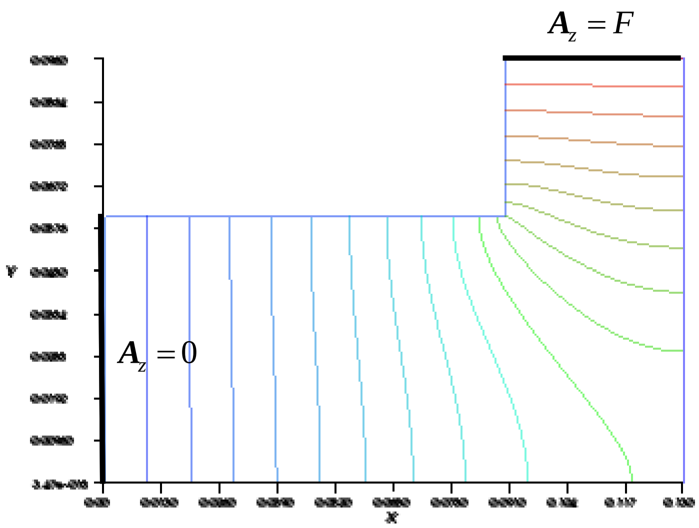
2022/08/19 A. Kameari

Complementarity（相補性）

Magneto-static 2D

On the side of ***b***



$$\operatorname{rot} a_z = \begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial a_z}{\partial y} \\ -\dfrac{\partial a_z}{\partial x} \end{bmatrix}$$

$$\mathsf{A}_z^{\,F} = \left\{ a_z \in \mathsf{L}^2_{\mathrm{rot}}\left(D\right) : a_z = 0 \ on\, S_0^b \,, a_z = F \ on\, S_1^b \right\}$$

$$find\ a_z \in \mathsf{A}_z^{\,F}\, such\, that \int_D \mu^{-1} \operatorname{rot} a_z \cdot \operatorname{rot} a_z' = 0\, \forall a_z' \in \mathsf{A}_z^{\,0}.$$

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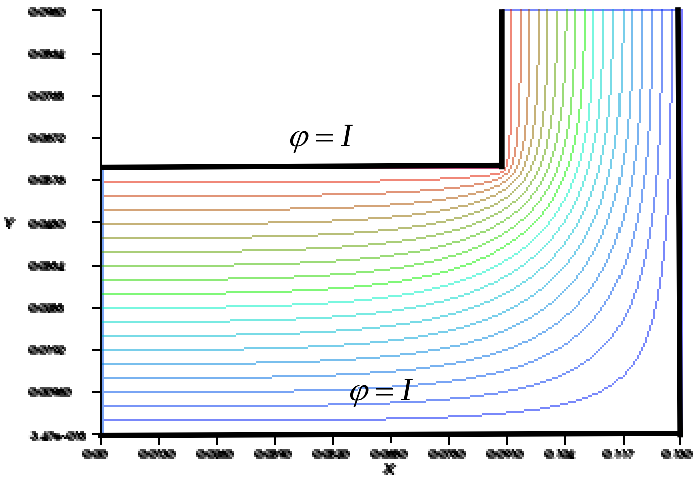
On the side of $\boldsymbol{h}$

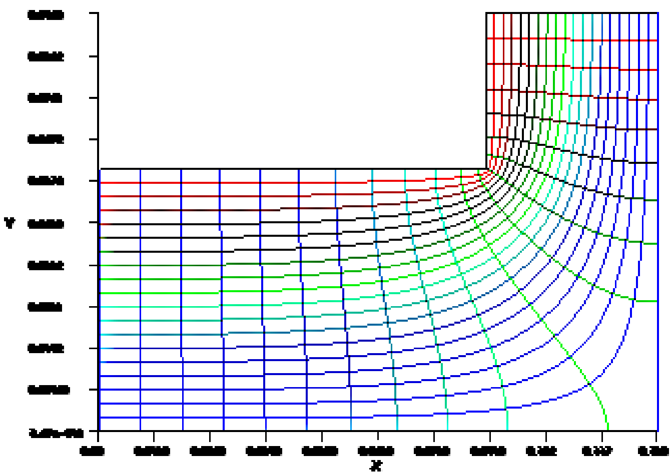


$$\text{-grad}\,\varphi = \begin{bmatrix} h_x \\ h_y \end{bmatrix} = -\begin{bmatrix} \dfrac{\partial \varphi}{\partial x} \\ \dfrac{\partial \varphi}{\partial y} \end{bmatrix}$$

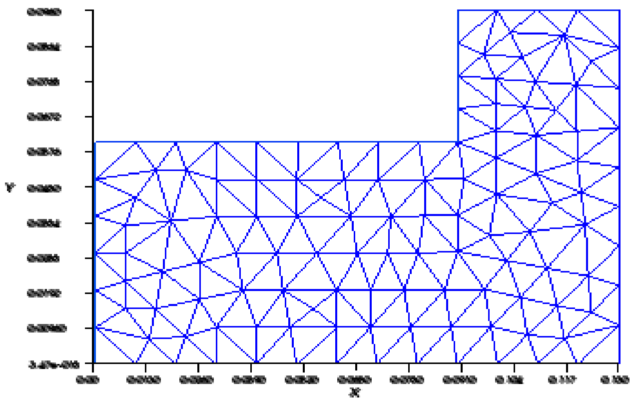
$$\Psi^I = \left\{ \psi \in \mathrm{L}^2_{\mathrm{grad}}\left(D\right) : \psi = I \ on\, S_0^h, \psi = 0 \ on\, S_1^h \right\}$$

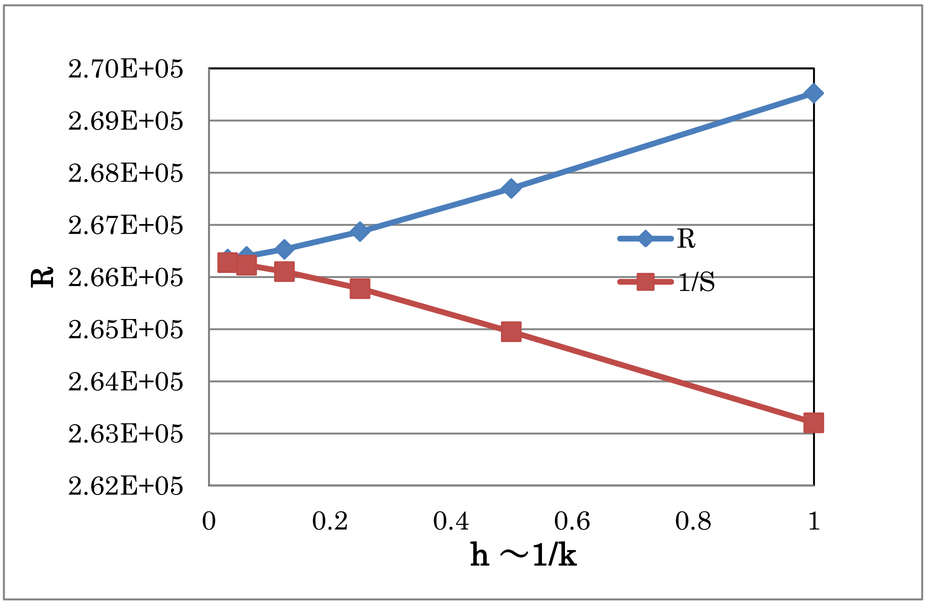
$$find\, \psi \in \Psi^I such\, that \int_D \mu\, \mathrm{grad}\, \psi \cdot \mathrm{grad}\, \psi' = 0\, \forall\, \psi' \in \Psi^0.$$

・Conformal transformation(等角写像)になる



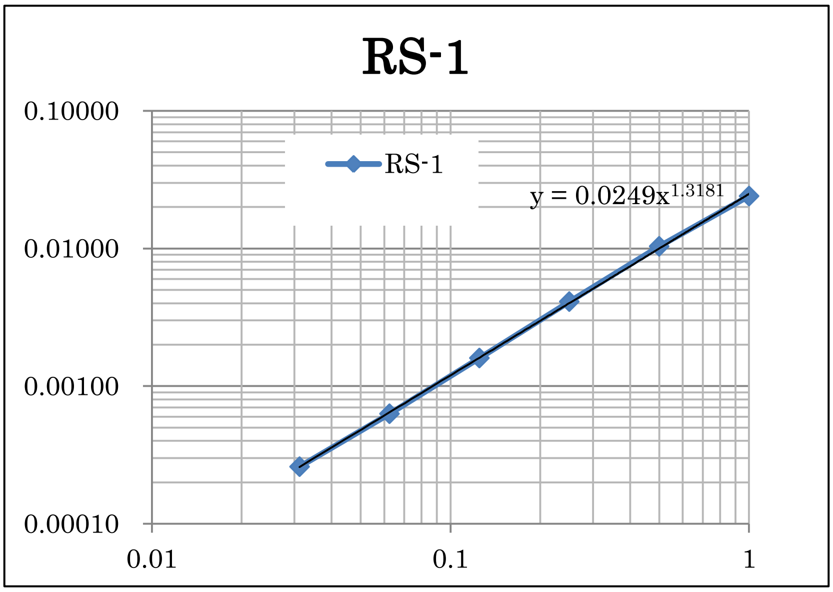
Mesh（k = 1）

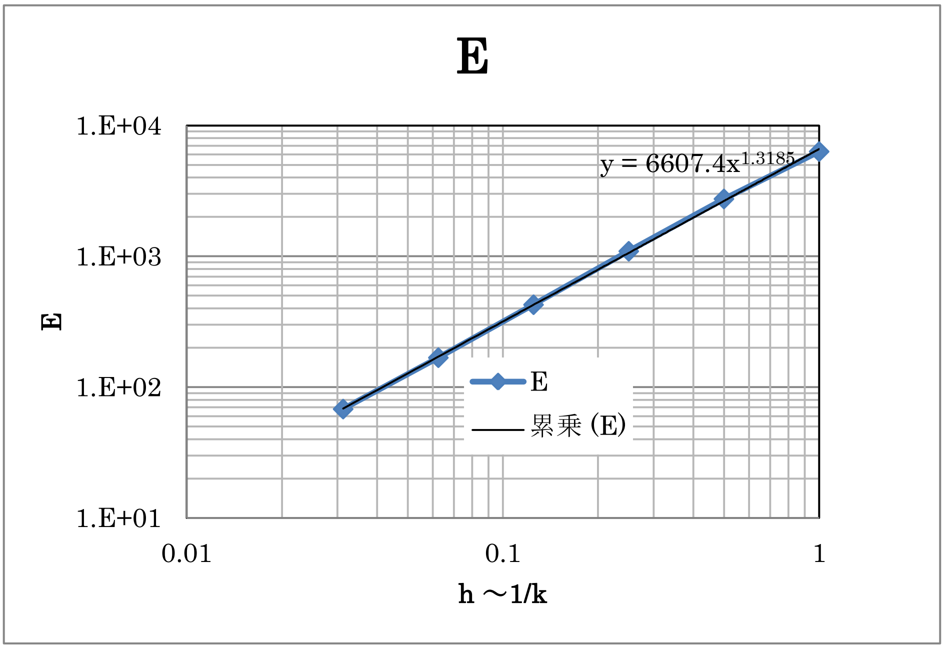




$$RF^2 = \int_D \mu_0^{-1} |\boldsymbol{b}|^2 \qquad SI^2 = \int_D \mu_0 |\boldsymbol{h}|^2$$

・R(relactance)の真値は上下から挟まれ誤差が推定できる。ほぼ中心値が真値。





$$E\left(\boldsymbol{b}_m,\boldsymbol{h}_m\right)=\int_D \mu_0^{-1}\left|\boldsymbol{b}_m-\mu\boldsymbol{h}_m\right|^2$$

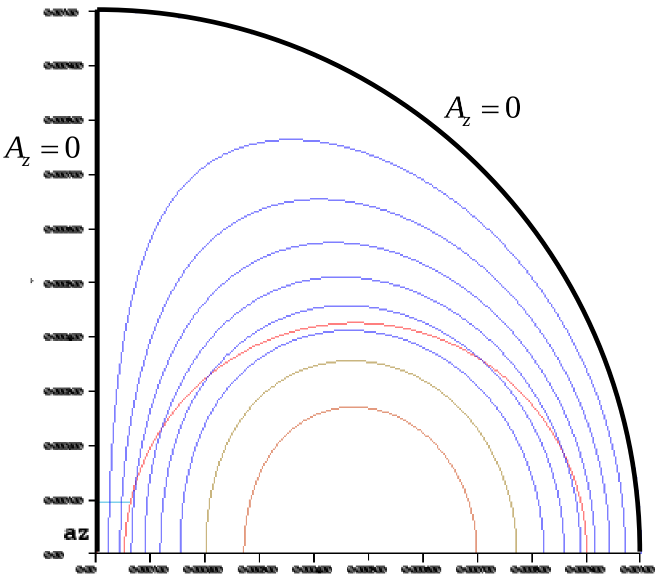
・$h^{1.32}$ の収束性

・二乗収束性が得られないのは、角の特異性があるからか？

2D Eddy current

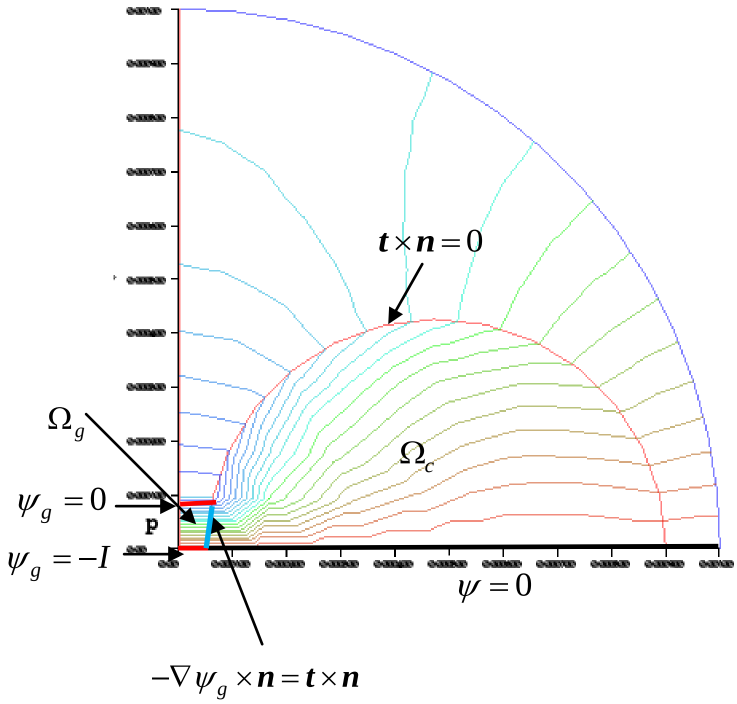
2 Conductor Model

s-domain



$$e_z = V - s a_z$$
$$\boldsymbol{b} = \nabla \times \boldsymbol{a}$$

$$find\ a_z \in \mathrm{A}_z^{\ F}\ such\ that \int_D \left(\mu^{-1} \operatorname{rot} a_z \cdot \operatorname{rot} a_z' - \sigma \left(V - s a_z \right) a_z' \right) = 0\ \forall a_z' \in \mathrm{A}_z^{\ 0}.$$



$$find\ \psi_g \in \Psi^I\ such\ that \int_{\Omega_g} \mu\,\mathrm{grad}\,\psi_g \cdot \mathrm{grad}\,\psi' = 0\ \forall\,\psi' \in \Psi^0.$$

$$find\ \boldsymbol{t} \in \mathrm{L}^2_{\mathrm{rot}}\left(\Omega_c\right) and\ \psi \in \Psi^0\left(D\right) such\ that$$

$$\int_{\Omega_c} 1/\left(s\sigma\right)\mathrm{rot}\,\boldsymbol{t}\cdot\mathrm{rot}\,\boldsymbol{t}' + \int_{\Omega_c} \mu(\boldsymbol{t}+\mathrm{grad}\,\psi)\cdot(\boldsymbol{t}'+\mathrm{grad}\,\psi')$$

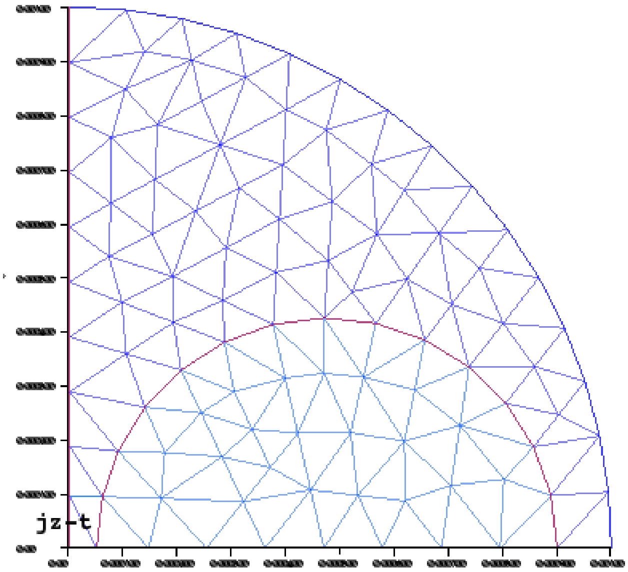
$$+\int_{\Omega_g} \mu\,\mathrm{grad}\,(\psi+\psi_g)\cdot\mathrm{grad}\,\psi' + \int_{D-\Omega_c-\Omega_g} \mu\,\mathrm{grad}\,\psi\cdot\mathrm{grad}\,\psi' = 0$$

$$\forall \boldsymbol{t} \in \mathrm{L}^2_{\mathrm{rot}}\left(\Omega_c\right) and\ \forall\,\psi \in \Psi^0\left(D\right).$$

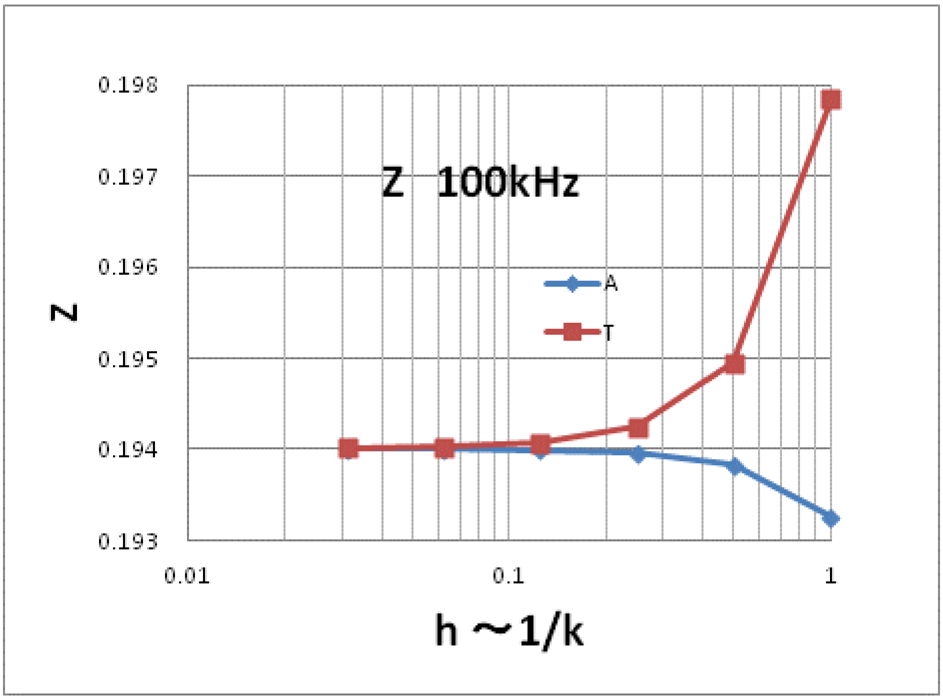
・方程式は不定、CG 法で解ける。

・Freefem＋＋では s＝jw の時解けない。

Mesh（k＝1）

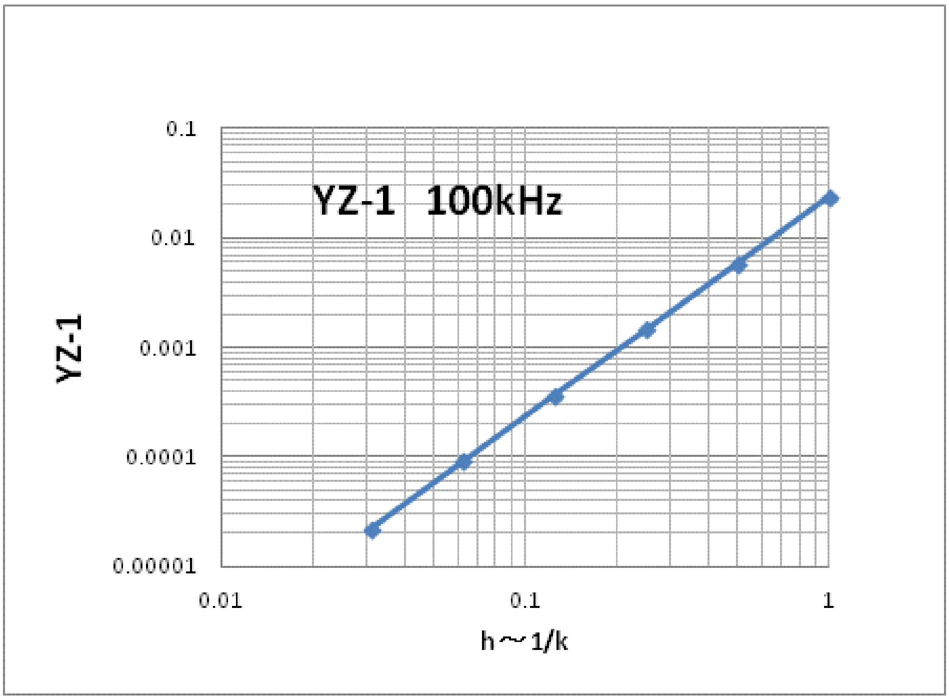


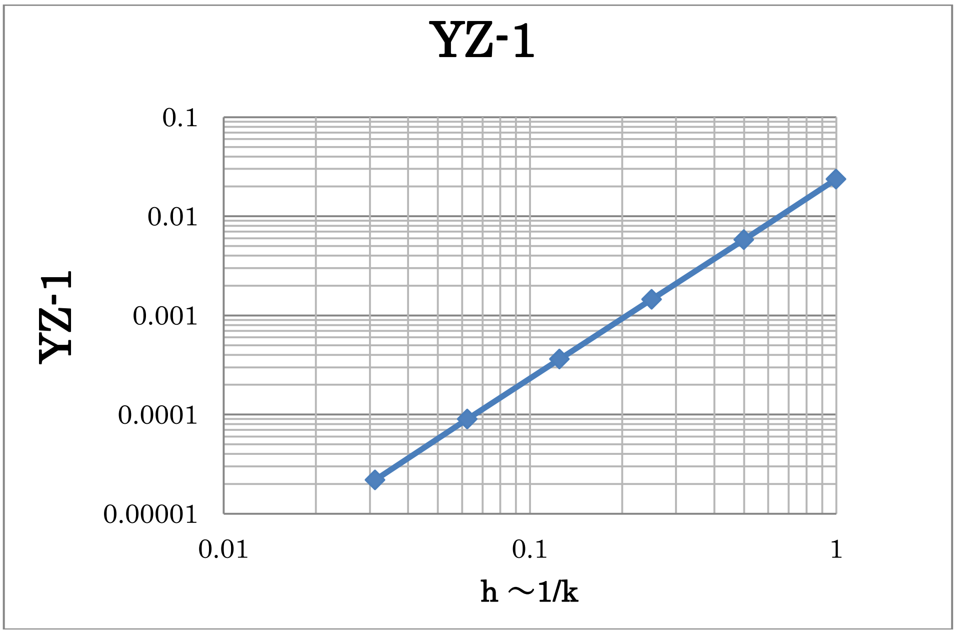
一次節点要素（P1）および一次辺要素（R0Ortho）



・真値は上下で挟み込まれる。

・A 法の方が T 法に比べ精度が良い。jz の形状函数の次数が高いからか。





$$\boxed{YV^2 = s\int_D \mu_0^{-1}|\boldsymbol{b}|^2 + s\int_D \sigma|\boldsymbol{e}|^2}$$

$$\boxed{ZI^2 = s\int_D \mu_0|\boldsymbol{h}|^2 + \int_D \sigma^{-1}|\boldsymbol{j}|^2}$$

・ほぼ二乗収束