→ SIMPLEST POSSIBLE DYNAMIC MODEL.

Synchronously dividing cells.
Each cell divides into FOUR, and fraction p of cells survives.
$n(t)$ = # cells in generation t.
$t = 1, 2, 3, 4 \ldots$

$n(1)$ [specify – start here]
$n(2) = 4\,n(1)\,p$

$n(3) = 4\,n(2)\cdot p$

$n(4) = 4\,n(3)\cdot p$

SOLUTION:

$= 4\,(4n(1)\,p)\cdot p = 4^2 p^2 n(1)$

$= 4\,(4^2 p^2)\,p = 4^3 p^3\, n(1)$

$\boxed{\text{RULE: } n(t) = 4\,n(t-1)\cdot p}$

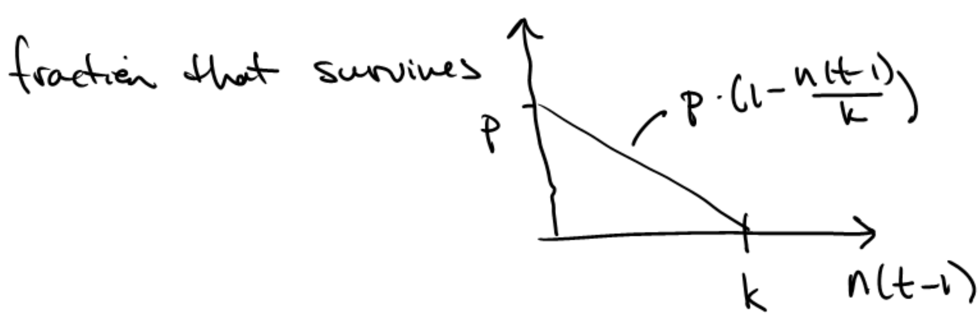
$\rightarrow \boxed{n(t) = (4p)^{t-1}\, n(1)}$

[Look at simulation in MATLAB]
cell-reproduction.m

use params:
$p = 0.15$
$p = 0.25$
$p = 0.95$

More complex model: resource-limited reproduction.

Say fraction $p\cdot\left(1 - \frac{n(t-1)}{K}\right)$ cells survive.



Interpretation: COMPETITION.
More cells ($n(t-1)$ larger) →
lower fraction of offspring survives.

New model:

$$n(t) = 4\,n(t-1)\cdot p\left(1 - \frac{n(t-1)}{K}\right)$$

code:

Cell-reproduction_nonlinear.m

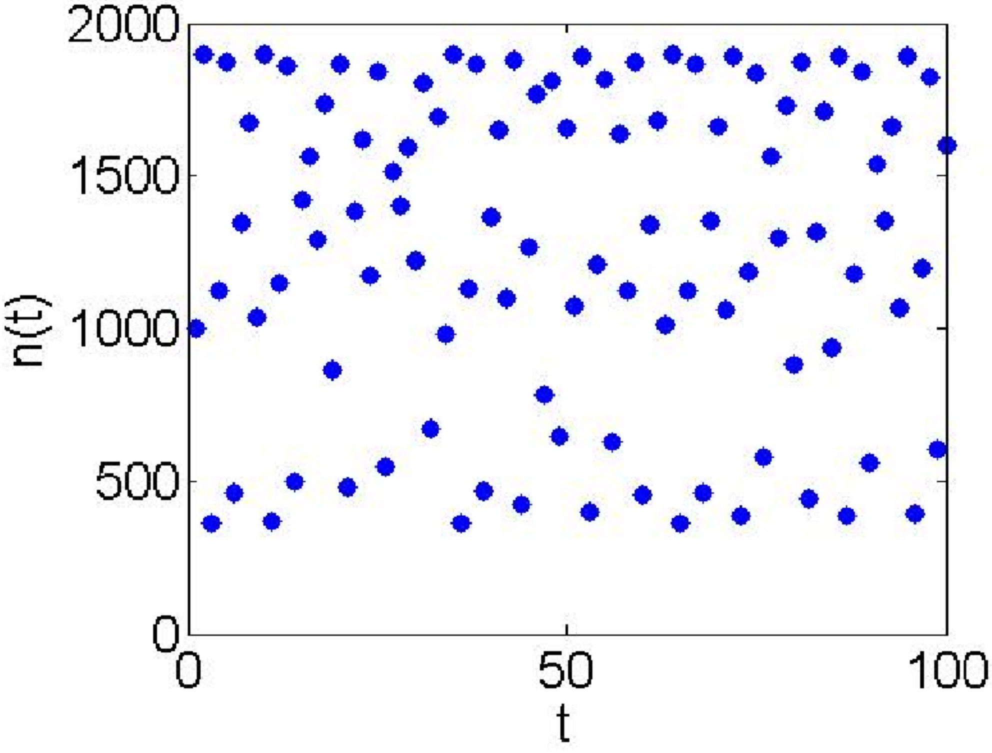
params: $p = 0.15$

$.95$

$.75$

solves this model.

Note: for $p \approx 0.9$, complex dynamics from simple model.



3 Components of a dynamic model: p.4

- STATE VARIABLES : The smallest set of variables that
  - a) Describe prop^s of system
  - b) Are sufficient for predicting how properties change over time

→ Combine into STATE VECTOR

- EXOGENOUS VARIABLES (PARAMETERS)
  - a) Affect how states change over time
  - b) Not affected by state variables

- DYNAMIC EQUATION

  How states evolve over time.