Surface Integration Free Combined Finite Element Method with Biot-Savart Law

Koji Fujiwara [1], Yoshifumi Okamoto [2], and Akihisa Kameari [3], *Member*, *IEEE*
[1] Department of Electrical and Electronic Engineering, Okayama University
3-1-1, Tsushima, Okayama 700-8530, Japan, E-mail: fujiwara@eplab.elec.okayama-u.ac.jp
[2] Advanced Center for Computing and Communication, The Institute of Physical and Chemical Research
2-1, Hirosawa, Wako-shi, Saitama 351-0198, Japan, E-mail: okamotoy@riken.jp
[3] Science Solutions International Laboratory, Inc.
2-21-7, Naka-cho, Meguro-ku, Tokyo 153-0065, Japan, E-mail: kamearia@ssil.co.jp

Abstract— This paper dealt with an efficient method for nonlinear magnetic field analysis utilizing the Biot-Savart law with the finite element method (FEM). The proposed method is surface integration free. This can lead a fairly simple formulation. When coils are located sufficiently far from magnetic bodies, it is not required for their regions to be included in the FEM region. Results obtained were verified using a box shield model established at one of the IEEJ investigation committees for technical meeting on electromagnetic field computation.

I. Introduction

Construction of electromagnetic devices becomes increasingly complicated. Therefore, it is important to develop novel techniques that can efficiently deal with difficult analyzed conditions. Relating to treatment of magnetic fields generated by exciting coils, several methods were already developed which utilize the Biot-Savart law. When the Biot-Savart law is combined with FEM, the FE mesh generation is not necessary in coil regions. In [1] and [2], the surface integration term should be calculated to satisfy continuity of magnetic field strength and flux density on interfaces between magnetic bodies and the other region. In [3], another method was proposed in which the surface integration is not required. However, only a few descriptions were assigned to the formulation and the application was limited to linear analysis. In this paper, features of the method were discussed and it was extended to a method using the reduced magnetic vector potential for nonlinear analysis. A box shield model was analyzed for verification.

II. Formulation

When the magnetic vector potential $\boldsymbol{A}$ is used, a fundamental equation is expressed as (1), where $\boldsymbol{J}_0$ is the exciting current density and ν is the reluctivity. When $\boldsymbol{A}_0$ is defined as the magnetic vector potential generated by $\boldsymbol{J}_0$, which can be calculated directly by means of the Biot-Savart law, (2) is obtained, where ν_0 is the reluctivity of vacuum.

$$\nabla\times(\nu\nabla\times\boldsymbol{A})=\boldsymbol{J}_0 \quad (1) \qquad \nabla\times(\nu_0\nabla\times\boldsymbol{A}_0)=\boldsymbol{J}_0 \quad (2)$$

After substituting (2) into (1), the following residual equation G_i is obtained from the Galerkin procedure:

$$G_i=\iiint_V \nabla\times\boldsymbol{N}_i\cdot(\nu\nabla\times\boldsymbol{A})\,\mathrm{d}V-\iiint_V \nabla\times\boldsymbol{N}_i\cdot(\nu_0\nabla\times\boldsymbol{A}_0)\,\mathrm{d}V$$
$$-\iint_S \boldsymbol{N}_i\cdot[\{\nu_0\nabla\times(\boldsymbol{A}-\boldsymbol{A}_0)\}\times\boldsymbol{n}]\,\mathrm{d}S=0 \quad (3)$$

where $\boldsymbol{N}_i$ is the edge shape function, $\boldsymbol{n}$ is the unit normal vector on the outermost boundary S, and V is the whole region. The surface integration term can be set at zero under the condition of $(\nu_0\nabla\times\boldsymbol{A})\times\boldsymbol{n}=(\nu_0\nabla\times\boldsymbol{A}_0)\times\boldsymbol{n}$ or $\boldsymbol{A}\times\boldsymbol{n}=\boldsymbol{A}_0\times\boldsymbol{n}$. When each of them is satisfied, namely that coils are located sufficiently far from magnetic bodies, their regions are not required to be included in the FEM region. In this case, the $\boldsymbol{A}\times\boldsymbol{n}$ free condition should be put on outermost boundaries.

When the reduced magnetic vector potential $\boldsymbol{A}_r$ (=$\boldsymbol{A}$-$\boldsymbol{A}_0$) is defined, the following equation is obtained:

$$G_i=\iiint_V \nabla\times\boldsymbol{N}_i\cdot(\nu\nabla\times\boldsymbol{A}_r)\,\mathrm{d}V-\iiint_{V_{mag}} \nabla\times\boldsymbol{N}_i\cdot\{(\nu_0-\nu)\nabla\times\boldsymbol{A}_0\}\,\mathrm{d}V$$
$$-\iint_S \boldsymbol{N}_i\cdot\{(\nu_0\nabla\times\boldsymbol{A}_r)\times\boldsymbol{n}\}\,\mathrm{d}S=0 \quad (4)$$

where V_{mag} is the region of magnetic body. The surface integration term can be set at zero under the condition of $(\nu_0\nabla\times\boldsymbol{A}_r)\times\boldsymbol{n}=\boldsymbol{0}$ or $\boldsymbol{A}_r\times\boldsymbol{n}=\boldsymbol{0}$.

III. Verification

A magnetic shielding model shown in Fig. 1 was analyzed for verification. The iron box is made of SS400 steel with nonlinear magnetic property. The inner space and thickness of the box are 200 mm cube and 1 mm, respectively.

Fig. 2 shows the z-component of flux density B_z on the z-axis. Results based on (3) and (4) denoting as $\boldsymbol{A}_0$-$\boldsymbol{A}$ and $\boldsymbol{A}_0$-$\boldsymbol{A}_r$, respectively, were fairly in good agreement with each other.

Details of the model and further comparison will be reported in the extended version of the paper.

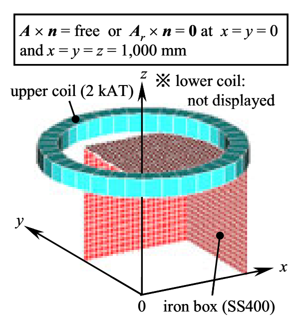

Fig. 1. Magnetic shielding model.

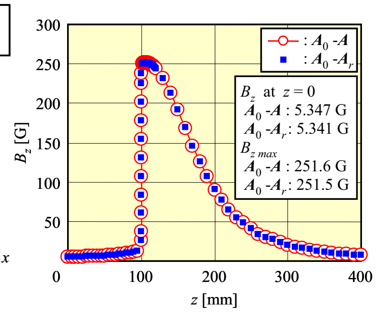

Fig. 2. B_z on z-axis.

IV. References

[1] C. W. Trowbridge, "Three-Dimensional Field Computation," *IEEE Trans. Magn.*, vol. MAG-18, no. 1, pp. 293-297, January 1982.

[2] A. Kameari, "Regularization on Ill-Posed Source Terms in FEM Computation Using Two Magnetic Vector Potentials," *ibid.*, vol. 40, no. 2, pp.1310-1313, March 2004.

[3] T. Ohnishi and N. Takahashi, "Effective Optimal Design of 3-D Magnetic Device Having Complicated Coil Using Edge Element and Biot-Savart Method," *ibid.*, vol. 38, no. 2, pp.1021-1024, March 2002.

