Predicting Climate Regime Change in Chaotic Convection

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Inception

Developed in the '60s by Edward N. Lorenz (1917–2008) to show the National Weather Service that linear methods are inadequate for the problem of short-term weather prediction. It is the simplest realistic model of convection.

First example of *deterministic chaos*:

"When the present determines the future, but the approximate present does not approximately determine the future."

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- sensitive dependence on initial conditions (ICs)
- small deviations grow exponentially with time
- aperiodic
- nonlinear
- ... but deterministic!

$\Longrightarrow$ It is difficult to forecast chaotic behavior even with knowledge of the governing equations.

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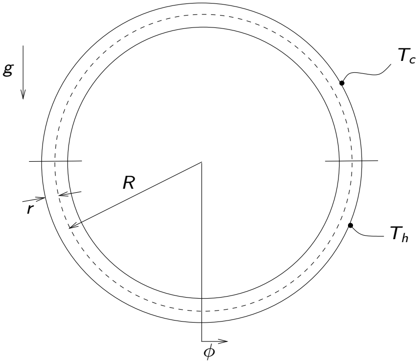
A physical analog

Lorenz derived his equations for fluid held between a lower, hot plate and an upper, cold plate (Rayleigh-Bénard). For certain parameters, his solution describes the dynamics of a *thermosyphon*.



Two systems of convective equations

Lorenz system

- Rayleigh-Bénard
- unconstrained
- many convection cells

$$\frac{dx}{dt} = s(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = xy - bz$$

Ehrhard-Müller (EM) system

- thermosyphon
- toroidal
- single convection cell

$$\frac{dx_1}{dt'} = \alpha\left(x_2 - x_1\right)$$
$$\frac{dx_2}{dt'} = \beta x_1 - x_2\left(1 + K|x_1|^{1/3}\right) - x_1 x_3$$
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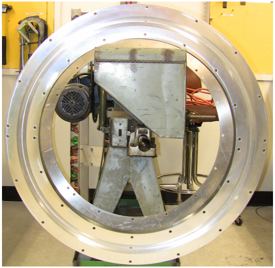
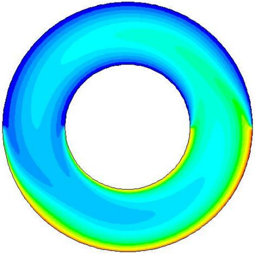
Experiment and simulation





- Experimental apparatus under construction
- 8 sites for temperature measurements
- Heating/cooling jackets

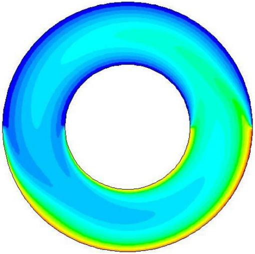
- Temperature profile for steady rotating fluid
- $\mathcal{O}(10^4)$ discretization of Navier-Stokes equations
- FLUENT: a computational fluid dynamics package

Image credit: Ridouane

Experiment and simulation

The plan:

FLUENT simulations represent the thermosyphon "truth"



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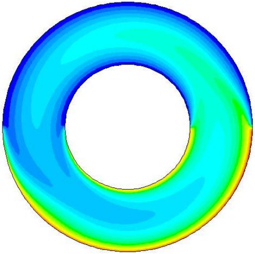

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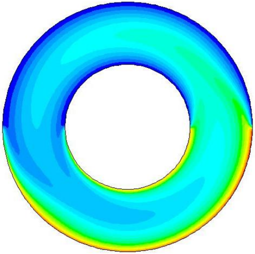
This is what we call the imperfect model forecasting scenario.



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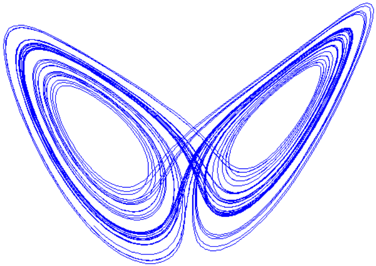
Regime changes in action

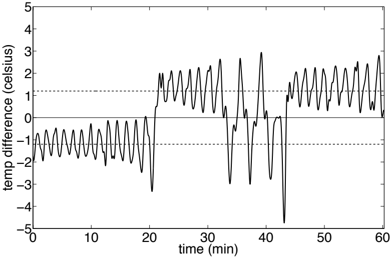
(LoadingMovie)

Credit: El Hassan Ridouane

Lorenz's chaotic attractor



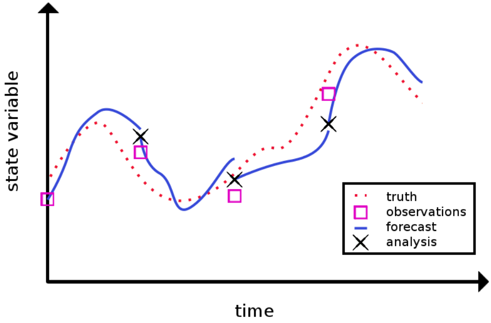
The system's attractor, which is the shape it traces out in state space



Timeseries for $\Delta T_{3-9} =$ horizontal temperature difference across the loop

The initial value problem

When observing a real system, we can never perfectly know its state. *Data assimilation* (DA) estimates this using forecasts and observations.



DA Algorithms

The optimal combination of background forecasts and observations depends on (estimated) background and observational error.

- 3D-Var: Constant background error. In operational use.
- Extended Kalman Filter: Update background error with linear model. Numerically prohibitive for large models.
- Ensemble Kalman Filter: Use an *ensemble* of states to represent the current state. Ensemble spread used to estimate background error. Numerically efficient in large models.

Ensemble filters are one type of candidate for next-generation operational weather forecasting

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DA results for Lorenz forecasting Lorenz (perfect model)

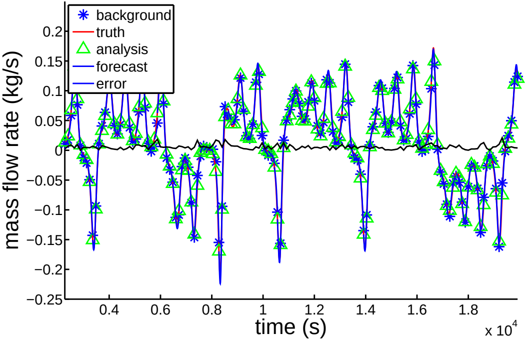
Rmse:	analysis	
Observed:	y	x, y, z
3D-Var	0.80	0.64
EKF $(\Delta = 0.05, \mu = 0.02)$	0.51	0.34
EnSRF $(\Delta = 0.04, N_{\mathrm{ens}} = 6)$	0.55	0.28
$(\Delta = 0.04, N_{\mathrm{ens}} = 3)$	0.77	0.65

Observational noise $= \sqrt{2} \approx 1.4$
Background error covariance inflation:

- multiply by $(1 + \Delta)$
- add randomly distributed errors between 1 and μ to diagonal

Similar errors as Yang et al.[1] and Kalnay et al.[2]

DA results for EM forecasting FLUENT (imperfect model) 120s assimilation window



Observational noise is 1% climatological mean ($\sqrt{\langle q^2 \rangle}$)

Error relative to climatological mean in observation space

Forecasting succeeds

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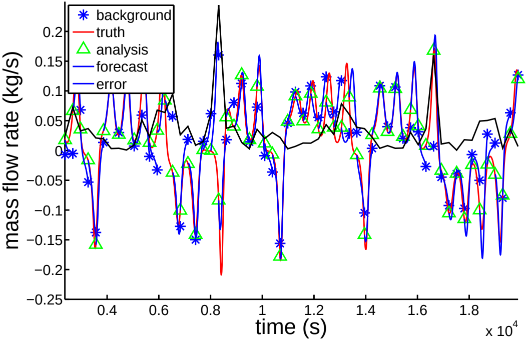
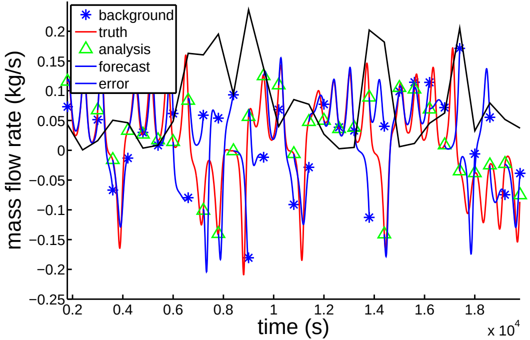


Observational noise is 1% climatological mean ($\sqrt{\langle q^2 \rangle}$)

Error relative to climatological mean in observation space

Forecasting (more or less) succeeds

DA results for EM forecasting FLUENT (imperfect model) 600s assimilation window

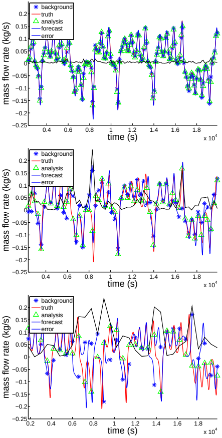


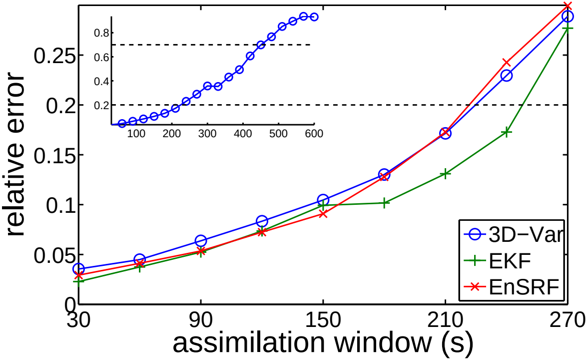
Observational noise is 1% climatological mean ($\sqrt{\langle q^2 \rangle}$)

Error relative to climatological mean in observation space

Forecasting fails!

DA results: accuracy degrades for longer windows





Error relative to climatological mean
Dashed lines for “perfect” and “useless” forecasts

Wrapping up

- DA is an effective way of coupling a low-dimensional, approximate model to a realistic physical simulation of the thermosyphon
- A combination of techniques should be able to quantitatively predict regime changes and duration (soon)
- Application to laboratory thermosyphon
- Part of a larger effort to improve predictive power of global weather and climate models . . .

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Global weather model simulations

(LoadingMovie)

Credit: Nick Allgaier

References

Shu-Chih Yang, Debra Baker, Hong Li, Katy Cordes, Morgan Huff, Geetika Nagpal, Ena Okereke, Josue Villafañe, Eugenia Kalnay, and Gregory S. Duane.
Data assimilation as synchronization of truth and model.
Journal of the Atmospheric Sciences, 63:2340–2354, 2006.

Eugenia Kalnay, Hong Li, Takemasa Miyoshi, Shu-Chih Yang, and Joaquim Ballabrera-Poy.
4-d-var or ensemble kalman filter?
Tellus, 59A:758–773, 2007.

Acknowledgments

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