NPTEL ONLINE CERTIFICATION COURSES

DIGITAL CONTROL IN SMPCs AND FPGA-BASED PROTOTYPING

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Module 04: Modeling Techniques and Mode Validation using MATLAB

Lecture 40: Model Accuracy with MATLAB Case Studies – Comparative Study

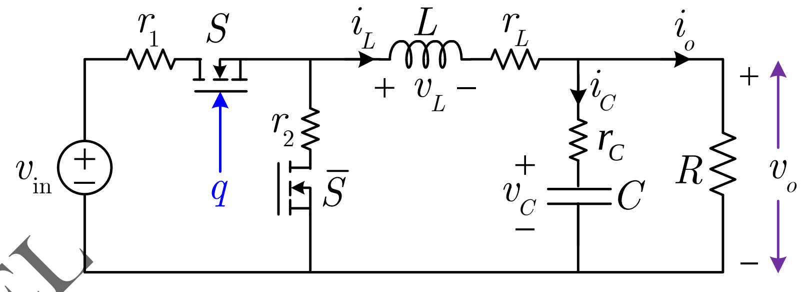
CONCEPTS COVERED

- CT control-to-output TF for synchronous buck converter with sampling delay
- DT control-to-output TF for synchronous buck converter
- DT and CT small-signal models – MATLAB implementation
- Small-signal model accuracy comparison – MATLAB case study

Buck Converter CT Control-to-Output TF for Sampled Data System

$$G_{vd}(s)=\frac{\tilde{v}_o(s)}{\tilde{d}(s)}=\frac{V_{IN}}{\frac{R+r_e}{R}}\frac{\left(1+r_C Cs\right)}{\left(1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}\right)}$$

V_e



$\omega_0 \approx \frac{1}{\sqrt{LC}}$

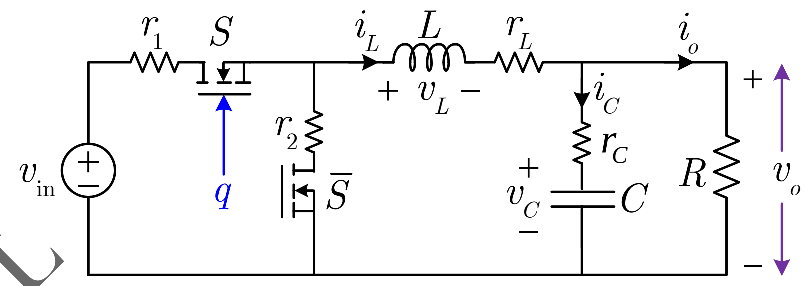
Total loop delay $\tau_d = t_s + DT$

$$G_{vd\_delay}(s)=e^{-s\tau_d}\times \boxed{G_{vd}(s)} \Rightarrow \boxed{G_{vd\_delay}(s)}=\frac{V_{IN}}{\frac{R+r_e}{R}}\frac{\left(1+r_C Cs\right)e^{-s\tau_d}}{\left(1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}\right)}$$

[For details, refer to Lecture~31, NPTEL "Digital Control in Switched…" course]

Buck Converter DT Control-to-Output TF

$$\tilde{x}_{n+1} = A_{\text{eq}}\tilde{x}_n + B_{\text{eq}}\tilde{d} \quad \textbf{and} \quad \tilde{v}_o[n] = C_{\text{eq}}\tilde{x}_n$$



where $A_{\text{eq}} = e^{AT}$ **and** $B_{\text{eq}} = e^{A(T-t_d)}B_1V_{\text{in}}T$

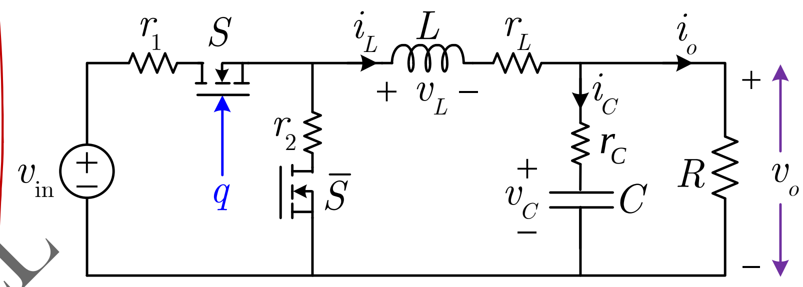
total loop delay $\tau_d = t_s + DT$ **and** $C_{\text{eq}} = \begin{bmatrix}\alpha r_C & \alpha\end{bmatrix}$

Applying Z-transformation $G_{\text{vd}}(z) = \dfrac{\tilde{v}_o(z)}{\tilde{d}(z)} = C_{\text{eq}}\left(zI - A_{\text{eq}}\right)^{-1}B_{\text{eq}}$

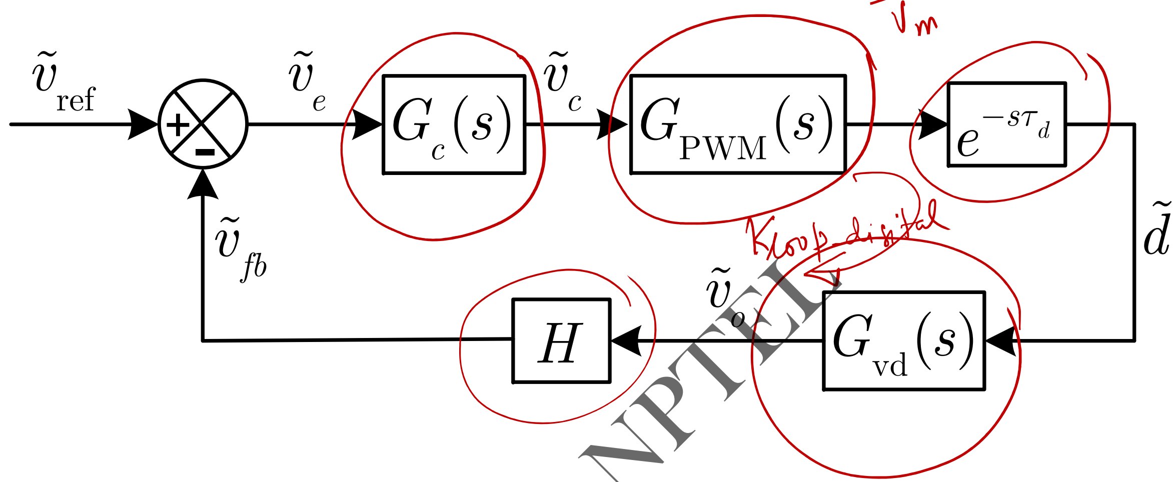
[**For details, refer to** Lecture~38, NPTEL "Digital Control in Switched…" course]

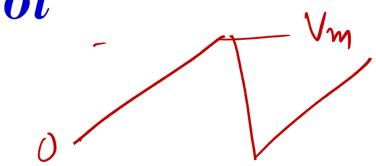
Synchronous Buck Converter Parameter

```
L=0.5e-6;          % output inductance
C=200e-6;          % output capacitance
T=2e-6;            % switching time period
r_L=5e-3;          % inductor DCR
r_1=5e-3;          % High-side MOSFET on resistance
r_2=5e-3;          % Low-side MOSFET on resistance
r_d=r_2;           % diode on resistance
v_d=0.55;          % diode voltage drop
r_C=3e-3;          % capacitor ESR
Vin=12;            % nominal input voltage
Vref=1;            % reference output voltage
Io_max =20;        % maximum load current
```



Approximate CT Small-Signal Model under Digital Control





$$K_{\text{loop, digital}}(s) = K_{\text{loop, analog}}(s) \times e^{-s\tau_d}$$

$$\tau_d = t_{\text{adc}} + t_{\text{DPWM}}$$

[R. Erickson and D. Maksimovic, "*Fundamentals of power electronics*", 3rd *Ed.*, Springer, 2020]

Buck Converter CT Control-to-Output TF

$t_d = t_s + DT$

ADC delay

DPWM delay

```
%% Define parameters
buck_parameter;
V_m=10; R=1;
D=Vref/Vin; r_eq=r_L+r_l;
Fm=1/V_m; t_s=0.1*T; t_d=t_s+(D*T);
%% Define poles
alpha=(R+r_eq)/R;
V_e=Vin/alpha;
z_c=sqrt(L/C);
w_o_ideal=1/sqrt(L*C);
w_o=w_o_ideal*(sqrt((R+r_eq)/(R+r_C)));
Q=alpha/(((r_C+r_eq)/z_c)+(z_c/R));
.....
```

Buck Converter CT Control-to-Output TF (cont….)

```
…..
%% Define zeros
w_z=1/(r_C*C);
%% Control-to-output TF Gvd
num_c=V_e*[1/w_z 1];
den_c=[1/(w_o^2) 1/(Q*w_o) 1];
Gvd=tf(num_c,den_c);
Gvd_delay=tf(num_c,den_c,'InputDelay',t_d);
Gvc=Fm*Gvd;
Gvc_delay=Fm*Gvd_delay;
figure(1)
bode(Gvc,'--b');    hold on;
bode(Gvc_delay,'g');    hold on;
```

$\tau_d = t_s + DT$

$\frac{\tilde{v}_o(s)}{\tilde{d}(s)}$

$$G_{vc}(s) = F_m G_{vd}(s)$$

$$G_{vc\_delay}(s) = F_m \boxed{G_{vd\_delay}(s)}$$

$G_{vd\_delay} = G_{vd} \times e^{-s\tau_d}$

Buck Converter DT Control-to-Output TF

```
%% Load system parameters
buck_parameter;
V_m=10;  R=1;
alpha=R/(R+r_C);  r_e=(r_l+r_L);
F_m=1/V_m;  D=Vref/Vin;  t_s=0.1*T;
%% Steady-state quantities
Io=Vref/R;  I_L_av=Io;
Xss=[I_L_av; Vref];
%% Define system, input and output matrices
A_on=[-(r_e+(alpha*r_C))/L -alpha/L; alpha/C -
alpha/(R*C)];
A_off=A_on;
B=[1/L; 0];
C_m=[r_C*alpha alpha];
.....
```

$C_m = [\alpha r_c \quad \alpha]$

Buck Converter DT Control-to-Output TF (cont….)

Lecture 38, 37

$\tilde{x}_{n+1} = A_{eq}\tilde{x}_n + B_{eq}\tilde{d}$

$\tilde{v}_o(n) = C_{eq}\tilde{x}_n + D_{eq}\cdot\tilde{d}$

```
…..
%% Obtain Aeq and Beq matrices
Aeq=(expm(A_on*T));
Beq=(expm(A_on*(T-(D*T)-t_s)))*B*Vin*T;
Ceq=C_m;
Deq=0;
%% DT Small-Signal Model
[num_vd,den_vd]=ss2tf(Aeq,Beq,Ceq,Deq);
G_vdd=tf(num_vd,den_vd,T);
G_vcd=F_m*G_vdd;
figure(1)
bode(G_vcd,'r');  hold on;
```

$G_{vcd}(w)$

$G_{vcd}(z)$

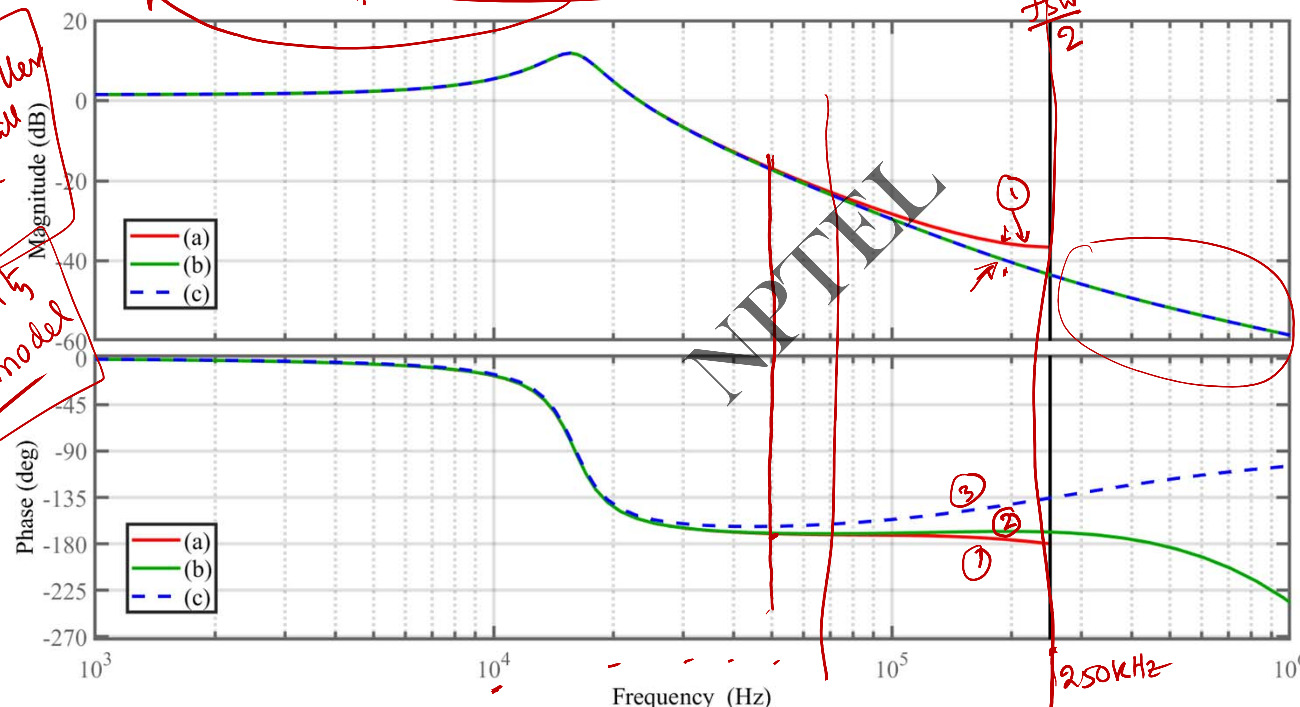
$$G_{vc}(z) = F_m G_{vd}(z)$$

Model Accuracy | Control-to-Output TF – Comparative Study

(a) DT SSM $G_{vc}(z)$, (b) $\boxed{\text{CT SSM } G_{vc\_delay}}(s)$ with delay $\tau_d = t_s + DT$, (c) CT SSM $G_{vc}(s)$

1) Design controller using CT small signal model with delay

2) Verify stability using DT model



$\frac{f_{sw}}{2}$

250kHz

$e^{-s\tau_d}$

$|e^{-j\omega\tau_d}| = 1$

CONCLUSION

- CT control-to-output TF for synchronous buck converter with sampling delay
- DT control-to-output TF for synchronous buck converter
- DT and CT small-signal models – MATLAB implementation
- Small-signal model accuracy comparison – MATLAB case study

THANK YOU !