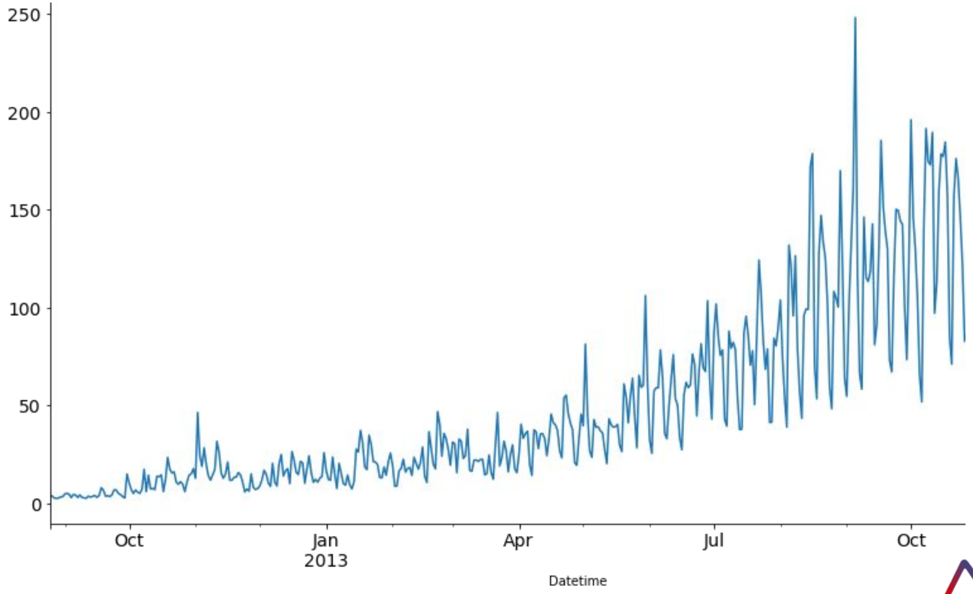
Evaluation Metrics for Time Series Forecasting

Defining the Problem Statement

Problem Statement: Forecast the number of passengers who will onboard the jetrail day in the next two quarter.

Historical Data



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Evaluation Metrics for Regression

- Regression metric
  - Mean Absolute Error
  - Mean Squared Error
  - Root Mean Squared Error
  - Root Mean Squared Log Error

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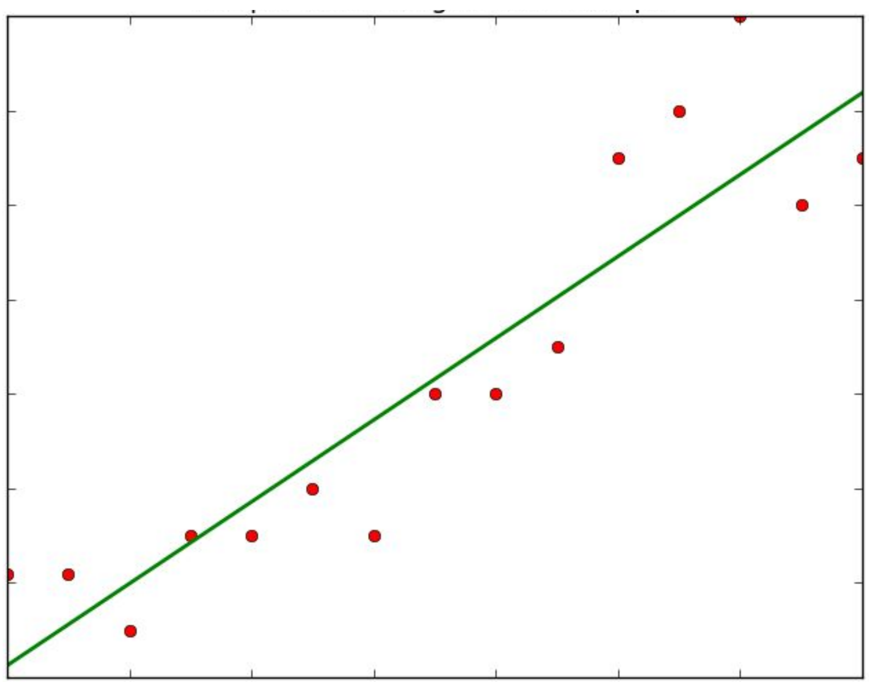
Evaluation Metrics for Time Series Forecasting

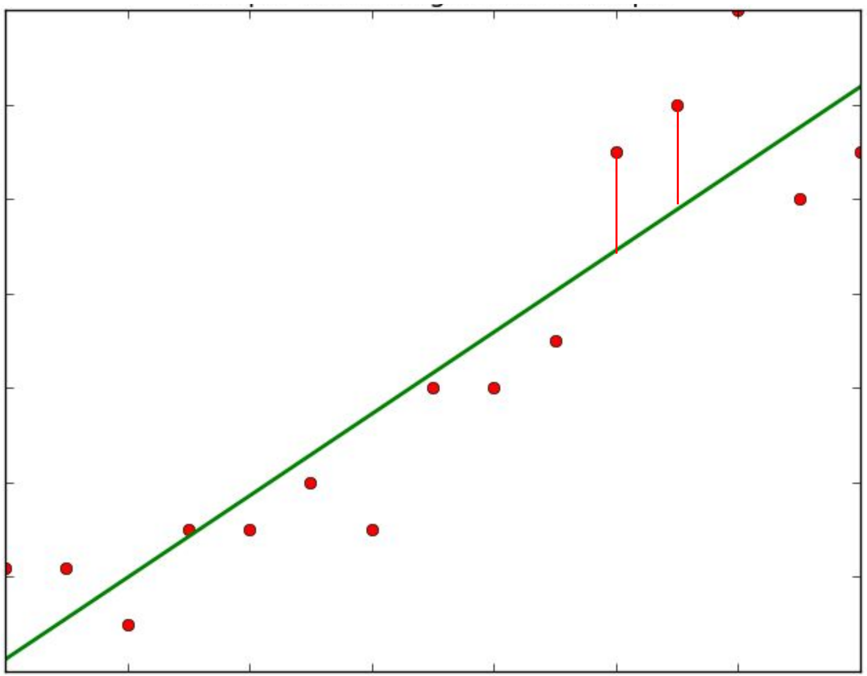
- Regression metric
    - Mean Absolute Error
    - Mean Squared Error
    - Root Mean Squared Error
    - Root Mean Squared Log Error
- Other Evaluation Metrics
    - Mean Absolute Percentage Error
    - Mean Absolute Scaled Error

What is Error?



What is Error?

Actual Values	Predicted Values
19	28
37	33
25	20
9	16
22	15



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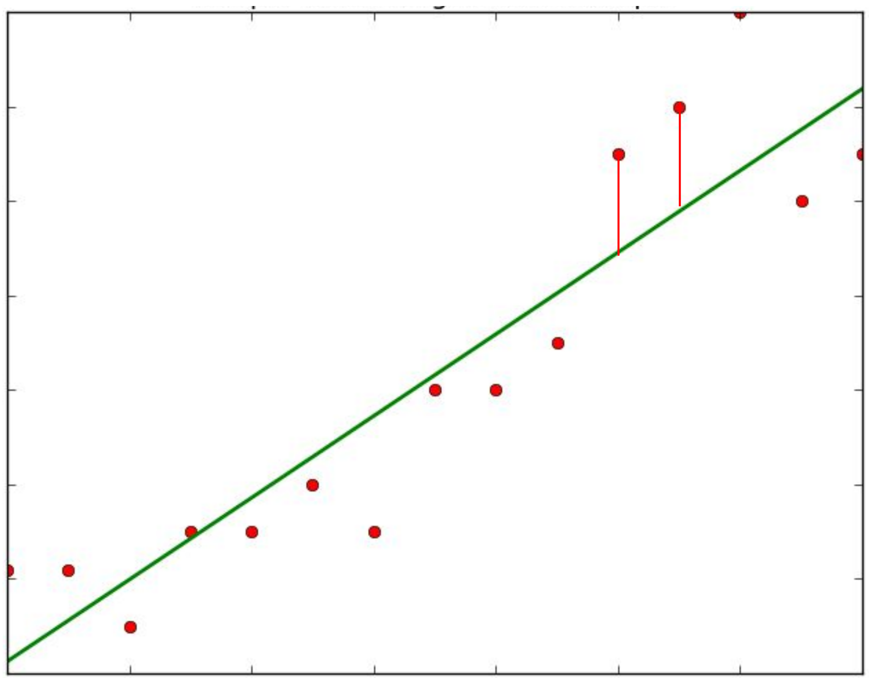
What is Error?

Actual Values	Predicted Values	Error
19	28	9
37	33	-4
25	20	-5
9	16	7
22	15	-7



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Mean Absolute Error

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N} |y_i - \hat{y}_i|$$

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Mean Absolute Error

$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i|$$

Actual Values	Predicted Values	Absolute Error
19	28	9
37	33	4
25	20	5
9	16	7
22	15	7

MAE = 6.4

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Mean Squared Error

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

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Mean Squared Error

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

MSE = 44 meter^2

Actual Values	Predicted Values	Squared Error
19	28	81
37	33	16
25	20	25
9	16	49
22	15	49

Root Mean Squared Error

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(Predicted_i - Actual_i)^2}{N}}$$

Root Mean Squared Error

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (Predicted_i - Actual_i)^2}{N}}$$

Actual Values	Predicted Values	Squared Error
19	28	81
37	33	16
25	20	25
9	16	49
22	15	49

MSE = 44 meter ^{2}

RMSE = 6.63 meters

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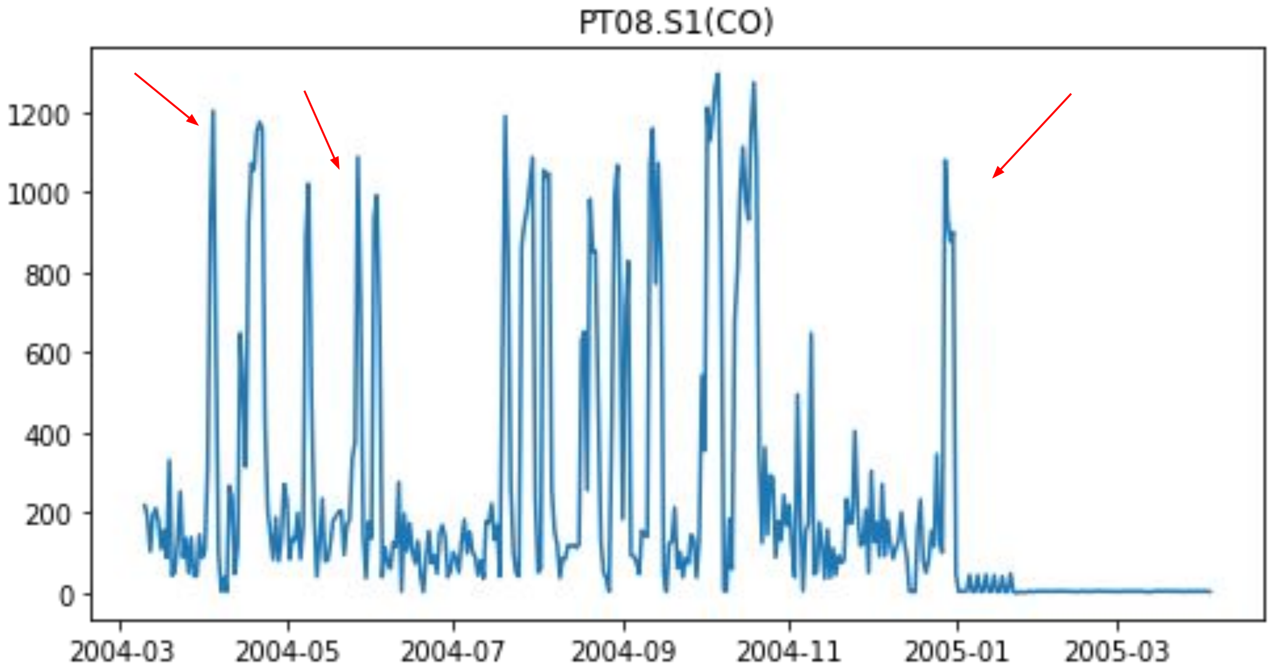
Problems with Root Mean Squared Error



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Problems with Root Mean Squared Error

Actual	Predicted
1	401

Actual	Predicted
10,001	10,401

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Problems with Root Mean Squared Error

Actual	Predicted
1	401

Error = 400

Actual	Predicted
10,001	10,401

Error = 400

Problems with Root Mean Squared Error

Actual	Predicted
1	401

RMSE = 400

Actual	Predicted
10,001	10,401

RMSE = 400

Root Mean Squared Log Error

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (Predicted_i - Actual_i)^2}{N}}$$

$$\text{RMSLE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(\log(Y_i + 1) - \log(\hat{Y}_i + 1) \right)^2}$$

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Root Mean Squared Log Error

Actual	Predicted
1	401

RMSE = 400

RMSLE = 5.3

Actual	Predicted
10,001	10,401

RMSE = 400

RMSLE = 0.039

Evaluation Metrics for Time Series Forecasting

- Mean Absolute Error
- Mean Squared Error
- Root Mean Squared Error
- Root Mean Squared Log Error
- Mean Absolute Percentage Error
- Mean Absolute Scaled Error

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Problems with Error Metrics

- Lower the error, better the model performance
- No defined range
- Compare the values with benchmark

Mean Absolute Percentage Error

- Calculates the relative error

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

Mean Absolute Percentage Error

- Calculates the relative error

$$\mathrm{MAPE} = \frac{\boxed{100\%}}{n} \sum_{i=1}^{n} \left| \boxed{\frac{y_i - \hat{y}_i}{y_i}} \right|$$

Mean Absolute Percentage Error

- Calculates the relative error

$$\text{MAPE} = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right|$$

mean

absolute

Mean Absolute Percentage Error

- Calculates the relative error

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

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Mean Absolute Percentage Error

- Calculates the relative error
- Cannot handle yi = 0

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

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Mean Absolute Scaled Error

- Alternative to MAPE
- Used for comparing forecast across different time series

Mean Absolute Scaled Error

- Alternative to MAPE
- Used for comparing forecast across different time series

$$\text{MASE} = \frac{\text{Mean Absolute Error}}{\text{Naive Forecast Error}}$$

Mean Absolute Scaled Error

Actual Values
19
37
25
9
22

Mean Absolute Scaled Error

	Actual Values	Naive Forecast
t_1	19	
t_2	37	
t_3	25	
t_4	9	
t_5	22	

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Mean Absolute Scaled Error

Actual Values	Naive Forecast
19	
37	19
25	
9	
22	

Mean Absolute Scaled Error

Actual Values	Naive Forecast
19	
37	19
25	37
9	
22	

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Mean Absolute Scaled Error

Actual Values	Naive Forecast
19	
37	19
25	37
9	25
22	9

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Mean Absolute Scaled Error

Actual Values	Naive Forecast	Predicted Values
19		28
37	19	33
25	37	20
9	25	16
22	9	15

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Mean Absolute Scaled Error

Actual Values	Naive Forecast	Predicted Values
19		28
37	19	33
25	37	20
9	25	16
22	9	15

$$\text{MASE} = \frac{\text{Mean Absolute Error (model)}}{\text{Naive Forecast MAE}}$$

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Mean Absolute Scaled Error

$$e_t = \mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i|$$

$$q_t = \frac{e_t}{\frac{1}{n-1}\sum_{i=2}^{n}|Y_i - Y_{i-1}|}$$

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Mean Absolute Scaled Error

$$e_t = \mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i|$$

$$q_t = \frac{e_t}{\boxed{\frac{1}{n-1}\sum_{i=2}^{n}|Y_i - Y_{i-1}|}}$$

Scaling factor

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Mean Absolute Scaled Error

- Alternative to MAPE
- Used for comparing forecast across different time series
  - If MASE >1 ---> worse than naive forecast
  - If MASE <1 ----> better than naive forecast

Thank You