NYU Tandon Bridge

Homework 6

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Question 5

a)

The goal is show that $5n^3 + 2n^2 + 3n = \Theta(n^3)$

<u>Step 1: Determine big O</u>

Using the exaggerate and simplify method:

$n \geq 1$	$for\ all\ n \geq 1$
$n^2 \geq n$	Multiply both sides by n. Since n is positive it doesn't affect the order of the inequality
$n^3 \geq n^2$	Multiply both sides by n. Since n is positive it doesn't affect the order of the inequality
$2n^3 \geq 2n^2$	Multiply both sides by 2

$n \geq 1$	$for\ all\ n \geq 1$
$n^2 \geq 1$	Square both sides
$n^3 \geq n$	Multiply both sides by n. Since n is positive it doesn't affect the order of the inequality
$3n^3 \geq 3n$	Multiply both sides by 3

$n \geq n$	$for\ all\ n$
$n^2 \geq n^2$	Square both sides
$n^3 \geq n^3$	Multiply both sides by n. Since n is positive it doesn't affect the order of the inequality
$5n^3 \geq 5n^3$	Multiply both sides by 5

Combining all three inequalities we get:

$5n^3 + 2n^3 + 3n^3 \geq 5n^3 + 2n^2 + 3n$	$for\ all\ n \geq 1$
$10n^3 \geq 5n^3 + 2n^2 + 3n$	Sum everything up
$10 * (n^3) \geq f(n)$	Replace the value of f(n) by its function name
$10\, g(n) \geq f(n)$	Replace n^3 with $g(n)$
$C_2 = 10$	By definition, we know $f(n) = O(g(n))$ if $c_2 \cdot g(n) \geq f(n)\ for\ all\ n \geq n_0$

Therefore, $f(n) = O(n^3)\ for\ c_2 = 10\ and\ n_0 = 1$

<u>Step 2: Determine Omega</u>

Let's find the value of n for which the lower order terms $2n^2 + 3n \geq 0$

$2n^2 + 3n \geq 0$	$for\ all\ n \geq ?$
$2n^2 \geq -3n$	Subtract both sides by 3n
$n \geq \frac{-3}{2}$	Divide both sides by 2n.
$n \geq 0$	Round to the nearest nonnegative integer

As such, we showed that $2n^2 + 3n \geq 0$ for all $n \geq 0$. We also have that:

$n \geq n$	$for\ all\ n$
$n^2 \geq n^2$	Square both sides
$n^3 \geq n^3$	Multiply both sides by n. Since n is positive it doesn't affect the inequality
$5n^3 \geq 5n^3$	Multiply both sides by 5

Combining those two inequalities we get:

$5n^3 + 2n^2 + 3n \geq 5n^3$	$for\ all\ n \geq 0$
$f(n) \geq 5(n^3)$	Replace the value of f (n) by its function name
$f(n) \geq 5g(n)$	Replace n^3 with $g(n)$
$C_1 = 5$	By definition, we know that $f(n) = \Omega(g(n))$ if $c_1 \cdot g(n) \leq f(n)\ for\ all\ n \geq n_0$

Therefore, $f(n) = \Omega(n^3)\ for\ c_1 = 5\ and\ n_0 = 0$

<u>Step 3: Determine theta</u>

We know that:

- f (n) is lower bound by $\Omega(n^3)$ with $c_1 = 5$ and $n_0 = 0$
- f (n) is upper bound by $O(n^3)$ with $c_2 = 10$ and $n_0 = 1$

By definition, $f(n) = \Theta(n^3)$ if there are positive constants $c_1\ and\ c_2\ and\ n_0$ such that $c_1\,(n^3) \leq f(n) \leq c_2(n^3)$ for all $n \geq n_0$.

If we use:

- $c_1 = 5$
- $c_2 = 10$
- $n_0 = 1$

Combining our results we get 5 $(n^3) \leq f(n) \leq 10\ (n^3)$ for all $n \geq 1$

Therefore, $5n^3 + 2n^2 + 3n = \Theta(n^3)$

b)

The goal is show that $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

<u>Step 1: Determine big Oh</u>

Using the exaggerate and simplify method:

$n \geq 1$	$for\ all\ n \geq 1$
$n^2 \geq n$	Multiply both sides by n. Since n is positive it doesn't affect the order of the inequality
$2n^2 \geq 2n$	Multiply both sides by 2

$0 \geq -8$	$true\ for\ all\ n$

$n \geq n$	$true\ for\ all\ n$
$n^2 \geq n^2$	Multiply both sides by n. Since n is positive it doesn't affect the order of the inequality
$7n^2 \geq 7n^2$	Multiply both sides by 7

Combining these three inequalities gives us:

$7n^2 + 2n^2 + 0 \geq 7n^2 + 2n - 8$	Since we know that $7n^2 \geq 7n^2$ and $0 \geq -8$ and $2n^2 \geq 2n$
$\sqrt{7n^2 + 2n^2} \geq \sqrt{7n^2 + 2n - 8}$	Take the square root of both sides
$\sqrt{9n^2} \geq \sqrt{7n^2 + 2n - 8}$	Sum the left side up together
$\sqrt{9} \cdot \sqrt{n^2} \geq \sqrt{7n^2 + 2n - 8}$	Square root product identity

$\sqrt{9} \cdot n \geq \sqrt{7n^2 + 2n - 8}$	The radical and the square for n cancel out
$3n \geq \sqrt{7n^2 + 2n - 8}$	Take the square root of 9
$3(n) \geq f(n)$	Replace the left side value by the function name
$3 \cdot g(n) \geq f(n)$	Replace (n) with $g(n)$
$C_2 = 3$	By definition, we know that $f(n) = O(g(n))\ if\ c_2 \cdot g(n) \geq f(n)$ $for\ all\ n \geq n_0$

Therefore, $f(n) = O(n)\ for\ c_2 = 3\ and\ n_0 = 1$

<u>Step 2: Determine Omega</u>

Let's find the value of n for which the lower order terms $2n - 8 \geq 0$

$2n - 8 \geq 0$	$for\ all\ n \geq ?$
$2n \geq 8$	Add 8 to both sides
$n \geq \frac{8}{2}$	Divide both sides by 2.
$n \geq 4$	Express as integer

As such, we showed that $2n - 8 \geq 0$ for all $n \geq 4$. We also have that:

$n \geq n$	$true\ for\ all\ n$
$n^2 \geq n^2$	Multiply both sides by n. Since n is positive it doesn't affect the order of the inequality
$7n^2 \geq 7n^2$	Multiply both sides by 7

Combining those two inequalities we get:

$7n^2 + 2n - 8 \geq 7n^2$	$for\ all\ n \geq 4$
$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7n^2}$	Take the square root of both sides. Can do this since square root is an increasing function.
$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7} \cdot \sqrt{n^2}$	Square root product identity
$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7} \cdot n$	Powers of n cancel out on left hand side
$f(n) \geq \sqrt{7} \cdot n$	Replace value by function name
$f(n) \geq \sqrt{7}\, g(n)$	Replace n by g(n)
$C_1 = \sqrt{7}$	By definition, we know that $f(n) = \Omega(g(n))$ if $c_1 \cdot g(n) \leq f(n)\ for\ all\ n \geq n_0$

Therefore, $f(n) = \Omega(n)\ for\ c_1 = \sqrt{7}\ and\ n_0 = 4$

Step 3: Determine theta

From the above steps we have:

- $f(n) = O(n)\ with\ c_2 = 3\ and\ n_0 = 1$
- $f(n) = \Omega(n)\ with\ c_1 = \sqrt{7}\ and\ n_0 = 4$

By definition, $f(n) = \Theta(n)$ if there are positive constants $c_1, c_2\ and\ n_0$ such that $c_1\,(n^3) \leq f(n) \leq c_2(n^3)$ for all $n \geq n_0$.

If we use:

- $c_1 = \sqrt{7}$
- $c_2 = 3$
- $n_0 = 4$

Combining our results we get $\sqrt{7}\,(n^3) \leq f(n) \leq 3\,(n^3)$ for all $n \geq 4$

$\boxed{\text{Therefore, } \sqrt{7n^2 + 2n - 8} = \Theta(n)}$