



静磁場方程式

$$\nabla \cdot \mathbf{B} = 0, \quad \mathbf{B} = \mu \mathbf{H}, \quad \nabla \times \mathbf{H} = 0 \quad (1)$$

磁気スカラーポテンシャル

$$\mathbf{H} = \nabla \omega \quad (2)$$

支配方程式

$$\nabla \cdot \mu \nabla \omega = 0 \quad \text{in } \Omega \quad (3)$$

境界条件

$$\begin{aligned} \omega &= \omega_s \quad \text{on } \Gamma_D \\ \mu \nabla \omega \cdot \mathbf{n} &= \mathbf{B} \cdot \mathbf{n} = B_s \quad \text{on } \Gamma_N \\ \Gamma_D + \Gamma_N &= \partial \Omega \end{aligned} \quad (4)$$

重み付き残差法

$$\begin{aligned} W &= - \int_{\Omega} \psi \nabla \cdot \mu \nabla \omega = \int_{\Omega} \nabla \psi \cdot \mu \nabla \omega - \int_{\Omega} \nabla \cdot (\psi \mu \nabla \omega) \\ &= \int_{\Omega} \nabla \psi \cdot \mu \nabla \omega - \int_{\partial \Omega} \mathbf{n} \cdot (\psi \mu \nabla \omega) \\ &= \int_{\Omega} \nabla \psi \cdot \mu \nabla \omega - \int_{\Gamma_N} \psi B_s \\ \psi &= 0 \quad \text{on } \Gamma_D \end{aligned} \quad (5)$$

離散化

$$\omega = \sum_{n \in \Omega} \omega_n N_n + \sum_{n \in \Gamma_N} \omega_n N_n + \sum_{n \in \Gamma_D} \omega_{sn} N_n$$

$$\psi = N_m, \, m \in \Omega + \Gamma_N$$

$$\begin{aligned} W &= \int_{\Omega} \nabla \psi \cdot \mu \nabla \omega - \int_{\Gamma_N} \psi B_s \\ &= \sum_{n \in \Omega + \Gamma_N} \omega_n \int_{\Omega} \nabla N_m \cdot \mu \nabla N_n + \sum_{n \in \Gamma_D} \omega_{sn} \int_{\Omega} \nabla N_m \cdot \mu \nabla N_n - \int_{\Gamma_N} N_m B_s \end{aligned}$$

$$\sum_{n \in \Omega + \Gamma_N} \omega_n \int_{\Omega} \nabla N_m \cdot \mu \nabla N_n = \int_{\Gamma_N} N_m B_s - \sum_{n \in \Gamma_D} \omega_{sn} \int_{\Omega} \nabla N_m \cdot \mu \nabla N_n$$