# Forecasting in a Chaotic Toy Climate

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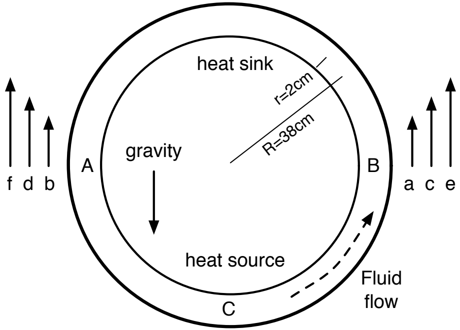
- The Lorenz system
  - History
  - Thermosyphon
  - Math and butterflies
- Chaos?
- Modern forecasting techniques
  - Data assimilation
  - Ensemble forecasting
- Preliminary results

# Inception

First studied by Edward N. Lorenz (1917–2008) in the '60s, he wanted to show that linear models were ineffective for describing the behavior of the weather.

# A physical analog

Lorenz derived the equations of motion by considering a truncation of the Rayleigh-Bénard equations for fluid flow. His solution describes the dynamics of a *thermosyphon*.



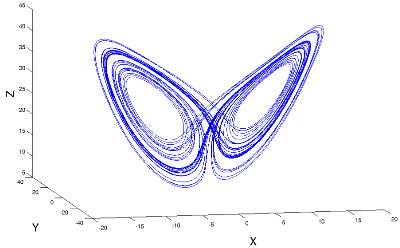
# His equations

Written in their dimensionless form, they read:

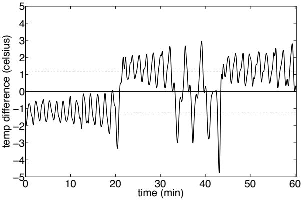
$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = \rho x - y - xz$$
$$\frac{dz}{dt} = xy - \beta z$$

This is an autonomous system of three coupled, nonlinear ODEs.

# A chaotic attractor




(a) attractor




(b) timeseries for y

# Defining chaos

Characteristics of chaotic systems

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It is difficult to forecast chaotic behavior even with knowledge of the governing equations.

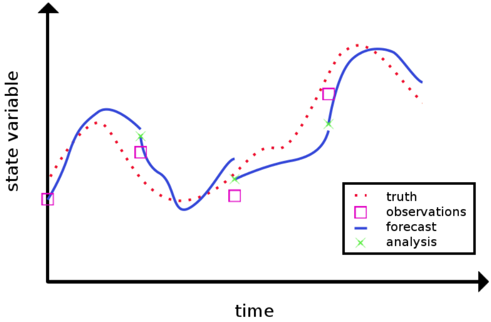
# Another definition

"When the present determines the future, but the approximate present does not approximately determine the future."

–Lorenz

# The initial value problem

When observing a real system, we can never perfectly know its state. *Data assimilation* is an *optimal interpolation* scheme that incorporates both our past forecasts and observations.



# Algorithms

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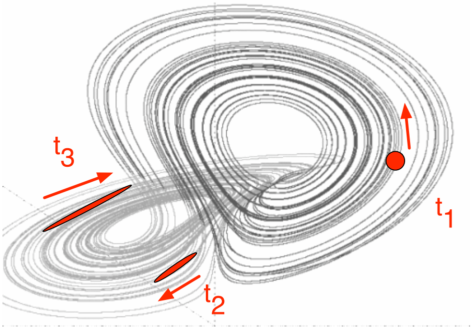
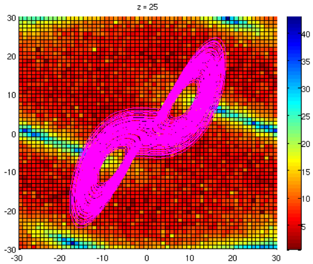
- Extended Kalman Filter: Similar to variational approach, only now we update the background error covariance with a linearized model.

# Sampling phase space

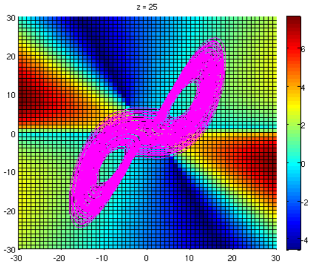
A complementary approach is to forecast an *ensemble* of initial conditions. If we choose them carefully, the forecast will tell us how the system stretches a ball in phase space and reveals information about the directions of expanding and contracting uncertainty.



# Attractor stability




(a)

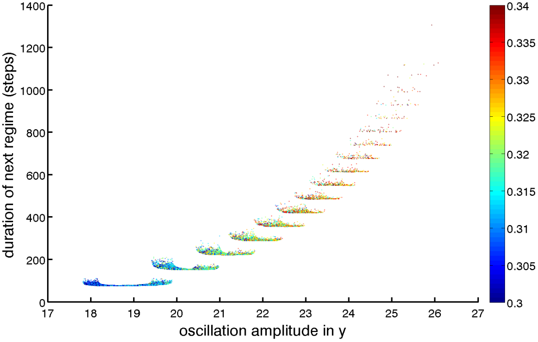



(b)

Figure: Stability calculated two ways: (a) directly measures divergence time of nearby orbits; (b) plots the largest real part of the eigenvalues of the Jacobian evaluated at each point.

# Information about the next regime?

The strength of the last oscillation before regime change indicates how long the system will remain in the next regime. This is correlated with bred vector growth rates (coloring).



## Hot pockets

(LoadingMovie)

Credit: El Hassan Ridouane

Once the thermosyphon is built, we anticipate being able to predict regime changes using a combination of data assimilation and ensemble forecasting.

Stay tuned for more . . .