Sheet 1

Question 1:

(a) 1^{st} Step: place $\lg\sqrt{n} = \frac{1}{2}\lg n$ on line

1	(h) $\lg(\lg n)$	(a) $\lg\sqrt{n}$	(e) $\lg n$		(b) $(\lg^2 n)$	(f) $\sqrt{n}$	(i) $\sqrt{n}\lg n$	(d) n	(c) $n\lg n$	(g) n^2

(b) $\lg^2 n \ ? \ \lg\sqrt{n}$

$(\lg n)^2 \ ? \ \frac{1}{2}\lg n$ $\quad \div(\lg n)$

$(\lg n) \ ? \ \frac{1}{2}$

$>$

$\therefore$ greater

(c) $n\lg n > \frac{1}{2}\lg n$

and

$n\lg n \ ? \ (\lg n)^2 \quad \div \lg n$

$n \ ? \ \lg n$

$>$

$\lg\sqrt{n} < (\lg n)^2 < n\lg n$

(d) $n > \lg\sqrt{n}$

$n < n\lg n$

$n \ ? \ (\lg n)^2$

Try ratio: @ $n = 1000$

$1000 > 100$

$\therefore n > (\lg n)^2$

(e) $\lg n$:

$\lg n \ ? \ \lg\sqrt{n}$

$\boxed{\lg n > \frac{1}{2}\lg n}$

and $\lg n < (\lg n)^2$

(f) $\sqrt{n} < n$

$\sqrt{n} \ ? \ (\lg n)^2$

@ $n = 10^6$

$\therefore 1000 \ ? \ 397$

$>$

make sure

$n = 10^8$

$10000 \ ? \ 706$

$>$

(g) $n^2 > n$.

$n^2 \ ? \ n\lg n \quad (\div n)$

$n \ ? \ \lg n$

$>$

$\therefore$ ~~$n\lg$~~ $n^2 > n\lg n$

(h) $\lg(\lg n)$

$\lg(\lg n) \ ? \ \lg\sqrt{n}$

@ $n = 10^6$

$4.3 \ ? \ 9.96$

$<$

@ $n = 10^{8}$

$4.7 \ ? \ 13.28$

$<$

(i) $\sqrt{n}\lg n$

$\sqrt{n}\lg n \ ? \ n$

from line $\leftarrow \lg n < \sqrt{n}$

and $\sqrt{n}\lg n > \sqrt{n}$.

$\therefore \sqrt{n} < \sqrt{n}\lg n < n$

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② Recurrences:

ⓐ $T(n) = 3T(\frac{n}{2}) + n.$

sol

$a=3$, $b=2$, $f(n)=n$

$n^{\log_b a} = n^{\log_2 3} \simeq n^{1.58} > f(n)$

$\therefore T(n) = \Theta(n^{\log_2 3})$

ⓑ $T(n) = 4T(\frac{n}{2}) + n\lg n.$

sol $a=4$, $b=2$, $f(n)=n\lg n.$

$n^{\log_b a} = n^{\log_2 4} = n^2 > n\lg n$

$\therefore T(n) = \Theta(n^2)$

ⓒ $T(n) = 8T(\frac{n}{3}) + n^2$

sol $a=8, b=3, f(n)=n^2$

$\therefore n^{\log_b a} = n^{\log_3 8} = n^{1.89} < n^2$

$\therefore T(n) = \Theta(n^2)$

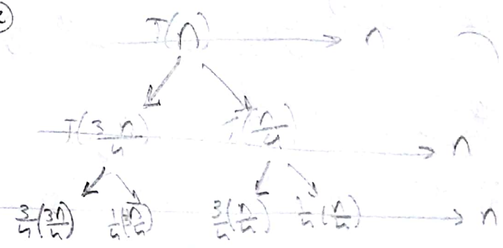
ⓓ $T(n) = 8T(\frac{n}{2}) + n^3$

sol $a=8, b=2, f(n)=n^3$

$n^{\log_b a} = n^{\log_2 8} = n^3 = f(n)$

$\therefore T(n) = \Theta(n^3 \lg n).$

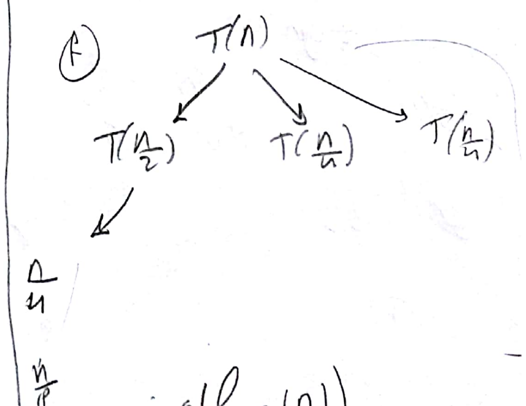
ⓔ



$\therefore \boxed{n * \log_{\frac{4}{3}} n}$

$\log_{\frac{4}{3}}(n)$

ⓕ



$T(n) = \Theta(\log_2(n))$

(g) $T(n) = T(n-1) + T(\sqrt{n}) + n$

(h) $T(n) = 2T(n-1) + 1 = 8T(n-3) + 7$

$T(n-1) = 2T(n-2)+1 = 4T(n-3) + 2 + 1$

$T(n-2) = 2T(n-3)+1$

$\cancel{T(n-k) = 2}$ $T(n) = 2^k T(n-k) + (2^k - 1)$

@ $k = n$, $T(0) = 1$

$T(n) = 2^n + 2^n - 1 = \boxed{2^{n+1} - 1}$

Question 3,

(i) (a) gets the identity of (n).

(b) $T(n) = T(n-1)+1$

(c) $T(n) = T(n-1)+1$

$T(n-1) = T(n-2)+1$

$T(n-2) = T(n-3)+1 \rightarrow \therefore T(n) = T(n-3)+3$

$\therefore T(n) = T(n-k) + k \rightarrow$ @ $k=n$ $T(n) = 0 + n$

$\rightarrow \boxed{T(n) = \Theta(n)}$

(ii) ⓐ returns $\sum_{i=0}^{n} i$.

ⓑ $T(n) = T(n-1) + \Theta(n)$

ⓒ $T(n) = T(n-1) + n$

$T(n-1) = T(n-2) + n - 1$

$T(n-2) = T(n-3) + n - 2$ $\rightarrow \therefore T(n) = T(n-k) + \sum_{i=n-k}^{n} (n-i)$

@ $\underline{k=n}$

$T(n) = 1 + \sum_{i=0}^{n} i = \frac{n(n-1)}{2}$

$T(n) = \Theta(n^2)$

(iii) ⓐ returns 2^n

ⓑ $T(n) = 2T(n-1) + 1$

ⓒ $T(n-1) = 2T(n-2) + 1$

$T(n-2) = 2T(n-3) + 1$ $\rightarrow T(n) = 2^3 T(n-3) + 3$

$\therefore T(n) = 2^k T(n-k) + k$

@ $n=k$ $\therefore T(n) = 2^n + n$

$\boxed{\therefore T(n) = \Theta(2^n).}$

(iv) ⓐ returns 2^n

ⓑ $T(n) = T(n-1) + 1$ $\rightarrow T(n) = T(n-k) + k$

$T(n-1) = T(n-2) + 1$

$T(n-2) = T(n-3) + 1$

@ $\underline{k=n}$: $T(n) = T(0) + n$

$= 1 + n$

$\boxed{\therefore T(n) = \Theta(n)}$