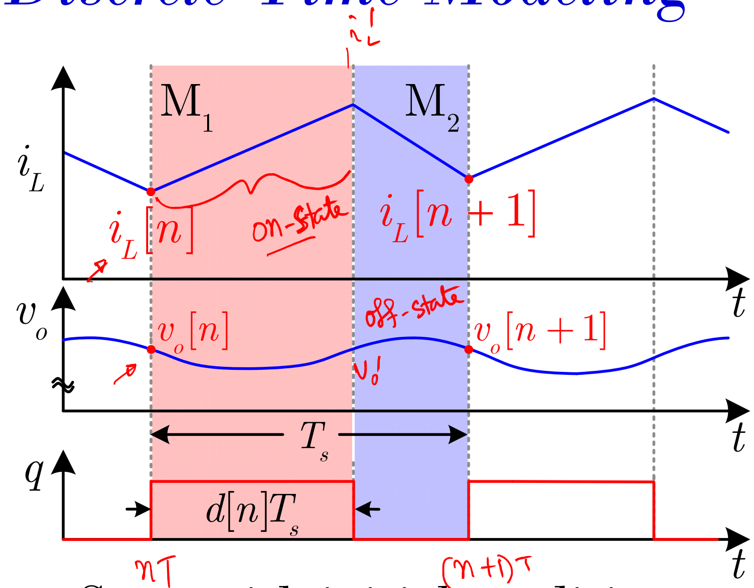
State-Space Matrices of a Synchronous Boost Converter

$$A_1 = \begin{bmatrix} -\dfrac{r_e}{L} & 0 \\ 0 & -\dfrac{\alpha}{RC} \end{bmatrix} \qquad B_1 = \begin{bmatrix} \dfrac{1}{L} & 0 \\ 0 & -\dfrac{\alpha}{C} \end{bmatrix} \qquad B_1 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\dfrac{(r_e + \alpha r_C)}{L} & -\dfrac{\alpha}{L} \\ \dfrac{\alpha}{C} & -\dfrac{\alpha}{RC} \end{bmatrix} \qquad B_2 = \begin{bmatrix} \dfrac{1}{L} & \dfrac{\alpha r_C}{L} \\ 0 & -\dfrac{\alpha}{C} \end{bmatrix} \qquad B_2 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}$$

in absence of current sink

Discrete-Time Modeling – Steps for Derivation



- Start with initial condition x_n at the beginning of n^{th} clock cycle
- Use the solution of state-space equation $\dot{x} = A_q x + B_q u$ for each mode
- Obtain the discrete-map $x_{n+1} = F(x_n)$ over a switching cycle

CONCLUSION

- State space modeling of buck and boost converters
- State space solution vectors
- State space solutions for individual switch configurations
- Guidelines for deriving discrete-time modeling of SMPCs

THANK YOU !