Averaged Overnight Rate Futures: Convexity Adjustment

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The convexity adjustment for averaged overnight rate futures, like SOFR 1m futures, is derived including the case where trading occurs during the reference period. These results are more general than previous work that relied solely on the HJM framework, and the results herein can easily incorporate and reuse previous derivations. Numerical results demonstrate that the averaged overnight rate futures convexity adjustment is close to the convexity adjustment for compounded overnight rate futures, which is due to the close proximity of daily compounding to the continuous compounding limit.

1 Introduction

The push for global transition away from Libor as a benchmark interest rate has promoted the importance of overnight rates as the planned source of primary interest rate benchmarks in major currencies. In USD the recently formalized secured overnight financing rate (SOFR) has been selected as the future interest rate benchmark in the event Libor is discontinued past 2021. SOFR is an overnight rate, and like Fed Funds it is only fixed on the following day based on observed transactions on the previous day. In order to price derivatives and payment obligations linked to SOFR at longer maturities beyond a single day's time, it is necessary to build forward-rate curves that reflect accurate, on-market pricing of SOFR-based futures, swaps, and basis swaps. Generally these instruments will pay the compounded or averaged SOFR rate, and the market quotes, such as futures prices, are used as inputs to forward-rate curve construction.

It is well known that in order to incorporate interest rate futures into forward-rate curve construction, it is necessary to account for a convexity adjustment between the implied futures rate and the forward rate [1, 2]. In the case of Eurodollar futures linked to Libor, a number of well-known results for the convexity adjustment have been derived which fall into the general category of Heath-Jarrow-Morton (HJM) framework of interest rate models [3, 4]. Recently these results have been extended to compounded overnight rate futures (CONF) [5]. In the case of averaged overnight rate futures (AONF), the convexity adjustment was previously derived only in the case where they are traded prior to the start of the reference period [6]. The following two sections extend these results to generalize the AONF convexity adjustment, showing it is a combination of standard Eurodollar convexity adjustments from previous works, and includes the case where trading occurs during the reference period. Several approximation techniques are discussed such as continuous compounding, and a numerical example using the Vasicek model is shown in the final section.

2 Averaged Overnight Rate Futures Instrument

AONF are currently traded on major exchanges such as CME, and typically have shorter maturities than Eurodollar or CONF. Generally these instruments are some of the most important for market-based discovery of the overnight rate due to high volume and liquidity as well as transparency of the futures price during trading hours. In USD, historically the most important of these futures are for the Fed Funds overnight rate, though more recently SOFR has started to become relevant to the transition away from Libor. Due to the SOFR rate being fairly new, the SOFR-linked derivatives markets are still developing, and the emergence of SOFR 1m futures in May of 2018 was an important milestone in developing the SOFR interest rate benchmark.

SOFR 3m futures are CONF, with quarterly reference period, and available for 20 consecutive quarters, with the reference period beginning on the third Wednesday IMM date of the contract month. Fed Funds and SOFR 1m futures are instead AONF, with monthly reference period beginning on the start of the contract month, and available for 7 consecutive months for spot trading. One of the interesting features of Fed Funds or SOFR 1m and 3m futures is the possibility for trading during the reference period. In contrast to Eurodollar futures with underlying Libor rate fixed at the start of the reference period, averaged and compounded overnight rate futures are not fixed until the end of the reference period.

Averaged and compounded overnight rate futures prices provide a vital market data source for the forward-rate curve construction on the underlying overnight rate. As discussed in [5], the ability to trade these futures during the reference period where the underlying rate is only partially fixed, introduces some complication on how to include the impact of interest rate volatility in the stripping of forward rates from the futures prices. The fundamental need to tackle this issue for forward-rate projection on overnight rates points to the need for derivation of the convexity adjustment for AONF, including the impact for trading during the reference period.

3 Futures Convexity Adjustment

The arithmetic average rate can be expressed as:

$$F_{\text{avg}}(t,T_0,T_n) = \frac{\Sigma_{k=1}^{n}\delta_k r_k}{\Sigma_{k=1}^{n}\delta_k} = \frac{1}{\delta}\Sigma_{k=1}^{n}\left(\frac{P(t,T_{k-1})}{P(t,T_k)} - 1\right) \tag{1}$$

In Eq. (1) the function $P(t,T)$ is the price of a zero coupon bond that is assumed to be funded at the overnight rate, with unit principal and maturity T, as observed at time t. The times T_0 and T_n are the start and end, respectively, of the AONF reference period, and δ_k is the daily overnight rate accrual fraction with δ being the total accrual for the reference period ($\delta = T_n - T_0$). The standard approach for deriving the futures convexity adjustment for Eurodollar futures relies on the futures price being a martingale under the risk-neutral measure, so the convexity adjustment for forward-rate determination can be derived from the expectation of the forward-rate under the risk-neutral measure [1, 2]:

$$C_{\text{ED}}(T_0,T_n) = E^{Q}[\frac{1}{\delta}\left(\frac{P(T_0,T_0)}{P(T_0,T_n)} - 1\right)|\mathscr{F}_0] - \frac{1}{\delta}\left(\frac{P(0,T_0)}{P(0,T_n)} - 1\right) = \frac{1}{\delta}\left(E^{Q}[\frac{P(T_0,T_0)}{P(T_0,T_n)}|\mathscr{F}_0] - \frac{P(0,T_0)}{P(0,T_n)}\right) \tag{2}$$

As this is linear in the summation of Eq. (1) we have the immediate result that:

$$C_{\text{avg}}(T_0,T_n) = \frac{1}{\delta}\Sigma_{k=1}^{n}\left(E^{Q}[\frac{P(T_{k-1},T_{k-1})}{P(T_{k-1},T_k)}|\mathscr{F}_0] - \frac{P(0,T_{k-1})}{P(0,T_k)}\right) = \Sigma_{k=1}^{n}\frac{\delta_k}{\delta}C_{\text{ED}}(T_{k-1},T_k) \tag{3}$$

As previously discussed in the context of forward rate obligations, a very accurate approximation of the arithmetic average of overnight rates is given by [7]:

$$F_{\text{avg}}(t,T_0,T_n) = \frac{1}{\delta}\Sigma_{k=1}^{n}\delta_k r_k \approx \frac{1}{\delta}\log\Pi_{k=1}^{n}(1+\delta_k r_k) = \frac{1}{\delta}\Sigma_{k=1}^{n}\log(1+\delta_k r_k) = \frac{1}{\delta}\Sigma_{k=1}^{n}\log\left(\frac{P(t,T_{k-1})}{P(t,T_k)}\right) \tag{4}$$

This can be compared with the continuously compounded forward rate defined as:

$$F_{\text{cont}}(t,T_0,T_n) = \frac{1}{\delta}\log\left(\frac{P(t,T_0)}{P(t,T_n)}\right) = \frac{1}{\delta}\Sigma_{k=1}^{n}\log\left(\frac{P(t,T_{k-1})}{P(t,T_k)}\right) \approx F_{\text{avg}}(t,T_0,T_n) \tag{5}$$

This demonstrates another important result that the continuous compounded rate is a good approximation for the average overnight rate, and the AONF convexity adjustment can be closely approximated as a sum of terms with the same form as the continuous compounded rate with 1-day tenor. This immediately leads to the main result that the average overnight rate convexity adjustment can be closely approximated by the daily weighted average of convexity adjustments for the continuously compounded rate over the full reference period:

$$C_{\text{avg}}(T_0,T_n) = \Sigma_{k=1}^{n}\frac{\delta_k}{\delta}C_{\text{ED}}(T_{k-1},T_k) \approx \Sigma_{k=1}^{n}\frac{\delta_k}{\delta}C_{\text{cont}}(T_{k-1},T_k) = C_{\text{cont}}(T_0,T_n) \approx C_{\text{cmpd}}(T_0,T_n) \tag{6}$$

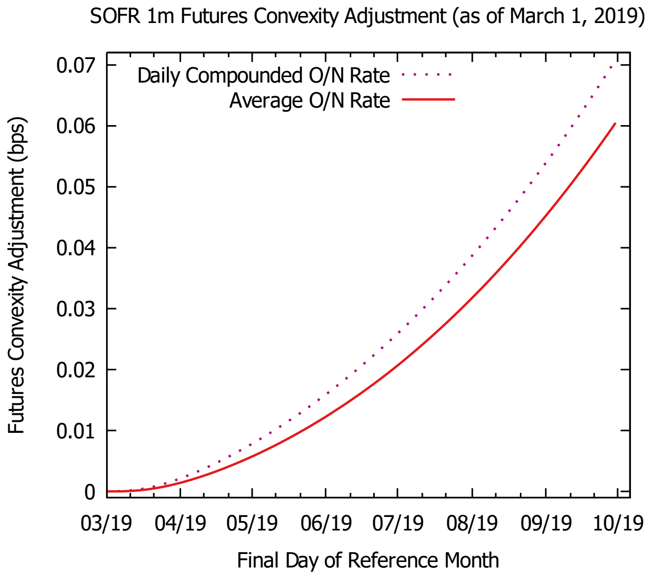
This result demonstrates the AONF convexity adjustment can be obtained from a summation of daily convexity adjustments of the form of the Eurodollar futures convexity adjustment, and closely approximated by derivations which use the continuously compounded rate. Due to the high-frequency of daily compounding used in the CONF discussed in [5], the present result also shows that there is a close connection between the convexity adjustments for averaged and daily-compounded overnight rates for the same period. This result in Eq. (6) is straightforward, but also more general than the HJM interest rate model framework which was required in the majority of previous work on futures convexity adjustments [3, 4, 5, 6], and the present derivation highlights the close connection between AONF convexity adjustment with any previous derivation for continuous compounded rates or Eurodollar rates, as well as a reasonable approximation for the case of CONF.

A further advantage of the present derivation is to provide a result for the convexity adjustment which is valid for trading during the reference period, which extends previous work on AONF convexity adjustments [6]. Eq. (6) expresses the convexity adjustment as a summation over the remaining time in the reference period. Before the reference period where $0 < T_0$, the summation in Eq. (6) covers the entire reference period. As time advances and trading begins to occur during the reference period, then throughout we set $T_0 = 0$, and subsequently a term will be dropped from the summation on a daily basis as overnight rate fixings become available, which will lead to the trend of a diminishing convexity adjustment as trading moves through the reference period.

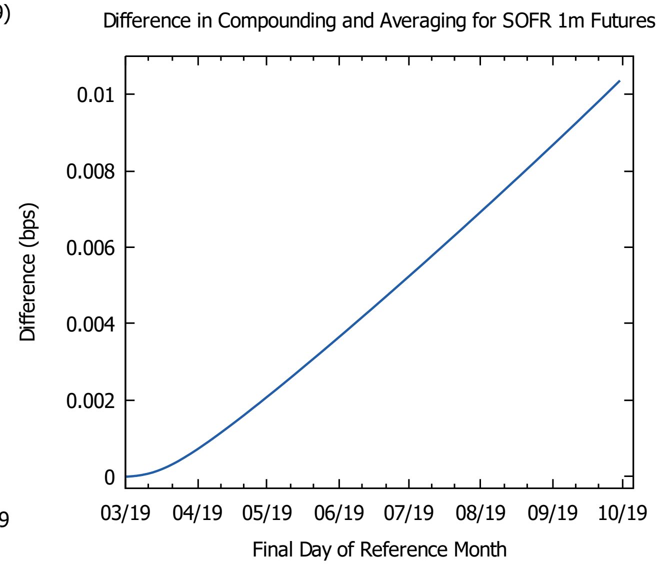
4 Numerical Example: SOFR 1m and 3m Futures

Previous closed-form derivations of the Eurodollar convexity adjustment can be used in Eq. (6) to calculate the AONF convexity adjustment numerically [4]. Using the Vasicek model specified in [5] with volatility of 65 bps and mean reversion of 3%, the CONF convexity adjustment can be compared with the present work for SOFR 1m AONF, as shown in Figs. 1a and 1b. The results of [5] are a very close approximation to the present derivation and may also be applicable to AONF. Under the Vasicek model the compounded convexity adjustment is larger than in the case of averaging, but at realistic volatility and mean reversion levels the difference is less than 0.01 basis points for the entire range of SOFR 1m futures contracts which are traded.

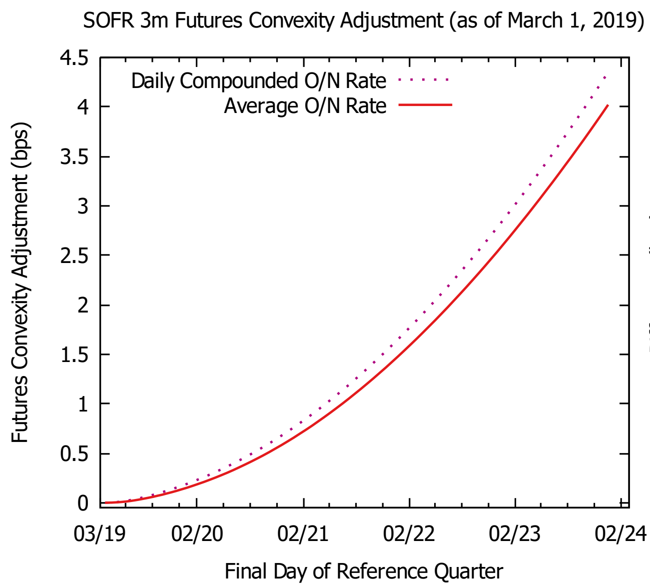
A similar calculation is repeated for SOFR 3m futures in Figs. 1c and 1d. These contracts extend much further in maturity, and at the long-end experience a larger convexity adjustment than for any SOFR 1m futures contracts, with the difference between the averaging and compounding convexity adjustment also being more pronounced for longer-maturity SOFR 3m futures. However the AONF convexity adjustment is still a good approximation for the CONF convexity adjustment, with the largest discrepancy between the two only reaching about a third of a basis point.



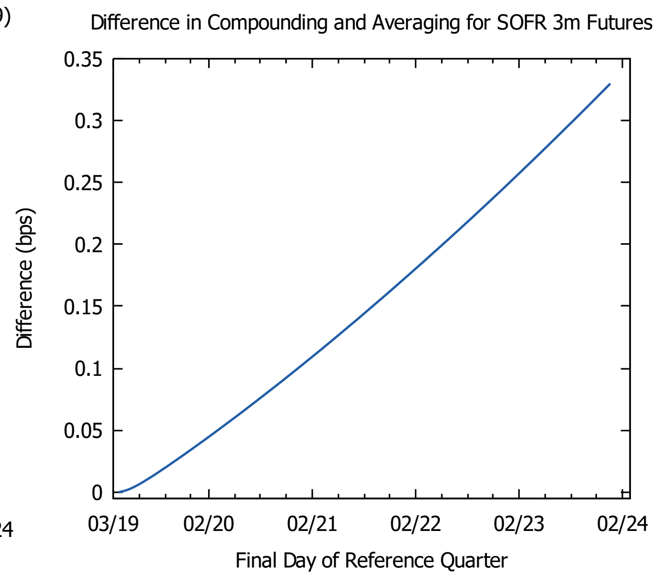
(a) SOFR 1m futures convexity adjustment under Vasicek model, with results derived in [5] compared with Eq. (6).



(b) Difference between compounded and averaged convexity adjustments shown in Fig. 1a for SOFR 1m futures.



(c) SOFR 3m futures convexity adjustment under Vasicek model, with results derived in [5] compared with Eq. (6).



(d) Difference between compounded and averaged convexity adjustments shown in Fig. 1c for SOFR 3m futures.

Figure 1

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