



**NPTEL ONLINE CERTIFICATION COURSES**

# **CONTROL AND TUNING METHODS IN SMPCs**

**Dr. Santanu Kapat**

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**Module 02: Modulation Techniques in SMPCs**

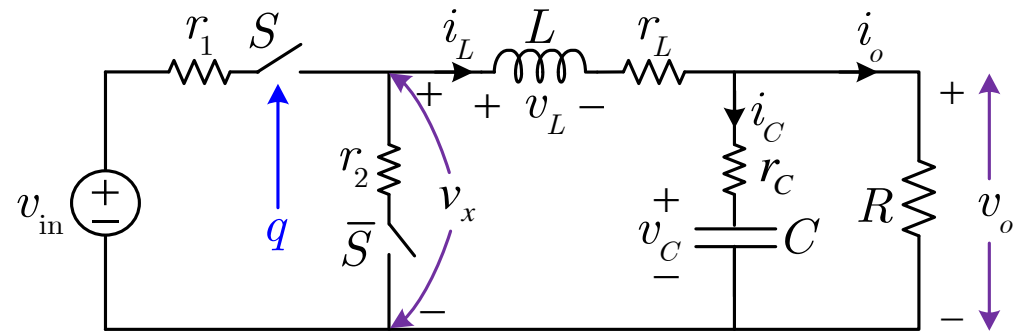
**Lecture 07: Power Stage Design of Basic SMPCs: Summary**

# Concepts Covered

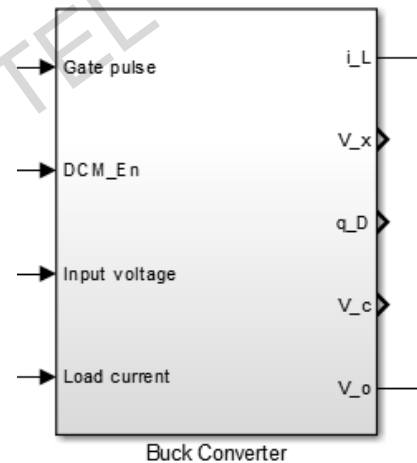
- Steady-state ripple parameters
- Derivation of RMS quantities
- Selection of inductor
- Selection of capacitor
- Simulation case studies

# Synchronous Buck Converter

$L=2\text{e-}6;$       % output inductor  
 $C=500\text{e-}6;$     % output capacitor  
 $T=1\text{e-}6;$         % switching time period  
 $r_L=10\text{e-}3;$      % inductor DCR  
 $v_d=0*0.7;$      % diode voltage drop  
 $r_1=5\text{e-}3;$       % High-side MOSFET on resistance  
 $r_2=5\text{e-}3;$       % Low-side MOSFET on resistance  
                     or diode resistance (in case diode)  
 $r_C=5\text{e-}3;$       % capacitor ESR  
 $I_{L\_int}=1;$      % initial inductor current  
 $V_{c\_int}=3.4;$  % initial capacitor voltage  
 $V_{up}=10;$       % ramp peak voltage  
 $V_b=0;$          % ramp base voltage  
 $V_{in}=12;$       % input voltage  
 $V_{ref}=3.3;$      % reference output voltage  
 $R=1;$             % load resistance



*Synchronous buck converter*

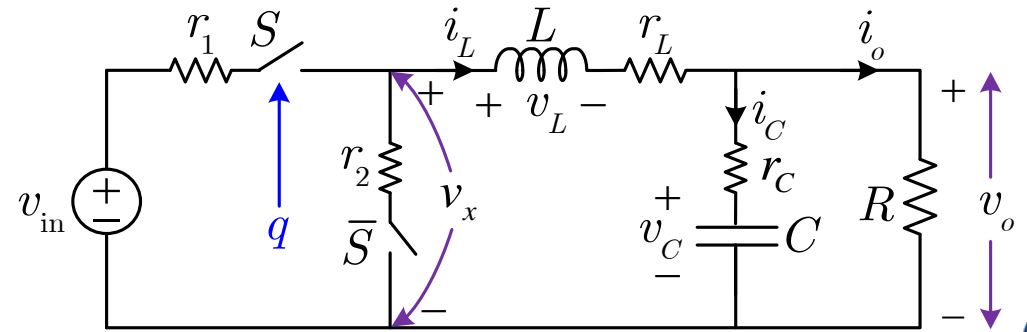


Buck Converter

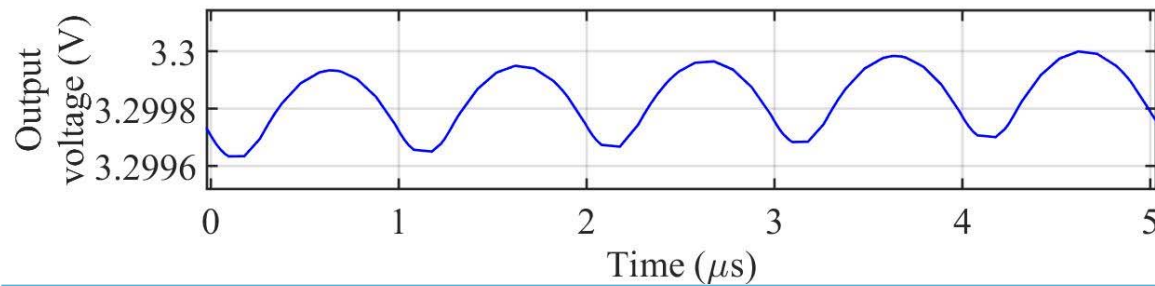
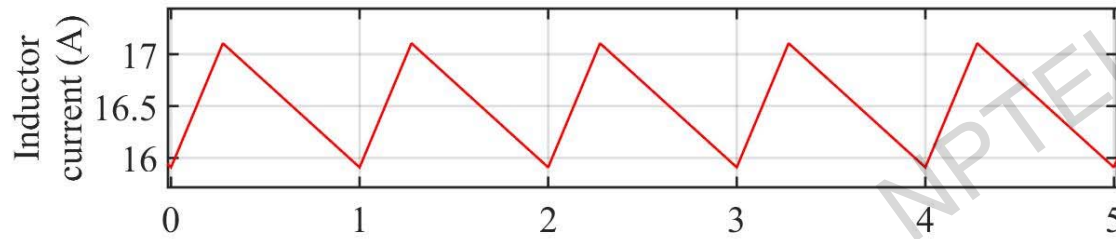
**MATLAB model**

## Ideal Buck Converter Simulation

$$r_L = r_1 = r_2 = r_C = 0$$

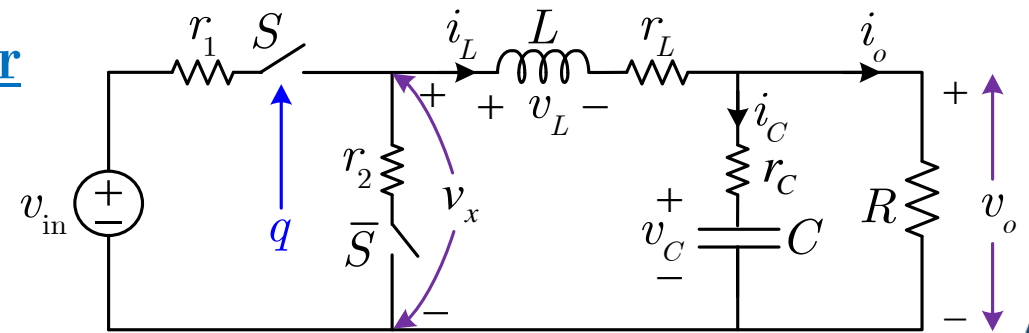
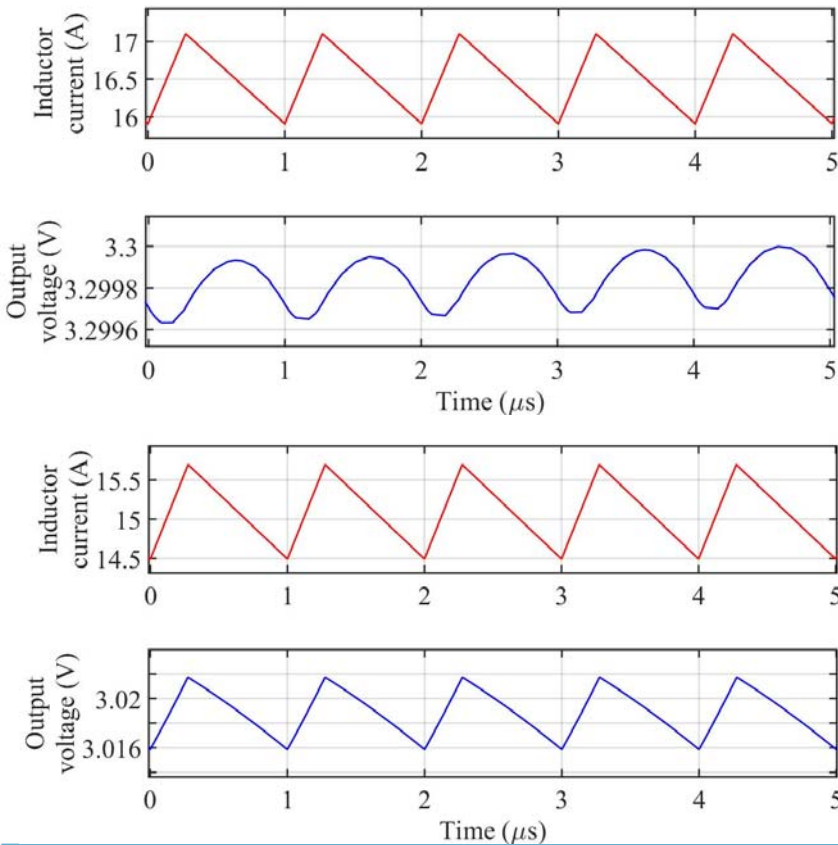


*Synchronous buck converter*



- ☐ Ripple current
- ☐ Ripple voltage
- ☐ Average current
- ☐ Average voltage

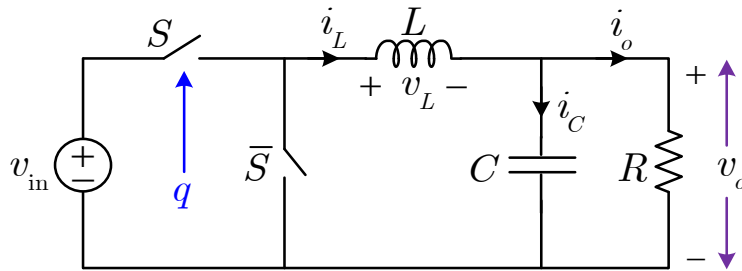
## Ideal vs Practical Buck Converter



*Synchronous buck converter*

- ☐ Ripple current – insignificant effect
- ☐ Ripple voltage – **significant effect**
- ☐ Average current, average voltage – **significantly affected**

## Buck Converter Ripple Parameters

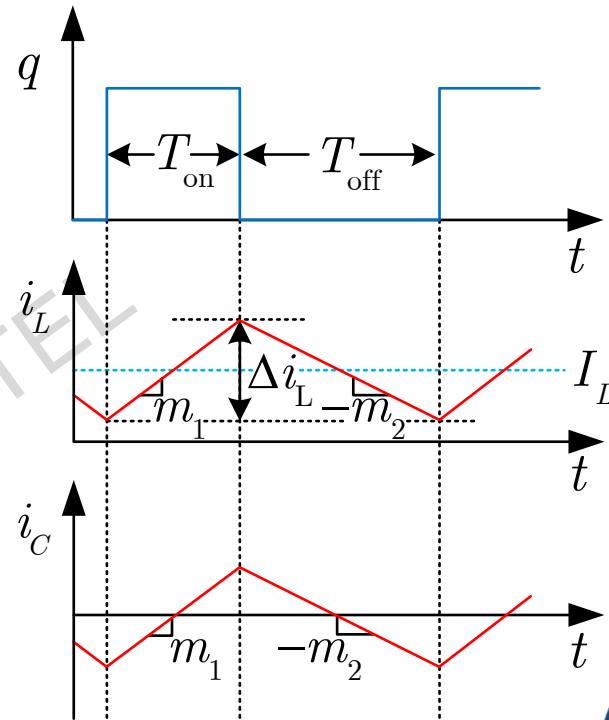


*Buck Converter*

- Inductor current ripple ( $\Delta i_L$ ) of a buck converter

$$\Delta i_L = m_1 T_{\text{on}} \quad \text{where } m_1 = \frac{V_{\text{IN}} - V_O}{L}$$

$$\therefore \Delta i_L = \frac{V_{\text{IN}} - V_O}{L} \times T_{\text{on}}$$



*Steady-state waveforms*

## Buck Converter Ripple Parameters (contd...)

- Write ripple current in terms of  $T_{\text{on}}$ ,  $T_{\text{off}}$  and  $V_o$  ( $V_o$  is constant for a VR)

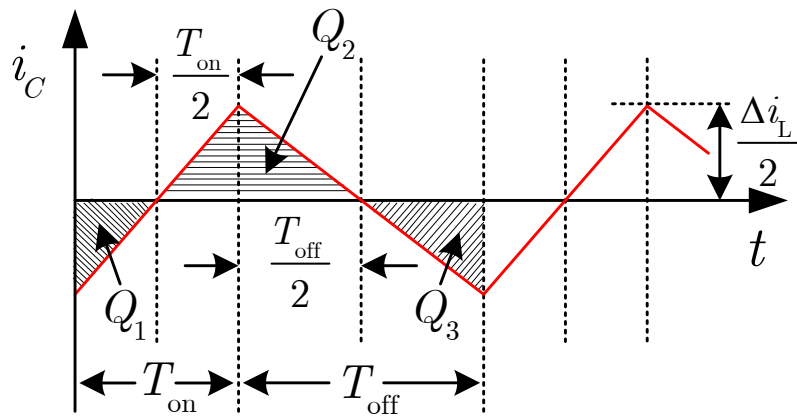
- Voltage gain 
$$K_V = \frac{V_o}{V_{\text{IN}}} = \frac{T_{\text{on}}}{T_{\text{on}} + T_{\text{off}}}$$

$$V_{\text{IN}} = \left( \frac{T_{\text{on}} + T_{\text{off}}}{T_{\text{on}}} \right) \times V_o = \left( 1 + \frac{T_{\text{off}}}{T_{\text{on}}} \right) \times V_o$$

$$\therefore \Delta i_L = \frac{V_{\text{IN}} - V_o}{L} \times T_{\text{on}} = \frac{T_{\text{on}}}{L} \times \left[ \left( 1 + \frac{T_{\text{off}}}{T_{\text{on}}} \right) - 1 \right] \times V_o$$

$$= \frac{V_o}{L} \times T_{\text{off}} \quad \square \text{ Current ripple/off-time trade-off}$$

## Capacitor Voltage Ripple – Ideal Buck Converter



*Capacitor current waveform*

$$Q_2 = \frac{1}{2} \times \frac{\Delta i_L}{2} \times \left( \frac{T_{\text{on}} + T_{\text{off}}}{2} \right)$$

$$= \frac{1}{8} \times (T_{\text{on}} + T_{\text{off}}) \times \Delta i_L$$

Substituting  $\Delta i_L = \frac{V_o}{L} \times T_{\text{off}}$

$$Q_2 = \frac{V_o}{8L} \times (T_{\text{on}} + T_{\text{off}}) \times T_{\text{off}}$$

Again  $Q_2 = C \times \Delta v_o$

$$\Delta v_o = \frac{V_o}{8LC} \times (T_{\text{on}} + T_{\text{off}}) \times T_{\text{off}}$$

□ **Voltage ripple impact?**



## Ripple Parameters of a Buck Converter under PWM

### ■ Under PWM

$$T_{\text{on}} + T_{\text{off}} = T_{\text{sw}} = \frac{1}{f_{\text{sw}}} \quad (\text{fixed})$$

$$T_{\text{on}} = D \times T_{\text{sw}}$$

$$T_{\text{off}} = T_{\text{sw}} - T_{\text{on}} = (1 - D) \times T_{\text{sw}}$$

$$\Delta v_o = \frac{V_o}{8LC} \times T_{\text{sw}} \times (1 - D) T_{\text{sw}}$$

$$\Delta v_o = \left( \frac{V_o}{8LCf_{\text{sw}}^2} \right) \times (1 - D)$$

→ Voltage ripple is maximum at minimum  $D \rightarrow$  highest  $v_{\text{in}}$

$$\Delta i_L = \frac{V_o}{L} \times (1 - D) T_{\text{sw}}$$

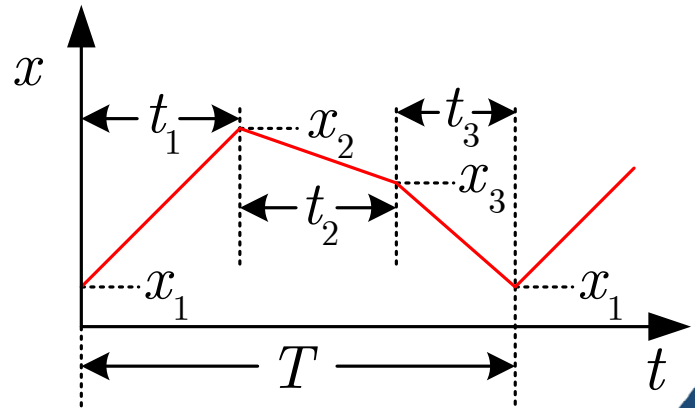
$$\Rightarrow \Delta i_L = \frac{V_o}{Lf_{\text{sw}}} \times (1 - D)$$

Current ripple is maximum at minimum  $D \rightarrow$  highest  $v_{\text{in}}$

## RMS Value of a Periodic Piecewise Linear Waveform

$$(x_{\text{rms}})^2 = \frac{1}{T} \left[ \left( \frac{x_1^2 + x_1 x_2 + x_2^2}{3} \right) t_1 + \left( \frac{x_2^2 + x_2 x_3 + x_3^2}{3} \right) t_2 + \left( \frac{x_3^2 + x_3 x_1 + x_1^2}{3} \right) (T - t_1 - t_2) \right]$$

Prove it



**Hint:**

$$(x_{\text{rms}})^2 = \frac{1}{T} \left[ \int_0^T x^2(t) dt \right] = \frac{1}{T} \left[ \int_0^{t_1} x^2(t) dt + \int_{t_1}^{t_1+t_2} x^2(t) dt + \int_{t_1+t_2}^T x^2(t) dt \right]$$

$$= \frac{1}{T} \left[ \int_0^{t_1} x^2(t) dt + \int_0^{t_2} x^2(t + t_1) dt + \int_0^{T-(t_1+t_2)} x^2(t + t_1 + t_2) dt \right]$$

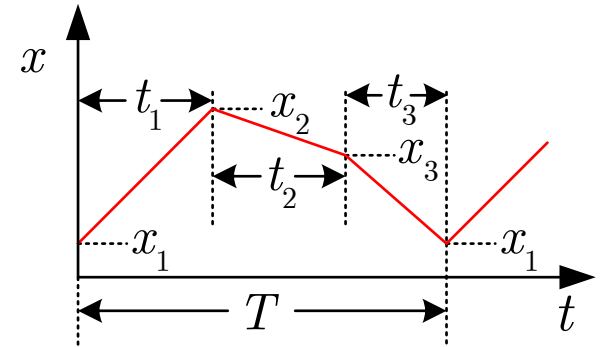
## RMS Formulation – Proof

$$\begin{aligned}(x_{\text{rms}})^2 &= \frac{1}{T} \left[ \int_0^T x^2(t) dt \right] \\ &= \frac{1}{T} \left[ \int_0^{t_1} x^2(t) dt + \int_0^{t_2} x^2(t + t_1) dt + \int_0^{T-(t_1+t_2)} x^2(t + t_1 + t_2) dt \right]\end{aligned}$$

□ Define

$$I_k = \int_0^{t_k} x^2(t) dt = \int_0^{t_k} \left[ x_k + \left( \frac{x_{k+1} - x_k}{t_k} \right) \tau \right]^2 d\tau$$

$$I_k = \int_0^{t_k} \left[ x_k^2 + 2x_k \left( \frac{x_{k+1} - x_k}{t_k} \right) \tau + \left( \frac{x_{k+1} - x_k}{t_k} \right)^2 \tau^2 \right] d\tau$$

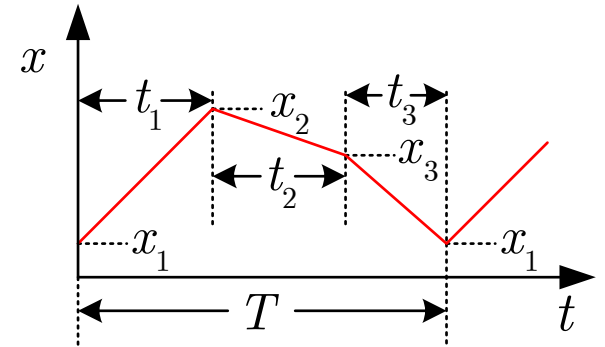


## RMS Formulation – Proof Contd...

$$I_k = \int_0^{t_k} \left[ x_k^2 + 2x_k \left( \frac{x_{k+1} - x_k}{t_k} \right) \tau + \left( \frac{x_{k+1} - x_k}{t_k} \right)^2 \tau^2 \right] d\tau$$

$$= \left[ x_k^2 \tau + 2x_k \left( \frac{x_{k+1} - x_k}{t_k} \right) \frac{\tau^2}{2} + \left( \frac{x_{k+1} - x_k}{t_k} \right)^2 \frac{\tau^3}{3} \right]_0^{t_k}$$

$$= \left( \frac{x_k^2 + x_k x_{k+1} + x_{k+1}^2}{3} \right) t_k$$

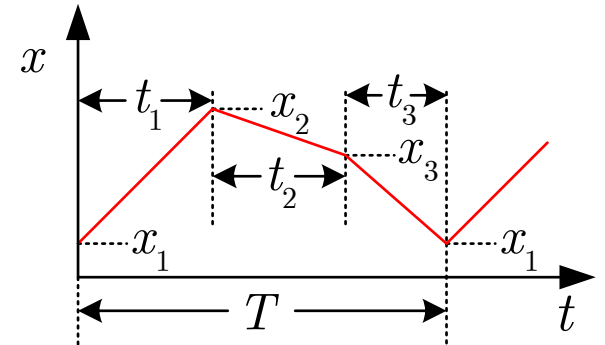


## RMS Formulation – Proof Contd...

$$I_k = \left( \frac{x_k^2 + x_k x_{k+1} + x_{k+1}^2}{3} \right) t_k$$

$$(x_{\text{rms}})^2 = \frac{1}{T} \left[ \int_0^T x^2(t) dt \right]$$

$$\begin{aligned} &= \frac{1}{T} \left[ \left( \frac{x_1^2 + x_1 x_2 + x_2^2}{3} \right) t_1 + \left( \frac{x_2^2 + x_2 x_3 + x_3^2}{3} \right) t_2 \right. \\ &\quad \left. + \left( \frac{x_3^2 + x_3 x_1 + x_1^2}{3} \right) (T - t_1 - t_2) \right] \end{aligned}$$



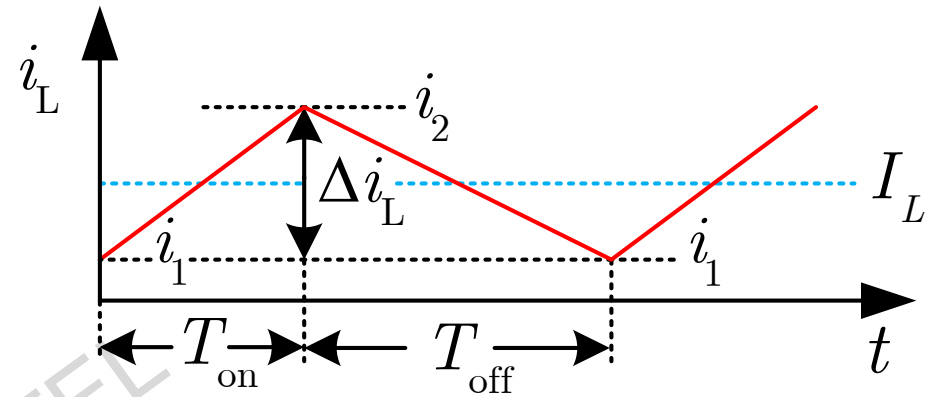
## RMS Value of Inductor Current

$$\Delta i_L = i_2 - i_1 \Rightarrow i_2 - i_1 = \Delta i_L$$

$$I_L = \frac{i_1 + i_2}{2} \Rightarrow i_1 + i_2 = 2I_L$$

$$i_1 = I_L - \frac{\Delta i_L}{2}$$

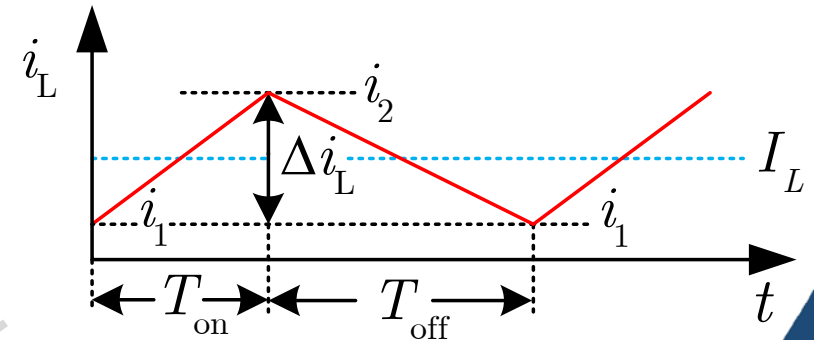
$$i_2 = I_L + \frac{\Delta i_L}{2}$$



*Inductor current waveform*

## RMS Value of Inductor Current (contd...)

$$\begin{aligned} (i_{L,\text{RMS}})^2 &= \frac{1}{T_{\text{on}} + T_{\text{off}}} \times \left[ \left( \frac{i_1^2 + i_1 i_2 + i_2^2}{3} \right) (T_{\text{on}} + T_{\text{off}}) \right] \\ &= \frac{i_1^2 + i_1 i_2 + i_2^2}{3} \end{aligned}$$



*Inductor current waveform*

Substituting,  $i_1 = I_L - \frac{\Delta i_L}{2}$  and  $i_2 = I_L + \frac{\Delta i_L}{2}$

$$(i_{L,\text{RMS}})^2 = I_L^2 + \frac{\Delta i_L^2}{12} = I_o^2 + \frac{\Delta i_L^2}{12}$$

## Summary

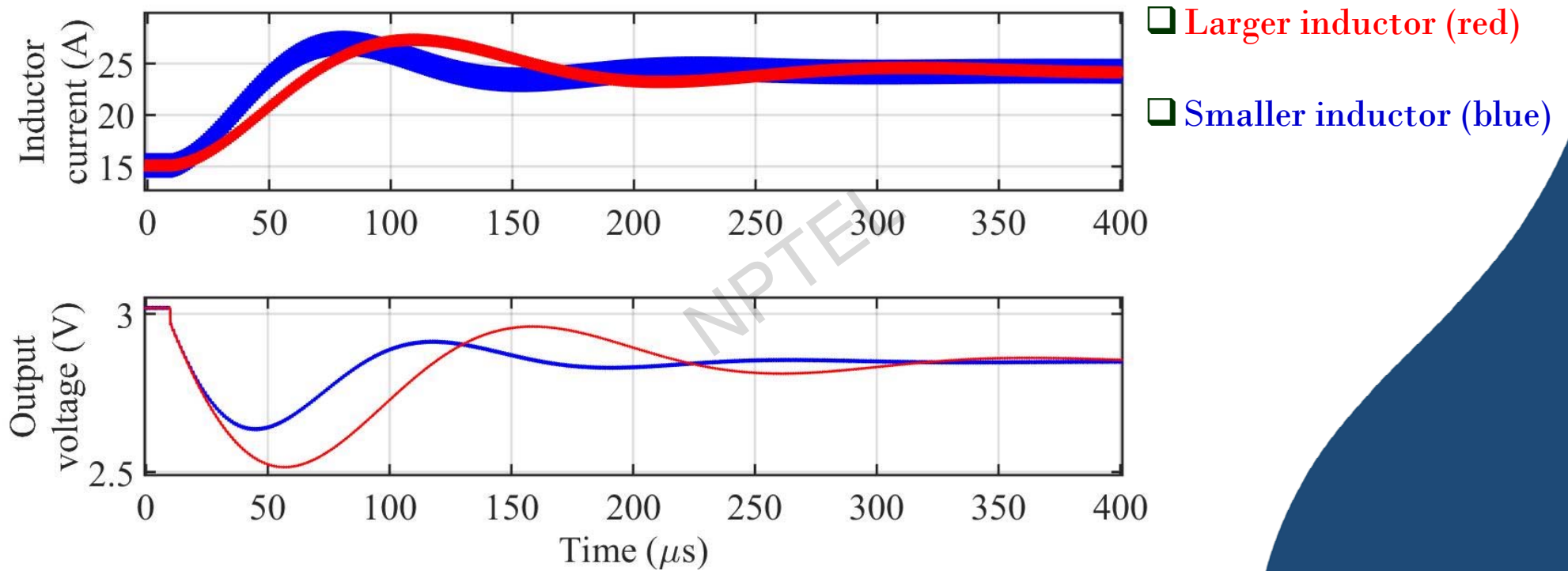
- For a given load current,
  - $i_{L,\text{RMS}}$  increases with increasing  $\Delta i_L$
  - $i_{L,\text{RMS}}$  is maximum at maximum  $v_{in}$
  - Higher  $i_{L,\text{RMS}}$  implies higher conduction loss
- For a given input voltage,
  - $i_{L,\text{RMS}}$  increases with increasing  $\Delta i_L$
  - Higher conduction loss at higher load current

$$\left(i_{L,\text{RMS}}\right)^2 = I_o^2 + \frac{\Delta i_L^2}{12}$$

Worst case RMS current (also conduction loss) at  
**highest input voltage and highest load current**



## Design Consideration (Inductor)



## Design Consideration (Inductor) Large Inductor

### Advantages

Smaller ripple current

$$\Delta i_L = \frac{V_o(1-D)}{f_{sw}} \times \frac{1}{L}$$

Smaller RMS current

$$(i_{L,RMS})^2 = I_o^2 + \frac{\Delta i_L^2}{12}$$

Lower conduction loss

Smaller voltage ripple

$$\Delta v_o = \frac{V_o(1-D)}{8Cf_{sw}^2} \times \frac{1}{L}$$

### Disadvantages

Larger size  
(bulky inductor)

Slower transient response!!

Higher voltage overshoot/  
undershoot!!

*Inductor should be  
carefully designed*

## Design Consideration (Capacitor) Large Capacitor

### Advantages

Smaller output voltage ripple

$$\Delta v_o = \frac{V_o(1-D)}{8Cf_{sw}^2} \times \frac{1}{L}$$

Smaller output voltage undershoot/ overshoot

### Disadvantages

Larger size and poor reliability

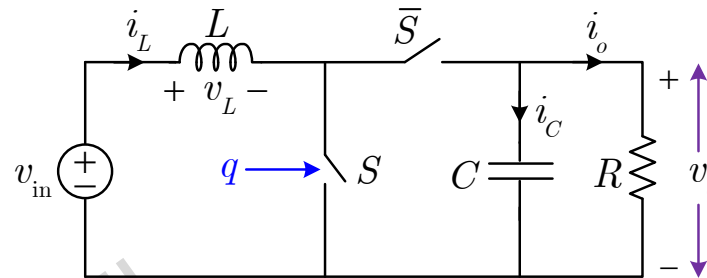
Higher time and energy overhead during reference voltage transient

*Capacitor should be carefully selected*

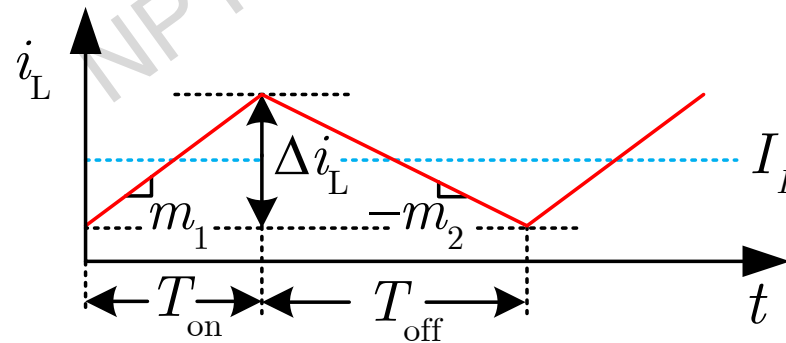
## Ripple Inductor Current- Boost Converter

$$\Delta i_L = m_1 \times T_{\text{on}} = \frac{V_{\text{IN}}}{L} \times T_{\text{on}}$$

Express  $V_{\text{IN}}$  in terms of  $V_o$   
since  $V_o$  is constant for a VR



*Boost converter*



*Inductor current waveform*

## For a Boost Converter

$$V_o = \frac{T_{\text{on}} + T_{\text{off}}}{T_{\text{off}}} V_{\text{IN}} \Rightarrow V_{\text{IN}} = \frac{T_{\text{off}}}{T_{\text{on}} + T_{\text{off}}} V_o$$

$$\Delta i_L = \frac{V_{\text{IN}}}{L} \times T_{\text{on}} \Rightarrow \Delta i_L = \frac{V_o}{L} \times \left( \frac{T_{\text{on}} T_{\text{off}}}{T_{\text{on}} + T_{\text{off}}} \right)$$

$$\therefore \Delta i_L = \frac{V_o}{L f_{\text{sw}}} \times [D(1 - D)]$$

$$\frac{\partial \Delta i_L}{\partial D} = \frac{V_o}{L f_{\text{sw}}} (1 - 2D) = 0$$

$$\Rightarrow D = 0.5$$

$\Delta i_L$  is maximum at  $D=0.5$

### Under PWM

$$T_{\text{on}} + T_{\text{off}} = T_{\text{sw}} = \frac{1}{f_{\text{sw}}}$$

$$T_{\text{on}} = D T_{\text{sw}}$$

$$T_{\text{off}} = (1 - D) T_{\text{sw}}$$

$$\frac{\partial^2 \Delta i_L}{\partial D^2} = -\frac{2V_o}{L f_{\text{sw}}} < 0$$

## Ripple Output Voltage – Boost Converter

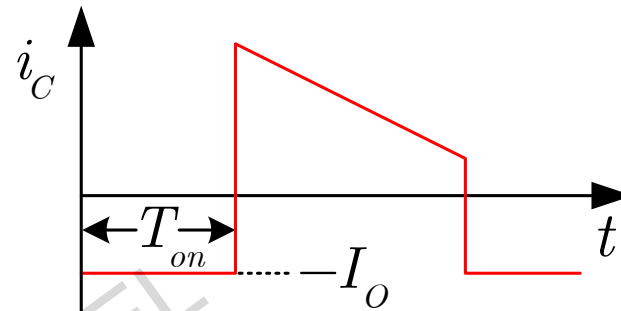
$$\Delta v_o \times C = I_o T_{\text{on}}$$

$$\Delta v_o = \frac{I_o}{C} \times T_{\text{on}}$$

Under PWM       $T_{\text{on}} = DT_{\text{sw}}$

$$\therefore \Delta v_o = \frac{I_o}{C f_{\text{sw}}} \times D$$

**Worst-case voltage ripple at**  
***lowest input voltage and highest load current***



*Capacitor current waveform*

- Voltage ripple is maximum when
  - Load current is maximum and
  - Duty ratio is maximum

# Summary

- Steady-state ripple parameters – important design constraint
- RMS current – direct impact on efficiency
- Selection of inductor – ripple/RSM parameters vs transient response
- Selection of capacitor – ripple parameter, reliability, etc.
- Power stage design – critical for steady-state and transient



**THANK  
YOU !**





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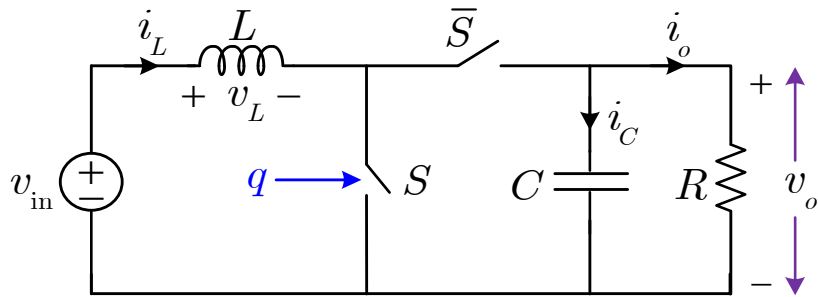
**Module 02: Modulation Techniques in SMPCs**

**Lecture 08 : Fixed Frequency Modulation Techniques**

# Concepts Covered

- Pulse width modulation (PWM) techniques
- Trailing-edge PWM and implementation
- Leading-edge PWM and implementation
- Dual-edge PWM and implementation
- Phase shift modulation

# Pulse Width Modulation

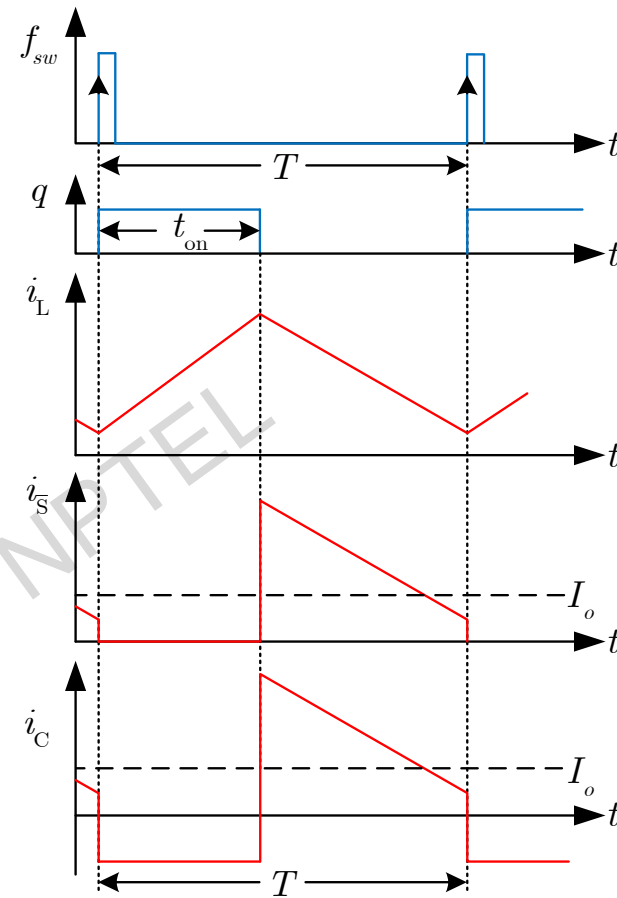


*Ideal Synchronous Boost Converter*

$$t_{on} = dT$$

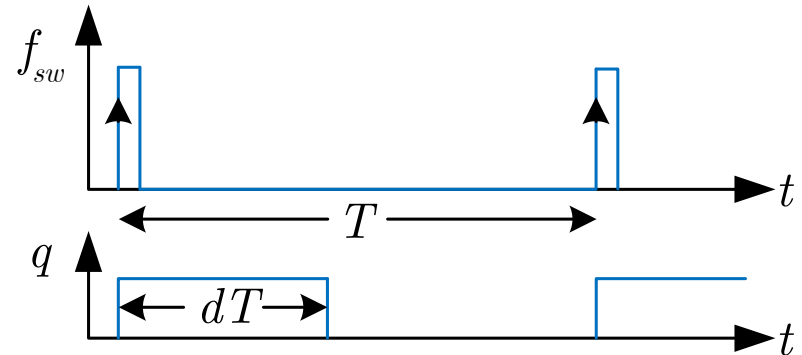
Duty ratio  **$d$**  is the control variable

## Switching waveforms



## Features under PWM

- Switching period  $T$  is constant
  - Fixed frequency operation
  - Predictable ripple parameters throughout
  - Easy to design input filter
- Synchronized with an external fixed frequency clock  $f_{sw}$ 
  - Switching frequency programmable using an external clock
  - Synchronization among other converters through clock sharing

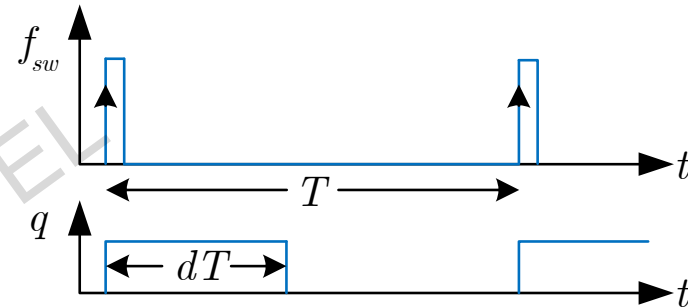


## Features under PWM (contd...)

- Switch state cannot change its state more than once

in between two subsequent edges of  $f_{sw}$

- Operation insensitive to switching noise
- False triggering can be avoided
- But introduces a transient detection delay



## Possible Configurations under PWM

- Control on-time  $\rightarrow$  duty ratio control

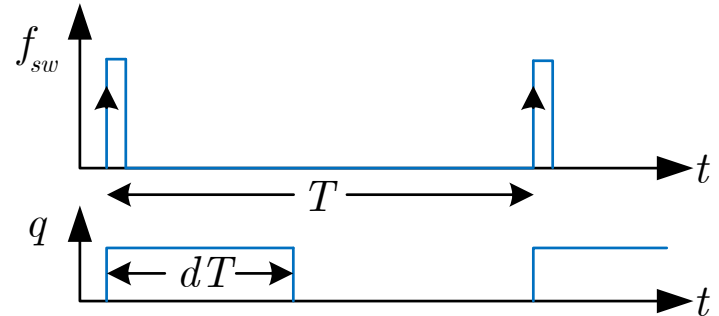
$$t_{\text{on}} = dt \quad (\text{known as trailing edge PWM})$$

- Control off-time  $\rightarrow$  control  $1-d$

$$t_{\text{off}} = (1-d)t \quad (\text{known as leading edge PWM})$$

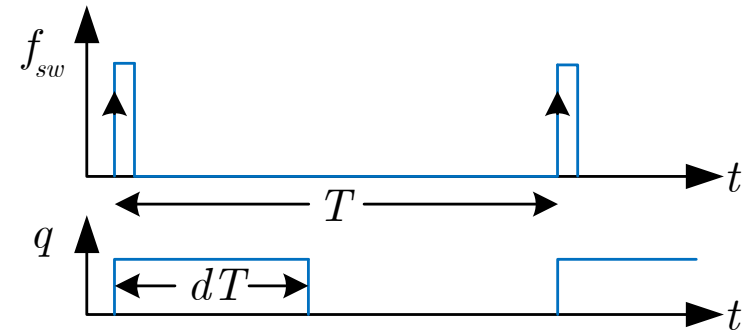
- Control both on and off-times  $t_{\text{on}}$  and  $t_{\text{off}}$  subject to the constraint

$$t_{\text{on}} + t_{\text{off}} = T \quad (\text{Example – dual edge PWM})$$



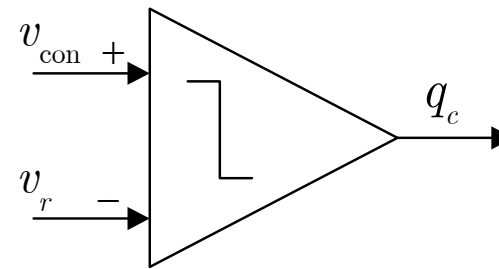
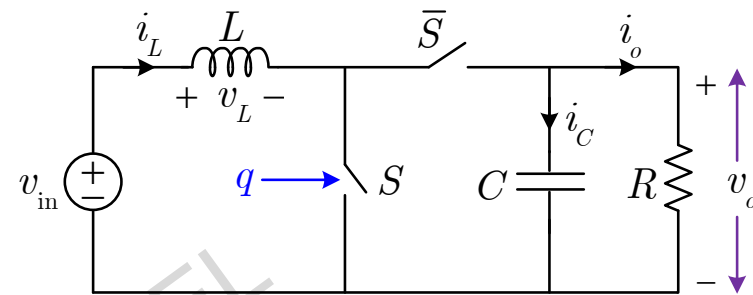
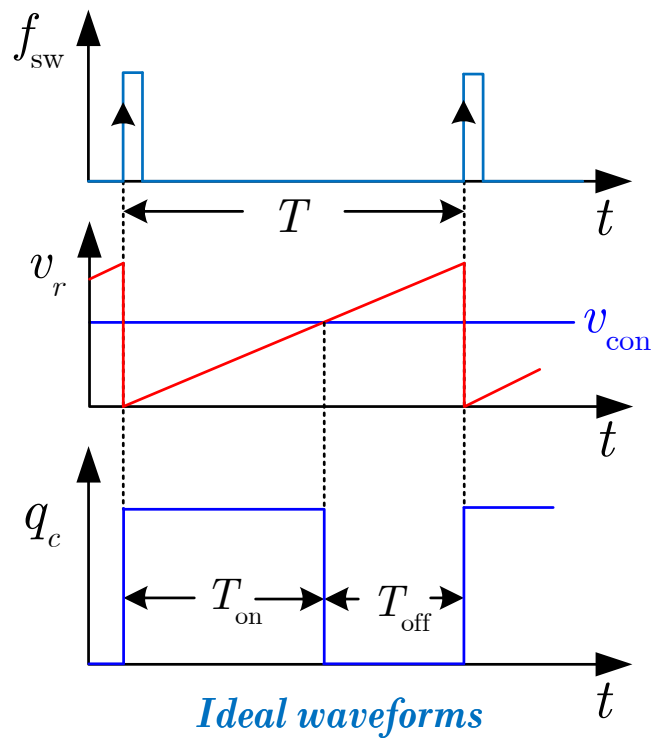
## Possible Configurations under PWM

- In all cases the periodic operation is synchronized with the external fixed frequency clock  $f_{sw}$



- How to implement different PWM techniques?
- What are the use of different PWM techniques?

## Trailing – edge PWM and Implementation

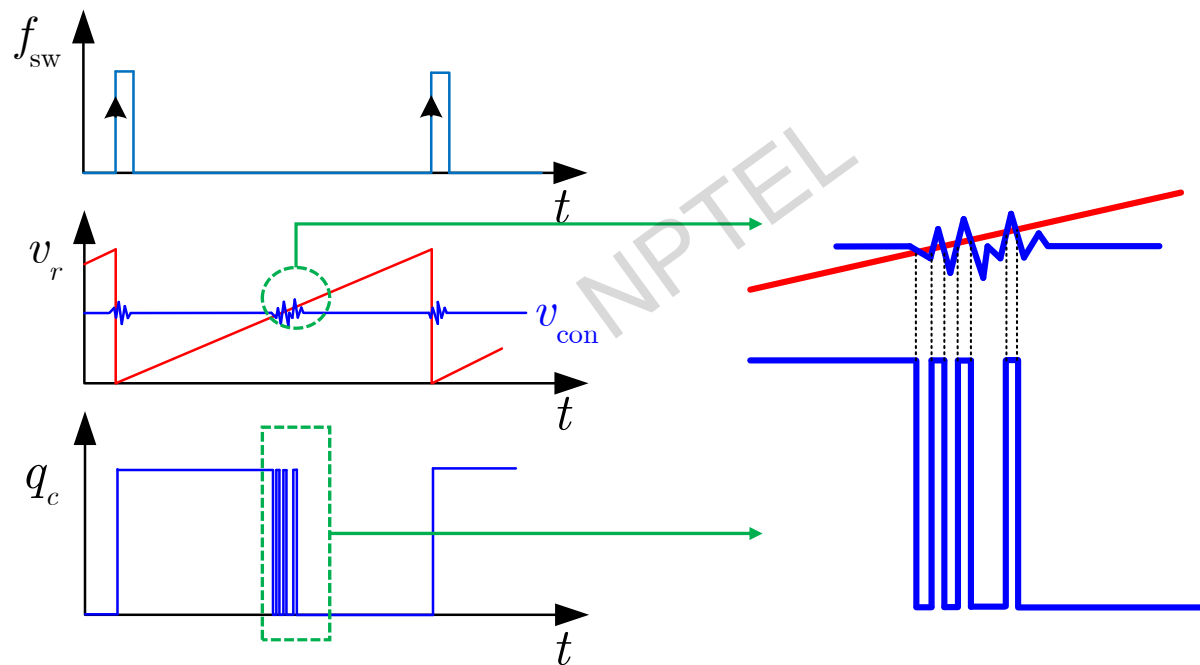


*Comparator*



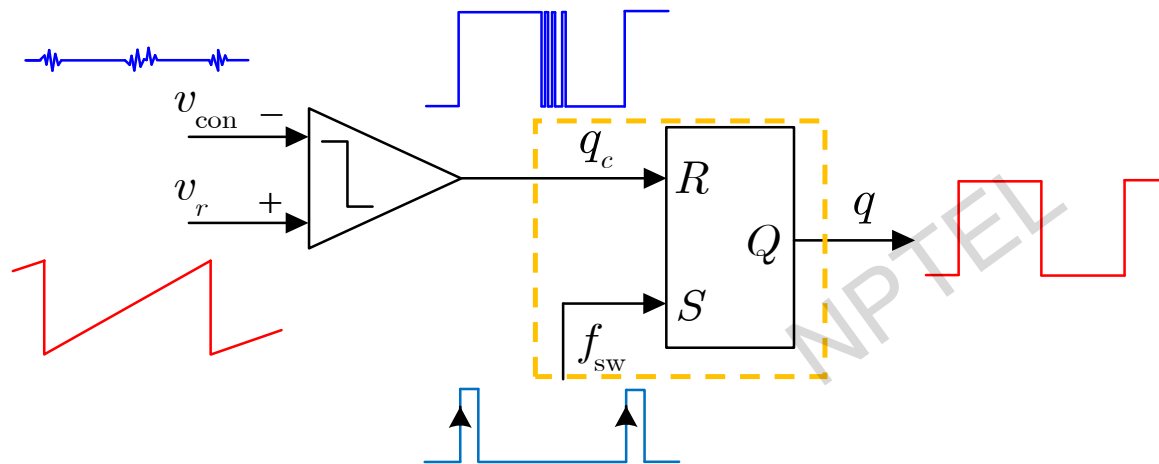
## Impact of Switching Noise

- Question: What will be the impact on  $q_c$  for noisy  $v_{con}$  ?



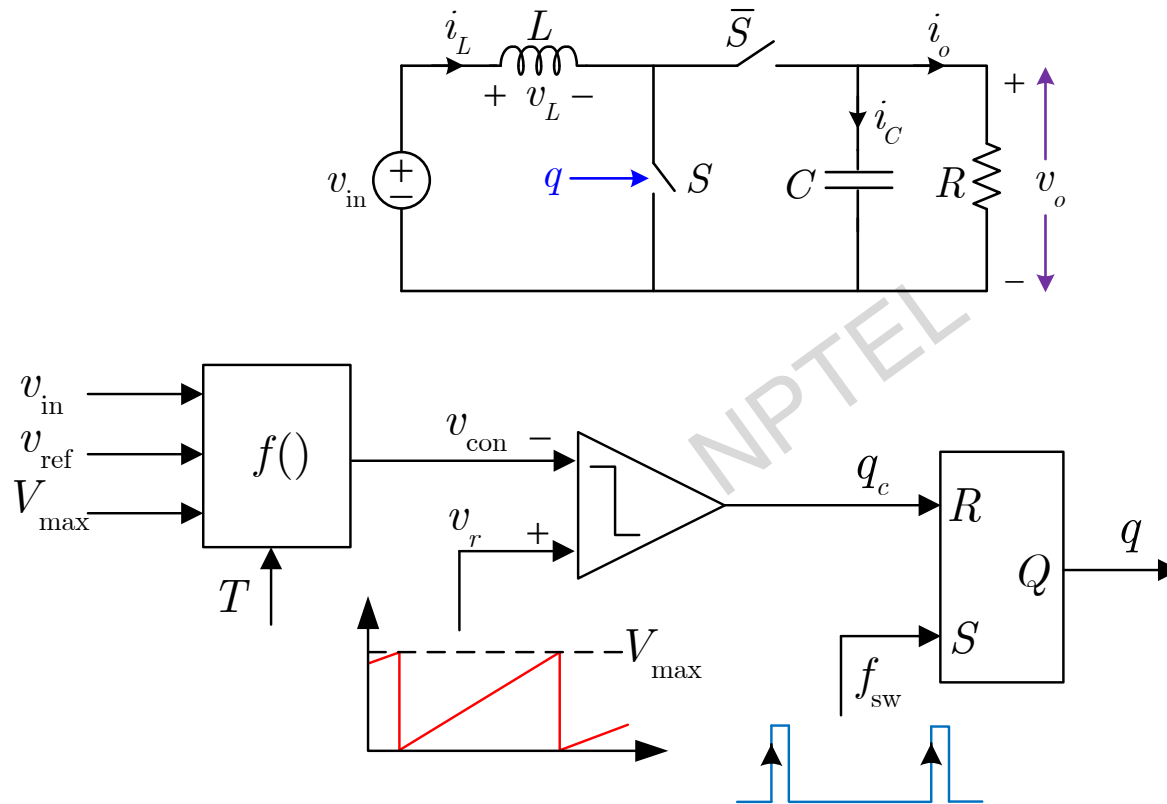
## Noise Insensitive PWM Operation

- Need to add a latch circuit



- For reset (R) dominated – full 0 to 100% duty ratio achievable
- For set (S) dominated – 0% duty ratio not achievable

## Implementation



## Setting Control Voltage for Open-Loop Simulation

- Using steady-state equations of an ideal boost converter, calculate  $v_{\text{con}}$  such that  $v_o$  can be maintained at  $v_{\text{ref}}$  (reference output voltage).

- Step 1:

$$V_o = \frac{V_{\text{IN}}}{(1 - D)}$$

- Find  $D_r$  for given  $V_{\text{IN}}$  and  $V_{\text{ref}}$ .

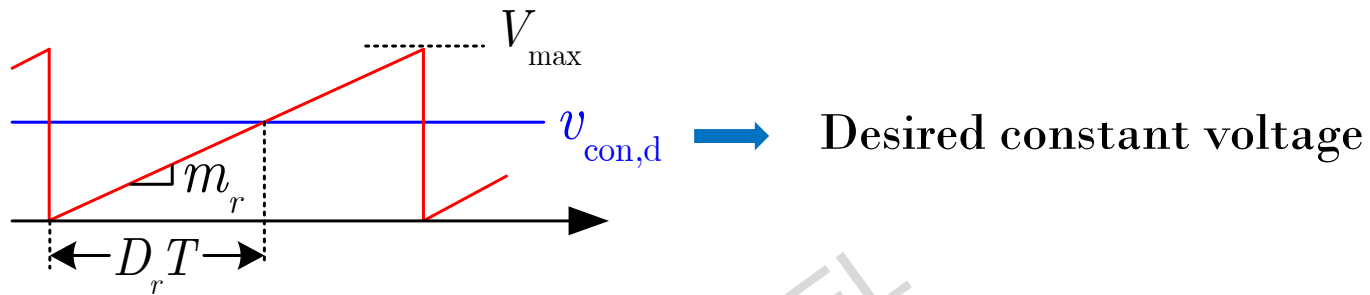
$$(1 - D_r) = \frac{V_{\text{IN}}}{V_{\text{ref}}}$$

$$D_r = 1 - \frac{V_{\text{IN}}}{V_{\text{ref}}} = \left( \frac{V_{\text{ref}} - V_{\text{IN}}}{V_{\text{ref}}} \right)$$

$$\underbrace{D_r \times T}_\text{Desired on-time} = \left( \frac{V_{\text{ref}} - V_{\text{IN}}}{V_{\text{ref}}} \right) \times T$$

Desired on-time  
(under trailing-edge PWM)

## Setting Control Voltage Contd...

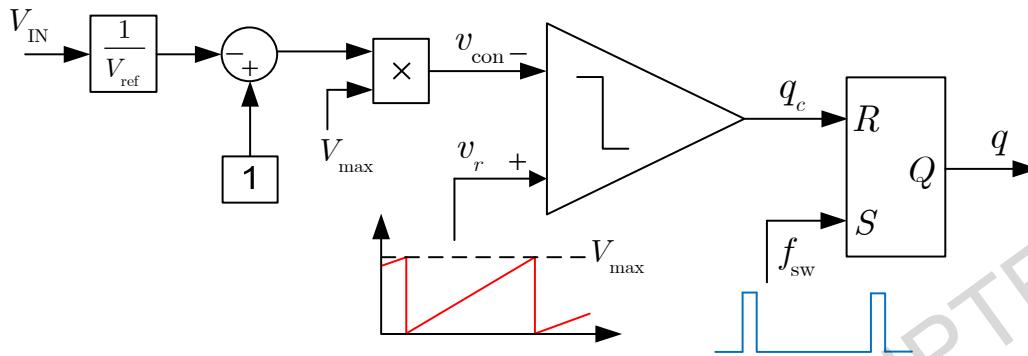


$$v_{\text{con,d}} = m_r \times (D_r \times T) \quad m_r = \frac{V_{\max}}{T}, \quad D_r \times T = \left( \frac{V_{\text{ref}} - V_{\text{IN}}}{V_{\text{ref}}} \right) \times T$$

$$v_{\text{con,d}} = V_{\max} \times \left( \frac{V_{\text{ref}} - V_{\text{IN}}}{V_{\text{ref}}} \right)$$

$$\therefore v_{\text{con,d}} = V_{\max} \times \left( 1 - \frac{V_{\text{IN}}}{V_{\text{ref}}} \right)$$

## MATLAB Simulation Case Study



- **Implement this in a boost converter.**

- For a case study  
With  $V_{IN} = 4.5V$ ,  $V_{ref} = 5V$
- Show steady-state results
- Apply a transient in  $v_{in}$   
 $v_{in}$  changes from  $4V$  to  $3V$
- Show the effect

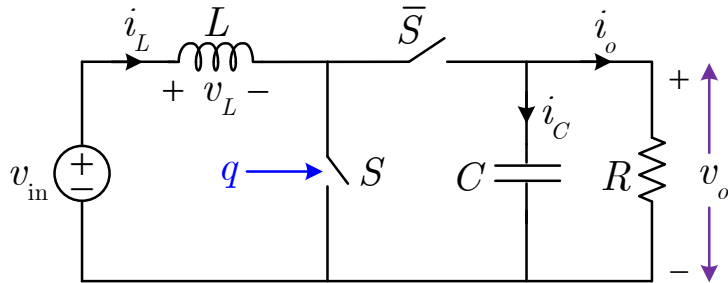
# MATLAB Simulations

# Trialing-edge vs Leading-edge Modulation a Boost Converter

MATLAB  
Simulations

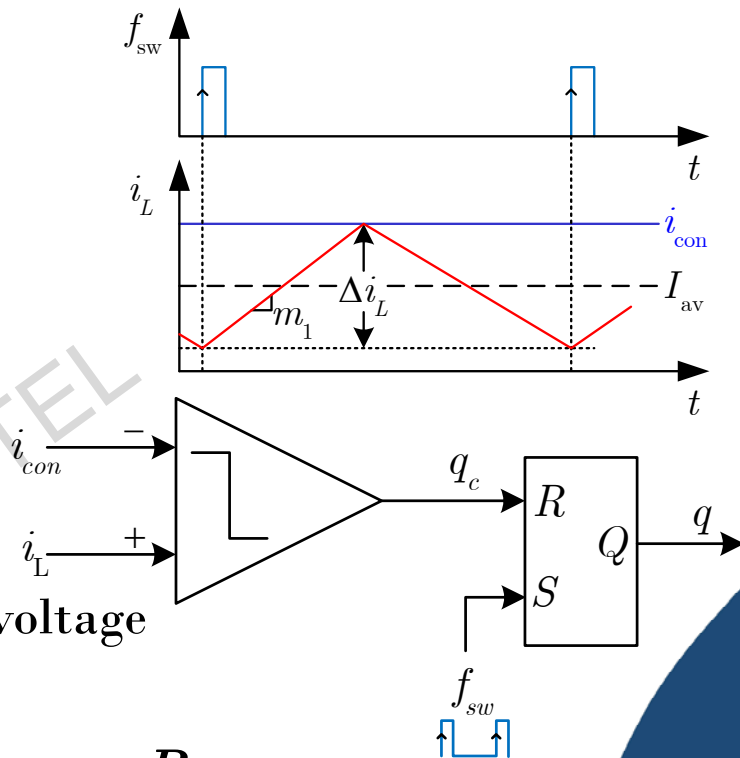
NPTEL

# Trailing Edge Current Control



- Control peak value of inductor current (known as Peak current mode control)
- $i_{con}$  can be arbitrarily set
- How to set  $i_{con}$  to achieve desired output voltage ( $V_o$ ) at  $V_{ref}$  for the following conditions?  
 $V_o = V_{ref}$  for a given  $V_{in}$  and the load resistance  $R$

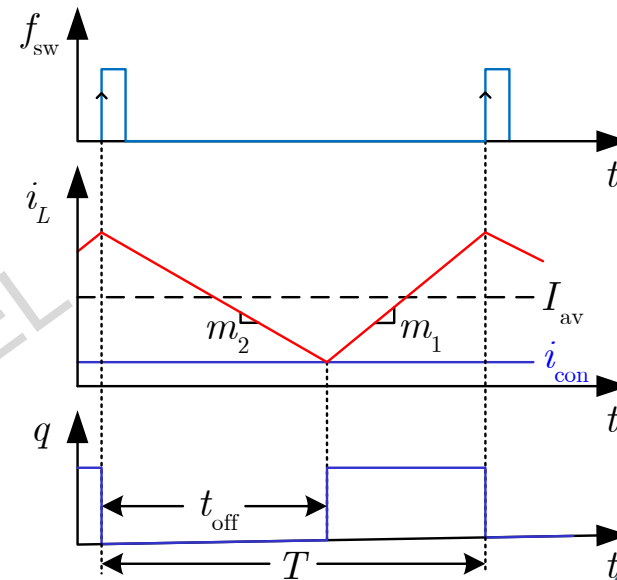
Will do it later !!!



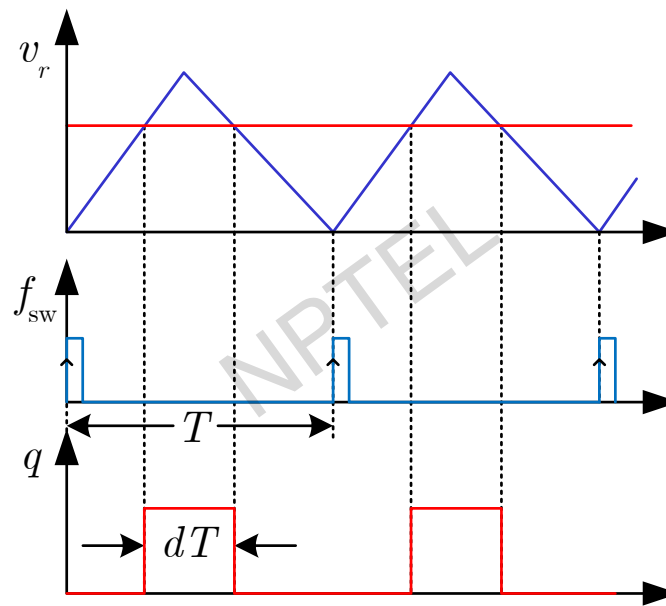


## Leading Edge Current Control

- Control valley (or lower peak) current
- Also known as Valley current mode control
- Why and when is it used over Peak current mode control?



## Dual Edge PWM



## Steady State Parameters under PWM

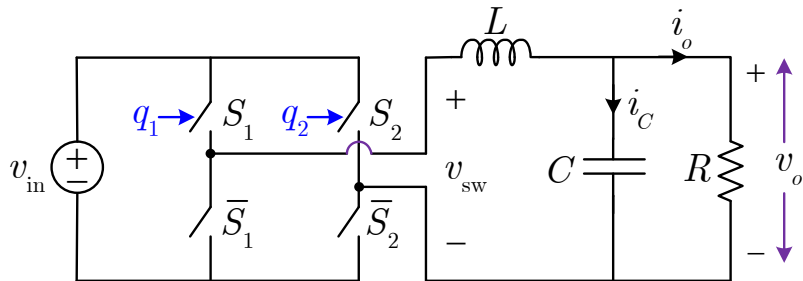
### Buck converter in CCM

- Inductor current ripple  $\Delta i_L$
- Output voltage ripple  $\Delta v_o$
- RMS current
  - Inductor current RMS value
  - Input current RMS value
- Find worst case scenario

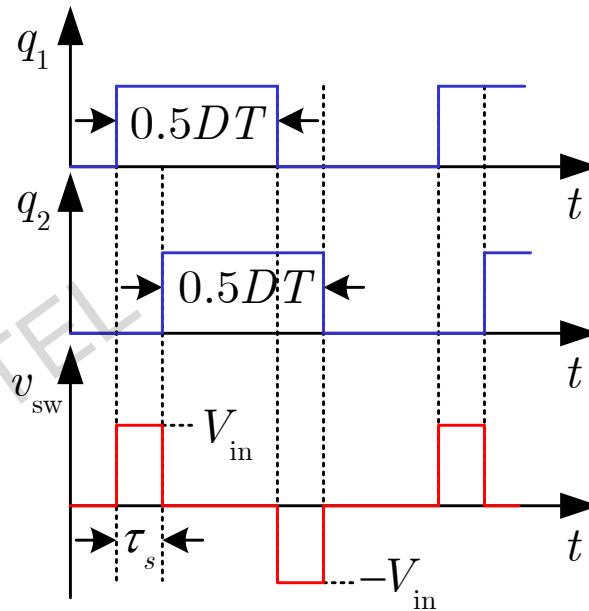
### Boost converter in CCM

- Inductor current ripple  $\Delta i_L$
- Output voltage ripple  $\Delta v_o$
- RMS current
  - Inductor current RMS value
  - Diode current RMS value
- Find worst case scenario

## Phase Shift Modulation

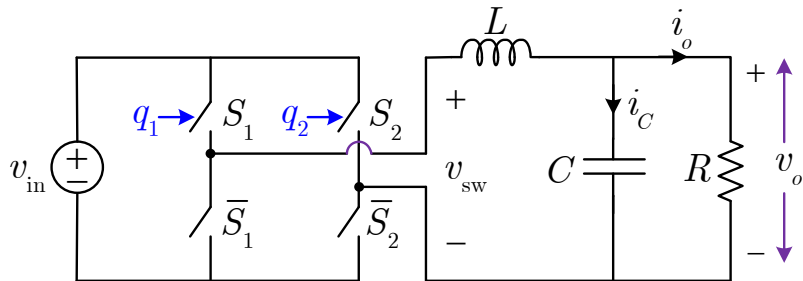


- What will happen with  $v_o$ ?
- How does it look like?



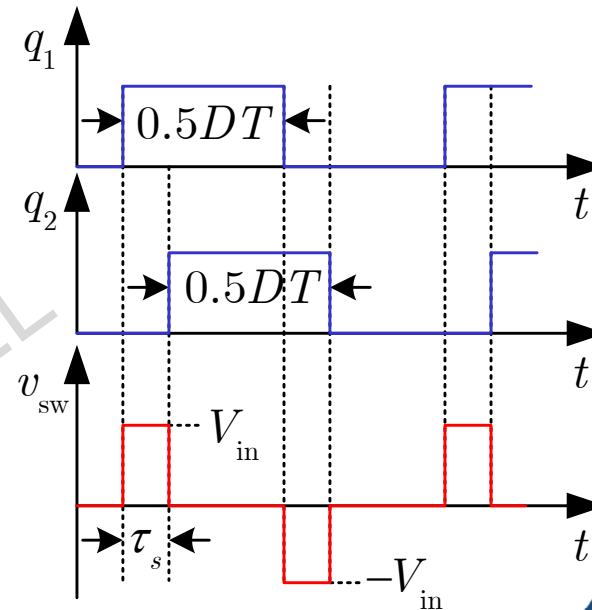
$$\tau_s = \underbrace{\phi}_\text{phase shift} T$$

## Phase Shift Modulation



### ■ Where do you use such techniques?

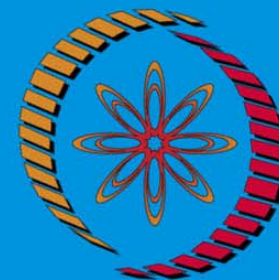
- Class-D audio
- Inverter
- Full bridge, dual active bridge converters
- Switch cap converter



$$\tau_s = \underbrace{\phi}_\text{phase shift} T$$

# Summary

- Pulse width modulation (PWM) techniques
- Trailing-edge PWM and implementation
- Leading-edge PWM and implementation
- Dual-edge PWM and implementation
- Phase shift modulation



**THANK  
YOU !**



**NPTEL ONLINE CERTIFICATION COURSES**

# **CONTROL AND TUNING METHODS IN SMPCs**

**Dr. Santanu Kapat**

**Electrical Engineering Department, IIT KHARAGPUR**

**Module 02: Modulation Techniques in SMPCs**

**Lecture 09: Variable Frequency Modulation Techniques**

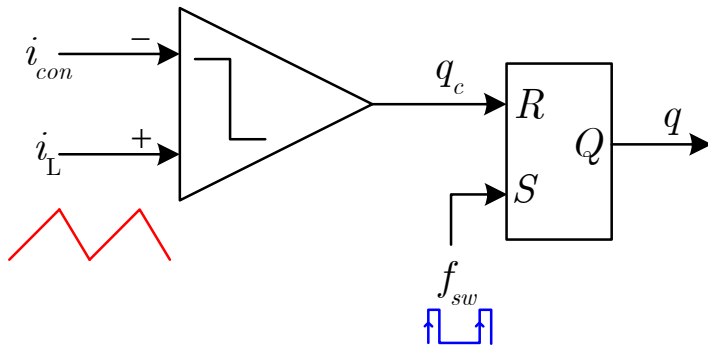


# Concepts Covered

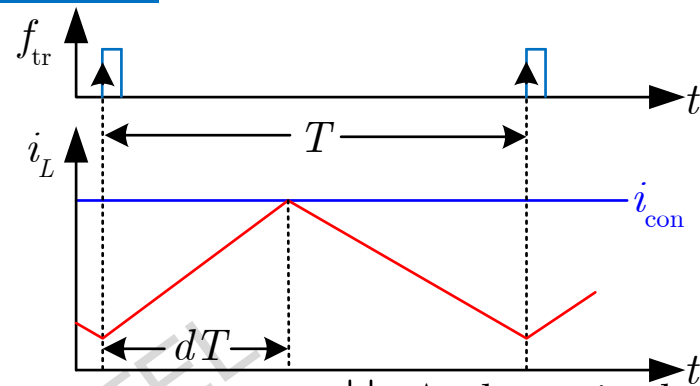
- Variable frequency control
- Constant on-time modulation
- Constant off-time modulation
- Hysteresis control
- Steady-state analysis

# Variable Frequency Control Method

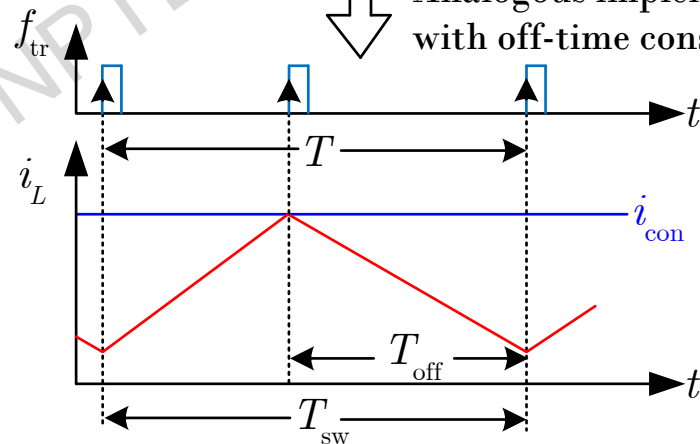
Recall fixed frequency Peak CMC



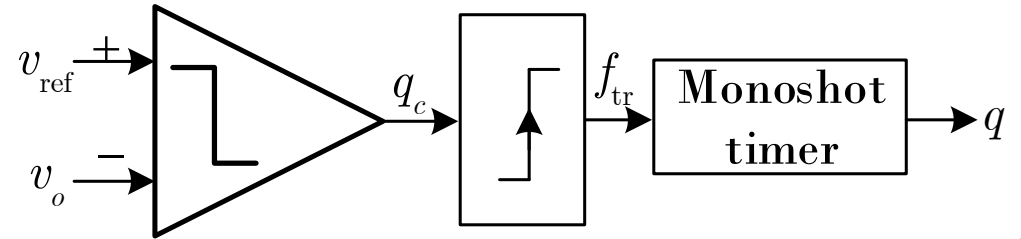
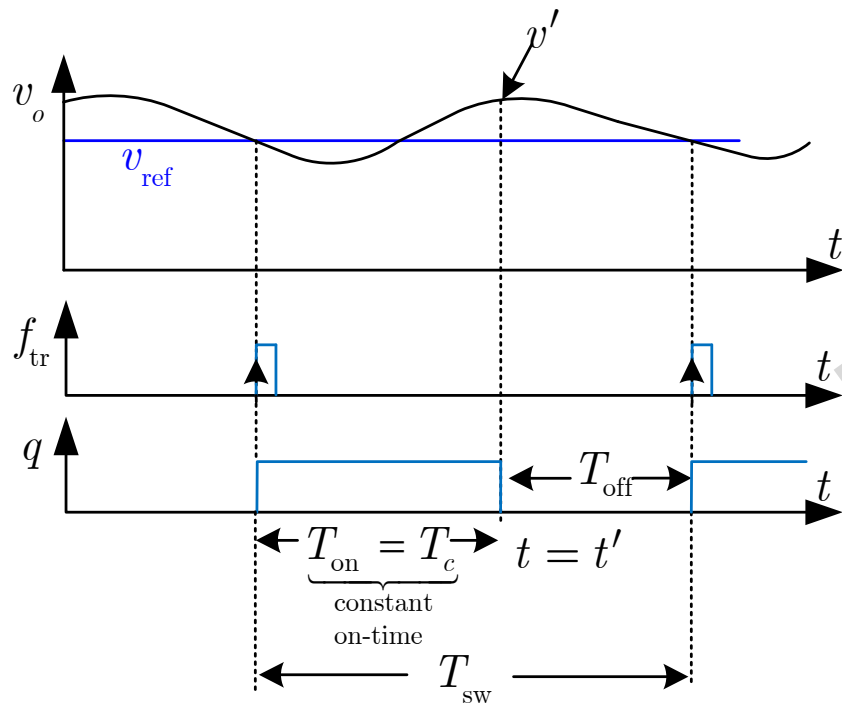
- $T_{\text{off}} \rightarrow$  constant, but unlike trailing-edge PWM,  $T_{\text{sw}}$  is not constant
- $f_{\text{tr}} \rightarrow$  trigger pulses (edge detection)
- Who generates  $f_{\text{tr}}$  ?



Analogous implementation but with off-time constant



## Constant ON-time Modulation



## Problem with Constant ON-time Implementation

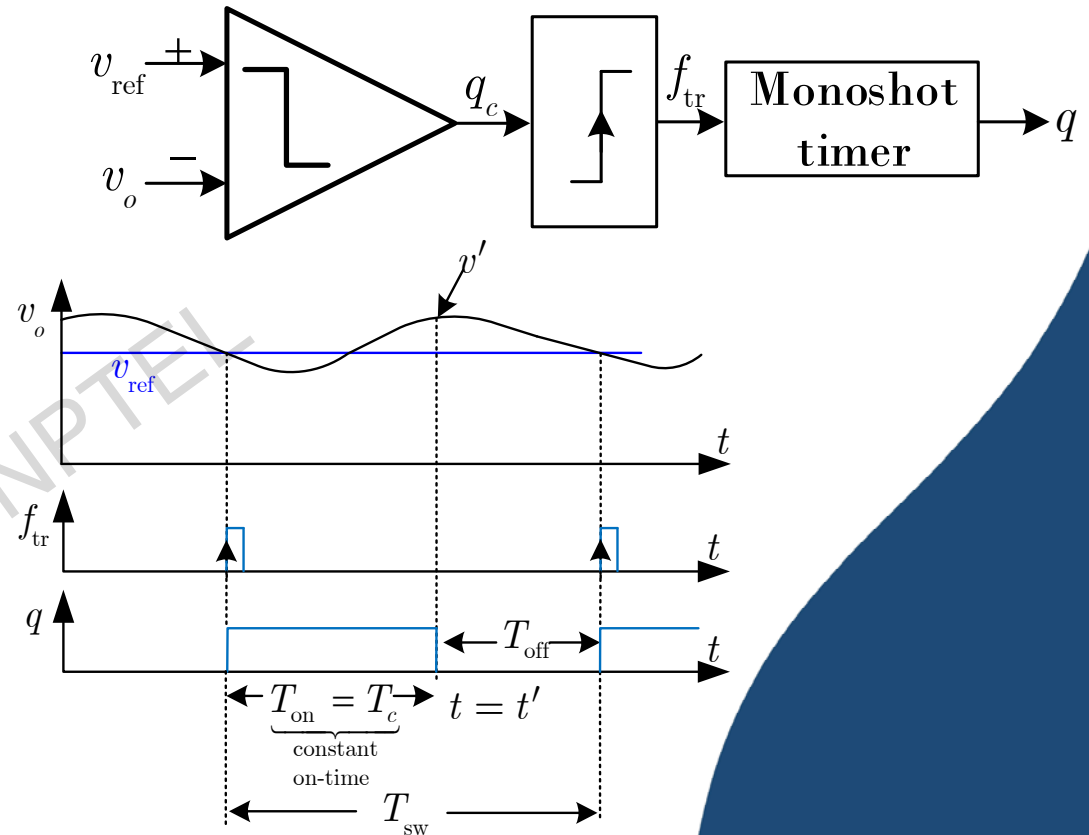
- What happens if  $v_o < v_{\text{ref}}$  after the switch S turns off?

$v' < v_{\text{ref}}$  where  $v' = v_o(t = t')$

$q_c$  continues to remain high and no rising-edge is detected at  $f_{\text{tr}}$

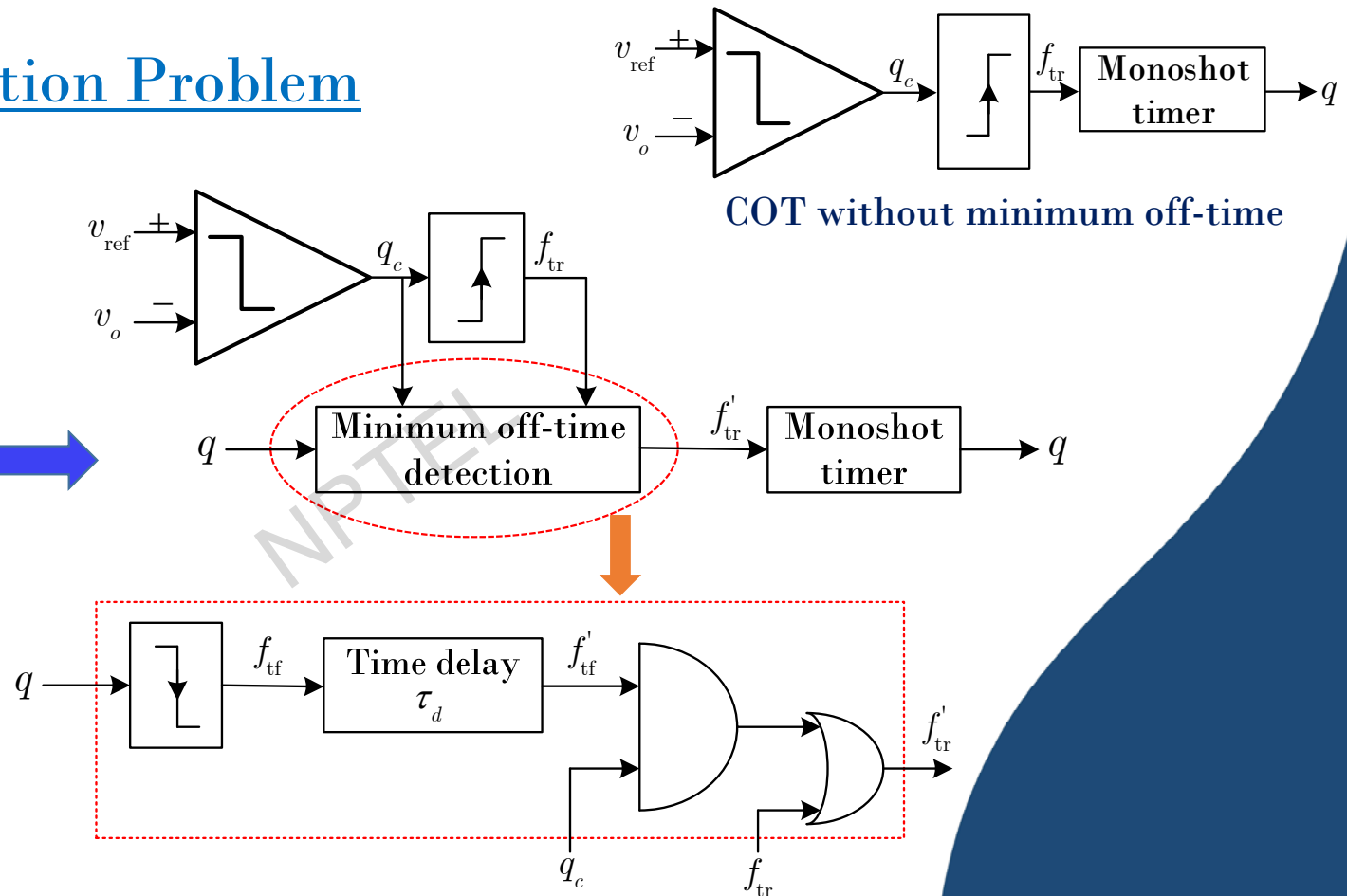
$q$  - remains off and voltage  $v_o$  further decreases

Output voltage completely collapses



## Solving Implementation Problem

- Turn ON problem  
in Constant on-time
- Solution:  
Introduce a  
minimum off-time  
in Constant on-time  
modulation



## Minimum Off-Time in Constant On-Time Control

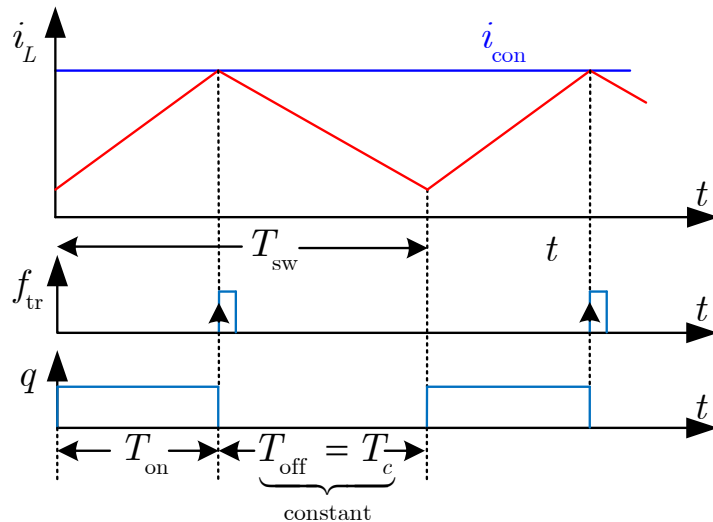
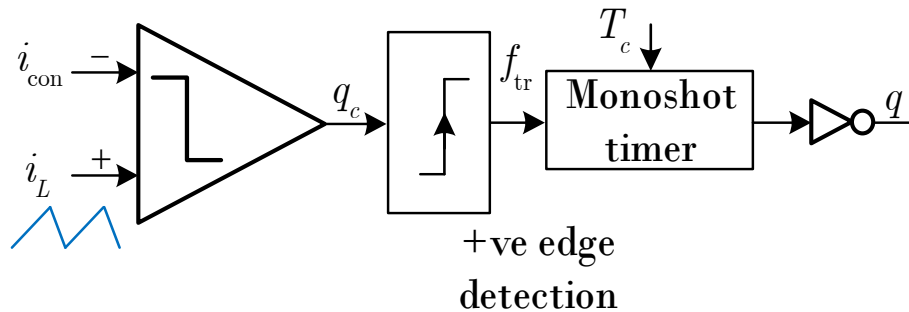
Objective: To solve turn ON problem in Constant on-time control

1. Identify the falling-edge ( $f_{tf}$ ) of  $q$  (output of monoshot timer)
2. Delay  $f_{tf}$  by a time delay  $\tau_d$
3. Compare delayed-edge  $f_{tf}'$  with  $q_c$  (output of voltage comparator)

Make an 'AND' operation

4. The output of 'AND' gate 'OR'-ed with  $f_{tr}$  to generate  $f_{tr}'$
  5. Use  $f_{tr}'$  as the trigger pulse for the monoshot timer
- Try a similar approach in Constant off-time

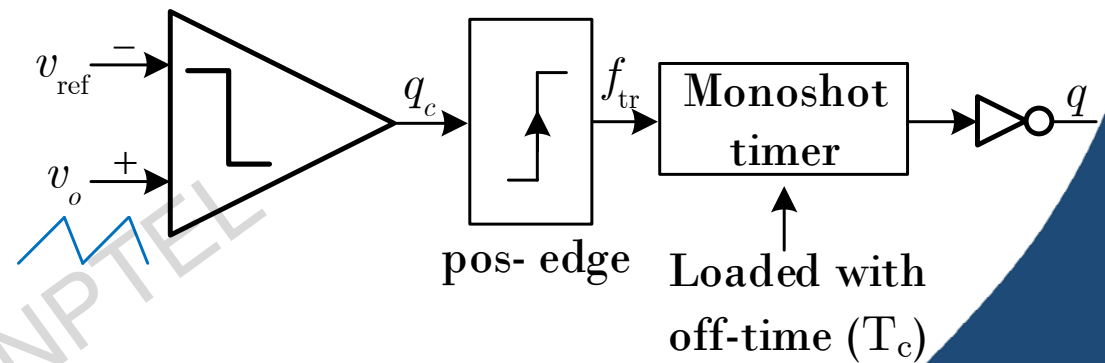
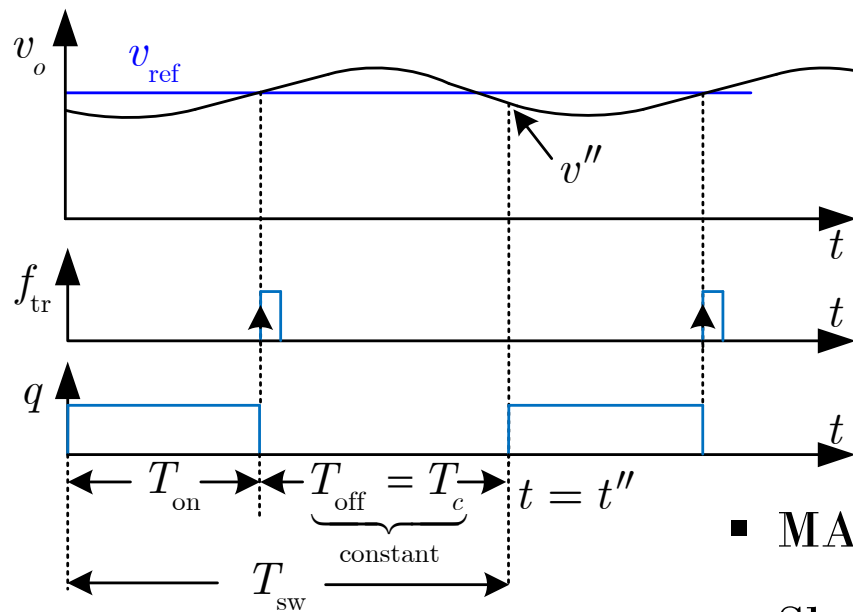
## Constant Off-Time Modulation



- Known as constant off-time modulation
- In constant off-time, off-time is constant, whereas in trailing-edge PWM, time period is constant
- Both techniques directly control peak inductor current

## Constant OFF-time Modulation

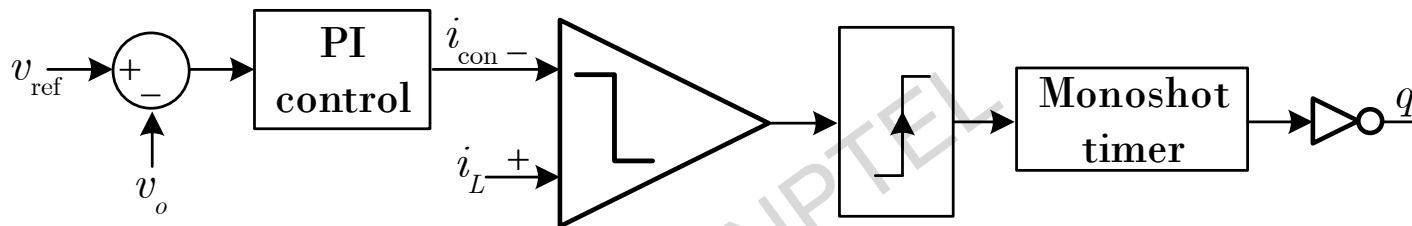
Voltage based implementation



- MATLAB Implementation
- Show transient cases?
- Problems with frequency variations?



## Constant OFF-time Control | Current based feedback control



Implement in MATLAB

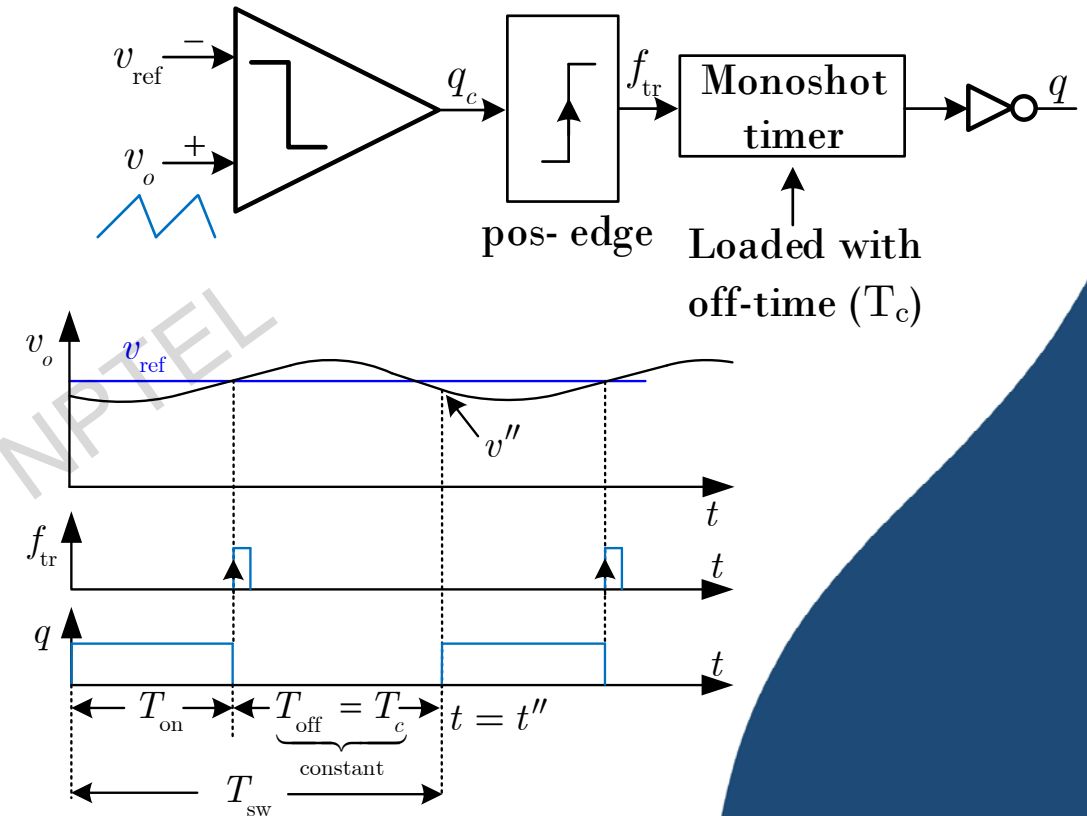
## Problem with Constant OFF-time Implementation

- What happens if  $v'' > v_{\text{ref}}$ ?

$q_c$  remains high, no rising-edge of  $f_{\text{tr}}$  detected

$q$  – remains on and voltage  $v_o$  continues build

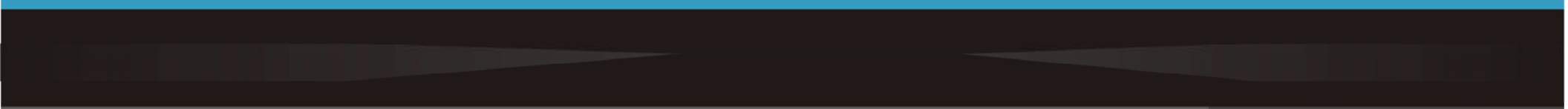
Voltage and current exceed limits !!



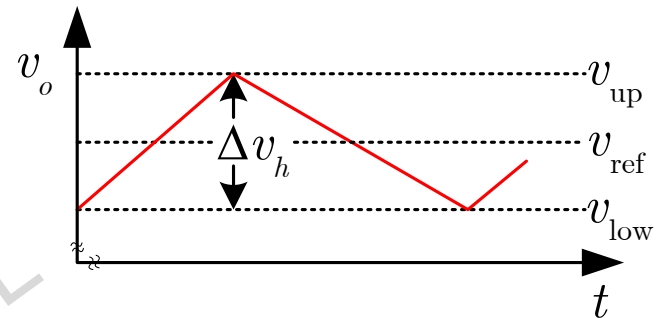
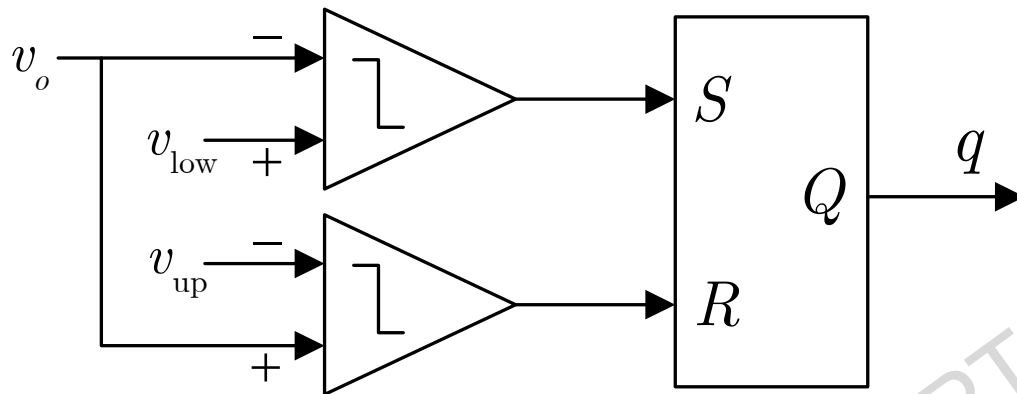


# Voltage Regulation Issue – Simulation Case Study

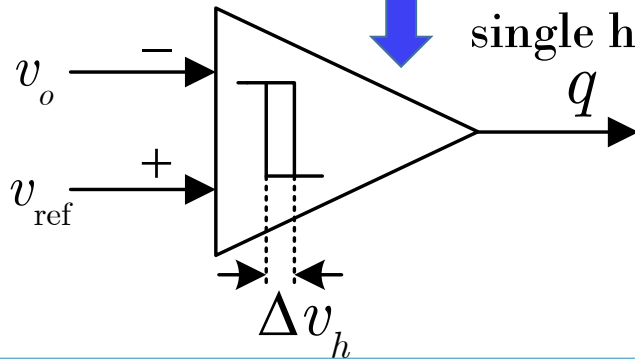
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## Hysteresis Control Techniques



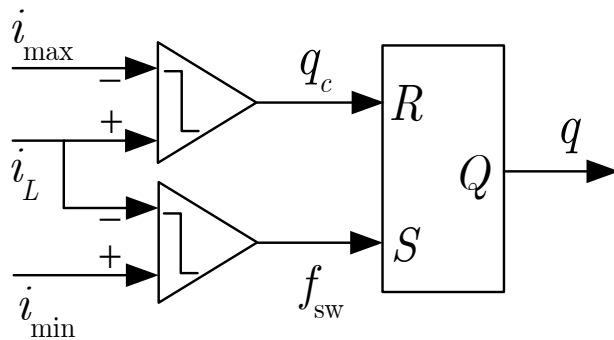
Can also be implemented using a single hysteresis comparator



**What happens for a Boost converter?**

**It may not work during a transient !!**

## Current Hysteresis Control

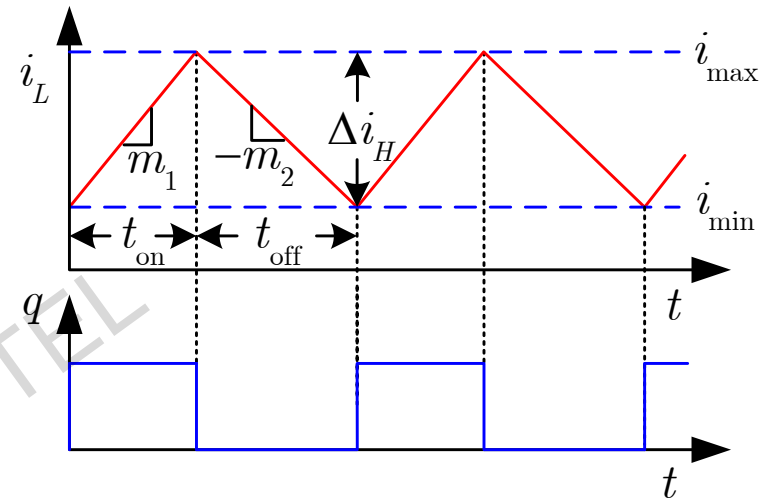


- For constant  $i_{\min}$  and  $i_{\max}$

$$\underbrace{\Delta i_L}_{\text{Current ripple}} = \underbrace{\Delta i_H}_{\text{Current Hysteresis band}}$$



Current ripple  
independent of  $v_{in}$ ,  $i_o$



## Current Hysteresis Control (contd...)

$$m_1 t_{\text{on}} = m_2 t_{\text{off}} \quad \Rightarrow \quad t_{\text{off}} = \left( \frac{m_1}{m_2} \right) t_{\text{on}}$$

$$m_1 t_{\text{on}} = \Delta i_H \quad \Rightarrow \quad t_{\text{on}} = \frac{\Delta i_H}{m_1}$$

$$T_{\text{sw}} = t_{\text{on}} + t_{\text{off}} = \left( 1 + \frac{m_1}{m_2} \right) t_{\text{on}}$$

$$T_{\text{sw}} = \left( \frac{m_1 + m_2}{m_1 m_2} \right) \times \Delta i_H$$

$$f_{\text{sw}} = \left( \frac{m_1 m_2}{m_1 + m_2} \right) \times \frac{1}{\Delta i_H}$$

# Current Hysteresis Control

## Buck converter

$$m_1 = \frac{V_{\text{IN}} - V_o}{L} \quad m_2 = \frac{V_o}{L}$$

$$\frac{m_1 m_2}{m_1 + m_2} = \frac{(V_{\text{IN}} - V_o) V_o}{L V_{\text{IN}}}$$

$$f_{\text{sw}} = \left( \frac{1}{L} \right) \times \left( \frac{1}{\Delta i_H} \right) \times \frac{(V_{\text{IN}} - V_o) V_o}{V_{\text{IN}}}$$

### ■ Switching frequency depends on

- input/output voltages
- hysteresis band
- inductance value

sensitive to non-linear BH  
curve of inductor core!!!

# Current Hysteresis Control in a Boost Converter

## Boost converter

$$m_1 = \frac{V_{\text{IN}}}{L} \qquad m_2 = \frac{V_o - V_{\text{IN}}}{L}$$

$$\frac{m_1 m_2}{m_1 + m_2} = \frac{V_{\text{IN}} (V_o - V_{\text{IN}})}{L V_o}$$

$$f_{\text{sw}} = \left( \frac{1}{L} \right) \times \left( \frac{1}{\Delta i_H} \right) \times \frac{V_{\text{IN}} (V_o - V_{\text{IN}})}{V_o}$$



## Steady-state Parameters under Constant ON-time

### Buck converter

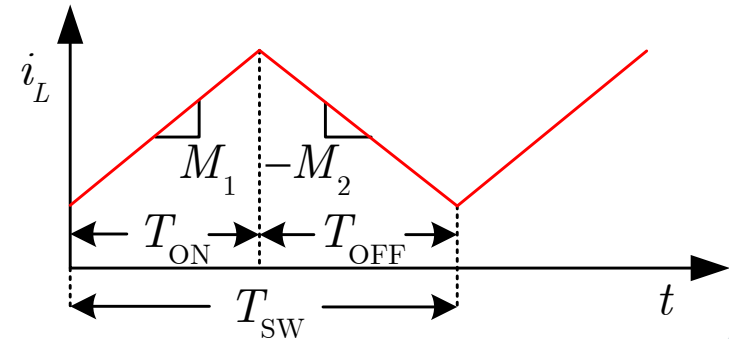
$$M_1 = \frac{V_{\text{IN}} - V_o}{L}$$

$$M_2 = \frac{V_o}{L}$$

$$M_1 T_{\text{ON}} = M_2 T_{\text{OFF}}$$

$$\Rightarrow T_{\text{OFF}} = \left( \frac{M_1}{M_2} \right) T_{\text{ON}} = \frac{(V_{\text{IN}} - V_o)}{V_o} \times T_{\text{ON}}$$

$$\therefore T_{\text{OFF}} = \frac{(V_{\text{IN}} - V_o)}{V_o} \times T_c$$



$T_{\text{ON}} \rightarrow \text{given}$

$$T_{\text{ON}} = T_c$$

## Constant ON-time – Buck Converter

$$T_{\text{SW}} = T_{\text{ON}} + T_{\text{OFF}} = \frac{V_{\text{IN}}}{V_o} \times T_c$$

$$f_{\text{SW}} = \left( \frac{V_o}{V_{\text{IN}}} \right) \times \frac{1}{T_c} \quad \text{Varying switching frequency}$$

$$\Delta i_L = \left( \frac{V_{\text{IN}} - V_o}{L} \right) \times T_c$$

- Current ripple is maximum at  $V_{\text{IN,max}}$  !!!
- Voltage ripple is maximum at  $V_{\text{IN,max}}$  !!!

## Steady-state Parameters under Constant ON-time

### Boost converter

$$M_1 = \frac{V_{\text{IN}}}{L} \qquad M_2 = \frac{V_o - V_{\text{IN}}}{L}$$

$$T_{\text{OFF}} = \left( \frac{M_1}{M_2} \right) T_{\text{ON}} = \left( \frac{V_{\text{IN}}}{V_o - V_{\text{IN}}} \right) \times T_c$$

$$T_{\text{SW}} = T_c + T_{\text{OFF}} = \left( \frac{V_o}{V_o - V_{\text{IN}}} \right) \times T_c$$

$$f_{\text{SW}} = \left( \frac{V_o - V_{\text{IN}}}{V_o} \right) \times \left( \frac{1}{T_c} \right) \qquad \text{Varying switching frequency}$$

## Constant ON-time – Boost Converter

$$\Delta i_L = M_1 \times T_C$$

$$\Delta i_L = \frac{V_{IN}}{L} \times T_C$$

- Current ripple is maximum at  $V_{IN,max}$  !!!
- Voltage ripple  $\Delta v_o = \frac{-I_o}{C} \times T_C$
- Voltage ripple is maximum at  $I_{O,max}$  !!!

## Steady-state Parameters under Constant OFF-time

- Calculate on-time  $T_{\text{ON}}$  in terms of  $T_C$

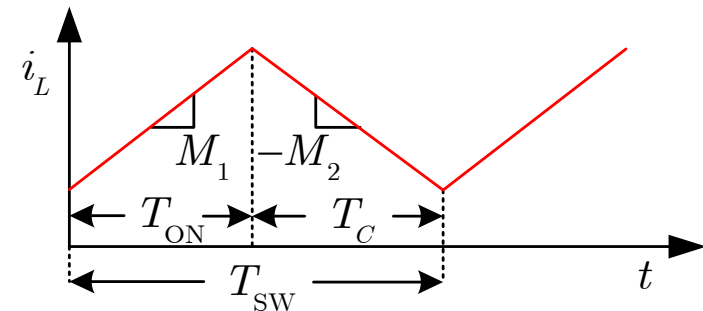
$$T_{\text{ON}} = \left( \frac{M_2}{M_1} \right) T_C$$

- Calculate time-period  $T_{\text{SW}}$  in terms of  $T_C$

$$T_{\text{SW}} = T_{\text{ON}} + T_C \Rightarrow T_{\text{SW}} = \left( \frac{M_1 + M_2}{M_1} \right) T_C$$

- Calculate time-period  $T_{\text{SW}}$  in terms of  $T_C$

$$\Delta i_L = M_2 \times T_C$$



- Off-time  $T_C$  is constant

## Current Ripple under Constant Off-Time Modulation

### Buck converter

$$M_1 = \frac{V_{\text{IN}} - V_o}{L} \quad M_2 = \frac{V_o}{L}$$

$$f_{\text{SW}} = \left( \frac{V_{\text{IN}} - V_o}{V_{\text{IN}}} \right) \times \frac{1}{T_c} \quad \text{Variable frequency}$$

$$\Delta i_L = M_2 \times T_c = \left( \frac{V_o}{L} \right) \times T_c$$

└ Independent of  $V_{\text{IN}}$

### Boost converter

$$M_1 = \frac{V_{\text{IN}}}{L} \quad M_2 = \frac{V_o - V_{\text{IN}}}{L}$$

$$f_{\text{SW}} = \left( \frac{V_{\text{IN}}}{V_o} \right) \times \left( \frac{1}{T_c} \right) \quad \text{Variable frequency}$$

$$\Delta i_L = M_2 \times T_c = \left( \frac{V_o - V_{\text{IN}}}{L} \right) \times T_c$$

└ Maximum at  $V_{\text{IN,min}}$  !!!

# Summary

- Variable frequency modulation – alternatives to PWM
- Constant on-time modulation – gaining popularity
- Variable frequency modulation – switching frequency variations
- Minimum off/on-time required for constant on/off-time modulation
- Comparative simulation studies to be shown later!!



**THANK  
YOU !**





## NPTEL ONLINE CERTIFICATION COURSES

### COURSE NAME

Dr. Santanu Kapat

Electrical Engineering Department, IIT KHARAGPUR

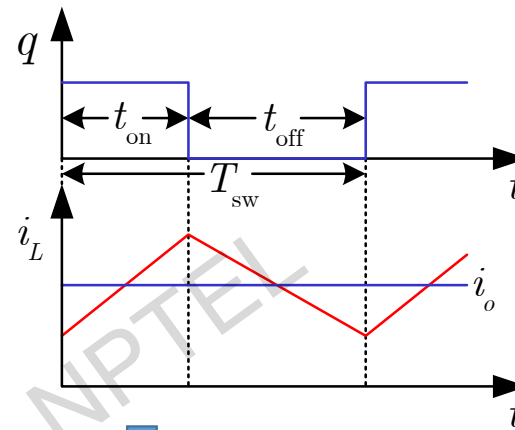
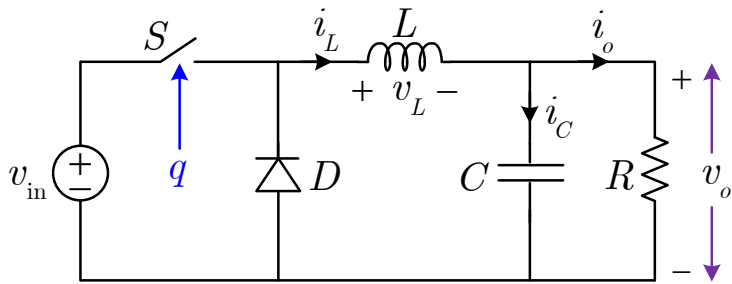
### Module 02: Modulation techniques in SMPCs

#### Lecture 10 : Modulation in Discontinuous Conduction Mode (DCM)

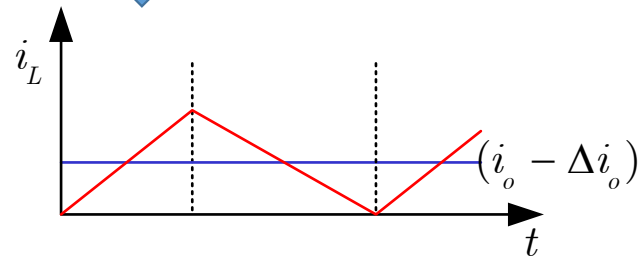
# Concepts Covered

- Discontinuous mode (DCM) operation
- Steady-state analysis in DCM
- Pulse width modulation in DCM
- Pulse frequency modulation in DCM
- Pulse skip modulation in DCM

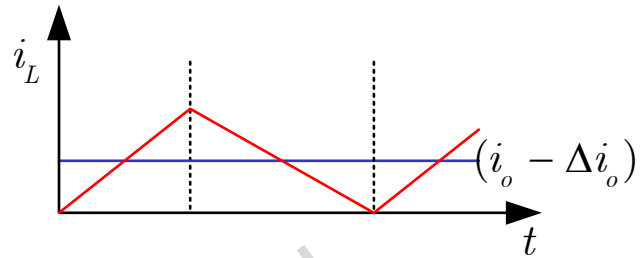
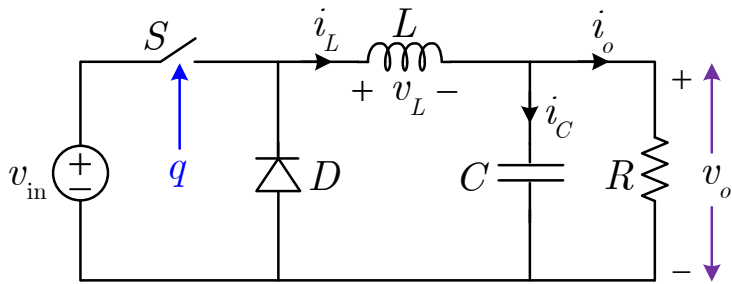
## Critical Conduction Mode (CrM)



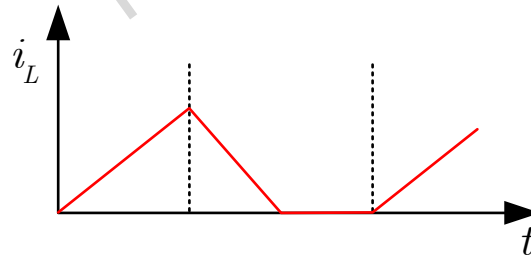
$i_o$  decreases to  $i_o - \Delta i_o$



## Discontinuous Conduction Mode (DCM)



$i_o$  further decreases to  $(i_o - 2\Delta i_o)$



## Conditions for DCM in a Buck Converter

$$I_{\text{av}} = \frac{V_o}{R} = i_o \quad \Delta i_L = m_1 t_{\text{on}} = \left( \frac{V_{\text{in}} - V_o}{L} \right) t_{\text{on}}$$

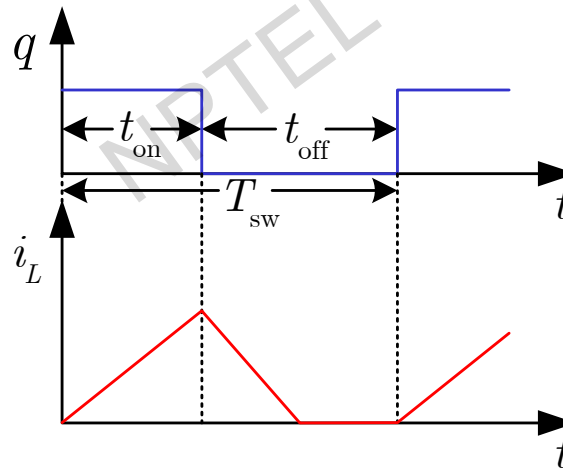
$I_{\text{av}} \rightarrow$  Average value of  $i_L$

$\Delta i_L \rightarrow$  ripple magnitude of  $i_L$

$$\left( I_{\text{av}} - \frac{\Delta i_L}{2} \right) > 0 \rightarrow \text{CCM}$$

$$\left( I_{\text{av}} - \frac{\Delta i_L}{2} \right) = 0 \rightarrow \text{CrM}$$

$$\left( I_{\text{av}} - \frac{\Delta i_L}{2} \right) < 0 \rightarrow \text{DCM}$$



$$I_{\text{av}} = \frac{V_o}{R} \quad \Delta i_L = m_1 t_{\text{on}} = \left( \frac{V_{\text{in}} - V_o}{L} \right) t_{\text{on}}$$

- Consider  $V_o$  to be constant — requirement for a voltage regulator

- Write  $V_{\text{in}}$  in terms of  $V_o$   $\rightarrow \frac{V_o}{V_{\text{in}}} = \frac{t_{\text{on}}}{t_{\text{on}} + t_{\text{off}}} \rightarrow V_{\text{in}} = \left( 1 + \frac{t_{\text{off}}}{t_{\text{on}}} \right) V_o$

$$m_1 = \frac{V_{\text{in}} - V_o}{L} = \frac{V_o}{L} \left[ \left( 1 + \frac{t_{\text{off}}}{t_{\text{on}}} \right) - 1 \right] \rightarrow \boxed{m_1 = \frac{V_o}{L} \times \frac{t_{\text{off}}}{t_{\text{on}}}}$$

$$\Delta i_L = m_1 t_{\text{on}} = \frac{V_o}{L} \times t_{\text{off}}$$

- Now write  $\left(I_{\text{av}} - \frac{\Delta i_L}{2}\right) \Rightarrow \left(I_{\text{av}} - \frac{\Delta i_L}{2}\right) = \frac{V_o}{R} - \left(\frac{V_o}{L} \times t_{\text{off}}\right)$

$$\left(I_{\text{av}} - \frac{\Delta i_L}{2}\right) = V_o \left(\frac{1}{R} - \frac{t_{\text{off}}}{L}\right)$$

- For CrM:  $\frac{1}{R_C} - \frac{t_{\text{off}}}{L} = 0 \Rightarrow R_C = \frac{L}{t_{\text{off}}} \Rightarrow R_C = \frac{L}{(T_{\text{sw}} - t_{\text{on}})}$

For DCM:  $R > R_C$

For CCM:  $R < R_C$

For CrM:  $R = R_C$

## DCM Operation of a Boost Converter

$$I_{\text{av}} = i_o \times \frac{T_{\text{sw}}}{t_{\text{off}}} = \frac{v_o}{R} \times \frac{T_{\text{sw}}}{t_{\text{off}}}$$

$$\Delta i_L = m_1 t_{\text{on}} = \frac{v_{\text{in}}}{L} t_{\text{on}}$$

- Represent  $v_{\text{in}}$  in terms of  $v_o$

$$\frac{v_o}{v_{\text{in}}} = \frac{T_{\text{sw}}}{t_{\text{off}}} \Rightarrow v_{\text{in}} = \left( \frac{t_{\text{off}}}{T_{\text{sw}}} \right) v_o$$

$$\therefore \Delta i_L = \left( \frac{t_{\text{on}} t_{\text{off}}}{T_{\text{sw}} L} \right) v_o$$

- Voltage gain  $K_V$  at CCM

$$K_V = \frac{v_o}{v_{\text{in}}} = \frac{T_{\text{sw}}}{t_{\text{off}}}$$



## DCM Operation of a Boost Converter (contd...)

Now write  $\left( I_{\text{av}} - \frac{\Delta i_L}{2} \right)$

$$\begin{aligned} I_{\text{av}} - \frac{\Delta i_L}{2} &= \left( \frac{v_o}{R} \times \frac{T_{\text{sw}}}{t_{\text{off}}} \right) - \left( \frac{t_{\text{on}} t_{\text{off}}}{2T_{\text{sw}} L} v_o \right) \\ &= \left( \frac{v_o T_{\text{sw}}}{t_{\text{off}}} \right) \left( \frac{1}{R} - \frac{t_{\text{on}} t_{\text{off}}^2}{2L T_{\text{sw}}^2} \right) \end{aligned}$$

Critical Resistance:

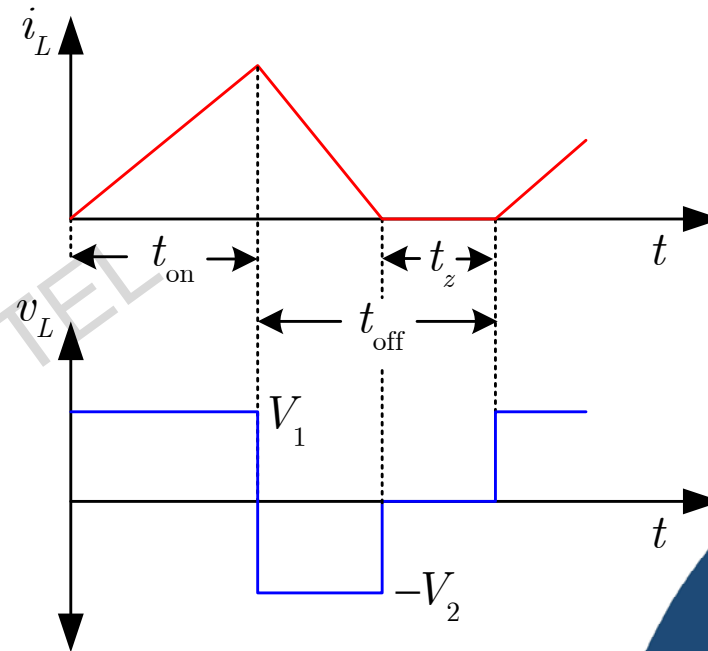
$$R_C = \left( \frac{2L}{t_{\text{on}}} \right) \times \left( \frac{T_{\text{sw}}}{t_{\text{off}}} \right)^2$$

## Volt-second Balance in DCM

$$V_1 \times t_{\text{on}} - V_2 \times (t_{\text{off}} - t_z) = 0$$

$$V_1 = \begin{cases} (v_{\text{in}} - v_o) & \text{buck converter} \\ v_{\text{in}} & \text{boost converter} \end{cases}$$

$$V_2 = \begin{cases} v_o & \text{buck converter} \\ (v_o - v_{\text{in}}) & \text{boost converter} \end{cases}$$



## Volt-second Balance in DCM (contd...)

### Buck converter

$$(v_{\text{in}} - v_o) t_{\text{on}} = v_o (t_{\text{off}} - t_z)$$

$$\Rightarrow v_o \underbrace{(t_{\text{on}} + t_{\text{off}})}_{T_{\text{sw}}} - t_z = v_{\text{in}} t_{\text{on}}$$

$$\Rightarrow K_V = \frac{v_o}{v_{\text{in}}} = \frac{t_{\text{on}}}{(T_{\text{sw}} - t_z)}$$

### Boost converter

$$v_{\text{in}} t_{\text{on}} = (v_o - v_{\text{in}}) (t_{\text{off}} - t_z)$$

$$\Rightarrow v_o (t_{\text{off}} - t_z) = v_{\text{in}} (T_{\text{sw}} - t_z)$$

$$\Rightarrow K_V = \frac{v_o}{v_{\text{in}}} = \frac{(T_{\text{sw}} - t_z)}{(t_{\text{off}} - t_z)}$$

## Pulse Width Modulation in DCM

$$I_{\text{av}} = \begin{cases} I_o \\ I_o \left( 1 + \frac{v_{\text{in}} D^2 T}{2I_o L} \right) \end{cases}$$

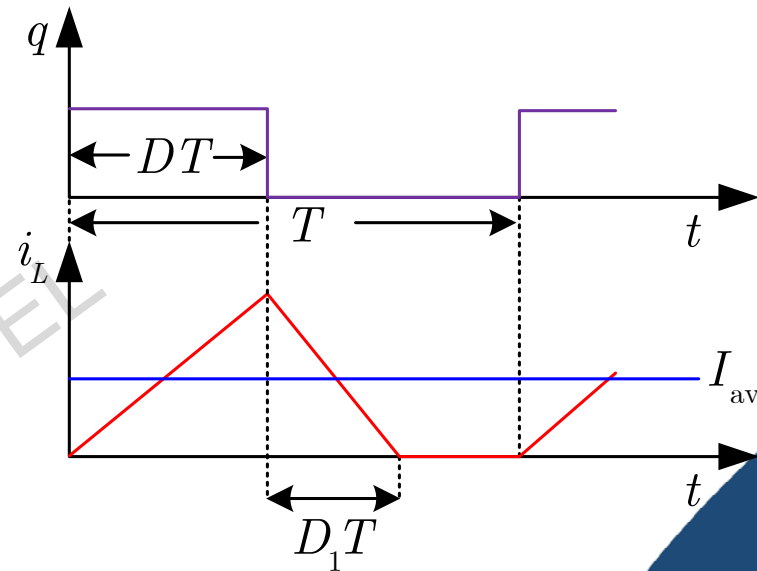
buck converter

boost converter

$$v_o = \begin{cases} \frac{v_{\text{in}}}{1 + \frac{2I_o L}{v_{\text{in}} D^2 T}} \\ v_{\text{in}} \left( 1 + \frac{v_{\text{in}} D^2 T}{2I_o L} \right) \end{cases}$$

buck converter

boost converter



## Pulse Width Modulation in DCM(contd...)

### In DCM

Inductor voltage:

$$v_L = \begin{cases} V_1 & 0 < t \leq DT \\ -V_2 & DT < t \leq (D + D_1)T \\ 0 & (D + D_1)T < t \leq T \end{cases}$$

Voltages	Buck	Boost
$V_1$	$(V_{\text{IN}} - V_o)$	$V_{\text{IN}}$
$V_2$	$V_o$	$(V_o - V_{\text{IN}})$

Capacitor current:

$$i_C = \begin{cases} I_1 & 0 < t \leq DT \\ I_2 & DT < t \leq (D + D_1)T \\ I_3 & (D + D_1)T < t \leq T \end{cases}$$

Currents	Buck	Boost
$I_1$	$I_L - I_o$	$-I_o$
$I_2$	$I_L - I_o$	$I_L - I_o$
$I_3$	$-I_o$	$-I_o$

## Pulse Width Modulation in DCM(contd...)

Using volt-second balance under PWM

$$V_1 D - V_2 D_1 = 0 \quad \Rightarrow V_1 D = V_2 D_1$$

Buck converter

$$(V_{\text{IN}} - V_o) D = V_o D_1$$

$$\Rightarrow \frac{V_o}{V_{\text{IN}}} = \frac{D}{D + D_1}$$

Boost converter

$$V_{\text{IN}} D = (V_o - V_{\text{IN}}) D_1$$

$$\Rightarrow \frac{V_o}{V_{\text{IN}}} = \frac{D + D_1}{D_1}$$

## Capacitor Charge Balance under DCM

$$m_1 DT = m_2 D_1 T \Rightarrow D_1 = \left( \frac{m_1}{m_2} \right) D$$

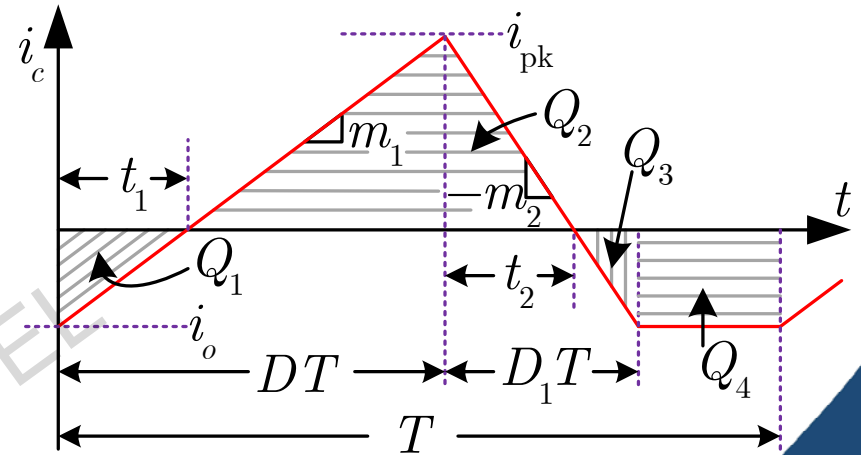
$$m_1 t_1 = i_o \Rightarrow t_1 = \frac{i_o}{m_1}$$

$$m_2 t_2 = i_o \Rightarrow t_2 = \frac{i_o}{m_2}$$

$$i_{pk} = -i_o + m_1 DT$$

$$Q_2 = \frac{1}{2} (i_{pk}) \times \left[ (D_1 + D) - (t_1 + t_2) \right]$$

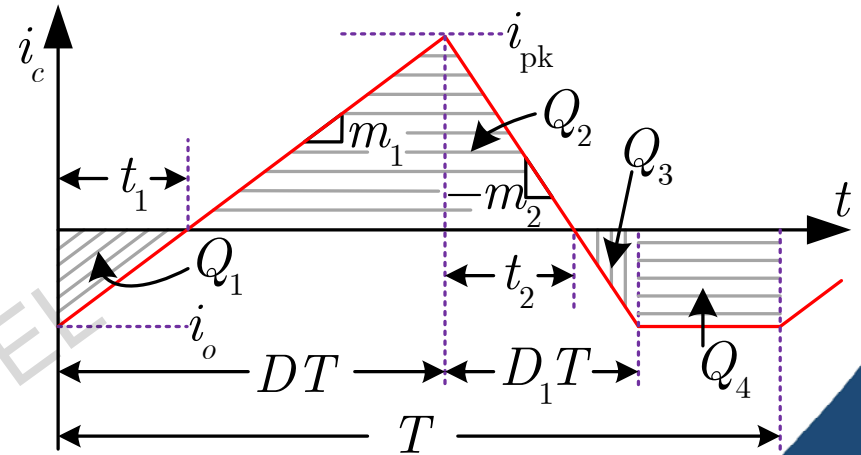
$$D_1 + D = \left( \frac{m_2}{m_1} + 1 \right) D = \left( \frac{m_1 + m_2}{m_2} \right)$$



## Capacitor Charge Balance under DCM (contd...)

$$t_1 + t_2 = i_o \left( \frac{1}{m_1} + \frac{1}{m_2} \right) = \left( \frac{m_1 + m_2}{m_1 m_2} \right) i_o$$

$$\begin{aligned} \therefore (D + D_1)T - (t_1 + t_2) &= \left( \frac{m_1 + m_2}{m_2} \right) \left( DT - \frac{i_o}{m_1} \right) \\ &= \left( \frac{m_1 + m_2}{m_1 m_2} \right) (m_1 DT - i_o) \end{aligned}$$





$$Q_1 = -\frac{1}{2}t_1 i_o = -\frac{i_o^2}{2m_1} \quad Q_2 = \frac{1}{2}\left(\frac{m_1 + m_2}{m_1 m_2}\right) \times (m_1 DT - i_o)^2 \quad Q_3 = -\frac{i_o^2}{2m_2}$$

$$Q_4 = -i_o (1 - D - D_1)T$$

Charge  
balance  $\sum Q = 0$

$$\Rightarrow \frac{1}{2}\left(\frac{m_1 + m_2}{m_1 m_2}\right)(m_1 DT - i_o)^2 = \frac{1}{2}i_o^2\left(\frac{m_1 + m_2}{m_1 m_2}\right) + i_o (1 - D - D_1)T$$

$$\Rightarrow \frac{1}{2}\left(\frac{m_1 + m_2}{m_1 m_2}\right)\left[\cancel{i_o^2} - 2m_1 DT i_o + m_1^2 D^2 T^2 - \cancel{i_o^2}\right] = i_o (1 - D - D_1)T$$

$$\Rightarrow \left(\frac{m_1}{2m_2}\right)(m_1 + m_2)D^2 T^2 - \frac{(m_1 + m_2)DT}{m_2}i_o = i_o (1 - D - D_1)T$$

$$D + D_1 = \left( \frac{m_1 + m_2}{m_2} \right) D$$

$$\frac{(m_1 + m_2)m_1}{2m_2} D^2 T^2 - \cancel{\frac{(m_1 + m_2)m_1}{m_2} i_o} = i_o T - \cancel{\frac{(m_1 + m_2)m_1}{m_2}}$$

$$D^2 = \frac{2m_2}{m_1(m_1 + m_2)T} i_o$$

For a Buck  
converter,

$$m_1 + m_2 = \frac{V_{in}}{L}, m_2 = \frac{V_o}{L}$$

$$D^2 = \frac{2LV_o}{(V_{in} - V_o)V_{in}T} i_o \Rightarrow D = \sqrt{\frac{V_o i_o}{(V_{in} - V_o)V_{in}}} \times \frac{L}{T}$$

## Constant ON-time Modulation in DCM

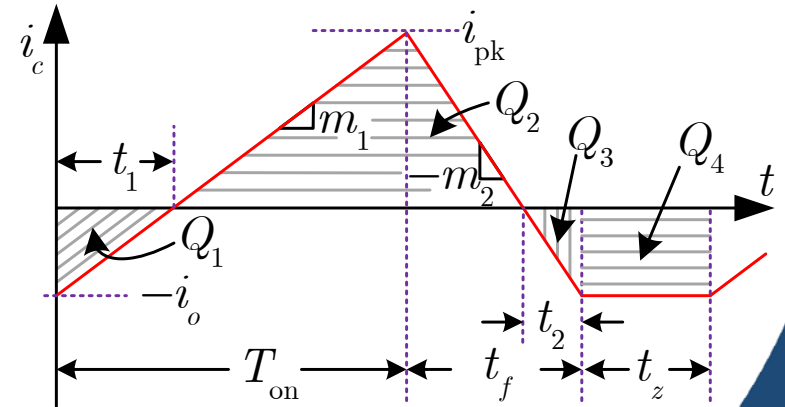
$$m_1 T_{\text{on}} = m_2 t_f \Rightarrow t_f = \left( \frac{m_1}{m_2} \right) T_{\text{on}}$$

$$t_1 = \frac{i_o}{m_1} \quad t_2 = \frac{i_o}{m_2}$$

$$Q_2 = \frac{1}{2} i_{\text{pk}} (T_{\text{on}} + t_f - t_1 - t_2)$$

$$T_{\text{on}} + t_f = \left( 1 + \frac{m_1}{m_2} \right) T_{\text{on}} = \left( \frac{m_1 + m_2}{m_2} \right) T_{\text{on}}$$

$$t_1 + t_2 = i_o \left( \frac{1}{m_1} + \frac{1}{m_2} \right) = \left( \frac{m_1 + m_2}{m_2} \right) i_o$$



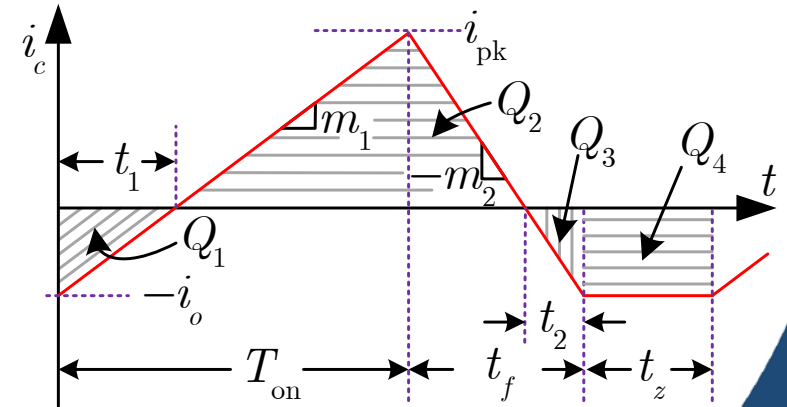
$$T_{\text{on}} + t_f = \left(1 + \frac{m_1}{m_2}\right) T_{\text{on}} = \left(\frac{m_1 + m_2}{m_2}\right) T_{\text{on}}$$

$$t_1 + t_2 = i_o \left(\frac{1}{m_1} + \frac{1}{m_2}\right) = \left(\frac{m_1 + m_2}{m_2}\right) i_o$$

$$(T_{\text{on}} + t_f) - (t_1 + t_2) = \left(\frac{m_1 + m_2}{m_2}\right) \left(T_{\text{on}} - \frac{i_o}{m_1}\right) = \left(\frac{m_1 + m_2}{m_1 m_2}\right) (m_1 T_{\text{on}} - i_o)$$

$$i_{\text{pk}} = -i_o + m_1 T_{\text{on}} = (m_1 T_{\text{on}} - i_o)$$

$$Q_2 = \frac{1}{2} \left(\frac{m_1 + m_2}{m_1 m_2}\right) (m_1 T_{\text{on}} - i_o)^2$$



$$Q_1 = -\frac{i_o^2}{2m_1} \quad Q_2 = \frac{1}{2} \left( \frac{m_1 + m_2}{m_1 m_2} \right) (m_1 T_{\text{on}} - i_o)^2 \quad Q_3 = -\frac{i_o^2}{2m_2} \quad Q_4 = -i_o t_z$$

$$Q_1 + Q_3 + Q_4 = -\frac{i_o^2}{2m_1 m_2} (m_1 + m_2) - i_o t_z$$

Charge  
balance  $\sum Q = 0 \Rightarrow$

$$\frac{1}{2} \left( \frac{m_1 + m_2}{m_1 m_2} \right) (m_1 T_{\text{on}} - i_o)^2 = \left( \frac{m_1 + m_2}{2m_1 m_2} \right) i_o^2 + i_o t_z$$

$$\Rightarrow \left( \frac{m_1 + m_2}{2m_1 m_2} \right) (m_1^2 T_{\text{on}}^2 - 2m_1 T_{\text{on}} i_o + i_o^2) = \left( \frac{m_1 + m_2}{2m_1 m_2} \right) i_o^2 + i_o t_z$$

$$t_z = T_{\text{sw}} - T_{\text{on}} - t_f \Rightarrow t_z = T_{\text{sw}} - \left( \frac{m_1 + m_2}{m_2} \right) T_{\text{on}}$$

$$\frac{(m_1 + m_2)m_1}{2m_2} T_{\text{on}}^2 - \frac{(m_1 + m_2)}{m_2} T_{\text{on}} i_o = i_o T_{\text{sw}} - \frac{(m_1 + m_2)}{m_2} T_{\text{on}} i_o$$

$$T_{\text{sw}} = T_{\text{on}}^2 \times \frac{(m_1 + m_2)m_1}{m_2} \times \frac{1}{i_o}$$

$$f_{\text{sw}} = \frac{1}{T_{\text{sw}}} = \frac{2m_2}{\underbrace{(m_1 + m_2)m_1}} \times \frac{1}{T_{\text{on}}^2} \times i_o$$

For a Buck  
converter,

$$m_1 + m_2 = \frac{V_{\text{in}}}{L}, m_2 = \frac{V_o}{L}$$

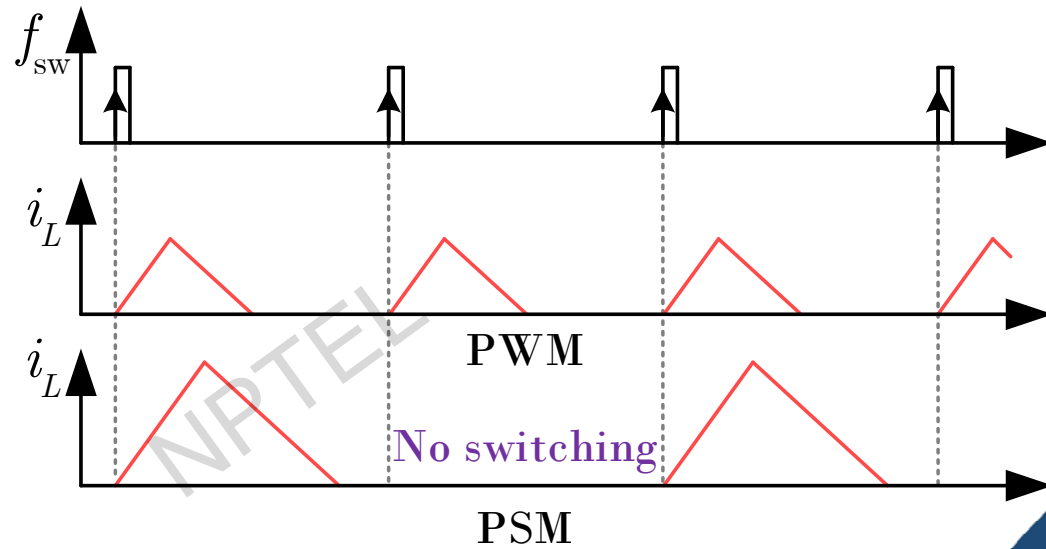
$$\frac{2V_o L}{V_{\text{in}} (V_{\text{in}} - V_o)}$$

$$f_{\text{sw}} = \left[ \frac{2V_o L}{V_{\text{in}} (V_{\text{in}} - V_o)} \right] \times \left( \frac{1}{T_{\text{on}}} \right)^2 \times i_o$$

For given  $V_{\text{in}}, V_o, T_{\text{on}}, L$

$$f_{\text{sw}} \propto i_o$$

## Pulse Skip Modulation





# Summary

- Voltage gain in DCM – load dependent
- Under PWM, duty ratio decreases with load current in DCM
- Constant on-time – switching frequency decreases with load current
- Constant on-time modulation – frequently used under light load
- Pulse frequency modulation improves light load efficiency



**THANK  
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**NPTEL ONLINE CERTIFICATION COURSES**

# **CONTROL AND TUNING METHODS IN SMPCs**

**Dr. Santanu Kapat**

**Electrical Engineering Department, IIT KHARAGPUR**

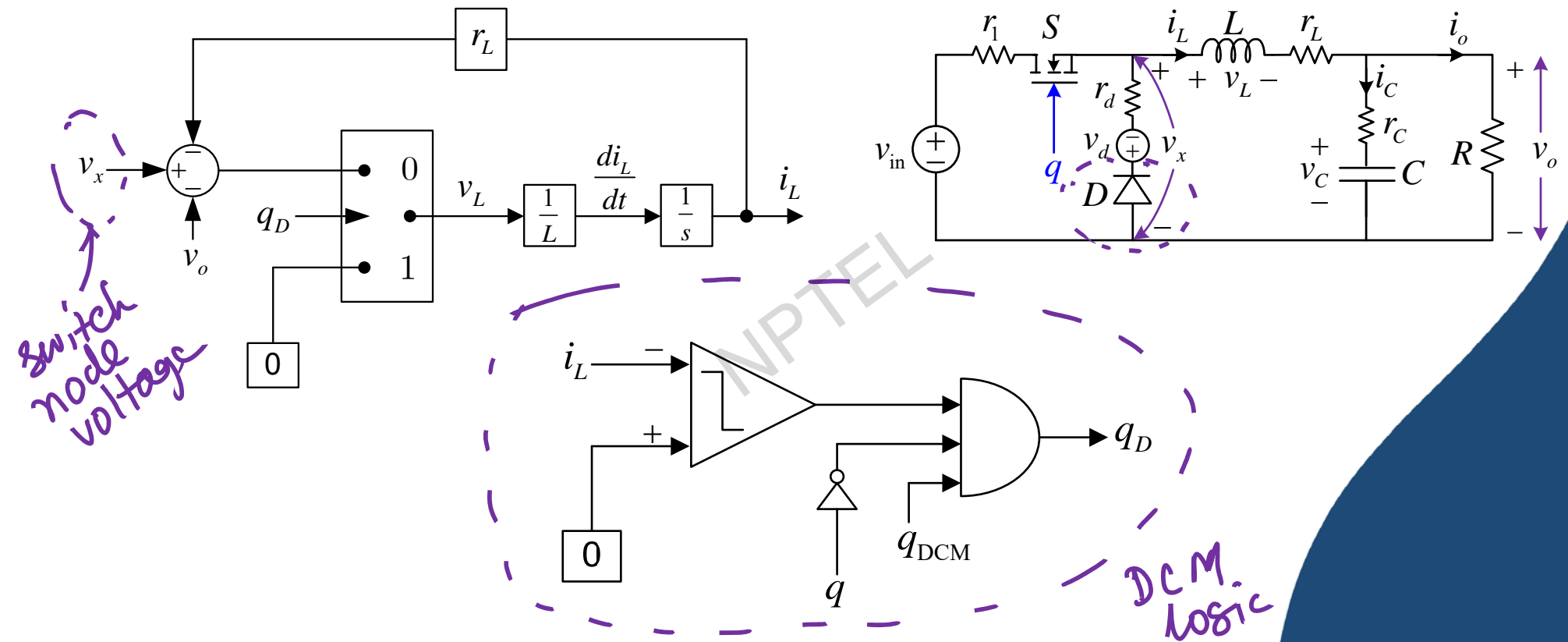
**Module 02: Modulation techniques in SMPCs**

**Lecture 11 : Synchronizing Simulation and Script files in MATLAB**

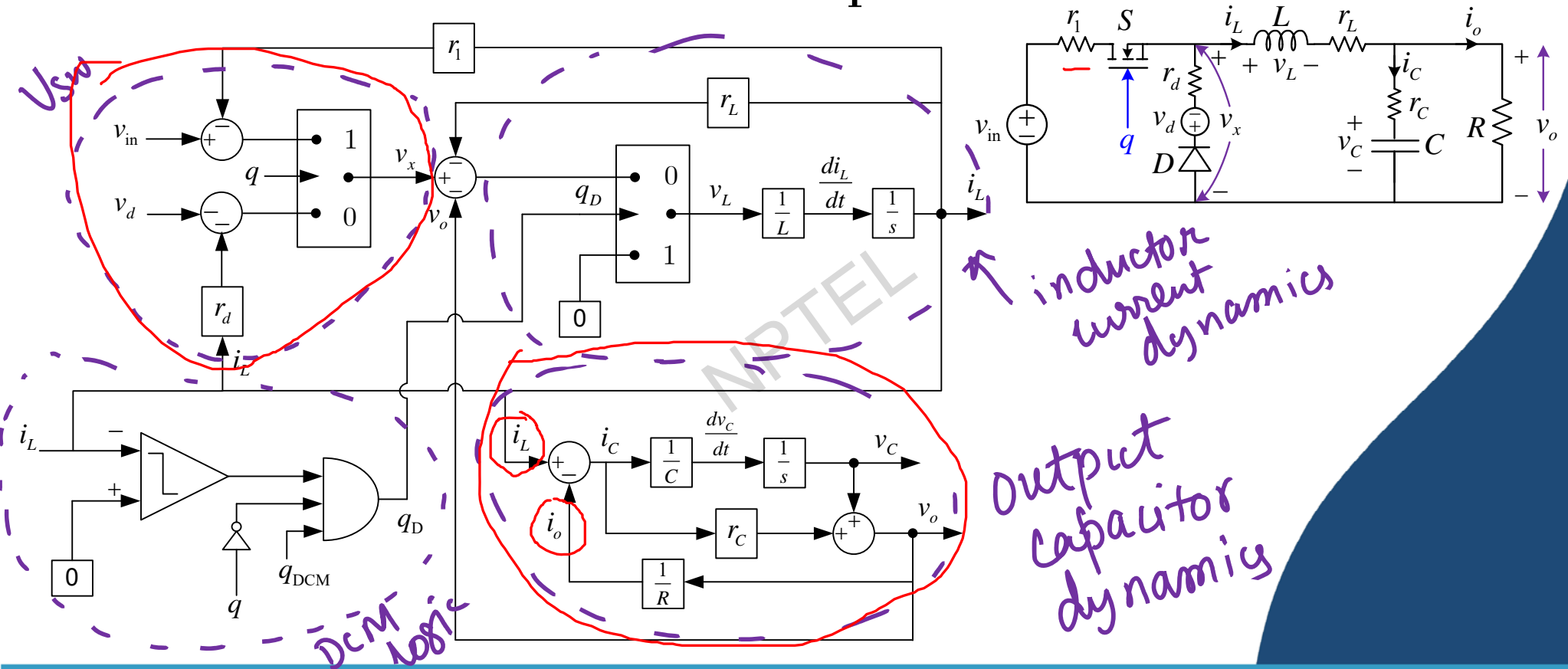
# Concepts Covered

- Model development of SMPCs with DCM Enable
- Simulink model development of SMPCs with DCM Enable
- Synchronizing Simulink and script files in MATLAB
- Interactive simulation case studies

## Block Diagram of a Conventional Buck Converter

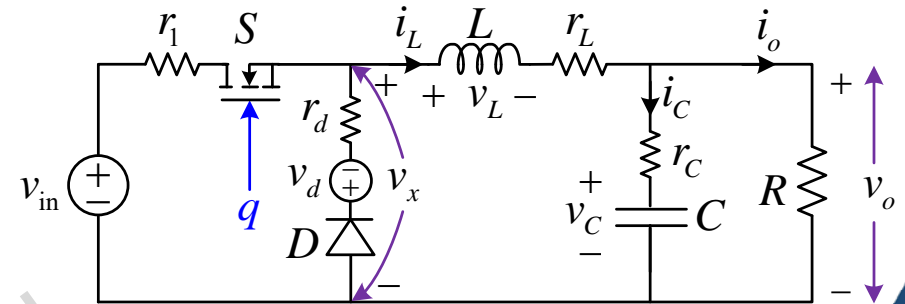


# Conventional Buck Converter – Complete Model

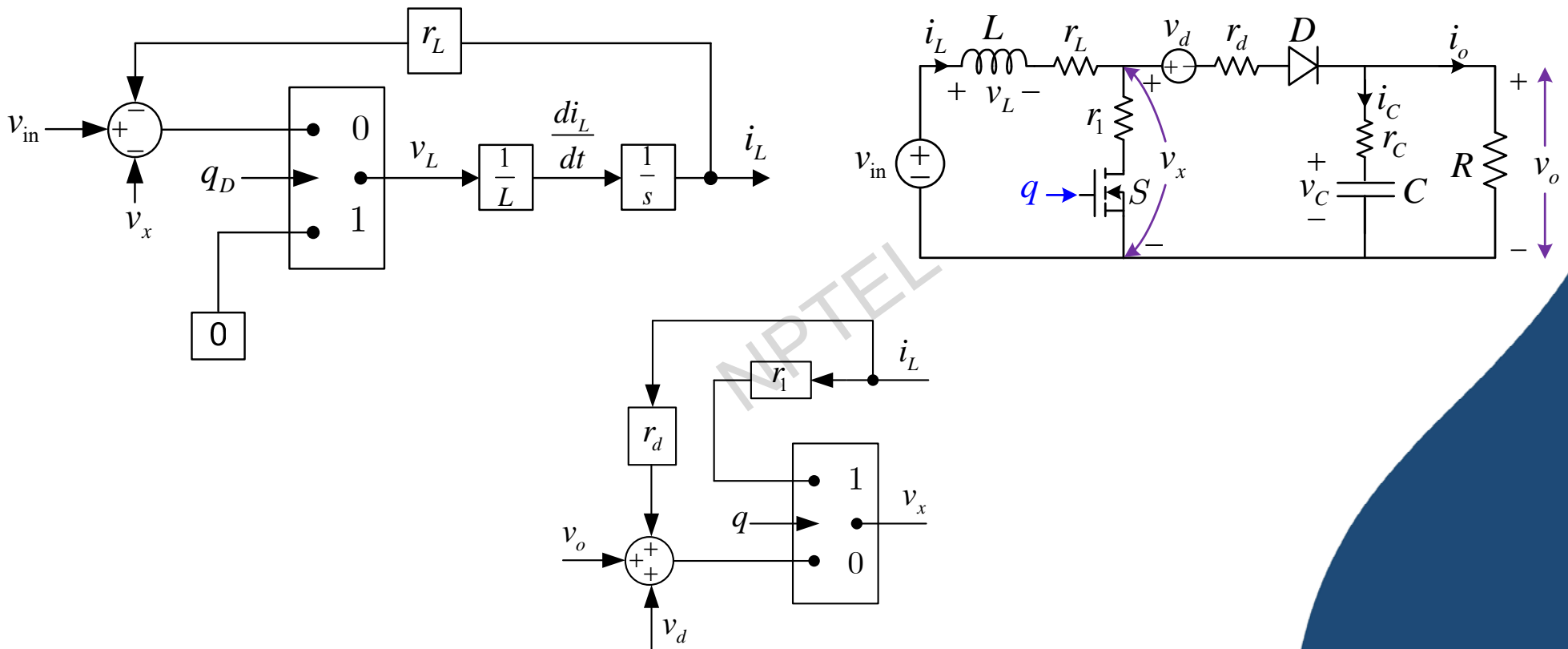


# Conventional Buck Converter – Simulation Parameters

$L=1\text{e-}6;$  % output inductance  
 $C=100\text{e-}6;$  % output capacitance  
 $T=1\text{e-}6;$  % switching time period  
 $r_L=10\text{e-}3;$  % inductor DCR  
 $r_d=10\text{e-}3;$  % diode resistance  
 $v_d=0*0.7;$  % diode voltage drop  
 $r_1=5\text{e-}3;$  % Low-side MOSFET on resistance  
 $r_2=5\text{e-}3;$  % High-side MOSFET on resistance  
 $r_C=5\text{e-}3;$  % buck converter - capacitor ESR  
 $V_{up}=10;$  % Ramp peak voltage  
 $V_b=0;$  % Ramp base voltage  
 $V_{in}=12;$  % Input voltage  
 $V_{ref}=3.3;$  % Reference output voltage in volt

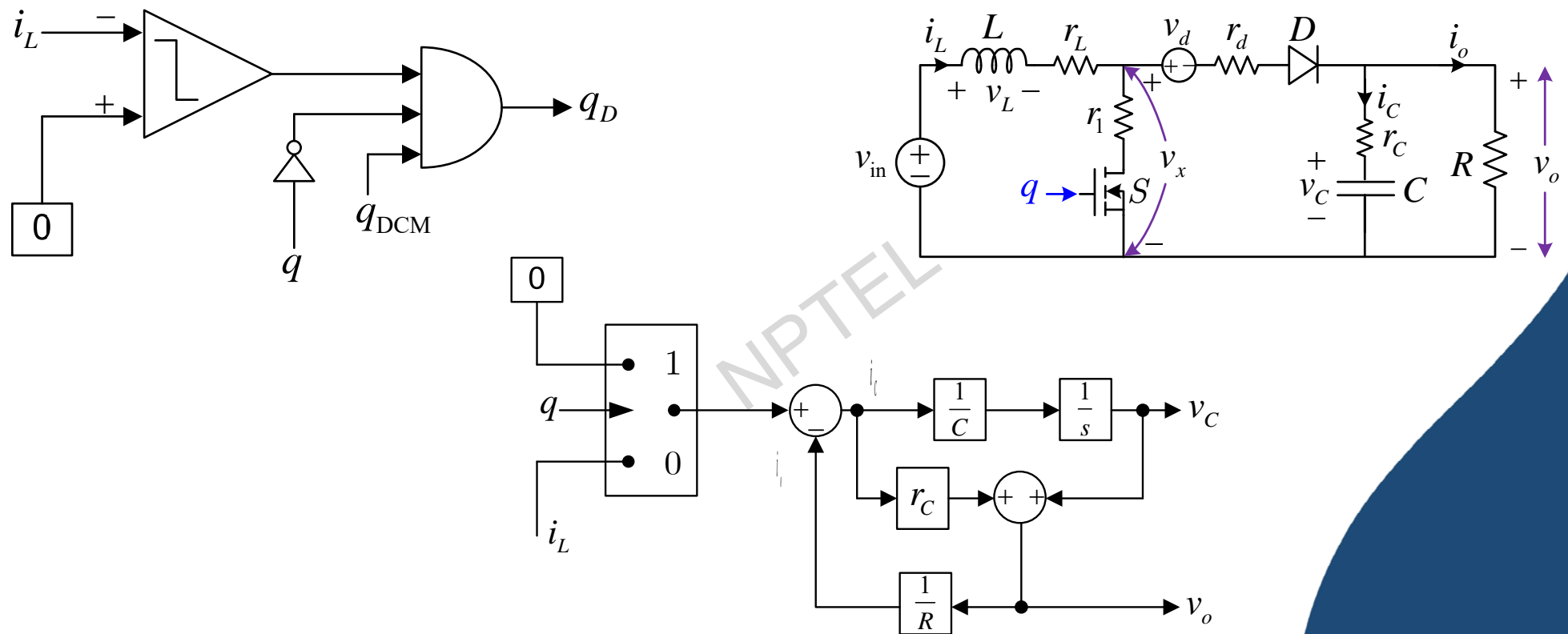


# Block Diagram of a Conventional Boost Converter



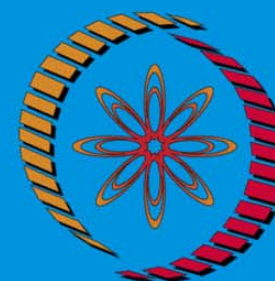


# Block Diagram of a Conventional Boost Converter



# Summary

- Simulink models developed for conventional DC-DC converters
- Demonstrated DCM mode activation and deactivation features
- Demonstrated interactive MATLAB case studies
- Modulation techniques to be shown next



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**NPTEL ONLINE CERTIFICATION COURSES**

# **CONTROL AND TUNING METHODS IN SMPCs**

**Dr. Santanu Kapat**

**Electrical Engineering Department, IIT KHARAGPUR**

**Module 02: Modulation techniques in SMPCs**

**Lecture 12 : Interactive MATLAB Simulation and Case Studies**

# Concepts Covered

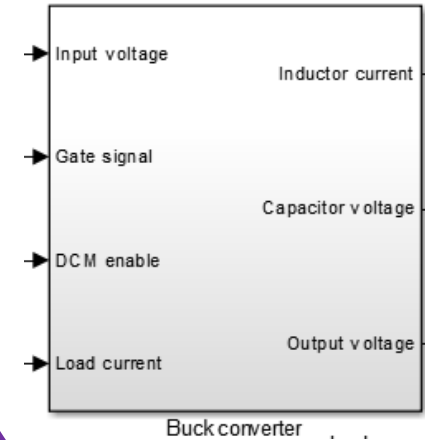
- Simulink and Script file Interactive Simulation
- Creation of Various Transient Test Cases
- Fixed Frequency and Variable Frequency Modulation

## Buck Converter Parameters for Simulation

$L=0.5\text{e-}6$       % output inductance  
 $C=200\text{e-}6$ ;      % output capacitance  
 $T=2\text{e-}6$ ;      % switching time period  
 $r_L=5\text{e-}3$ ;      % inductor DCR  
 $v_d=0.55$ ;      % diode voltage drop  
 $r_1=5\text{e-}3$ ;      % High-side MOSFET on resistance  
 $r_d=5\text{e-}3$ ;      % Low-side MOSFET on resistance  
 $r_C=3\text{e-}3$ ;      % capacitor ESR  
 $V_{in}=12$ ;      % input voltage  
 $V_{ref}=1$ ;      % reference output voltage  
 $I_{o\_max}=20$ ;      % maximum load current

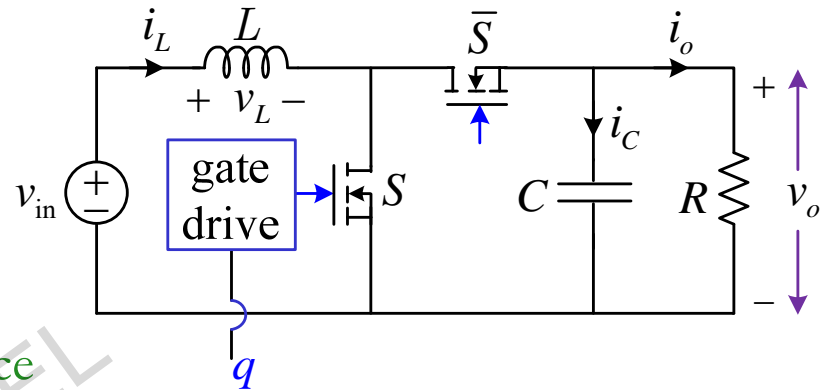
$f_{sw} = 500 \text{ kHz}$

12 V to 1 V  
POL



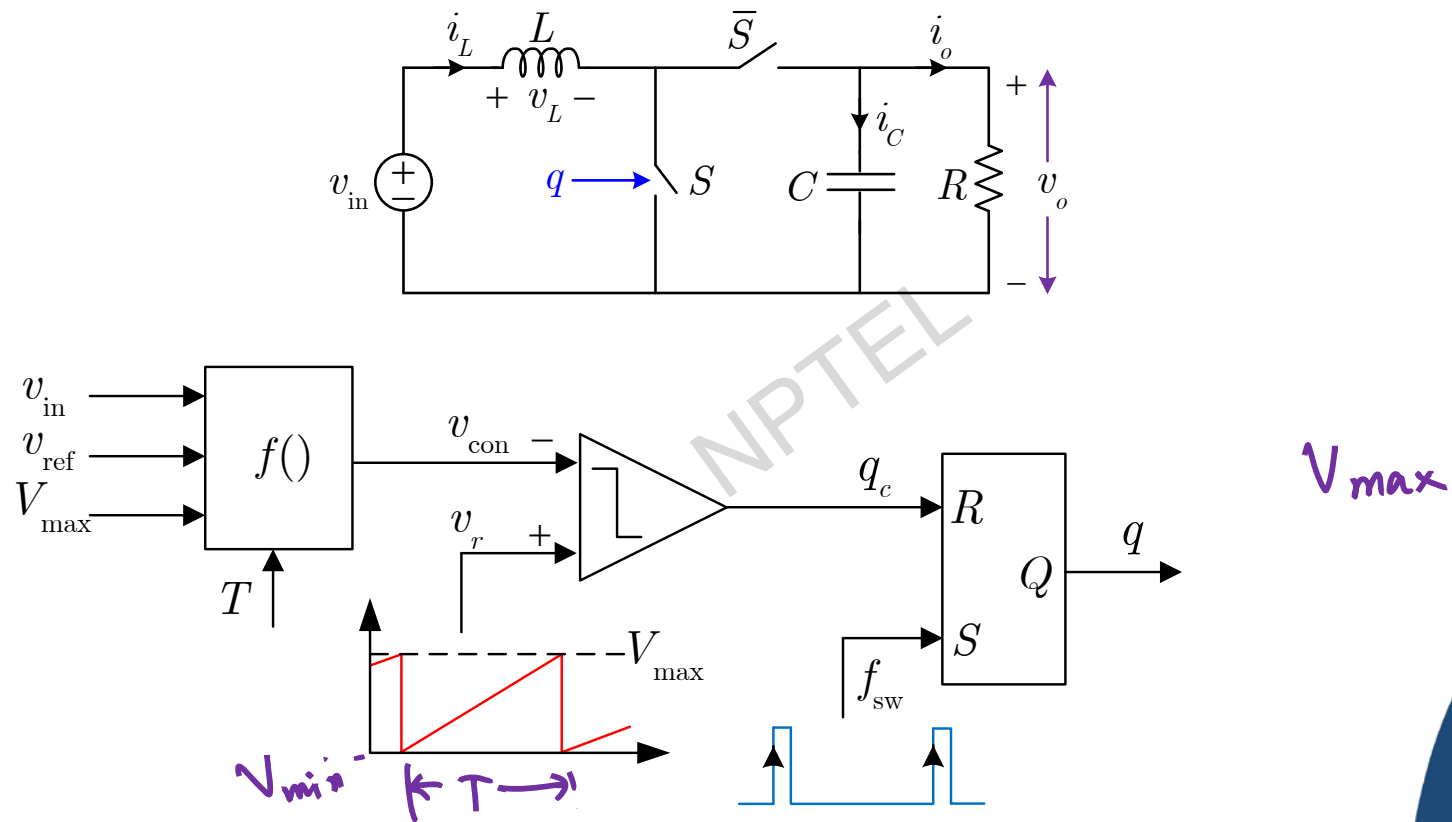
## Boost Converter Parameters for Simulation

$L=1\text{e-}6$       % output inductance  
 $C=47\text{e-}6$ ;      % output capacitance  
 $T=2\text{e-}6$ ;      % switching time period  
 $r_L=5\text{e-}3$ ;      % inductor DCR  
 $r_1=5\text{e-}3$ ;      % High-side MOSFET on resistance  
 $r_2=5\text{e-}3$ ;      % Low-side MOSFET on resistance  
 $r_C=5\text{e-}3$ ;      % capacitor ESR  
 $V_{in}=3.3$ ;      % input voltage (range – 2.5 to 4 V)  
 $V_{ref}=5$ ;      % reference output voltage  
 $I_{o\_max}=5$ ;      % maximum load current



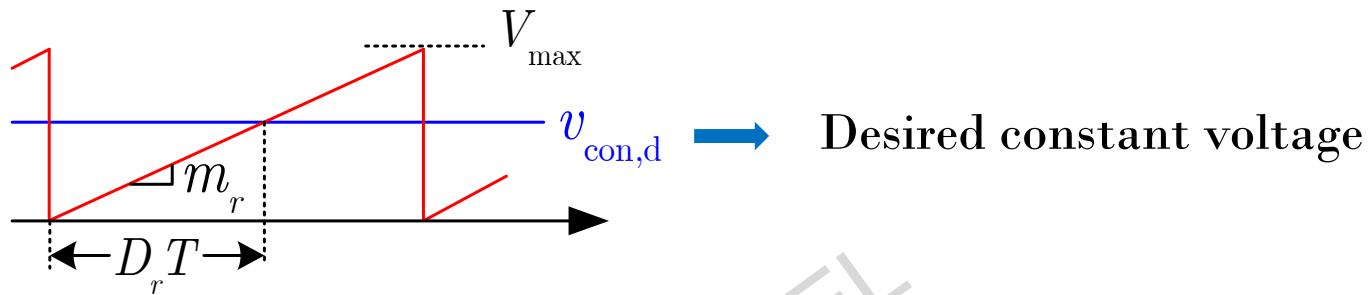
3.3 V to 5 V  
boost converter

## Trailing-edge PWM Control – Simulation





## Setting Control Voltage Contd...



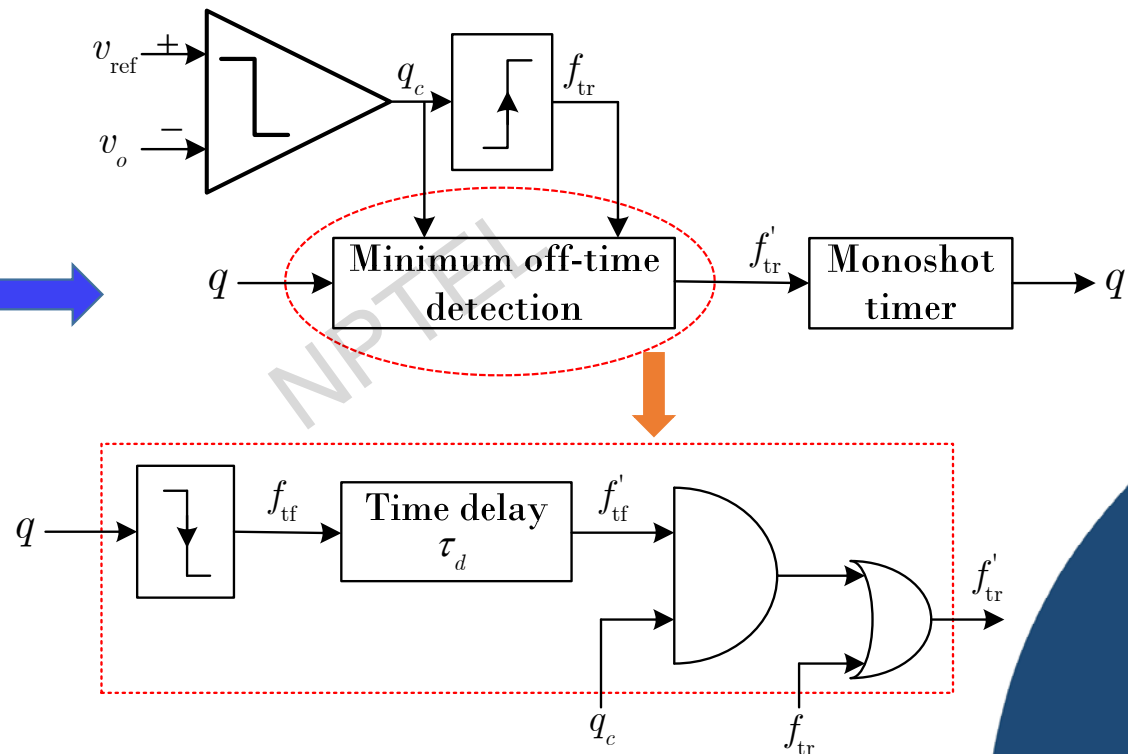
$$v_{\text{con,d}} = m_r \times (D_r \times T) \quad m_r = \frac{V_{\text{max}}}{T}, \quad D_r \times T = \left( \frac{V_{\text{ref}} - V_{\text{IN}}}{V_{\text{ref}}} \right) \times T$$

$$v_{\text{con,d}} = V_{\text{max}} \times \left( \frac{V_{\text{ref}} - V_{\text{IN}}}{V_{\text{ref}}} \right)$$

$$\therefore v_{\text{con,d}} = V_{\text{max}} \times \left( 1 - \frac{V_{\text{IN}}}{V_{\text{ref}}} \right)$$

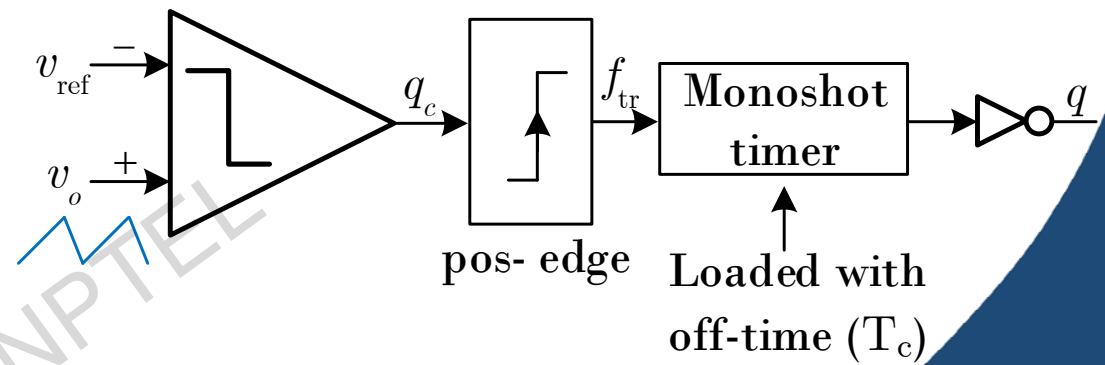
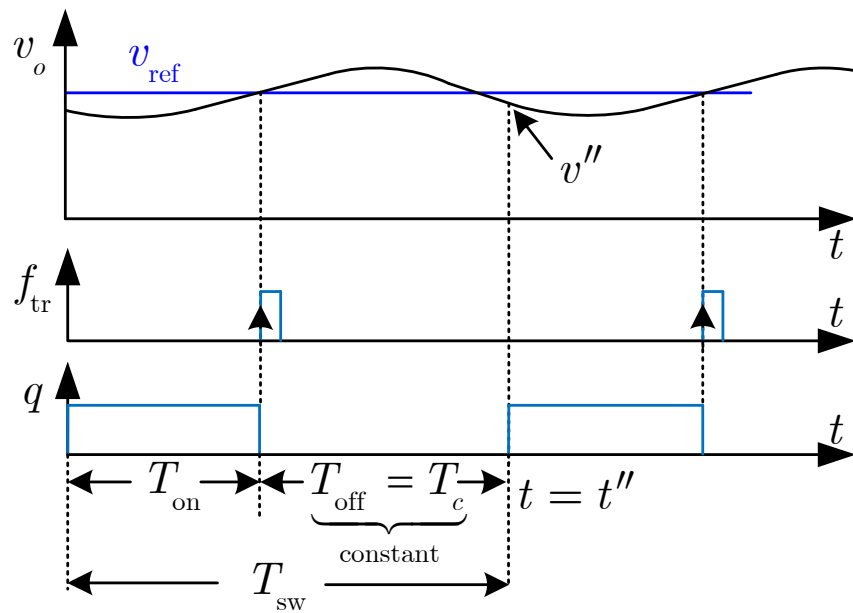
## Constant On-Time Modulation

- Turn ON problem  
in Constant on-time
- Solution:  
Introduce  
minimum off-time  
in Constant on-time



## Constant OFF-time Modulation

Voltage based implementation



# Summary

- Simulink model development of fixed-frequency and variable-frequency modulators
- MATLAB simulation of buck and boost converters



**THANK  
YOU !**