$$\mathcal{M} = \int_\Omega I_{\bar\Omega} c \, d\Omega + \int_\Omega \varphi_{zu}\left(-\rho(uu_x+vu_y) - p_x + \mu(u_{xx}+u_{yy}) + f_{(x)}\right) + \varphi_u\left(-\mu\nabla^2 z_u + \frac{\partial z_p}{\partial x} - z_c c_x + \rho\left(-u\frac{\partial z_u}{\partial x} - \frac{\partial (z_u v)}{\partial y}\right)\right)$$

$$-\varphi_{zv}\left(-\rho(uv_x+vv_y) - p_y + \mu(v_{xx}+v_{yy}) + f_{(y)}\right) + \varphi_v\left(-\mu\nabla^2 z_v + \frac{\partial z_p}{\partial y} - z_c c_y + \rho\left(-v\frac{\partial z_v}{\partial y} - \frac{\partial (z_v u)}{\partial x}\right)\right)$$

$$-\varphi_{zp}(u_x+v_y) \qquad +\varphi_p\left(-\frac{\partial z_u}{\partial x} - \frac{\partial z_v}{\partial y}\right)$$

$$-\varphi_{zc}(-\nabla^2 c + \vec{v}\cdot\nabla c - f^c) \qquad +\varphi_c\left(\nabla^2 z_c + (\vec{v}\cdot\nabla z_c + z_c(\nabla\cdot\vec{v})) + (c-c^*)\right)$$

$$+\varphi_{fc}\left(\beta(-\nabla^2 f^c) + z_c\right) d\Omega$$

$$\frac{\delta\mathcal{M}}{\delta u} = \int_\Omega -\varphi_{zu}\left(-\rho(\hat{u}u_x + u\hat{u}_x + v\hat{u}_y) + \mu\nabla^2\hat{u}\right) + \varphi_{zv}(\rho\hat{u}v_x)$$

$$-\varphi_{zp}\hat{u}_x - \varphi_{zc}\hat{u}c_x - \varphi_u\rho\hat{u}\frac{\partial z_u}{\partial x} - \varphi_v\rho\left(\hat{u}\frac{\partial z_v}{\partial x} + z_v\hat{u}_x\right) + \varphi_c\left(\hat{u}\frac{\partial z_c}{\partial x} + z_c\hat{u}_x\right) d\Omega$$

$$= \int_\Omega \hat{u}\left(\rho\varphi_{zu}u_x - \rho\frac{\partial(\varphi_{zu}u)}{\partial x} - \rho\frac{\partial(\varphi_{zu}v)}{\partial y} - \mu\nabla^2\varphi_{zu}\right) d\Omega + \cancelto{0}{\int_\Gamma \rho\hat{u}(\hat{n}_{(x)}\varphi_{zu}u + \hat{n}_{(y)}\varphi_{zu}v) + \mu\left(\varphi_{zu}\frac{\partial\hat{u}}{\partial n} - \hat{u}\frac{\partial\varphi_{zu}}{\partial n}\right) d\Gamma}$$

$$+\int_\Omega \hat{u}(\rho\varphi_{zv}v_x) d\Omega + \int_\Omega \hat{u}\frac{\partial\varphi_{zp}}{\partial x} d\Omega - \cancelto{0}{\int_\Gamma \hat{u}(\hat{n}_{(x)}\varphi_{zp}) d\Gamma} - \int_\Omega \hat{u}(\varphi_{zc}c_x) d\Omega - \int_\Omega \hat{u}\left(\rho\varphi_u\frac{\partial z_u}{\partial x}\right) d\Omega$$

$$+\int_\Omega \hat{u}\Big(\underbrace{-\rho\varphi_v\frac{\partial z_v}{\partial x} + \rho\frac{\partial(\varphi_v z_v)}{\partial x}}_{=+\rho z_v\frac{\partial\varphi_v}{\partial x}}\Big) d\Omega - \cancelto{0}{\int_\Gamma \hat{u}\hat{n}_{(x)}(\rho\varphi_v z_v) d\Gamma} + \int_\Omega \hat{u}\Big(\underbrace{\varphi_c\frac{\partial z_c}{\partial x} - \frac{\partial(\varphi_c z_c)}{\partial x}}_{=-z_c\frac{\partial\varphi_c}{\partial x}}\Big) d\Omega + \cancelto{0}{\int_\Gamma \hat{u}\hat{n}_{(x)}\varphi_c z_c d\Gamma}$$

$$\rightsquigarrow \boxed{\rho\left(-u\frac{\partial\varphi_{zu}}{\partial x} - \frac{\partial(\varphi_{zu}v)}{\partial y} - \mu\nabla^2\varphi_{zu}\right) + \rho\varphi_{zv}v_x + \frac{\partial\varphi_{zp}}{\partial x} - \varphi_{zc}c_x - \rho\varphi_u\frac{\partial z_u}{\partial x} + \rho z_v\frac{\partial\varphi_v}{\partial x} - z_c\frac{\partial\varphi_c}{\partial x} = 0 \text{ in } \Omega,\ \varphi_{zu} = 0 \text{ on } \Gamma}$$

$$\frac{\delta\mathcal{M}}{\delta v} \rightsquigarrow \boxed{\rho\left(-v\frac{\partial\varphi_{zv}}{\partial y} - \frac{\partial(\varphi_{zv}u)}{\partial x} - \mu\nabla^2\varphi_{zv}\right) + \rho\varphi_{zu}u_y + \frac{\partial\varphi_{zp}}{\partial y} - \varphi_{zc}c_y - \rho\varphi_v\frac{\partial z_v}{\partial y} + \rho z_u\frac{\partial\varphi_u}{\partial y} - z_c\frac{\partial\varphi_c}{\partial y} = 0 \text{ in } \Omega,\ \varphi_{zv} = 0 \text{ on } \Gamma}$$

$$\frac{\delta\mathcal{M}}{\delta p} = \int_\Omega \varphi_{zu}\hat{p}_x + \varphi_{zv}\hat{p}_y \, d\Omega \rightsquigarrow \boxed{-\frac{\partial\varphi_{zu}}{\partial x} - \frac{\partial\varphi_{zv}}{\partial y} = 0}$$

$$\frac{\delta\mathcal{M}}{\delta c} = \int_\Omega \varphi_{zc}\nabla^2\hat{c} - \varphi_{zc}\vec{v}\cdot\nabla\hat{c} - z_c\varphi_u\hat{c}_x - z_c\varphi_v\hat{c}_y + \varphi_c\hat{c} \, d\Omega + \int_\Omega I_{\bar\Omega}\hat{c} \, d\Omega$$

$$= \int_\Omega \hat{c}\left(\nabla^2\varphi_{zc} + \nabla\cdot(\vec{v}\varphi_{zc})\right) d\Omega + \cancelto{0}{\int_\Gamma \hat{c}\frac{\partial\varphi_{zc}}{\partial n} - \varphi_{zc}\frac{\partial\hat{c}}{\partial n} - \hat{c}(\vec{v}\cdot\hat{n}\varphi_c) d\Gamma} + \int_\Omega \hat{c}\left(\frac{\partial(z_c\varphi_u)}{\partial x} + \frac{\partial(z_c\varphi_v)}{\partial y}\right) d\Omega - \cancelto{0}{\int_\Gamma \hat{c}(\hat{n}_{(x)}\varphi_u + \hat{n}_{(y)}\varphi_v) d\Gamma} + \int_\Omega \hat{c}\varphi_c \, d\Omega + \int_\Omega \hat{c}I_{\bar\Omega} \, d\Omega$$

$$\rightsquigarrow \boxed{\nabla^2\varphi_{zc} + \underbrace{\nabla\cdot(\vec{v}\varphi_{zc})}_{=\vec{v}\cdot\nabla\varphi_{zc} + \varphi_{zc}(\nabla\cdot\vec{v})} + \frac{\partial(z_c\varphi_u)}{\partial x} + \frac{\partial(z_c\varphi_v)}{\partial y} + \varphi_c + I_{\bar\Omega} = 0 \text{ in } \Omega,\ \varphi_{zc} = 0 \text{ on } \Gamma}$$

$$\frac{\delta\mathcal{M}}{\delta z_u} = \int_\Omega -\mu\varphi_u\nabla^2\hat{z}_u - \rho\varphi_u\left(u\frac{\partial\hat{z}_u}{\partial x} + v\frac{\partial\hat{z}_u}{\partial y} + \hat{z}_u v_y\right) - \varphi_p\frac{\partial\hat{z}_u}{\partial x} \, d\Omega$$

$$= \int_\Omega \hat{z}_u\left(-\mu\nabla^2\varphi_u + \rho\left(-\varphi_u v_y + \frac{\partial(\varphi_u u)}{\partial x} + \frac{\partial(\varphi_u v)}{\partial y}\right)\right) d\Omega + \cancelto{0}{\int_\Gamma \hat{z}_u\frac{\partial\varphi_u}{\partial n} - \varphi_u\frac{\partial\hat{z}_u}{\partial n} - \hat{z}_u\rho(\hat{n}_{(x)}\varphi_u u + \hat{n}_{(y)}\varphi_u v) d\Gamma} + \int_\Omega \hat{z}_u\frac{\partial\varphi_p}{\partial x} d\Omega - \cancelto{0}{\int_\Gamma \hat{n}_{(x)}\hat{z}_u\varphi_p \, d\Gamma}$$

$$\rightsquigarrow \boxed{-\mu\nabla^2\varphi_u + \rho\left(\frac{\partial(\varphi_u u)}{\partial x} + v\frac{\partial\varphi_u}{\partial y}\right) + \frac{\partial\varphi_p}{\partial x} = 0 \text{ in } \Omega,\ \varphi_u = 0 \text{ on } \Gamma}$$

$$\frac{\delta\mathcal{M}}{\delta z_v} \rightsquigarrow \boxed{-\mu\nabla^2\varphi_v + \rho\left(\frac{\partial(\varphi_v v)}{\partial y} + u\frac{\partial\varphi_v}{\partial x}\right) + \frac{\partial\varphi_p}{\partial y} = 0 \text{ in } \Omega,\ \varphi_v = 0 \text{ on } \Gamma}$$

$$\frac{\delta\mathcal{M}}{\delta z_p} = \int_\Omega \varphi_u\frac{\partial\hat{z}_p}{\partial x} + \varphi_v\frac{\partial\hat{z}_p}{\partial y} d\Omega = \int_\Omega \hat{z}_p\left(-\frac{\partial\varphi_u}{\partial x} - \frac{\partial\varphi_v}{\partial y}\right) d\Omega + \cancelto{0}{\int_\Gamma \hat{z}_p(\hat{n}_{(x)}\varphi_u + \hat{n}_{(y)}\varphi_v) d\Gamma} \rightsquigarrow \boxed{-\frac{\partial\varphi_u}{\partial x} - \frac{\partial\varphi_v}{\partial y} = 0 \text{ in } \Omega}$$

$$\frac{\delta\mathcal{M}}{\delta z_c} = \int_\Omega -\hat{z}_c(\varphi_u c_x + \varphi_v c_y) + \varphi_c\nabla^2\hat{z}_c + \varphi_c(\vec{v}\cdot\nabla\hat{z}_c + \hat{z}_c(\nabla\cdot\vec{v})) + \varphi_{fc}\hat{z}_c \, d\Omega$$

$$= \int_\Omega \hat{z}_c\left(-\varphi_u c_x - \varphi_v c_y + \nabla^2\varphi_c + \varphi_c(\nabla\cdot\vec{v}) - (\varphi_c(\nabla\cdot\vec{v}) + \vec{v}\cdot(\nabla\varphi_c)) + \varphi_{fc}\right) d\Omega + \cancelto{0}{\int_\Gamma -\hat{z}_c\frac{\partial\varphi_c}{\partial n} + \varphi_c\frac{\partial\hat{z}_c}{\partial n} + \hat{z}_c(\vec{v}\cdot\hat{n}\varphi_c) d\Gamma}$$

$$\rightsquigarrow \boxed{-\varphi_u c_x - \varphi_v c_y + \nabla^2\varphi_c - \vec{v}\cdot(\nabla\varphi_c) + \varphi_{fc} = 0 \text{ in } \Omega,\ \varphi_c = 0 \text{ on } \Gamma}$$

$$\frac{\delta\mathcal{M}}{\delta f^c} = \int_\Omega \varphi_{zc}\hat{f}^c - \beta\varphi_{fc}\nabla^2\hat{f}^c \, d\Omega = \int_\Omega \hat{f}^c(\varphi_{zc} - \beta\nabla^2\varphi_{fc}) d\Omega + \cancelto{0}{\int_\Gamma \hat{f}^c\frac{\partial\varphi_{fc}}{\partial n} - \varphi_{fc}\frac{\partial\hat{f}^c}{\partial n} d\Gamma} \rightsquigarrow \boxed{\varphi_{zc} - \beta\nabla^2\varphi_{fc} = 0 \text{ in } \Omega,\ \varphi_{fc} = 0 \text{ on } \Gamma}$$