NPTEL ONLINE CERTIFICATION COURSES

DIGITAL CONTROL IN SMPCs AND FPGA-BASED PROTOTYPING

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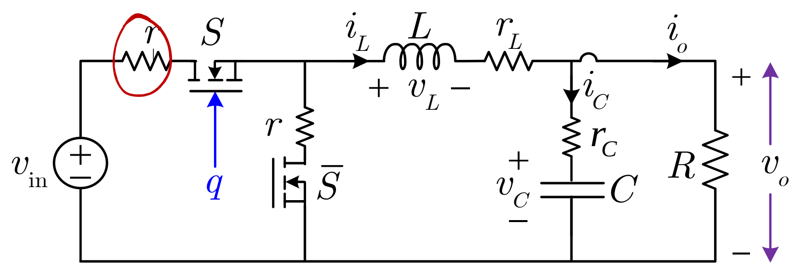
Module 04: Modeling Techniques and Model Validation using MATLAB

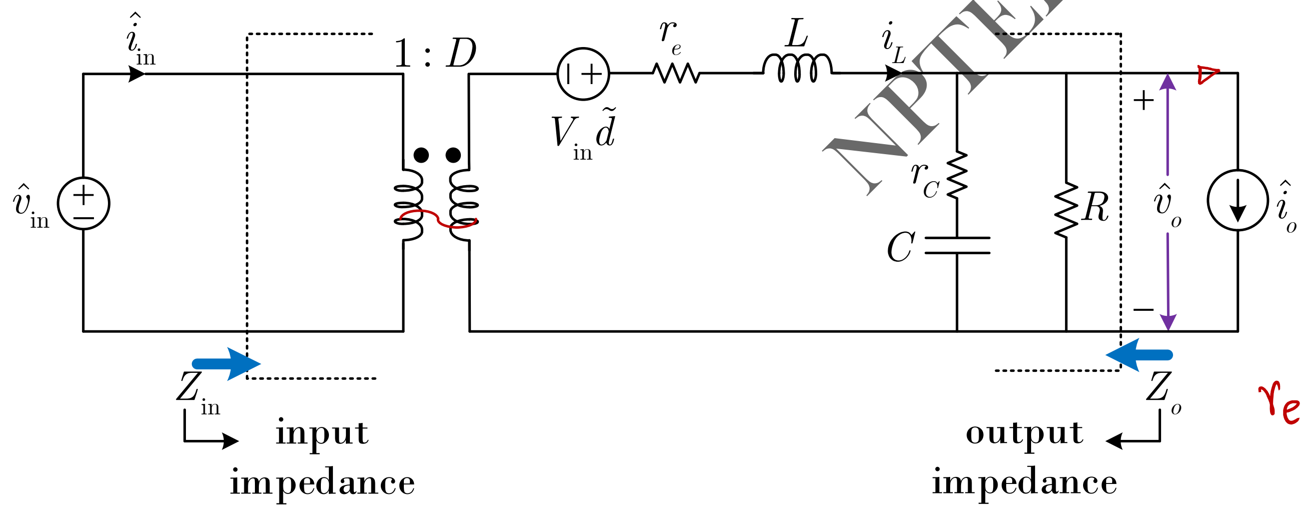
Lecture 31: Continuous-Time Small-Signal Modeling under Digital Control

CONCEPTS COVERED

- Recap of continuous-time small-signal modeling
- Modeling of sampled data-system
- Continuous-time small-signal modeling under digital control

AC Equivalent Circuit of a Practical Synchronous Buck Converter





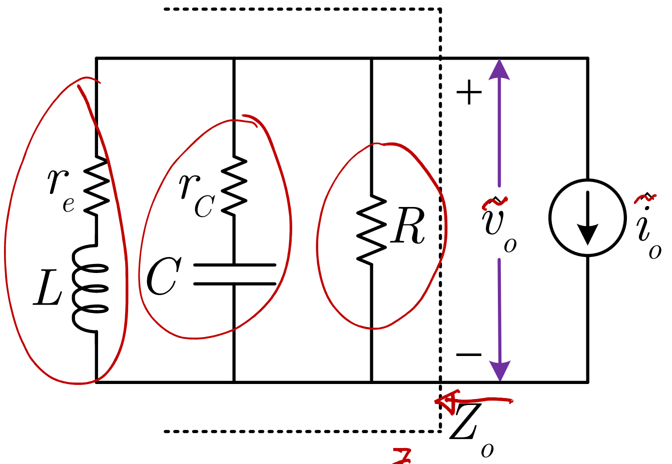
$r_e = r + r_L$

Output Impedance

Assumptions:

a) Converter in open-loop with $\hat{d} = 0$

b) No input voltage perturbation $\hat{v}_{\text{in}} = 0$



$$Z_o(s) = (r_e + sL) \parallel \left(r_C + \frac{1}{Cs}\right) \parallel R$$

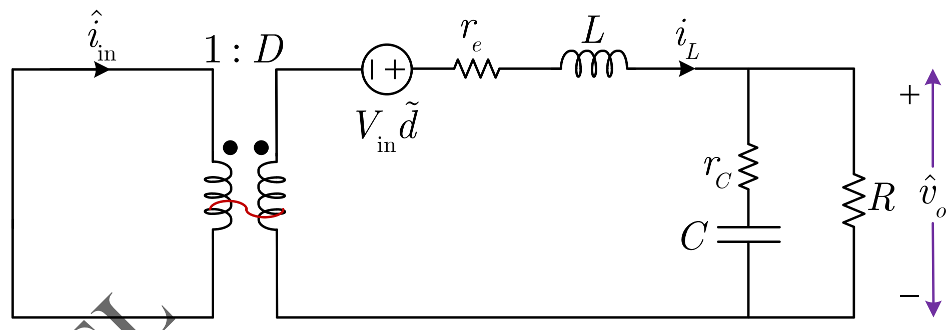
$$\tilde{v}_o = -Z_o \tilde{i}_o$$

[For details, refer to Lecture~33, NPTEL "Control and Tuning Methods …" course (Link)

Control-to-Output TF

$$G_{vd}(s) = \frac{V_{IN}}{\frac{R+r_e}{R}} \frac{(1+r_C Cs)}{\left(1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}\right)}$$

$\tilde{v}_{in}=0$ $\tilde{i}_o=0$



$$Q = \frac{R+r_e}{R}\left[\frac{r_C+r_e}{\sqrt{\frac{L}{C}}}+\frac{\sqrt{\frac{L}{C}}}{R}\right]^{-1}$$

$$\omega_o = \sqrt{\frac{R+r_e}{R+r_C}}\cdot\frac{1}{\sqrt{LC}}$$

[For details, refer to Lecture~33, NPTEL "Control and Tuning Methods …" course (Link)

Complete Small-Signal Block Diagram

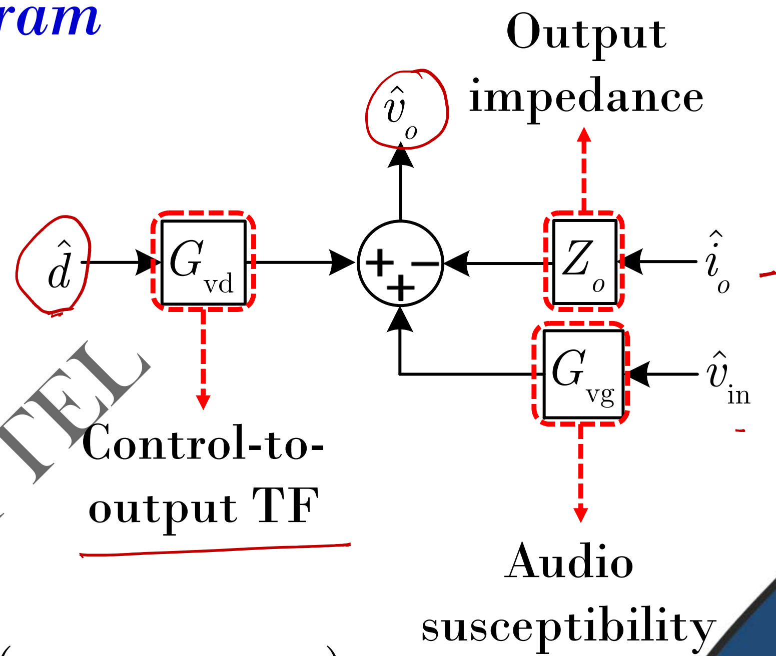
$$G_{vd} = \frac{V_{IN}}{\alpha}\frac{\left(1+\frac{s}{w_{ESR}}\right)}{\Delta(s)}$$

$$\alpha = \frac{R+r_e}{R}$$

$$G_{vg} = \frac{D}{\alpha}\frac{\left(1+\frac{s}{w_{ESR}}\right)}{\Delta(s)}$$

$$Z_o(s) = \frac{r_e}{\alpha}\frac{\left(1+\frac{s}{w_L}\right)\left(1+\frac{s}{w_{ESR}}\right)}{\Delta(s)}$$

$$\Delta(s) = \left(1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}\right)$$



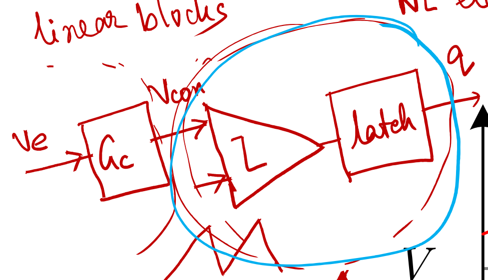
NPTEL

[For details, refer to Lecture~33, NPTEL "Control and Tuning Methods …" course (Link)

Modulator Gain – Pulse Width Modulator

Direct duty ratio control

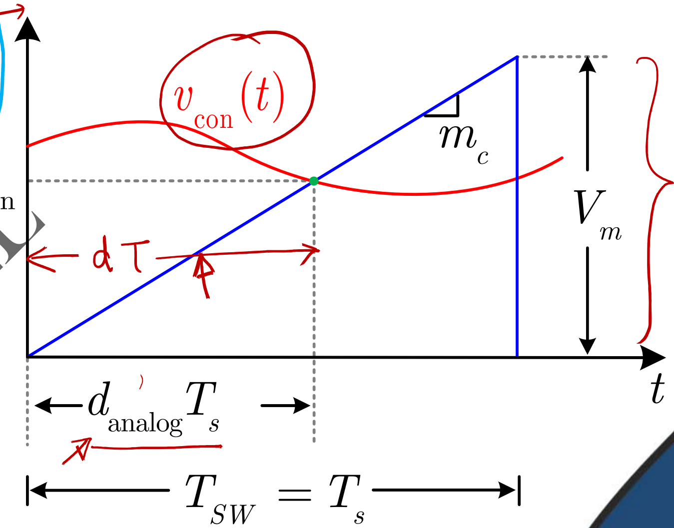
VMC



- In case of analog control

$$v_{con}(t)\Big|_{t=d_{analog}T_s} = V_{con}$$

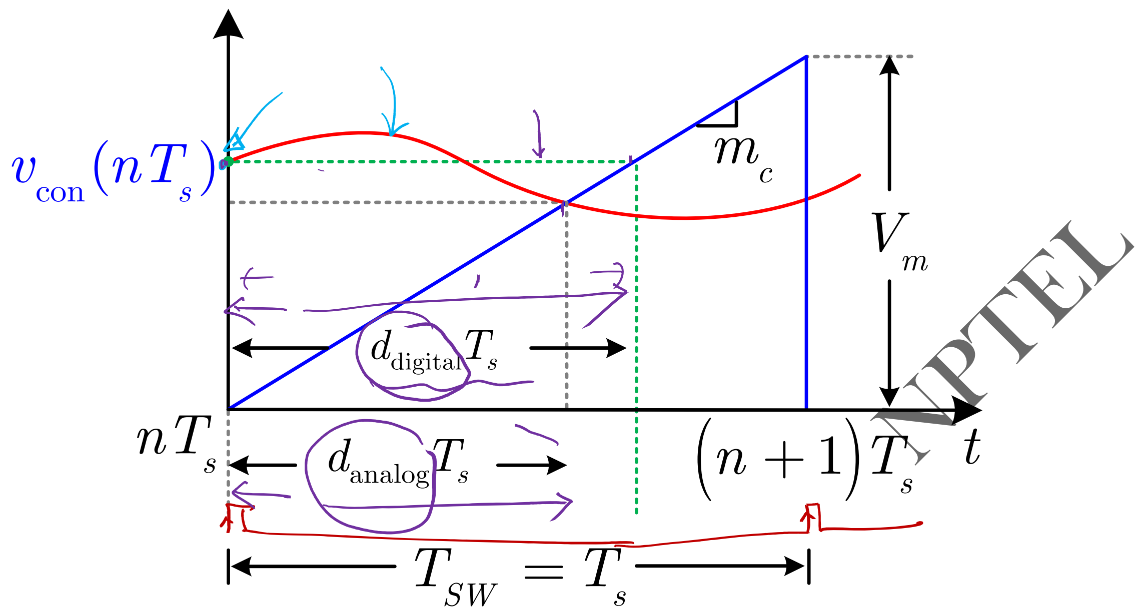
$$\frac{V_{con}}{V_m} = \frac{d_{analog}T_s}{T_s} \Rightarrow d_{analog} = \frac{V_{con}}{V_m}$$

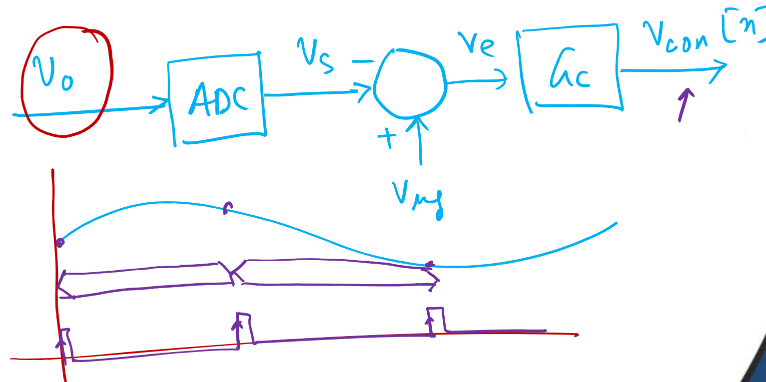


- Modular transfer Function

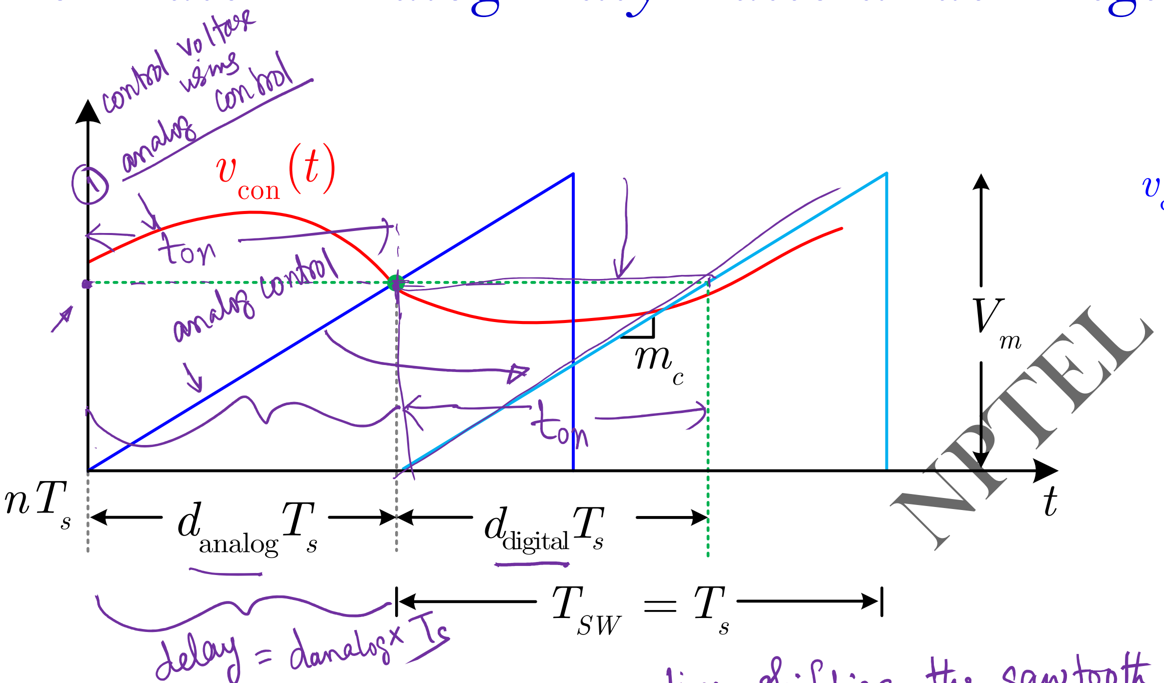
$$G_{PWM}(s) = \frac{1}{V_m} \triangleq F_m$$

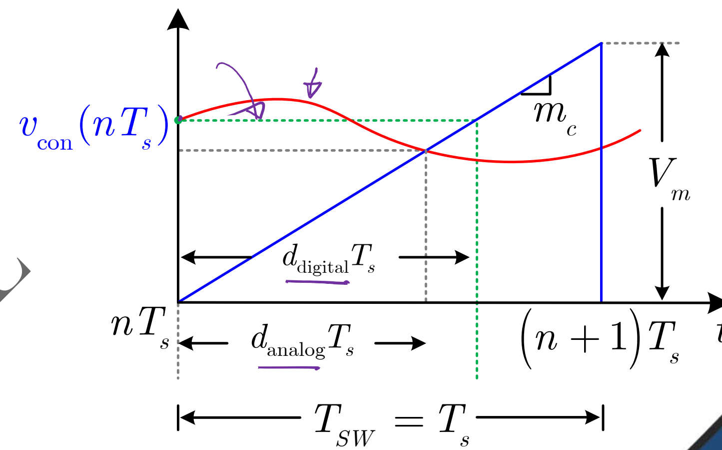
Modulator Gain – Pulse Width Modulator





To Match Analog Duty Ratio under Digital Control





time shifting the sawtooth waveform

$d_{digital} = d_{analog}$ **with a time delay of** d_{analog}T**!!**

Modulator Gain under Digital Control

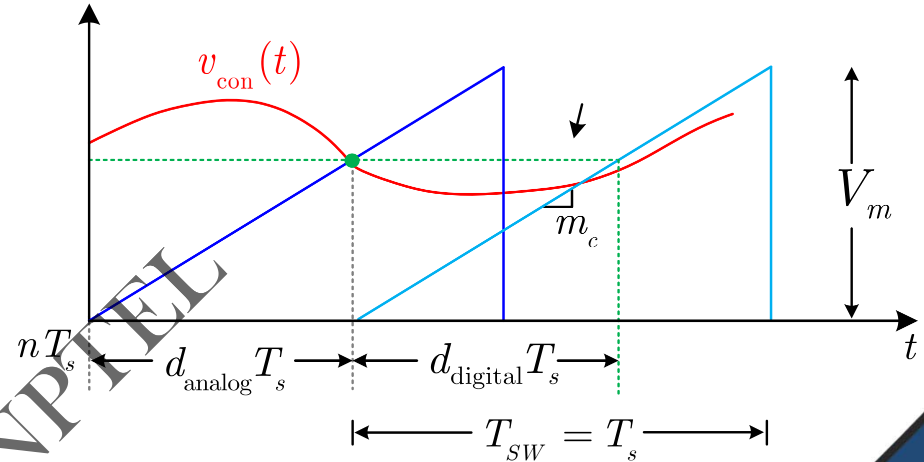
- **Modular transfer function (analog)**

$$G_{\text{analog}}(s) = \frac{1}{V_m}$$

- **Modular transfer function (digital)**

$$G_{\text{digital}}(s) = e^{-sDT_s} G_{\text{analog}}(s)$$

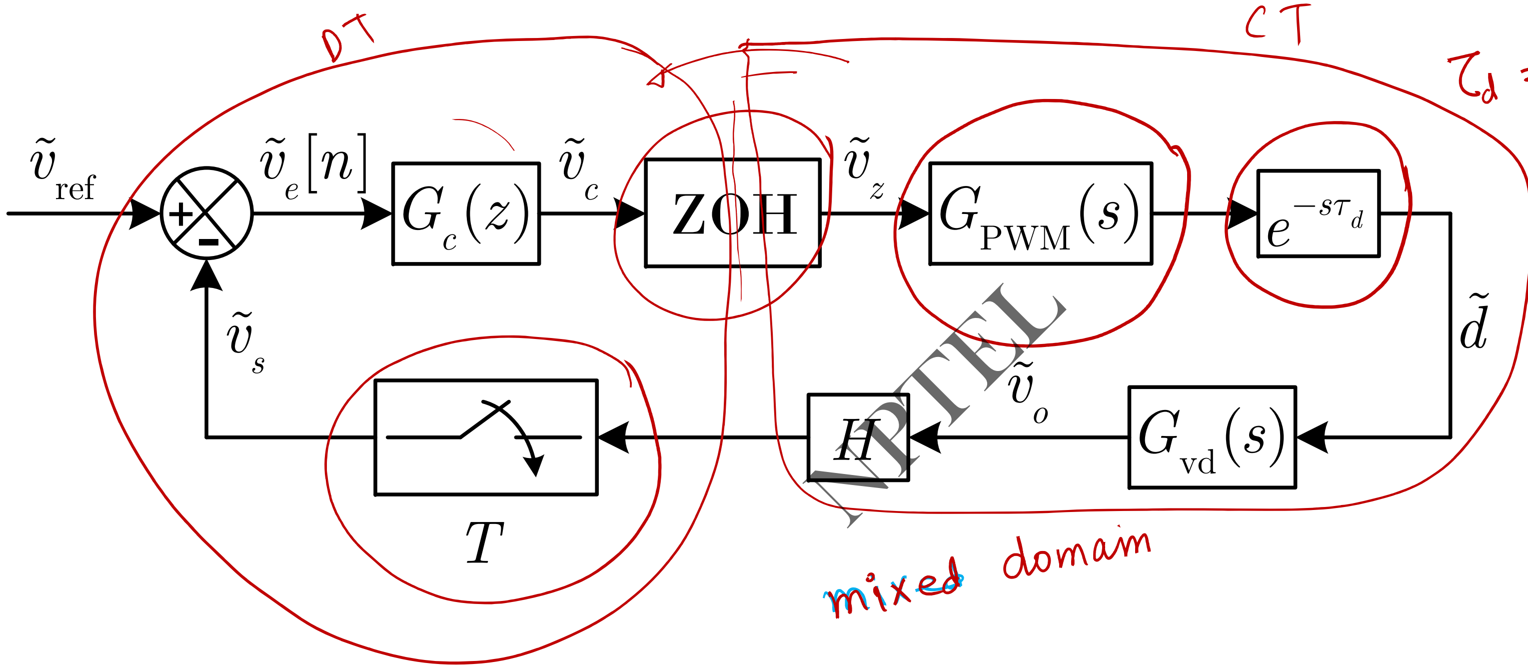
$$G_{\text{digital}}(s) = \frac{1}{V_m} e^{-sDT_s}$$



Digital PWM → introduces a delay = DT_s

Modular delay $t_d = DT_s$!!

Small-Signal Block Diagram under Digital Control

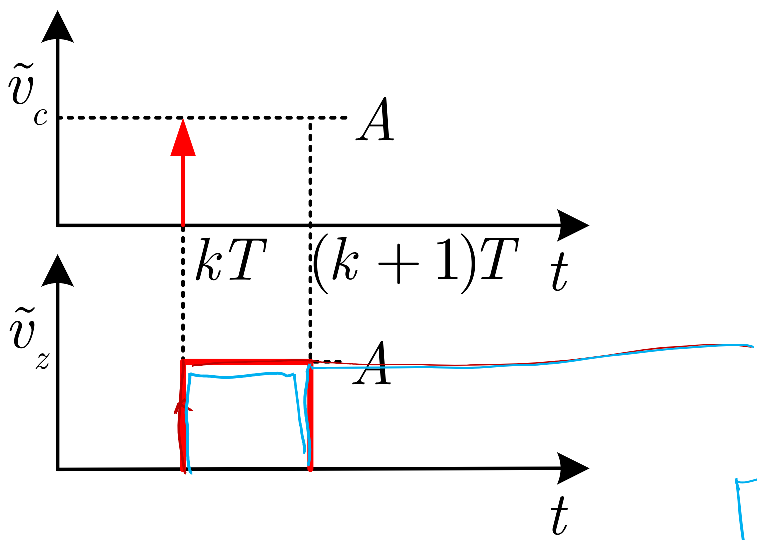


$\tau_d = \tau_{ADC} + DT$

DPWM delay

Modeling of ZOH Block

ZOH transfer function:



$$\tilde{v}_c = A\delta(t-kT)$$

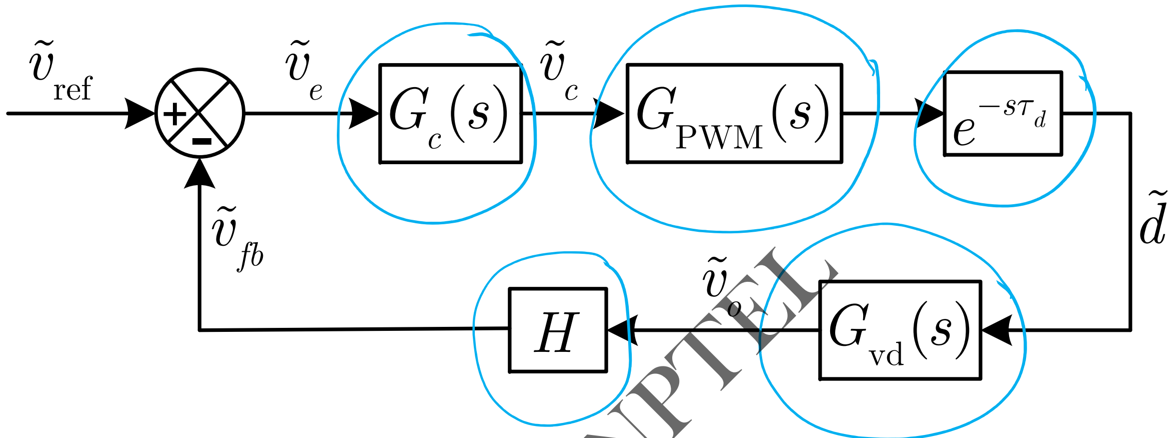
$$\tilde{v}_z = A\Big[u(t-kT) - u\big\{t-(k+1)T\big\}\Big]$$

$$\tilde{v}_c(s) = A$$

$$\tilde{v}_z(s) = \frac{A}{s}\Big[1-e^{-sT}\Big]$$

$$\boxed{G_{ZOH} = \frac{1-e^{-sT}}{s}}$$

Approximate CT Small-Signal Model under Digital Control



$$K_{\text{loop, digital}}(s) = K_{\text{loop, analog}}(s) \times e^{-s\tau_d} \qquad \tau_d = t_{\text{adc}} + t_{\text{DPWM}}$$

[R. Erickson and D. Maksimovic, "*Fundamentals of power electronics*", 3rd *Ed.*, Springer, 2020]

Control-to-Output TF with Delay

$$G_{\mathrm{vd}}(s) = \frac{V_{\mathrm{IN}}}{\left(\frac{R+r_e}{R}\right)} \frac{(1+r_C C s)}{\left(1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}\right)}$$

Total loop delay $\tau_d = t_{\mathrm{adc}} + t_{\mathrm{DPWM}}$

$$G_{\mathrm{vd\_delay}}(s) = e^{-s\tau_d} \times G_{\mathrm{vd}}(s)$$

$\tau_d = \tau_{adc} + DT$

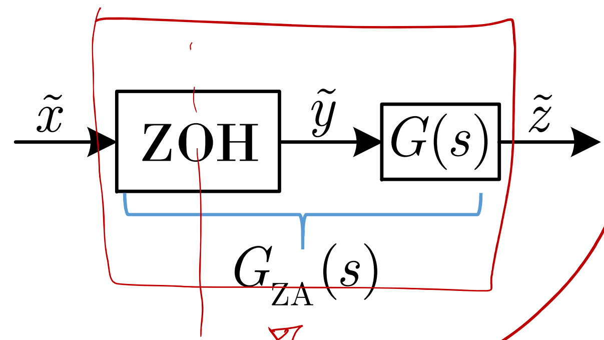
TE PWM

$$G_{\mathrm{vd\_delay}}(s) = \frac{V_{\mathrm{IN}}}{\left(\frac{R+r_e}{R}\right)} \frac{(1+r_C C s)e^{-s\tau_d}}{\left(1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}\right)}$$

ZOH Equivalent Modeling – Alternative Approach

DT equivalent model

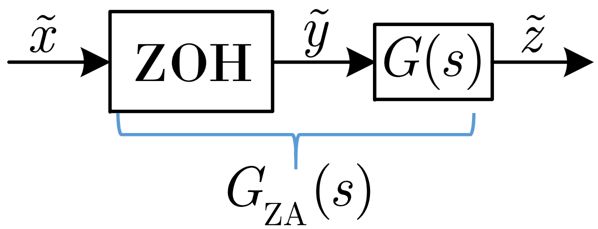
Consider a cascaded block

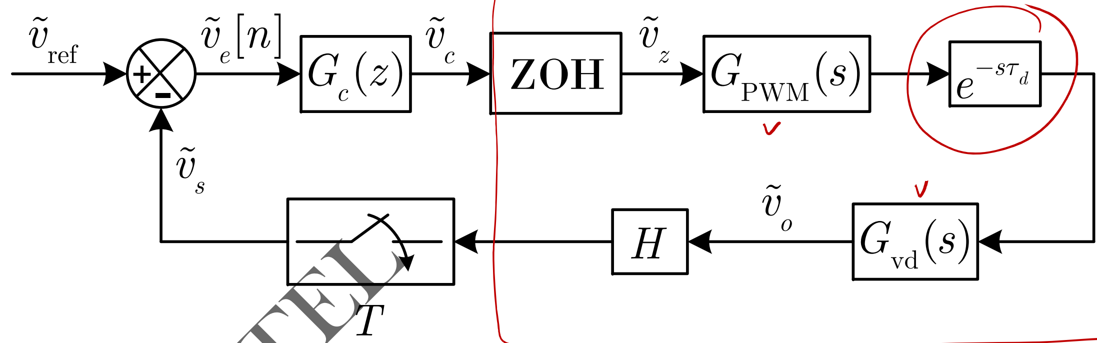


$$G_{ZA}(s) = G(s) \times G_{ZOH}(s) = \left(1 - e^{-sT}\right)\left(\frac{G(s)}{s}\right)$$

$$G_{ZA}(z) = \left(1 - z^{-1}\right)\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

ZOH Equivalent Modeling of SMPC – Alternative Approach





$G_{ZA}(z)$

$0 < \tau_d < T_s$

$$G_{ZA}(z) = \left(1 - z^{-1}\right) \mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

$$G(s) = G_{PWM}(s) \times G_{vd}(s) \times e^{-s\tau_d}$$

CONCLUSION

- Recap of continuous-time small-signal modeling
- Modeling of sampled data-system
- Continuous-time small-signal modeling under digital control

THANK YOU !