NPTEL ONLINE CERTIFICATION COURSES

DIGITAL CONTROL IN SMPCs AND FPGA-BASED PROTOTYPING

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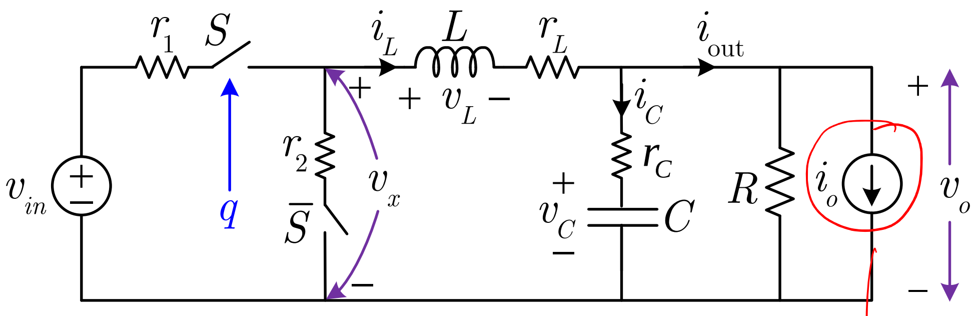
Module 04: Modeling Techniques and Mode Validation using MATLAB

Lecture 34: Derivation of Discrete-Time Large-Signal Models

CONCEPTS COVERED

- Discrete-time modeling in DC-DC converters
- Derivation of discrete-time large-signal models

State-Space Matrices of a Synchronous Buck Converter



Synchronous buck converter

$r_1 = r_2$

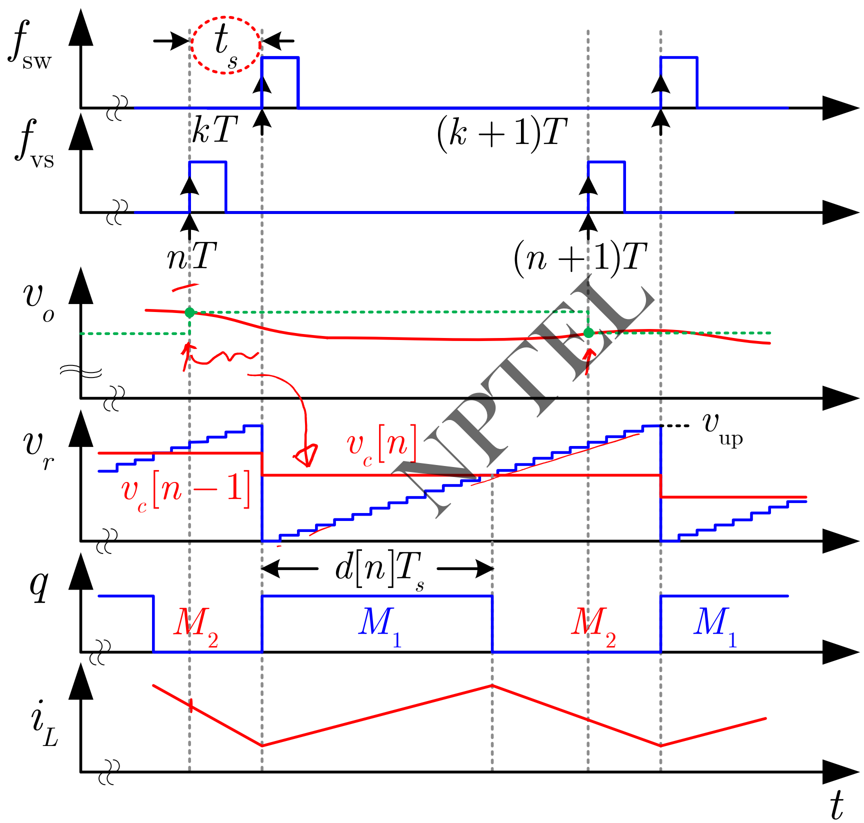
$$A_1 = A_2 = A = \begin{bmatrix} -\dfrac{(r_e + \alpha r_C)}{L} & -\dfrac{\alpha}{L} \\ \dfrac{\alpha}{C} & -\dfrac{\alpha}{RC} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \dfrac{1}{L} & \dfrac{\alpha r_C}{L} \\ 0 & -\dfrac{\alpha}{C} \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 & \dfrac{\alpha r_C}{L} \\ 0 & -\dfrac{\alpha}{C} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

in absence of current sink

Waveforms under Trailing-Edge Modulation with Interval-2 Sampling



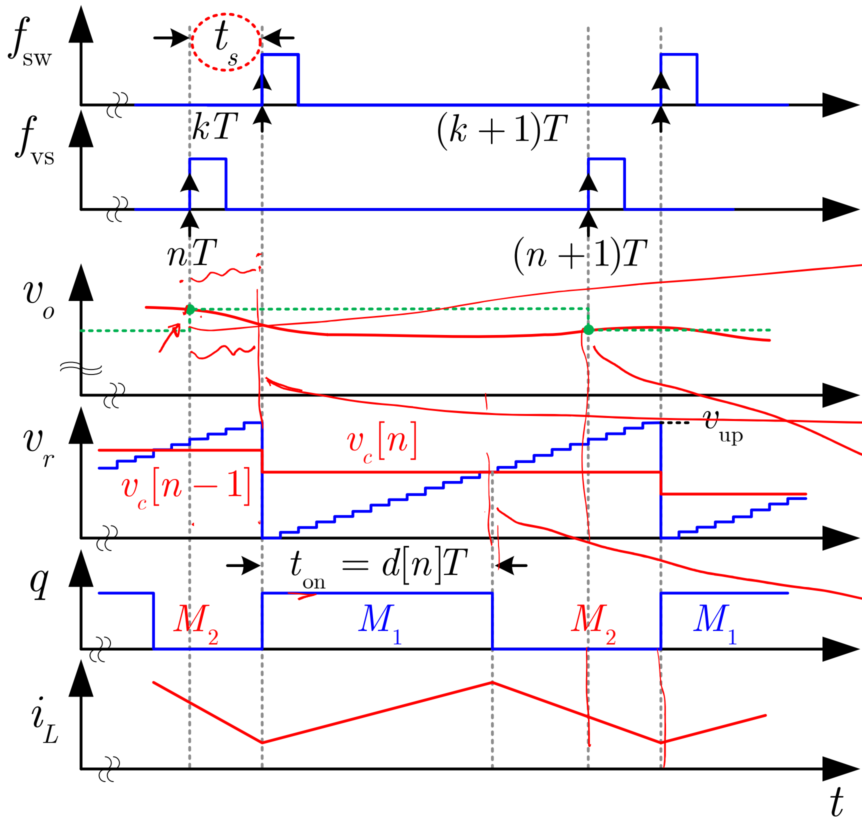
Trailing-edge PWM

Two subsequent sampling points

Periodic Evolution of State Vector over within a Sampling Cycle

$$x(t) = \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix}$$

$$v_o(t) = C_o\, x(t)$$



Let $x(t)\big|_{t=nT} = x_n$ initial state

$x(t)\big|_{t=nT+t_s} = x_1$

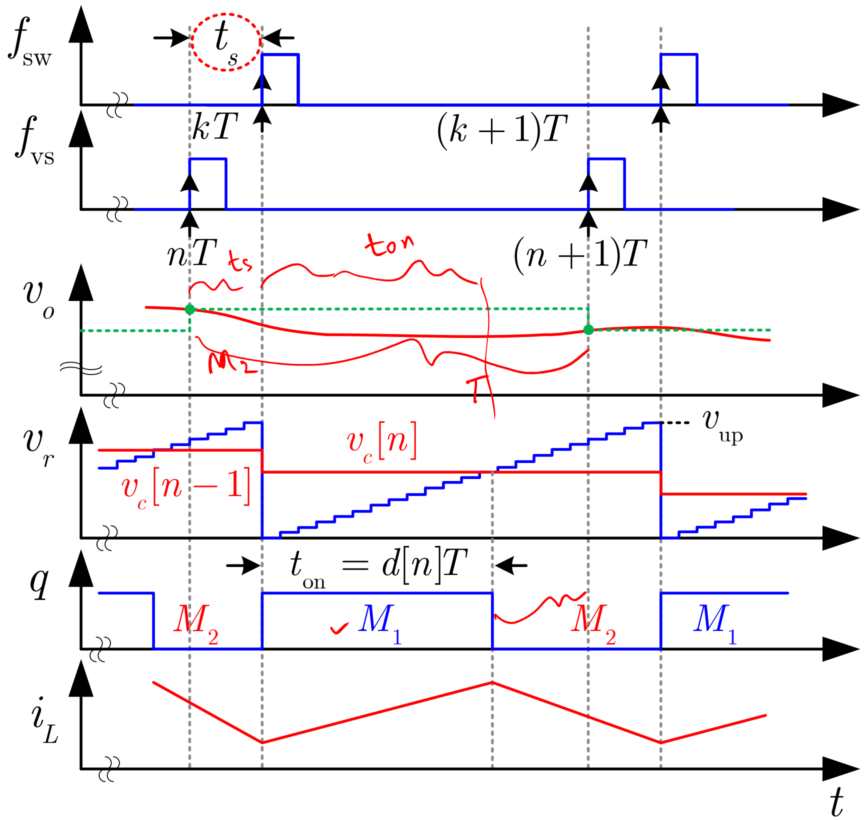
$x(t)\big|_{t=nT+t_s+t_{on}} = x_2$

intermediate states

$x(t)\big|_{t=(n+1)T} = x_{n+1}$

final state

Periodic Evolution of State Vector over within a Sampling Cycle (contd..)



$t = nT$

Mode M$_2$ $\quad t_s$ — duration

$t = nT + t_s$

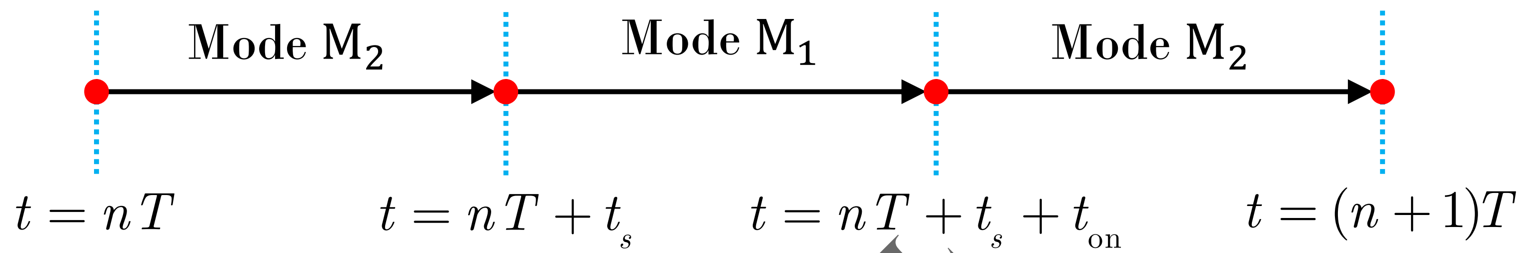
Mode M$_1$ $\quad t_{on}$

$t = nT + t_s + t_{on}$

Mode M$_2$ $\quad T - t_{on} - t_s$

$t = (n+1)T$

Periodic Evolution of State Vector over within a Sampling Cycle (contd..)



S. No.	Mode	Duration	Initial state vector	Final state vector	State matrices
1.	M_2	$nT \le t < nT + t_s$	x_n	x_1	A_2, B_2
2.	M_1	$nT + t_s \le t < nT + t_s + t_{on}$	x_1	x_2	A_1, B_1
3.	M_2	$nT + t_s + t_{on} \le t < (n+1)T$	x_2	x_{n+1}	A_2, B_2

[S. Kapat, "An Analytical Approach of Discrete-Time Modeling …" *IEEE APEC*, 2021]

Discrete-Time Large-Signal Model with Resistive Load

S. No.	Mode	Duration	Initial state vector	Final state vector	State matrices
1.	M_2	$nT \le t < nT + t_s$	x_n	x_1	$A_2 = A, B_2 = 0$
2.	M_1	$nT + t_s \le t < nT + t_s + t_{on}$	x_1	x_2	$A_1 = A, B_1$
3.	M_2	$nT + t_s + t_{on} \le t < (n+1)T$	x_2	x_{n+1}	$A_2 = A, B_2 = 0$

$t_0 = nT$

State-space solution

$$x(t) = e^{A_q(t-t_o)}x(t_o) + (e^{A_q(t-t_o)} - I)A_q^{-1}B_q u$$

$t = nT + ts$
$t_0 = nT$

$$t_o = nT \Rightarrow x(t_o) = x_n$$

$$t = \underline{nT + t_s} \Rightarrow x(t) = x_1$$

$$\boxed{\text{Mode } M_2 \Rightarrow x_1 = e^{At_s} x_n}$$

Discrete-Time Large-Signal Model with Resistive Load (contd..)

S. No.	Mode	Duration	Initial state vector	Final state vector	State matrices
1.	M_2	$nT \le t < nT + t_s$	x_n	x_1	$A_2 = A, B_2 = 0$
2.	M_1	$nT + t_s \le t < nT + t_s + t_{on}$	x_1	x_2	$A_1 = A, B_1$
3.	M_2	$nT + t_s + t_{on} \le t < (n+1)T$	x_2	x_{n+1}	$A_2 = A, B_2 = 0$

State-space solution

$$x(t) = e^{A_q(t-t_o)}x(t_o) + (e^{A_q(t-t_o)} - I)A_q^{-1}B_q u$$

$A_1 = A_2 = A$

$$t_o = nT + t_s \Rightarrow x(t_o) = x_1$$

$$t = nT + t_s + t_{on} \Rightarrow x(t) = x_2$$

$$\text{Mode } M_1 \Rightarrow x_2 = e^{At_{on}}x_1 + (e^{At_{on}} - I)A^{-1}B_1 v_{in}$$

Discrete-Time Large-Signal Model with Resistive Load (contd..)

S. No.	Mode	Duration	Initial state vector	Final state vector	State matrices
1.	M_2	$nT \le t < nT + t_s$	x_n	x_1	$A_2 = A, B_2 = 0$
2.	M_1	$nT + t_s \le t < nT + t_s + t_{on}$	x_1	x_2	$A_1 = A, B_1$
3.	M_2	$nT + t_s + t_{on} \le t < (n+1)T$	x_2	x_{n+1}	$A_2 = A, B_2 = 0$

State-space solution

$$x(t) = e^{A_q(t-t_o)}x(t_o) + (e^{A_q(t-t_o)} - I)A_q^{-1}B_q u$$

$T - t_{on} - t_s$

$$t_o = nT + t_s + t_{on} \Rightarrow x(t_o) = x_2$$

$$t = (n+1)T \Rightarrow x(t) = x_{n+1}$$

$$\text{Mode } M_2 \Rightarrow x_{n+1} = e^{A(T-t_{on}-t_s)}x_2 + (0)$$

Discrete-Time Large-Signal Model with Resistive Load (contd..)

S. No.	Mode	Duration	Initial state vector	Final state vector	State matrices
1.	M_2	$nT \le t < nT + t_s$	x_n	x_1	$A_2 = A, B_2 = 0$
2.	M_1	$nT + t_s \le t < nT + t_s + t_{on}$	x_1	x_2	$A_1 = A, B_1$
3.	M_2	$nT + t_s + t_{on} \le t < (n+1)T$	x_2	x_{n+1}	$A_2 = A, B_2 = 0$

$x_n \longrightarrow x_1 \longrightarrow x_2 \longrightarrow x_{n+1}$

$$\text{Mode } M_2\text{:} \quad x_1 = e^{At_s} x_n$$

$$\text{Mode } M_1\text{:} \quad x_2 = e^{At_{on}} x_1 + (e^{At_{on}} - I)A^{-1}B_1 v_{in}$$

$$\text{Mode } M_2\text{:} \quad x_{n+1} = e^{A(T - t_{on} - t_s)} x_2$$

[S. Kapat, "An Analytical Approach of Discrete-Time Modeling ..." *IEEE APEC*, 2021]

Discrete-Time Large-Signal Model with Resistive Load (contd..)

Eliminate x_1 and x_2 to obtain x_{n+1} in terms of x_n

Mode M_2: $x_1 = e^{At_s} x_n$

Mode M_1: $x_2 = e^{At_{on}} x_1 + (e^{At_{on}} - I)A^{-1}B_1 v_{in}$

in terms of x_n

Mode M_2: $x_{n+1} = e^{A(T-t_{on}-t_s)} x_2$

$e^{A_1 t_1} \cdot e^{A_2 t_2} \neq e^{(A_1 t_1 + A_2 t_2)}$

if $A_1 A_2 \neq A_2 A_1$

$e^{At_s} \cdot e^{A t_{on}} \cdot e^{A(T-t_{on}-t_s)}$

$= e^{A(t_s + t_{on} + T - t_{on} - t_s)}$

$= e^{AT}$

Large-Signal Discrete-Time Model

$$x_{n+1} = e^{AT} x_n + e^{A(T-t_{on}-t_s)}(e^{At_{on}} - I)A^{-1}B_1 v_{in}$$

Complete Discrete-Time Large-Signal Model with Resistive Load

$$x_{n+1} = \begin{bmatrix} i_L[n+1] \\ v_C[n+1] \end{bmatrix}$$

Large-Signal Discrete-Time Model

$$x_{n+1} = e^{AT}x_n + e^{A(T-t_{on}-t_s)}(e^{At_{on}} - I)A^{-1}B_1 v_{in}$$

$$\underbrace{x_{n+1}}_{2\times 1} = \underbrace{e^{AT}}_{2\times 2}x_n + \underbrace{e^{A(T-t_{on}-t_s)}}_{2\times 2}\underbrace{(e^{At_{on}} - I)}_{2\times 2}\underbrace{A^{-1}}_{2\times 2}\underbrace{B_1}_{2\times 1}\underbrace{v_{in}}_{1\times 1} \triangleq f(x_n, t_{on}) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

[S. Kapat, "An Analytical Approach of Discrete-Time Modeling ..." *IEEE APEC*, 2021]

CONCLUSION

- Discrete-time modeling in DC-DC converters
- Derivation of discrete-time large-signal models

THANK YOU !