NPTEL ONLINE CERTIFICATION COURSES

DIGITAL CONTROL IN SMPCs AND FPGA-BASED PROTOTYPING

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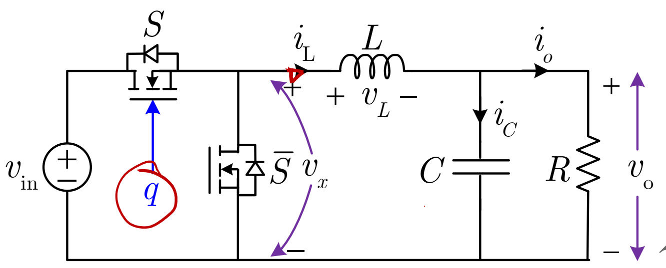
Module 01: Introduction to Digital Control in SMPCs

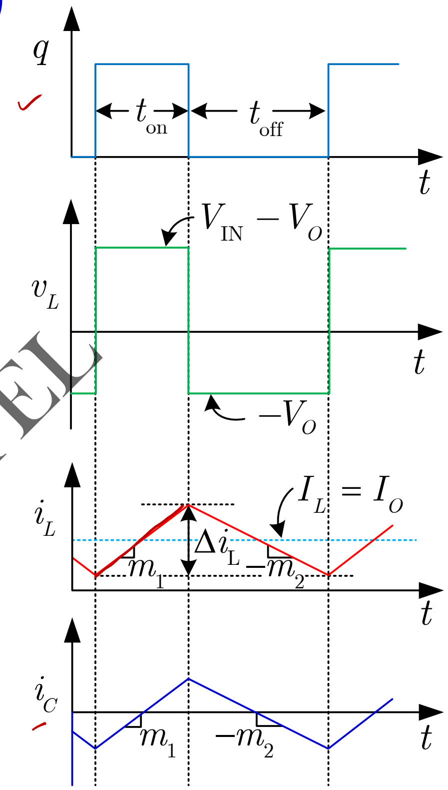
Lecture 10: SMPC Topologies and Power Stage Design for Hardware Demonstrations

CONCEPTS COVERED

- Power stage design aspects of buck and boost converters
- Buck and boost converters under steady-state in CCM and DCM
- Full-bridge LLC converter – basic operations for varying frequency
- Conventional boost PFC and totem-pole PFC for AC/DC conversion

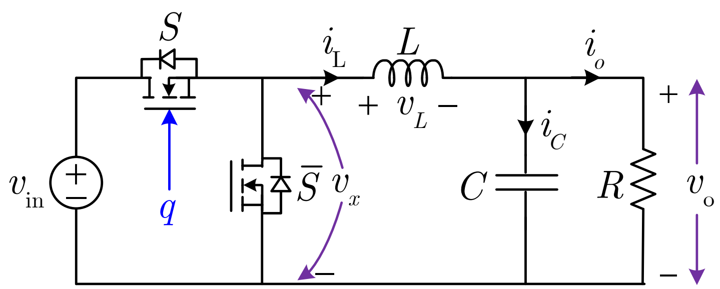
Synchronous Buck Converter (CCM)





Steady-state waveforms

Buck Converter Ripple Parameters



- **Inductor current ripple (Δi_L) of a buck converter**

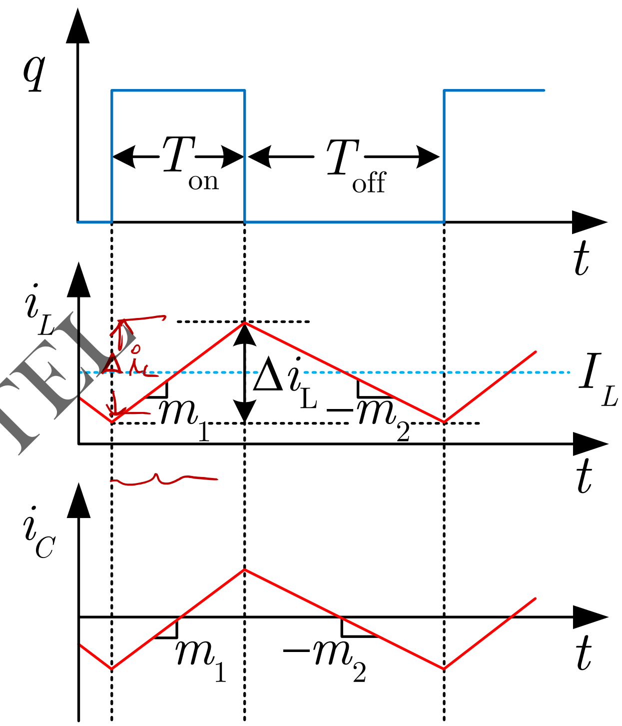
$$\Delta i_L = m_1 T_{on} \quad \text{where } m_1 = \frac{V_{IN} - V_O}{L}$$

$$\therefore \Delta i_L = \frac{V_{IN} - V_O}{L} \times T_{on}$$

$T_{on} = DT$

$V_o = DV_{IN}$

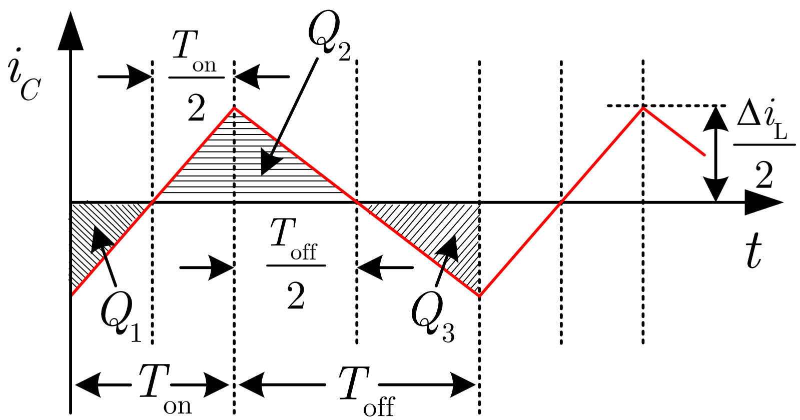
$$\boxed{\Delta i_L = \frac{D(1-D)V_{IN}T}{L}}$$



Steady-state waveforms

[For details, refer to Lecture~7, NPTEL "Power Stage Design of ..." course (link)]

Capacitor Voltage Ripple – Ideal Buck Converter



Capacitor current waveform

$$Q_2 = \frac{1}{2} \times \frac{\Delta i_L}{2} \times \left(\frac{T_{on} + T_{off}}{2} \right)$$

$$= \frac{1}{8} \times \left(T_{on} + T_{off} \right) \times \Delta i_L$$

Substituting $\Delta i_L = \frac{V_O}{L} \times T_{off}$

$$Q_2 = \frac{V_O}{8L} \times \left(T_{on} + T_{off} \right) \times T_{off}$$

Again $Q_2 = C \times \Delta v_o$

$$\Delta v_o = \frac{V_O}{8LC} \times \left(T_{on} + T_{off} \right) \times T_{off}$$

- Voltage ripple impact?

[For details, refer to Lecture~7, NPTEL "Power Stage Design of …" course (link)]

Ripple Parameters of a Buck Converter under PWM

- **Under PWM**

$$T_{on} + T_{off} = T_{sw} = \frac{1}{f_{sw}} \quad \text{(fixed)}$$

$$T_{on} = D \times T_{sw}$$

$$T_{off} = T_{sw} - T_{on} = (1-D) \times T_{sw}$$

$$\Delta v_o = \frac{V_O}{8LC} \times T_{sw} \times (1-D)T_{sw}$$

$$\boxed{\Delta v_o = \left(\frac{V_O}{8LCf_{sw}^2}\right) \times (1-D)}$$

Voltage ripple is maximum at minimum $D \rightarrow$ highest v_{in}

$$\Delta i_L = \frac{V_O}{L} \times (1-D)T_{sw}$$

$$\boxed{\Rightarrow \Delta i_L = \frac{V_O}{Lf_{sw}} \times (1-D)}$$

Current ripple is maximum at minimum $D \rightarrow$ highest v_{in}

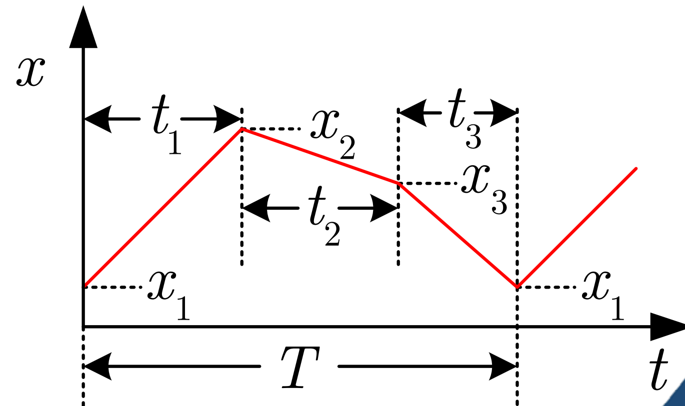
Const.

$V_0 = D V_{IN}$

[For details, refer to Lecture~7, NPTEL "Power Stage Design of …" course (link)]

RMS Value of a Periodic Piecewise Linear Waveform

$$x_{\mathrm{rms}}^2 = \frac{1}{T}\left[\left(\frac{x_1^{\ 2} + x_1x_2 + x_2^{\ 2}}{3}\right)t_1 + \left(\frac{x_2^{\ 2} + x_2x_3 + x_3^{\ 2}}{3}\right)t_2 + \left(\frac{x_3^{\ 2} + x_3x_1 + x_1^{\ 2}}{3}\right)\left(T - t_1 - t_2\right)\right]$$

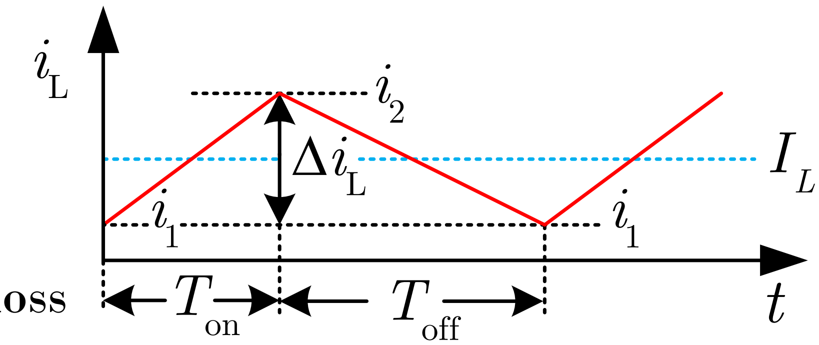


Hint: $$\left(x_{\mathrm{rms}}\right)^2 = \frac{1}{T}\left[\int_0^T x^2(t)dt\right] = \frac{1}{T}\left[\int_0^{t_1} x^2(t)dt + \int_{t_1}^{t_1+t_2} x^2(t)dt + \int_{t_1+t_2}^{T} x^2(t)dt\right]$$

[For details, refer to Lecture~7, NPTEL "Power Stage Design of …" course (link)]

Buck Converter RMS Current

- For a given load current,
  - $i_{L,\mathrm{RMS}}$ is maximum at maximum v_{in}
  - Higher $i_{L,\mathrm{RMS}}$ implies higher conduction loss



Inductor current waveform

- For a given input voltage,
  - $i_{L,\mathrm{RMS}}$ increases with increasing $\cancel{\Delta i_L}$ i_o
  - Higher conduction loss at higher load current

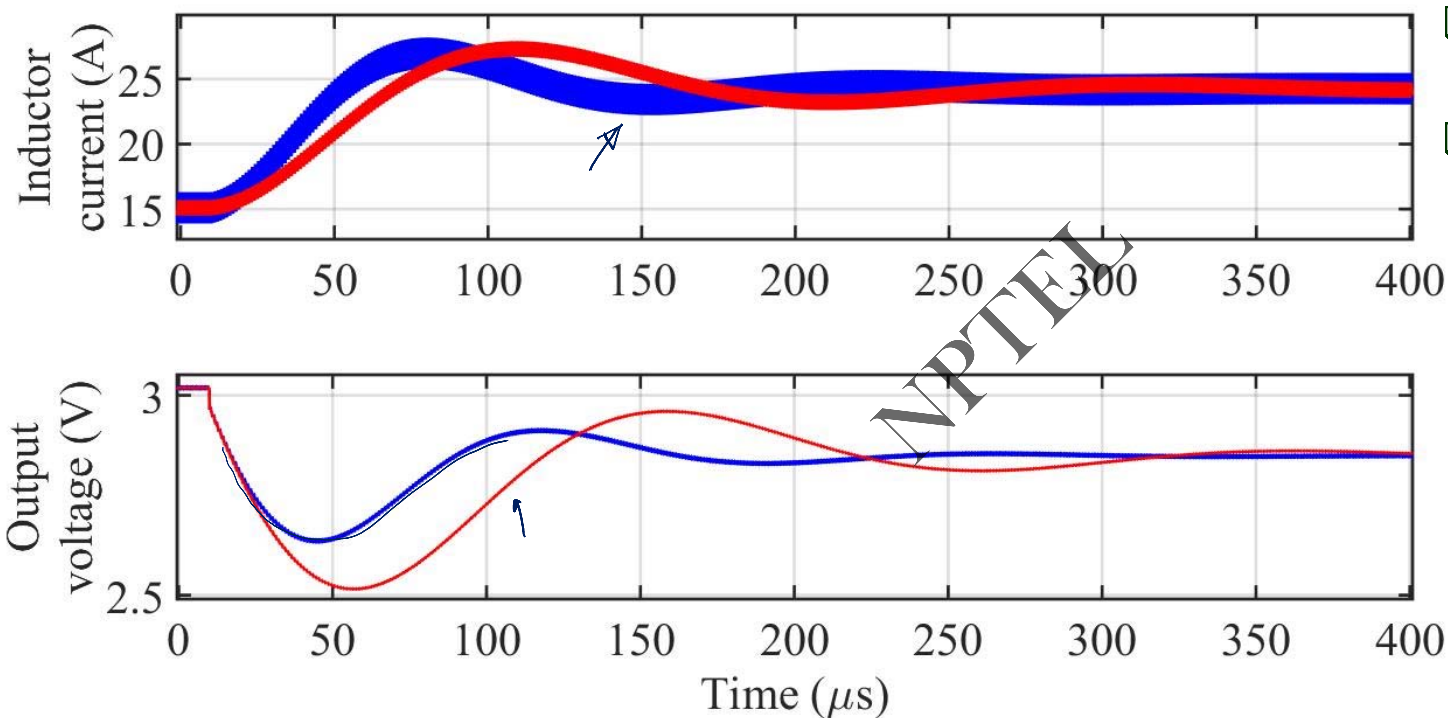
$$\boxed{i_{L,\mathrm{RMS}}^2 = I_O^2 + \frac{\Delta i_L^2}{12}}$$

i_o

Worst case RMS current (also conduction loss) at highest input voltage and highest load current

[For details, refer to Lecture~7, NPTEL “Power Stage Design of …” course (link)]

Design Consideration (Inductor)



- Larger inductor (red)
- Smaller inductor (blue)

Design Consideration (Inductor) Large Inductor

Advantages

Smaller ripple current

$$\Delta i_L = \frac{V_O(1-D)}{f_{sw}} \times \frac{1}{L}$$

Smaller RMS current

$$\left(i_{L,\mathrm{RMS}}\right)^2 = I_O^2 + \frac{\Delta i_L}{12}$$

Lower conduction loss

Smaller voltage ripple

$$\Delta v_o = \frac{V_O(1-D)}{8Cf_{sw}^2} \times \frac{1}{L}$$

Disadvantages

Larger size (bulky inductor)

Slower transient response!!

Higher voltage overshoot/ undershoot!!

Inductor should be carefully designed

[For details, refer to Lecture~7, NPTEL "Power Stage Design of …" course (link)]

Design Consideration (Capacitor) Large Capacitor

Advantages

Smaller output voltage ripple

$$\Delta v_o = \frac{V_o(1-D)}{8Cf_{sw}^2} \times \frac{1}{L}$$

Smaller output voltage undershoot/ overshoot

Disadvantages

Larger size and poor reliability

Higher time and energy overhead during reference voltage transient

Capacitor should be carefully selected

[For details, refer to Lecture~7, NPTEL "Power Stage Design of …" course (link)]

Worst Case Inductor Current Ripple – Buck Converter in CCM

Modulation Technique	Current Ripple (Δi_L)	Worst case scenario
Pulse width modulation	$\frac{T}{L} \times V_o \left(1 - \frac{V_o}{V_{in}}\right)$	Highest input voltage
Constant on-time modulation	$\frac{T_{on}}{L} \times \left(V_{in} - V_o\right)$	Highest input voltage
Constant off-time modulation	$\frac{T_{off}}{L} \times V_o$	Insensitive to operating conditions

Voltage reg.

$V_0 \rightarrow$ Const.

[For details, refer to Lecture~23, NPTEL "Stability and Performance …" course (link)]

Worst Case RMS Inductor Current – Buck Converter in CCM

Modulation Technique	RMS Current ($i_{L,rms}$)	Worst case scenario
Pulse width modulation	$\sqrt{I_o^2 + \frac{1}{12}\left[\frac{TV_o}{L}\left(1-\frac{V_o}{V_{\text{in}}}\right)\right]^2}$	Highest input voltage and highest load current
Constant on-time modulation	$\sqrt{I_o^2 + \frac{1}{12}\left[\frac{T_{\text{on}}}{L}\left(V_{\text{in}}-V_o\right)\right]^2}$	Highest input voltage and highest load current
Constant off-time modulation	$\sqrt{I_o^2 + \frac{1}{12}\left(\frac{V_oT_{\text{off}}}{L}\right)^2}$	Highest load current

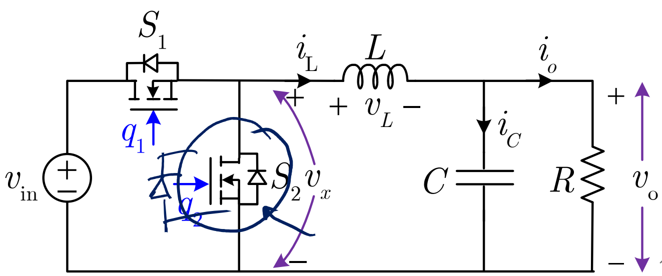
[For details, refer to Lecture~23, NPTEL "Stability and Performance …" course (link)]

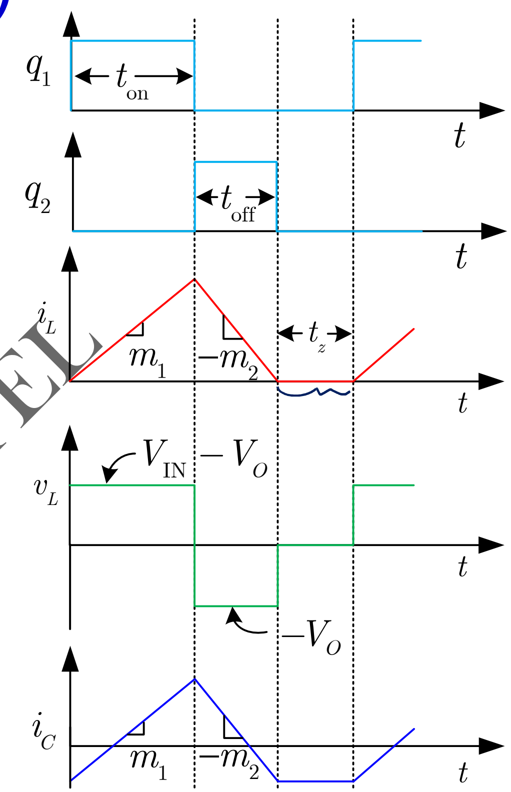
Switching Frequency – Buck Converter in CCM

Modulation Technique	Switching frequency (f_{sw})	Worst case scenario
Pulse width modulation	$f_{\text{sw}} = f_{\text{ext}}$	Insensitive to system and operating conditions
Constant on-time modulation	$f_{\text{sw}} = \dfrac{1}{T_{\text{on}}} \times \left(\dfrac{V_o}{V_{\text{in}}} \right)$	Highest switching frequency at lowest input voltage
Constant off-time modulation	$f_{\text{sw}} = \dfrac{1}{T_{\text{off}}} \times \left(1 - \dfrac{V_o}{V_{\text{in}}} \right)$	Highest switching frequency at highest input voltage

[For details, refer to Lecture~23, NPTEL "Stability and Performance …" course (link)]

Synchronous Buck Converter (DCM)





Steady-state waveforms

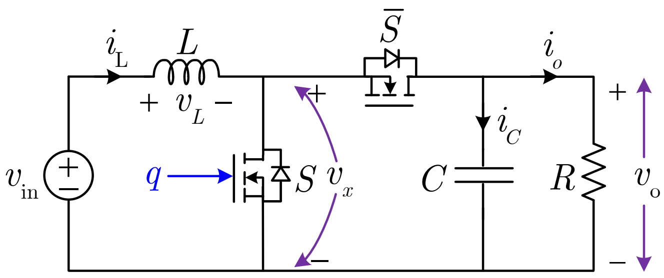
Steady State Characterization of a Buck Converter in DCM

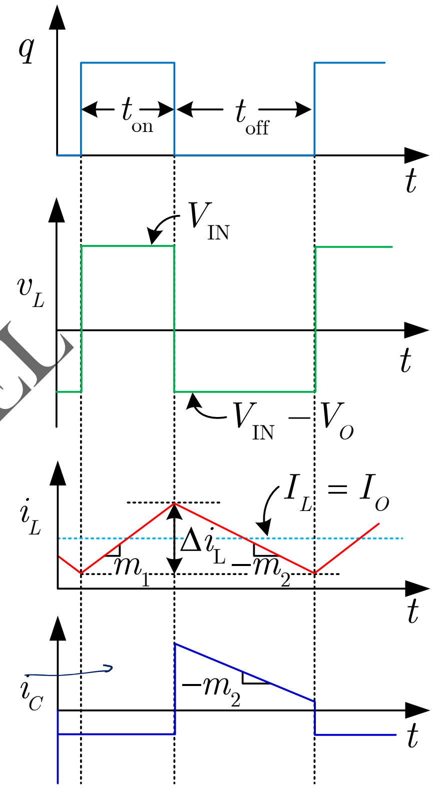
Modulation Technique	Voltage ripple (Δv_o)	Switching frequency (f_{sw})
Pulse width modulation	$\Delta v_o \approx \frac{1}{C} \times \left(\frac{V_{in}}{V_o}\right) \times \frac{i_o}{f_{sw}}$	Fixed f_{sw}, but varying duty ratio $D = \sqrt{\frac{2V_o i_o}{(V_{in}-V_o)V_{in}} \times \frac{L}{T}}$
Constant on-time modulation	$\boxed{\Delta v_o \approx \left[\frac{(V_{in}-V_o)V_{in}}{V_o}\right] \times \frac{T_{on}^2}{2LC}}$	$f_{sw} = \left[\frac{V_o}{V_{in}(V_{in}-V_o)}\right] \times \left(\frac{2L}{T_{on}^2}\right) \times i_o$

Ripple voltage increases with increasing input voltage

[For details, refer to Lecture~24, NPTEL "Light Load Control…" course (link)]

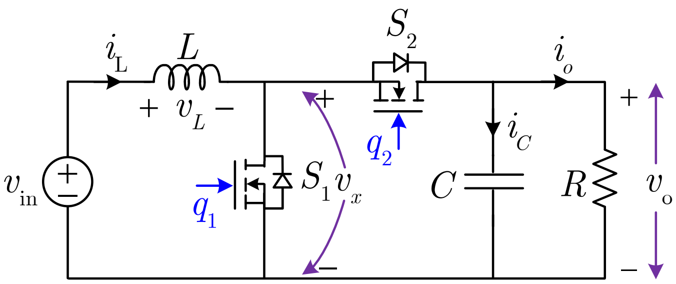
Synchronous Boost Converter (CCM)

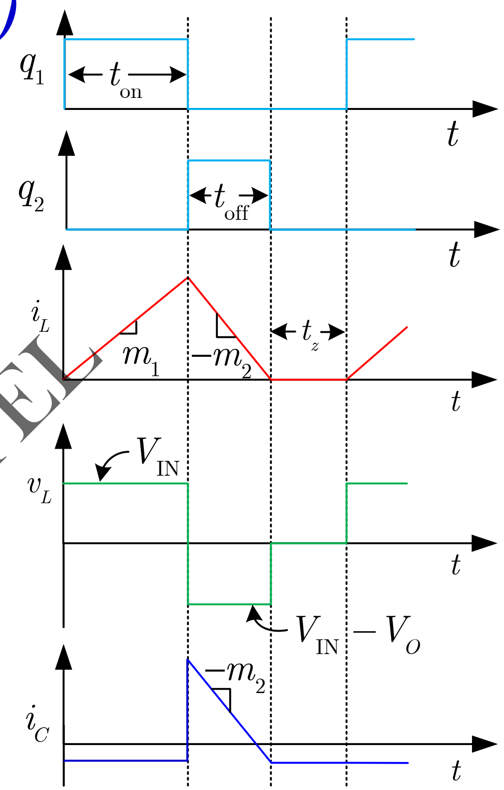




Steady-state waveforms

Synchronous Boost Converter (DCM)





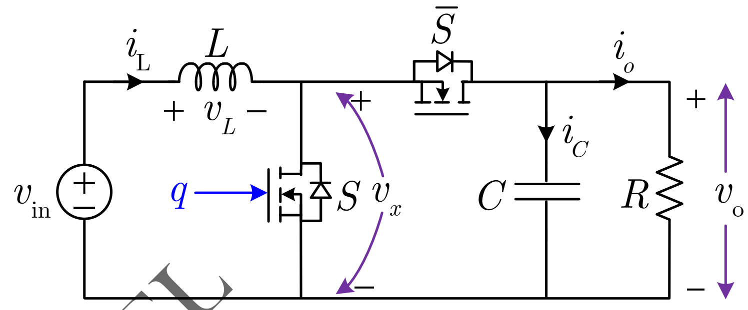
Steady-state waveforms

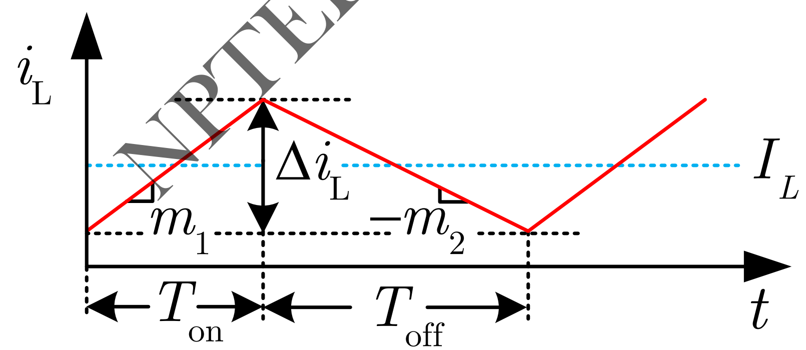
Ripple Inductor Current- Boost Converter

$$\Delta i_L = m_1 \times T_{on} = \frac{V_{IN}}{L} \times T_{on}$$

Express V_{IN} in terms of V_o

since V_o is constant for a VR





Inductor current waveform

[For details, refer to Lecture~7, NPTEL "Power Stage Design of …" course (link)]

For a Boost Converter

$$V_O = \frac{T_{\text{on}} + T_{\text{off}}}{T_{\text{off}}} V_{\text{IN}} \quad \Rightarrow V_{\text{IN}} = \frac{T_{\text{off}}}{T_{\text{on}} + T_{\text{off}}} V_O$$

$$\Delta i_L = \frac{V_{\text{IN}}}{L} \times T_{\text{on}} \quad \Rightarrow \Delta i_L = \frac{V_O}{L} \times \left(\frac{T_{\text{on}} T_{\text{off}}}{T_{\text{on}} + T_{\text{off}}} \right)$$

$$\therefore \boxed{\Delta i_L = \frac{V_O}{L f_{\text{sw}}} \times \left[D(1-D) \right]}$$

Δi_L is maximum at D=0.5

Under PWM

$$T_{\text{on}} + T_{\text{off}} = T_{\text{sw}} = \frac{1}{f_{\text{sw}}}$$

$$T_{\text{on}} = D T_{\text{sw}}$$

$$T_{\text{off}} = (1-D) T_{\text{sw}}$$

[For details, refer to Lecture~7, NPTEL "Power Stage Design of …" course (link)]

Ripple Output Voltage – Boost Converter

$$\Delta v_o \times C = I_O T_{\text{on}}$$

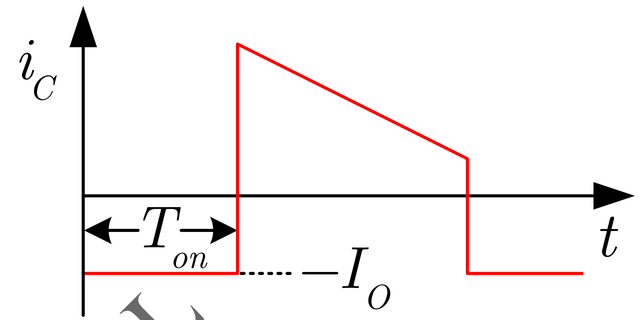
$$\Delta v_o = \frac{I_O}{C} \times T_{\text{on}}$$

<u>Under PWM</u> $\quad T_{\text{on}} = DT_{\text{sw}}$

$$\therefore \boxed{\Delta v_o = \frac{I_O}{Cf_{\text{sw}}} \times D}$$

Worst-case voltage ripple at

lowest input voltage and highest load current



Capacitor current waveform

- Voltage ripple is maximum when
  - Load current is maximum and
  - Duty ratio is maximum

[For details, refer to Lecture~7, NPTEL "Power Stage Design of …" course ([link]())]

Worst Case Inductor Current Ripple – Boost Converter in CCM

Modulation Technique	Current Ripple (Δi_L)	Worst case scenario
Pulse width modulation	$\frac{T}{L}\times\left[\frac{V_{\text{in}}}{V_o}\left(V_o - V_{\text{in}}\right)\right]$	Input voltage equals to half of the output voltage
Constant on-time modulation	$\frac{T_{\text{on}}}{L}\times V_{\text{in}}$	Highest input voltage
Constant off-time modulation	$\frac{T_{\text{off}}}{L}\times\left(V_o - V_{\text{in}}\right)$	Lowest input voltage

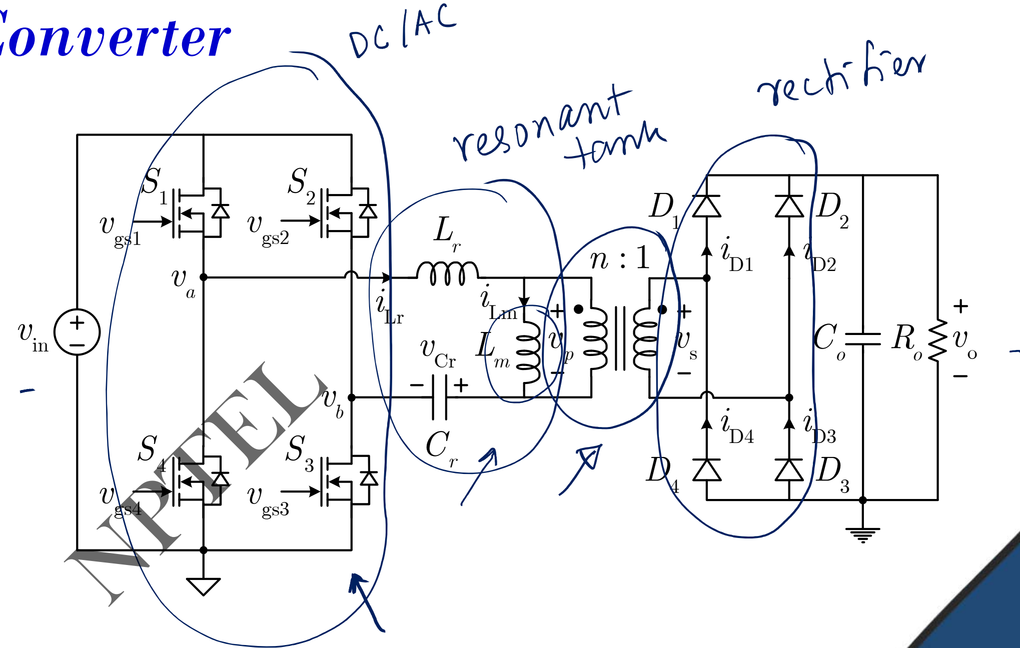
[For details, refer to Lecture~23, NPTEL “Stability and Performance …” course (link)]

Switching Frequency – Boost Converter in CCM

Modulation Technique	Switching frequency (f_{sw})	Worst case scenario
Pulse width modulation	$f_{\text{sw}} = f_{\text{ext}}$	Insensitive to system and operating conditions
Constant on-time modulation	$f_{\text{sw}} = \frac{1}{T_{\text{on}}} \times \left(1 - \frac{V_{\text{in}}}{V_o}\right)$	Highest switching frequency at lowest input voltage
Constant off-time modulation	$f_{\text{sw}} = \frac{1}{T_{\text{off}}} \times \frac{V_{\text{in}}}{V_o}$	Highest switching frequency at highest input voltage

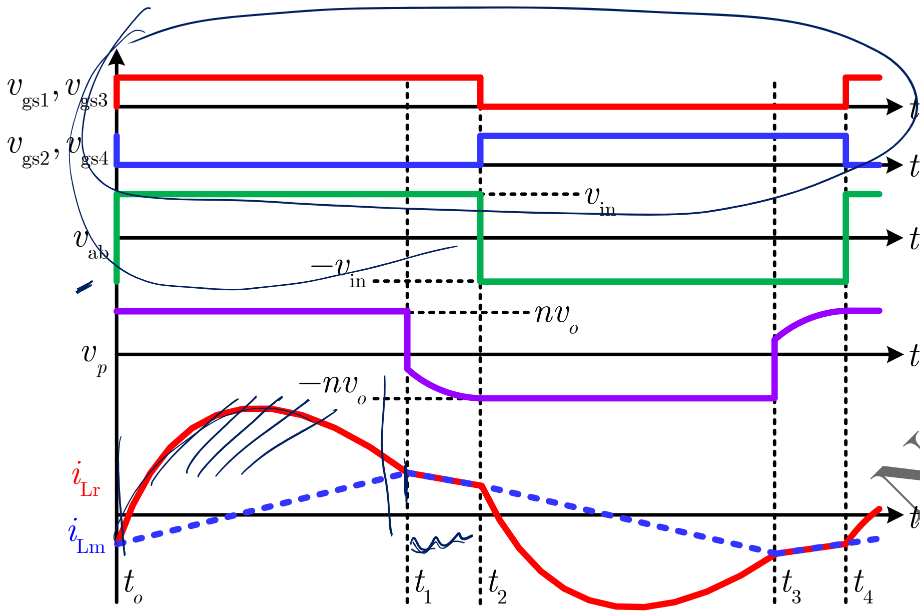
[For details, refer to Lecture~23, NPTEL "Stability and Performance …" course (link)]

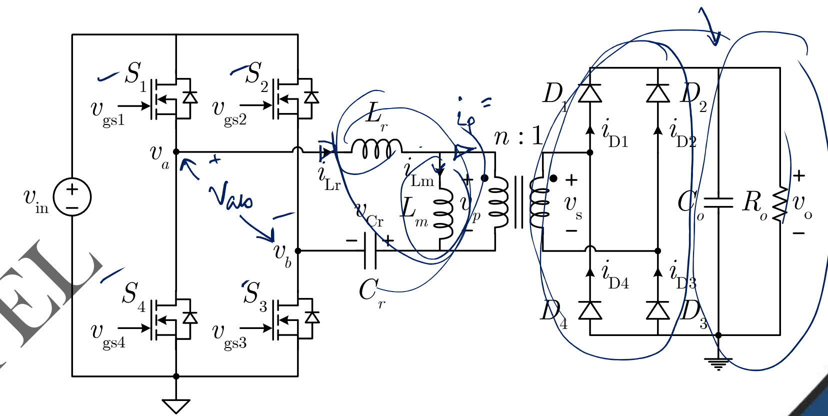
Full-Bridge LLC Resonant Converter



- High efficiency
- High power density
- Magnetic integration
- Low EMI
- High operation frequency
- Regulation of output over wide line and load variations with a relatively smaller variation of switching frequency

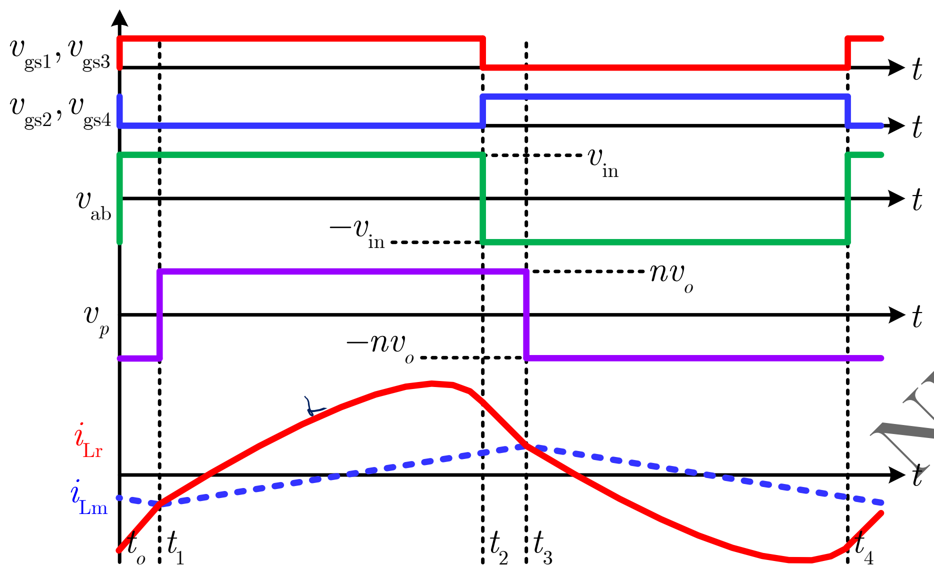
Switching Frequency Below Resonant Frequency (DCM)



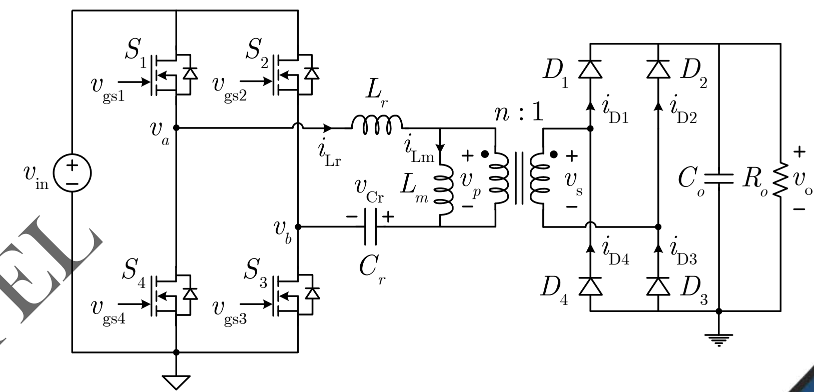


$$f_{sw} < f_r$$

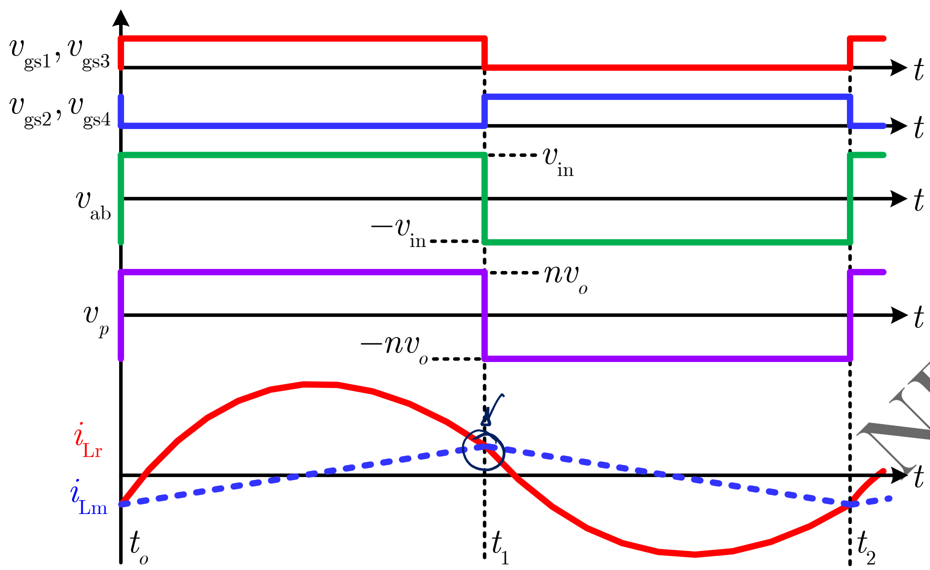
Switching Frequency Above Resonant Frequency (CCM)



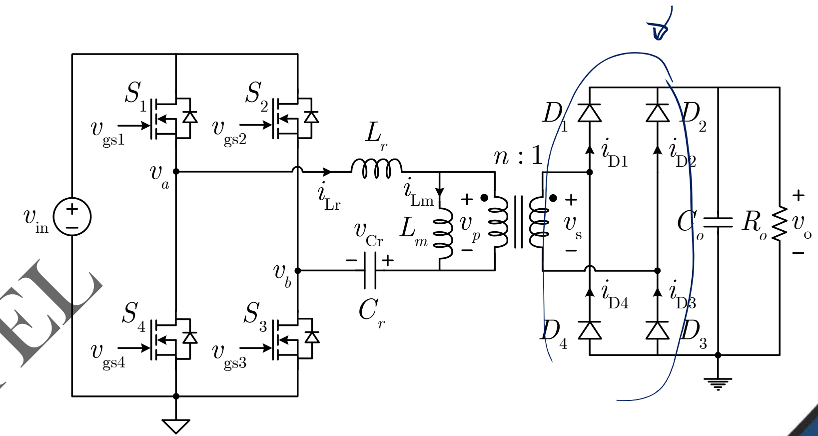
$$f_{sw} > f_r$$



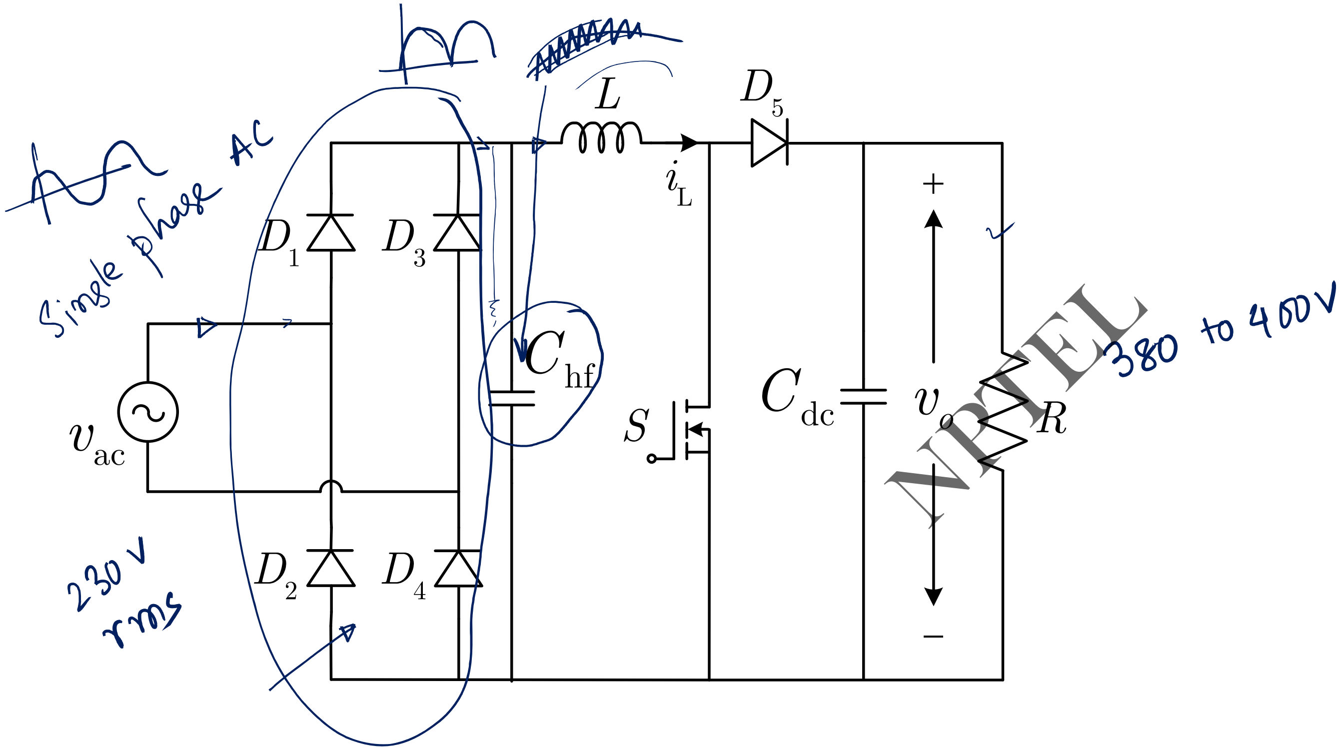
Switching Frequency Equals Resonant Frequency (CCM)



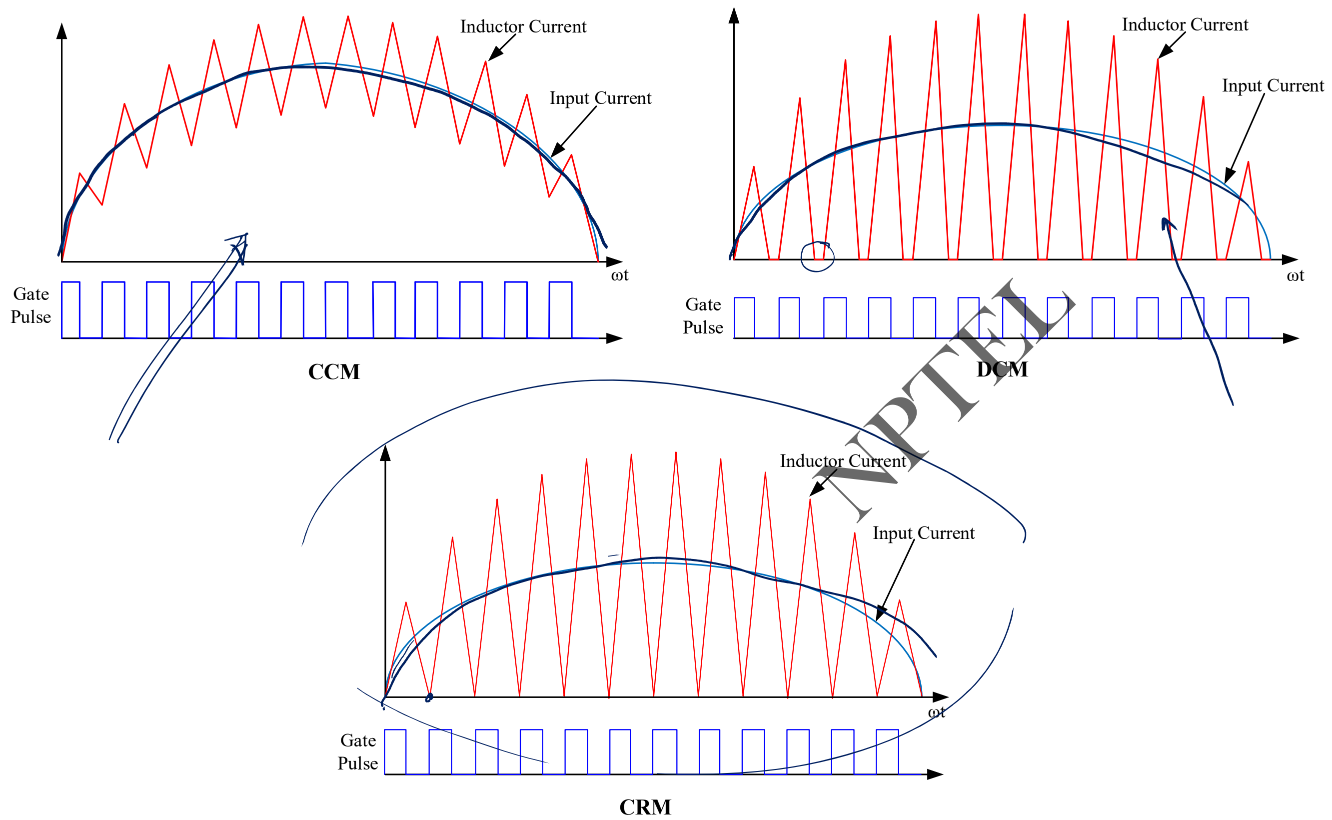
$$f_{sw} = f_r$$



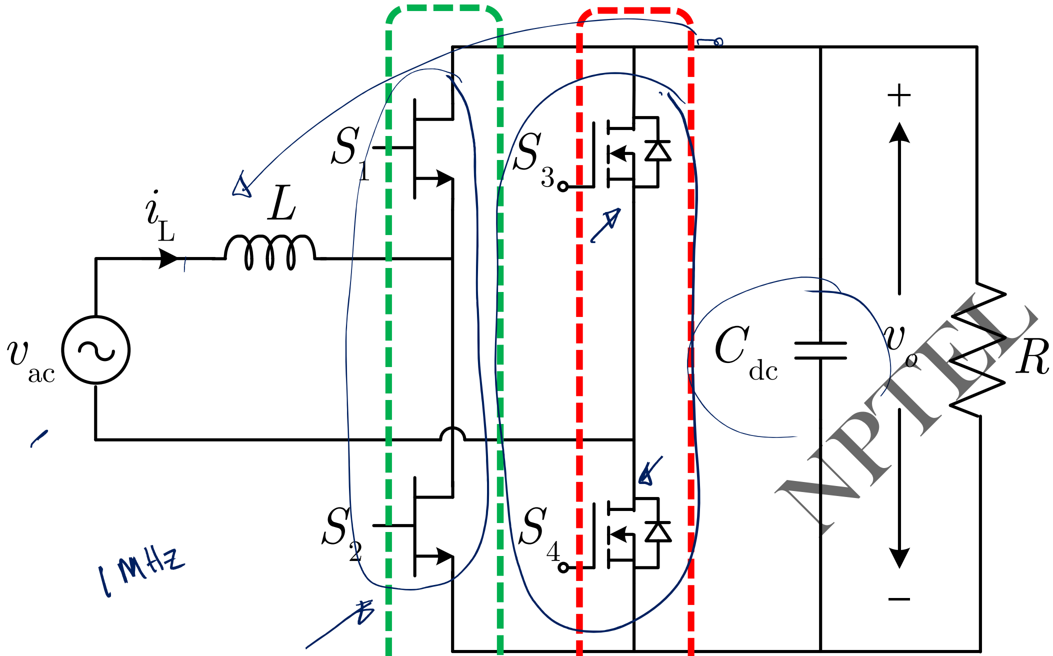
Full bridge with Boost PFC



Modes of Operation



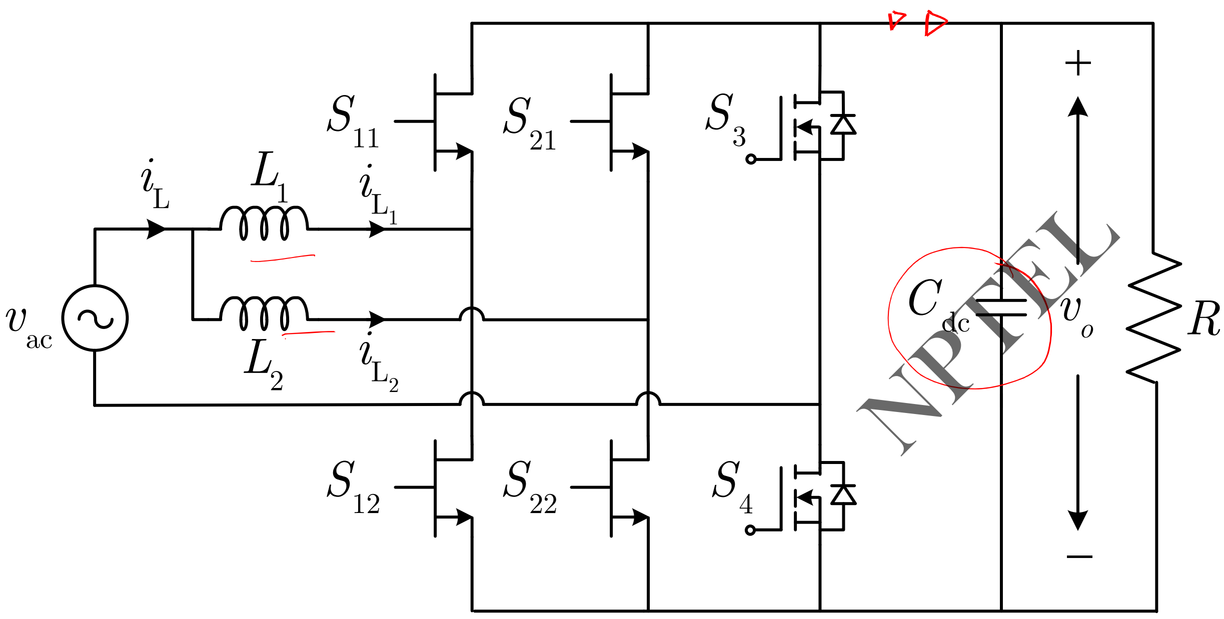
Single-Phase Totem-pole PFC

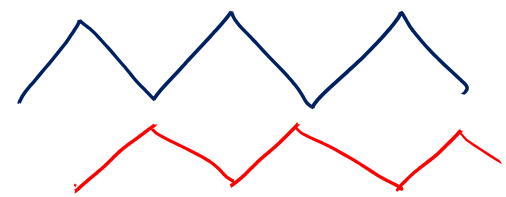


High switching frequency GaN leg

Line switching frequency Si leg

Single-Phase Interleaved Totem-pole PFC





CONCLUSION

- Power stage design aspects of buck and boost converters
- Buck and boost converters under steady-state in CCM and DCM
- Full-bridge LLC converter – basic operations for varying frequency
- Conventional boost PFC and totem-pole PFC for AC/DC conversion

THANK YOU !