Forecasting in a Chaotic Toy Climate

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Chaos?

The Lorenz system
- Mathematical formulation
- Thermosyphon

Modern forecasting techniques
- Data assimilation
- Ensemble forecasting

Preliminary results

Defining chaos

Characteristics of chaotic systems

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It is difficult to forecast chaotic behavior even with knowledge of the governing equations.

Another definition

"When the present determines the future, but the approximate present does not approximately determine the future."

–Lorenz

Inception

First studied by Edward N. Lorenz (1917–2008) in the '60s, he was looking for a more realistic weather model as an alternative to the linear ones then in use.

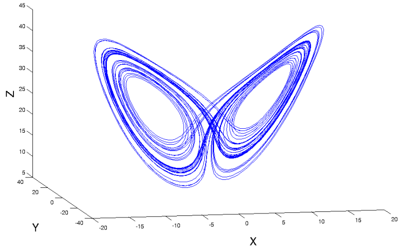
His equations

Written in their dimensionless form, they read:

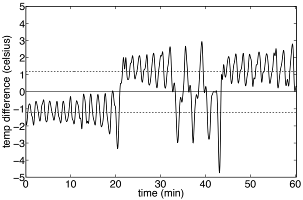
$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = \rho x - y - xz$$
$$\frac{dz}{dt} = xy - \beta z$$

This is a system of three coupled, nonlinear ODEs.

A chaotic attractor



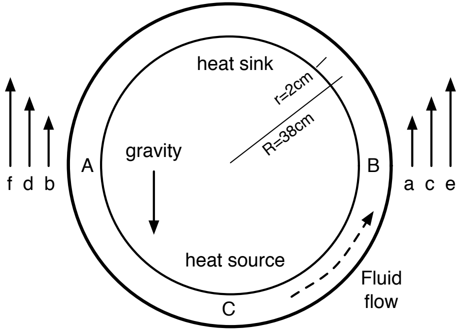
(a) attractor



(b) timeseries for y

A physical analog

Lorenz derived the equations of motion by considering a truncation of the Rayleigh-Bénard equations for fluid flow. His solution describes the dynamics of a *thermosyphon*.



The initial value problem

When observing a real system, we can never perfectly know its state. *Data assimilation* is an *optimal interpolation* scheme that incorporates both our past forecasts and observations.

- 3D-Var: Minimize a cost function of the analysis state vector. Depends on background and observational error covariance (assumed to be constant).

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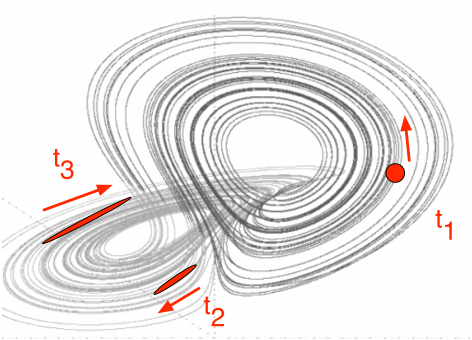
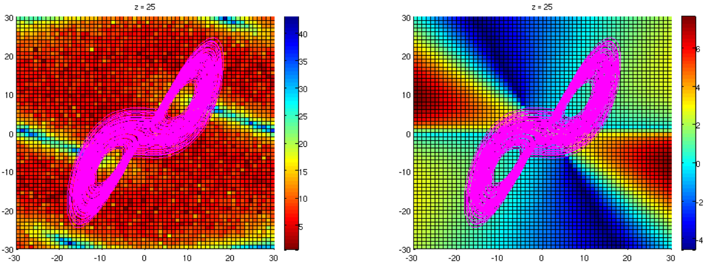
- Extended Kalman Filter: Similar to variational approach, only now we update the background error covariance from the linear tangent model and earlier error.

Sampling phase space

A complementary approach is to forecast an *ensemble* of initial conditions. If we choose them carefully, the forecast will tell us how the system stretches a ball in phase space and reveals information about the directions of expanding and contracting uncertainty.



Attractor stability



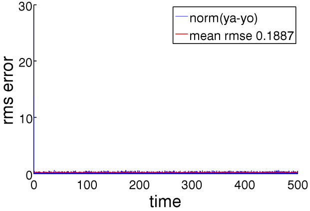
(a) (b)

Figure: Stability calculated two ways: (a) directly measures divergence time of nearby orbits; (b) plots the largest real part of the eigenvalues of the Jacobian evaluated at each point.

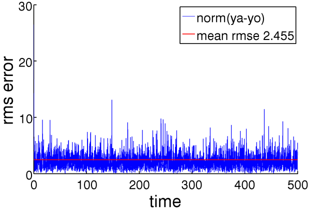
Bred vector growth on the attractor

(LoadingMovie)

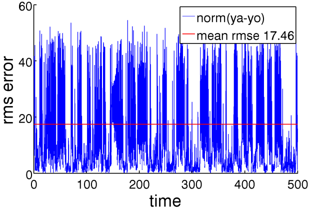
3D-Var numerical results



(a) xyz



(b) x



(c) z

Once the thermosyphon is built, we anticipate being able to predict regime changes using a combination of data assimilation and ensemble forecasting.

Stay tuned for more . . .