NPTEL ONLINE CERTIFICATION COURSES

DIGITAL CONTROL IN SMPCs AND FPGA-BASED PROTOTYPING

Dr. Santanu Kapat
Electrical Engineering Department, IIT KHARAGPUR

Module 04: Modeling Techniques and Mode Validation using MATLAB

Lecture 37: Derivation of Discrete-Time Small-Signal Models - I

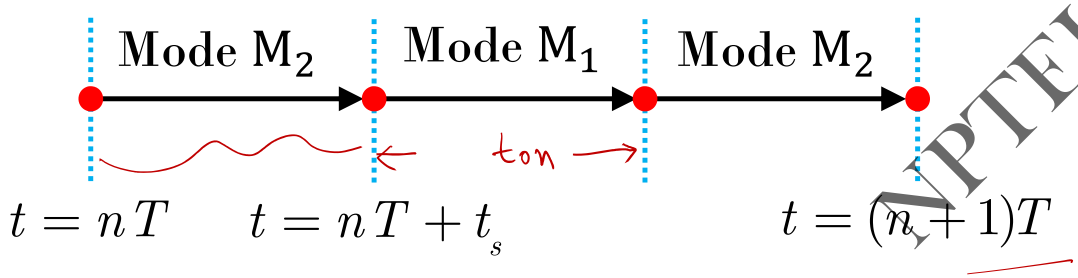
CONCEPTS COVERED

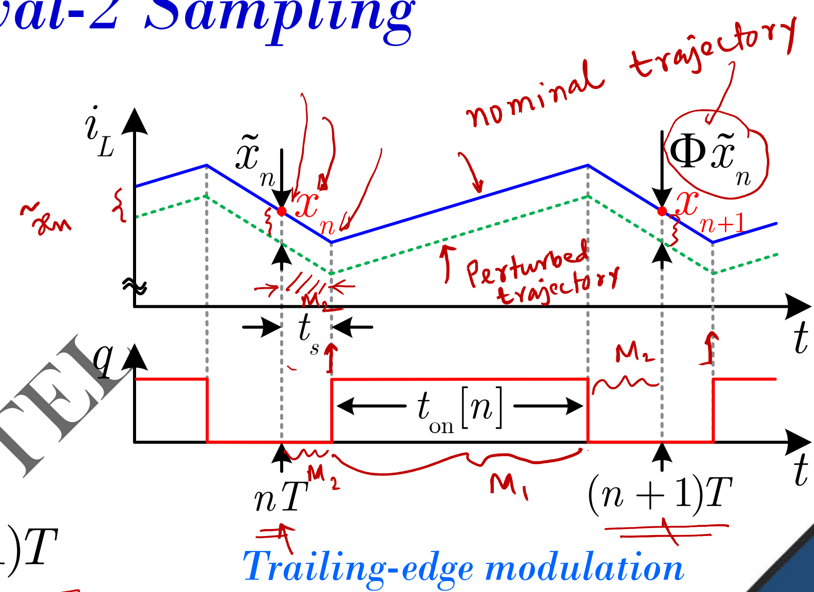
- Effect of perturbations in discrete-time modeling
- Zero-input and zero-state response
- Discrete-time small-signal modeling

Discrete-Time Modeling with Interval-2 Sampling

$$x(t)\Big|_{t=nT} = x_n \qquad x(t)\Big|_{t=(n+1)T} = x_{n+1}$$

$$t = nT + t_s + t_{on}$$





Mode M_1	Mode M_2
A_1, B_1, C_1	A_2, B_2, C_2

Discrete-Time Modeling : Initial State Perturbation

Considering the effect of perturbation of states $\tilde{x}_n$ and neglecting the effect of perturbation in duty cycle $\tilde{d} = 0$

$$\Rightarrow \tilde{x}_{n+1} = e^{A_2(T - t_{on} - t_s)} e^{A_1 t_{on}} e^{A_2 t_s} \tilde{x}_n$$

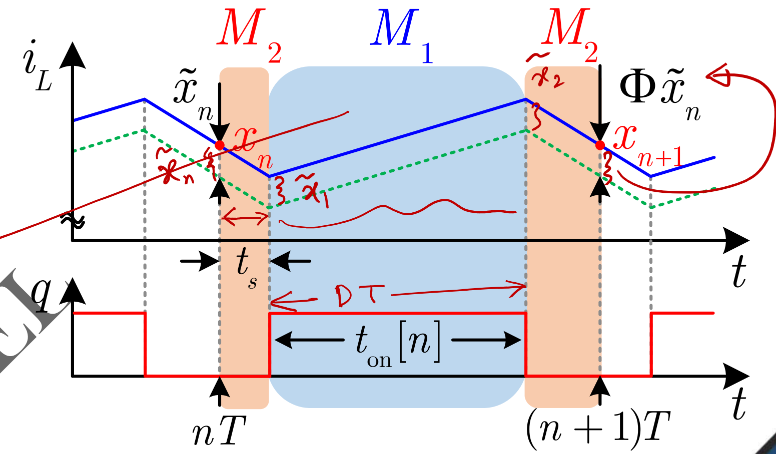
$\tilde{x}_1$

$\tilde{x}_2$

$$\Rightarrow \tilde{x}_{n+1} = \underbrace{e^{A_2((1-D)T - t_s)} e^{A_1 DT} e^{A_2 t_s}}_{\Phi} \tilde{x}_n$$

$t_{on} = DT$

$\tilde{x}_{n+1} = \Phi \tilde{x}_n$



Trailing-edge modulation

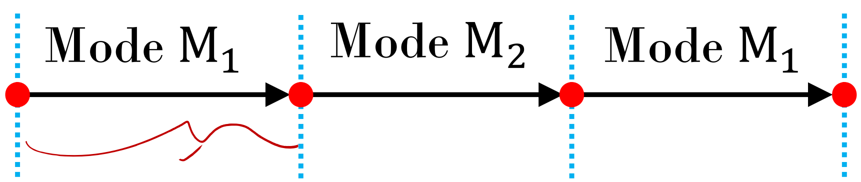
[D. Maksimovic and R. Zane, "Small-Signal Discrete-Time Modeling…..", *IEEE TPEL*, 2007]

Discrete-Time Modeling : Initial State Perturbation

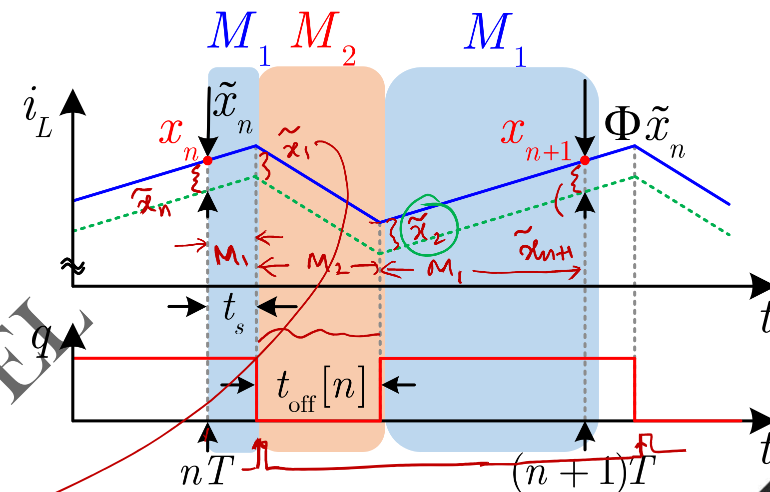
$\tilde{x}_{n+1} = \phi\, \tilde{x}_n$

Interval-1 sampling

$$t = nT + t_s + t_{\text{off}}$$



$t = nT$ $\quad t = nT + t_s$ $\quad t = (n+1)T$



Leading-edge modulation

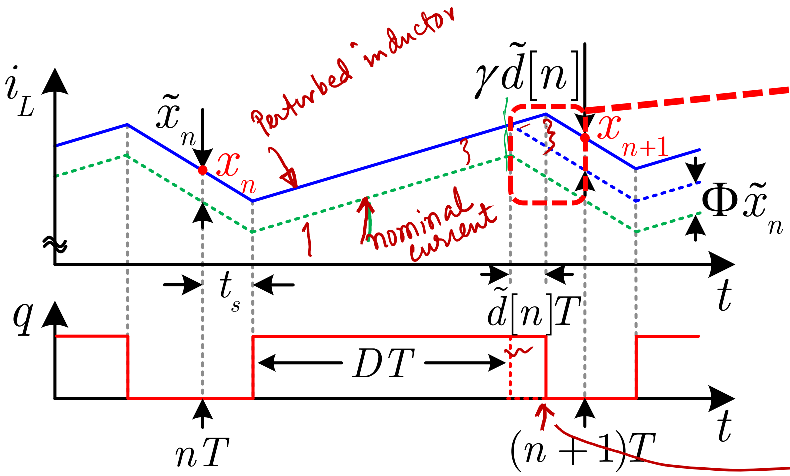
$$\Rightarrow \tilde{x}_{n+1} = e^{A_1(T-t_{\text{off}}-t_s)}\, e^{A_2 t_{\text{off}}}\, e^{A_1 t_s}\, \tilde{x}_n$$

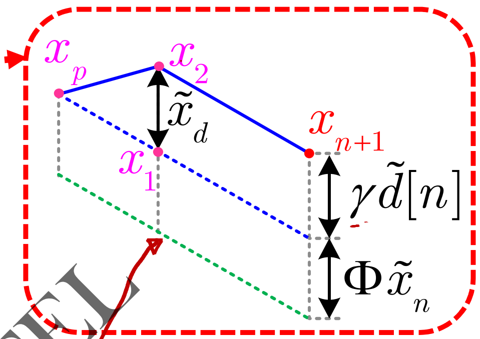
$$\Rightarrow \tilde{x}_{n+1} = \underbrace{e^{A_1(DT-t_s)}\, e^{A_2(1-D)T}\, e^{A_1 t_s}}_{\Phi}\, \tilde{x}_n$$

$\tilde{x}_{n+1} = \phi\, \tilde{x}_n$

[D. Maksimovic and R. Zane, "Small-Signal Discrete-Time Modeling…..", *IEEE TPEL*, 2007]

Discrete-Time Modeling : Initial State and Duty Ratio Perturbations





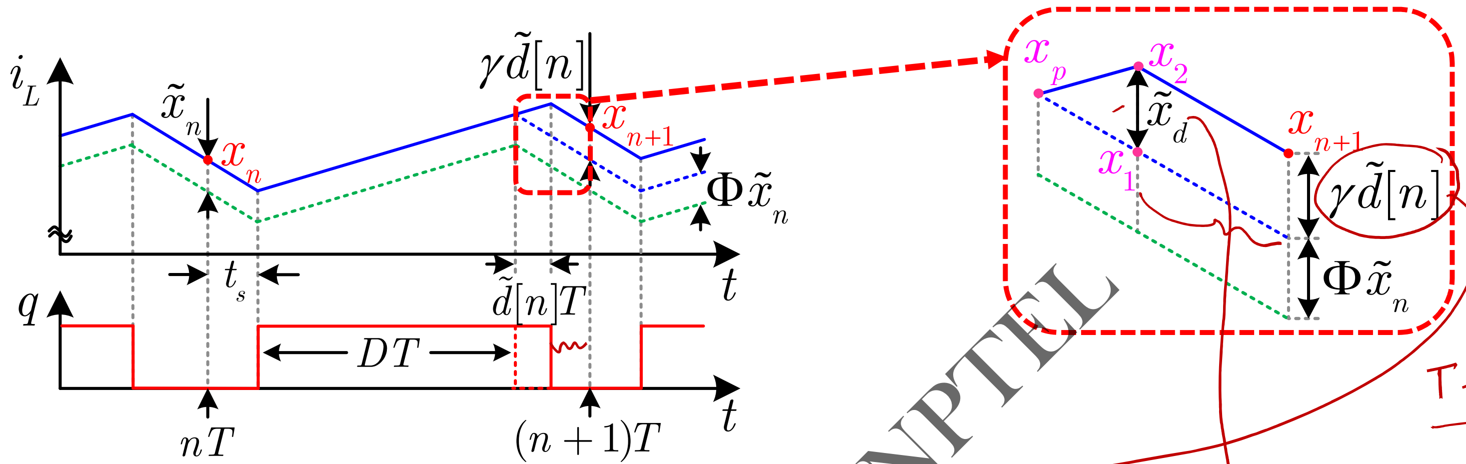
Trailing-edge modulation

Considering only the effect of perturbation in duty cycle $\tilde{d}[n]$

$$\tilde{x}_d = x_2 - x_1 \quad = \left[\left(A_1 - A_2\right)x_p + \left(B_1 - B_2\right)V_{in}\right]\tilde{d}[n]T \quad = \text{Constant} \times \tilde{d}(n)$$

[D. Maksimovic and R. Zane, "Small-Signal Discrete-Time Modeling…..", *IEEE TPEL*, 2007]

Discrete-Time Modeling : Initial State and Duty Ratio Perturbations



Trailing-edge modulation

$t_{on} = T_{on} + \tilde{t}_{on}$

$T - t_{on} - t_s \approx T - T_{on} - t_s$

Considering $\alpha = (A_1 - A_2)x_p + (B_1 - B_2)V_{in} \Rightarrow \tilde{x}_d = \alpha T\tilde{d}[n]$

$$\Rightarrow \tilde{x}_{n+1} = e^{A_2(T-t_{on}-t_s)}\tilde{x}_d = \boxed{e^{A_2(T-t_{on}-t_s)}}\alpha T\tilde{d}[n]$$

γ only due to duty ratio perturbation

Discrete-Time Modeling : Initial State and Duty Ratio Perturbations

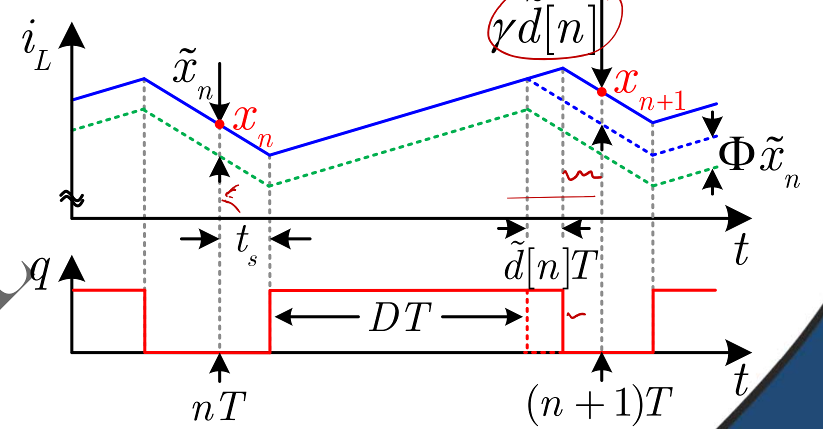
CCM buck conv. with $A_1 = A_2 = A$

$$\tilde{x}_{n+1} = \Phi \tilde{x}_n + \gamma \tilde{d}[n]$$

$\Phi = e^{AT}$

$$\Phi = e^{A_2((1-D)T - t_s)} e^{A_1 DT} e^{A_2 t_s}$$

$$\gamma = e^{A_2(T - t_{on} - t_s)} \alpha T = e^{A_2((1-D)T - t_s)} \alpha T$$



Trailing-edge modulation

Output state-space equation $\tilde{y}_n = C_{eq} \tilde{x}_n$

where $C_{eq} = \begin{bmatrix} \alpha r_C & \alpha \end{bmatrix}$

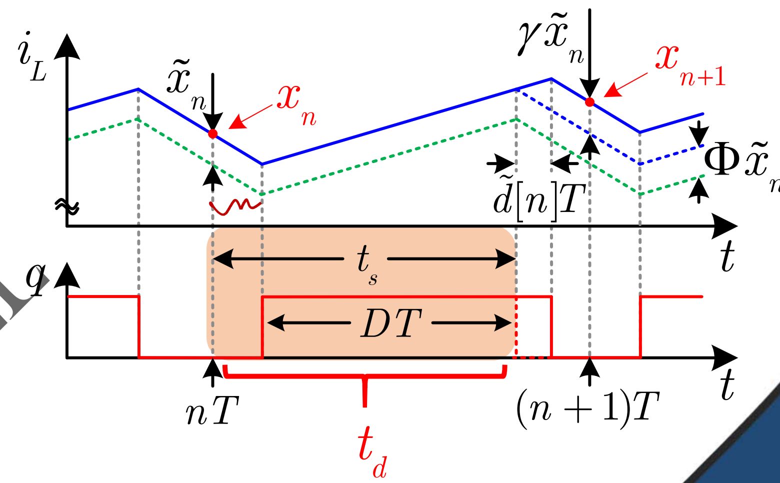
Using the approximation $e^{AT} \approx 1 + AT$ **and Z-transform**

$$G_{vd}(z) = \frac{\tilde{v}_{out}(z)}{\tilde{d}(z)}$$

Discrete-Time Small-Signal Model Parameters : Trailing-edge PWM

Considering Total delay

$$t_d = DT + t_s$$



Sampling	Duration	Φ	γ
Interval-1	$0 \leq t_s < DT$	$e^{A_1(DT-t_d)}e^{A_2DT}e^{A_1t_d}$	$e^{A_1(DT-t_d)}e^{A_2DT}\alpha T$
Interval−2	$DT \leq t_s < T$	$e^{A_2(T-t_d)}e^{A_1DT}e^{A_2(t_d-DT)}$	$e^{A_2(T-t_d)}\alpha T$

[For details, refer to "Digital Control of High-frequency Switched …", Wiley-IEEE Press, 2015]

Discrete-Time Small Signal Model : Synchronous Buck Converter

For synchronous buck converter $A_1 = A_2 = A,\ B_1 = \begin{bmatrix} \frac{1}{L} & 0 \end{bmatrix}^T$ **and** $B_2 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$

$e^{A_1 t_1} \cdot e^{A_2 t_2} \neq e^{A_1 t_1 + A_2 t_2}$

$$\tilde{x}_{n+1} = \Phi \tilde{x}_n + \gamma \tilde{d}[n] = \boxed{e^{A_2(T-t_d)} e^{A_1 DT} e^{A_2(t_d - DT)}} \tilde{x}_n + e^{A_2(T-t_d)} \alpha T \tilde{d}[n]$$

$$\textbf{where } \alpha = \underbrace{(A_1 - A_2)}_{=0} x_p + \underbrace{(B_1 - B_2)}_{=0} V_{in} = B_1 V_{in}$$

$$\Rightarrow \boxed{\tilde{x}_{n+1} = e^{AT} \tilde{x}_n + e^{A(T-t_d)} B_1 V_{in} T \tilde{d}[n]}$$

$A_1 A_2 = A^2$

If this holds $A_1 A_2 = A_2 A_1$

then $e^{A_1 t_1} \cdot e^{A_2 t_2} = e^{(A_1 t_1 + A_2 t_2)}$

Output Voltage $v_o[n] = C_{eq} x_n$ **where** $C_{eq} = \begin{bmatrix} \alpha r_C & \alpha \end{bmatrix}$

$$\Rightarrow \boxed{\tilde{v}_o[n] = C_{eq} \tilde{x}_n}$$

CONCLUSION

- Effect of perturbations in discrete-time modeling
- Zero-input and zero-state response
- Discrete-time small-signal modeling

THANK YOU !