Supplement 2

Experiment 1 – Measures of central tendency

Peter Kamerman

17 June 2018

Contents

In this experiment, there are two levels of correlated data:

1) Correlation at the level of the individual, with each individual tested at each stimulus intensity (the individual is the observational unit).
2) Correlation at each stimulus intensity within each individual, with each stimulus intensity being assessed several times in each individual (stimulus intensity is the observational unit).

This double layer of correlation creates problems for analysis. One solution is to remove the second layer of correlation (correlation at each stimulus intensity within each individual) by calculating a single summary measure of the repeated measurement at each stimulus intensity. But what is the best measure of location (central tendency) for these data?

Measures of central tendency provide information on what the *'average'* value of a set of numbers is; a useful summary measure of a dataset. Some examples of measures of central tendency are:

1. Arithmetic mean (`base::mean`)
2. Median (`stats::median`)
3. Geometric mean (`psych::geometric.mean`, mathematical definition below)
4. Tukey trimean (`user-defined::tri.mean`, mathematical definition and function specification below)

Each measure has strengths and limitations. The *arithmetic mean* is best suited to symmetrical distributions, and hence asymmetrical distributions (right or left-skewed data) reduced the usefulness of the measure as a measure of central tendency. The *geometric mean* is equivalent to the *arithmetic mean*, but the additive structure is on the logarithm of the original data [1]. As such, the *geometric mean* handles skewed data better than does the *arithmetic mean*, particularly right-skewed data. The *median* is the middle element in a sorted set of numbers (the centre-point of the data), and therefore it is not influenced by extreme values. However using the *median* means that information about the distribution of the data is lost, and it can be biased in small samples. The fourth measure is the *Tukey trimean*. The *Tukey trimean* combines the median's emphasis on centre values with the midhinge's (average of the 25th and

[1] Because the calculation of geometric mean requires values ≥ 0, `rating_positive` data (SPARS rating + 50) were used to calculate all measures of central tendency.

75th percentiles) attention to the extremes [2]. As such, the distribution of the data is incorporated to some extent into the measure of central tendency.

Judging the suitability of the four measures of central tendency was done by visually inspecting modified 'Raw, Descriptive, Inference (RDI)' plots [^3]. The plots included individual data points, smoothed densities, and the four measures of central tendency, but dropping the 'inference' component.

Definition of the geometric mean

The *geometric mean* is defined as the nth root of the product of a set of numbers.

$$G_{mean} = (\prod_{i=1}^{n} x_i)^{\frac{1}{n}} = \sqrt[n]{x_1 x_2 ... x_n}$$

Where:

- x_i = elements in a set of numbers
- n = number of elements in the set

Definition of the Tukey trimean (and function)

The *Tukey trimean* is defined as the weighted average of the distribution's median and its two quartiles.

$$T_{mean} = \frac{1}{2}(Q_2 + \frac{Q_1 + Q_3}{2})$$

Where:

- Q_1 = 25th percentile
- Q_2 = 50th percentile (median)
- Q_3 = 75th percentile

```r
# Specify tri.mean function
tri.mean <- function(x) {
  # Calculate quantiles
  q1 <- quantile(x, probs = 0.25, na.rm = TRUE)[[1]]
  q2 <- median(x, na.rm = TRUE)
  q3 <- quantile(x, probs = 0.75, na.rm = TRUE)[[1]]
  # Calculate trimean
  tm <- (q2 + ((q1 + q3) / 2)) / 2
  # Convert to integer
  tm <- as.integer(round(tm))
  return(tm)
}
```

[2] Weisberg, H. F. (1992). Central Tendency and Variability. Sage University. ISBN 0-8039-4007-6

Import and inspect data

```
# Import
data <- read_rds('./data-cleaned/SPARS_A.rds')

# Inspect
glimpse(data)
```

```
## Observations: 1,927
## Variables: 19
## $ PID               <chr> "ID01", "ID01", "ID01", "ID01", "ID01", "ID0...
## $ block             <chr> "A", "A", "A", "A", "A", "A", "A", "A", "A",...
## $ block_order       <dbl> 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,...
## $ trial_number      <dbl> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 1...
## $ intensity         <dbl> 3.75, 1.50, 3.25, 1.50, 3.00, 2.75, 1.00, 2....
## $ intensity_char    <chr> "3.75", "1.50", "3.25", "1.50", "3.00", "2.7...
## $ rating            <dbl> -10, -40, -10, -25, -20, -25, -40, 2, -40, -...
## $ rating_positive   <dbl> 40, 10, 40, 25, 30, 25, 10, 52, 10, 40, 54, ...
## $ EDA               <dbl> 18315.239, 13904.177, 11543.449, 20542.834, ...
## $ age               <dbl> 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, ...
## $ sex               <dbl> 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,...
## $ panas_positive    <dbl> 36, 36, 36, 36, 36, 36, 36, 36, 36, 36, 36, ...
## $ panas_negative    <dbl> 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, ...
## $ dass42_depression <dbl> 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,...
## $ dass42_anxiety    <dbl> 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,...
## $ dass42_stress     <dbl> 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,...
## $ pcs_magnification <dbl> 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,...
## $ pcs_rumination    <dbl> 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, ...
## $ pcs_helplessness  <dbl> 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, ...
```

Clean data

```
# Basic clean-up
data %<>%
  # Select required columns
  select(PID, block, block_order, intensity,
         intensity_char, rating, rating_positive)
```

Participant-level: Best measure of central tendency

Calculate measures of central tendency

```
# Calculate participant-level centrality measures
central <- data %>%
```

```
  group_by(PID, intensity_char) %>%
  summarise(mean = mean(rating_positive, na.rm = TRUE),
            median = median(rating_positive, na.rm = TRUE),
            geometric_mean = psych::geometric.mean(rating_positive),
            tri_mean = tri.mean(rating_positive)) %>%
  ungroup() %>%
  # Change from wide to long format for plotting
  gather(key = type,
         value = value,
         mean:tri_mean) %>%
  # Order measurement types
  mutate(type = fct_relevel(factor(type),
                            'mean', 'median',
                            'geometric_mean',
                            'tri_mean'))
```

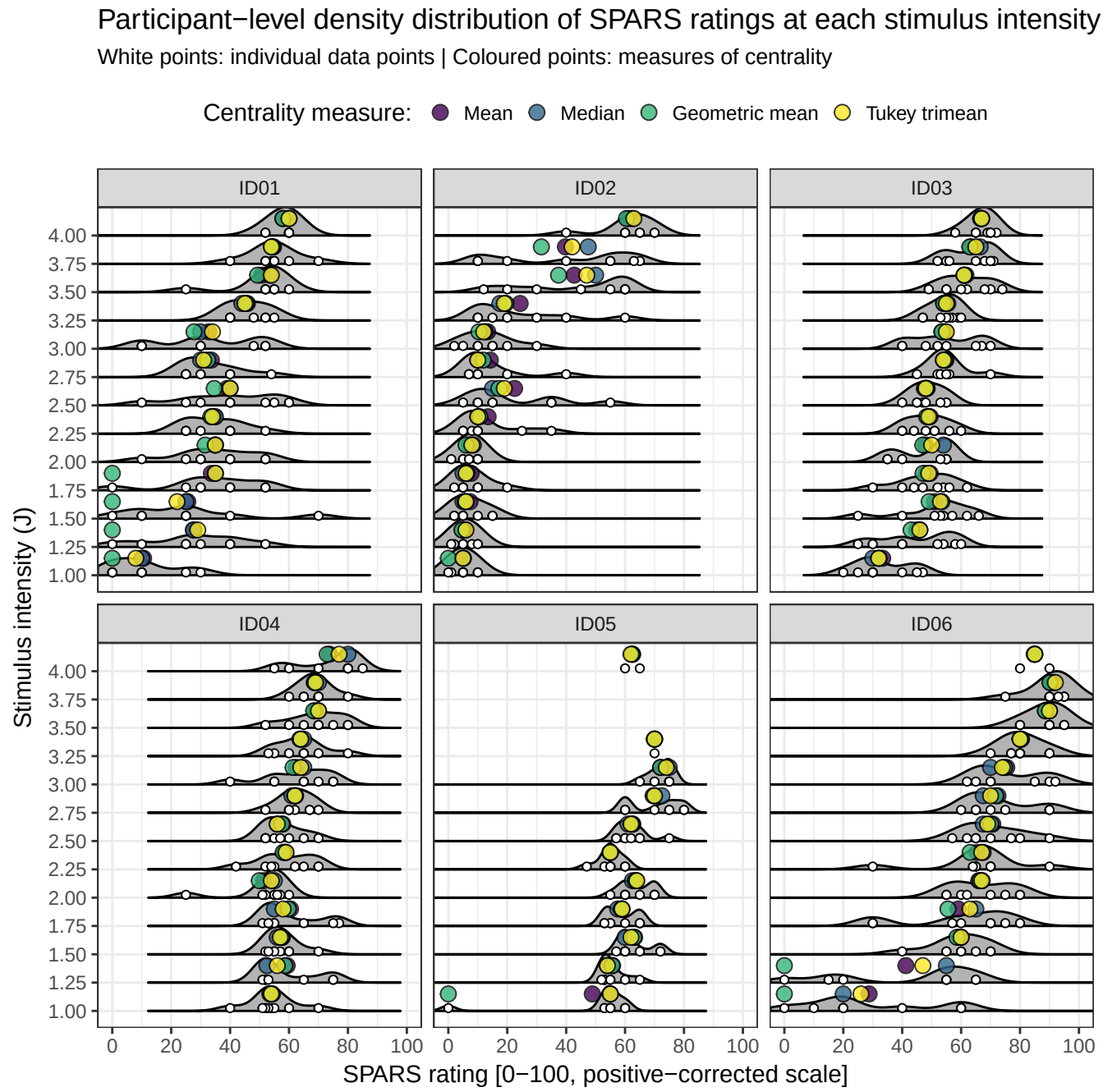
Plots

```
# Divide participants into batches of 6 (for better plot layout)
pid_a <- c(paste0('ID0', 1:6))
pid_b <- c(paste0('ID0', 7:9), 'ID10', 'ID11', 'ID12')
pid_c <- c(paste0('ID', 13:18))

# Plot first 6 participants
p_1to6 <- data %>%
  filter(PID %in% pid_a) %>%
  # Plot
  ggplot(data = .) +
  aes(x = rating_positive,
      y = intensity_char) +
  geom_density_ridges2(scale = 1) +
  geom_point(position = position_nudge(y = 0.1),
             shape = 21,
             fill = '#FFFFFF') +
  geom_point(data = central[central$PID %in% pid_a, ],
             aes(x = value, fill = type),
             shape = 21,
             size = 3,
             alpha = 0.8,
             position = position_nudge(y = 0.6)) +
  scale_fill_viridis_d(name = 'Centrality measure:',
                       labels = c('Mean', 'Median',
                                  'Geometric mean',
                                  'Tukey trimean')) +
  scale_x_continuous(limits = c(-5, 105),
                     breaks = seq(from = 0, to = 100, by = 20),
                     expand = c(0,0)) +
  labs(title = 'Participant-level density distribution of SPARS ratings at each stimulus i
       subtitle = 'White points: individual data points | Coloured points: measures of cen
       x = 'SPARS rating [0-100, positive-corrected scale]',
```

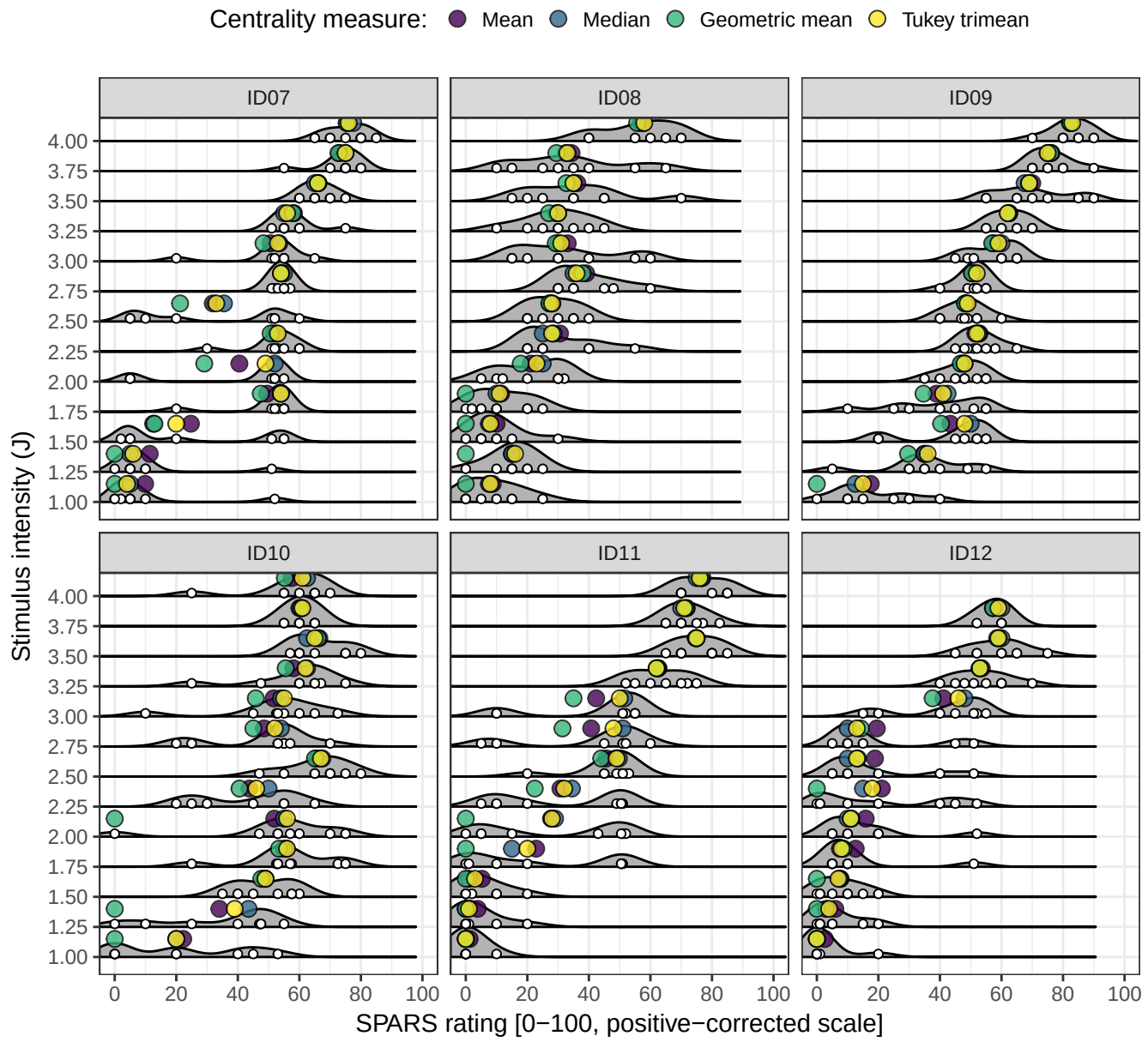
```
       y = 'Stimulus intensity (J)') +
  facet_wrap(~PID, ncol = 3) +
  theme_bw() +
  theme(legend.position = 'top')

# Plot participants 7-12
p_7to12 <- data %>%
  # Filter participants 7-12
  filter(PID %in% pid_b) %>%
  # Plot
  ggplot(data = .) +
  aes(x = rating_positive,
      y = intensity_char) +
  geom_density_ridges2(scale = 1) +
  geom_point(position = position_nudge(y = 0.1),
             shape = 21,
             fill = '#FFFFFF') +
  geom_point(data = central[central$PID %in% pid_b, ],
             aes(x = value, fill = type),
             shape = 21,
             size = 3,
             alpha = 0.8,
             position = position_nudge(y = 0.6)) +
  scale_fill_viridis_d(name = 'Centrality measure:',
                       labels = c('Mean', 'Median',
                                  'Geometric mean',
                                  'Tukey trimean')) +
  scale_x_continuous(limits = c(-5, 105),
                     breaks = seq(from = 0, to = 100, by = 20),
                     expand = c(0,0)) +
  labs(title = 'Participant-level density distribution of SPARS ratings at each stimulus i
       subtitle = 'White points: individual data points | Coloured points: measures of cen
       x = 'SPARS rating [0-100, positive-corrected scale]',
       y = 'Stimulus intensity (J)') +
  facet_wrap(~PID, ncol = 3) +
  theme_bw() +
  theme(legend.position = 'top')

# Plot participants 13-18
p_13to18 <- data %>%
  # Filter participants 13-18
  filter(PID %in% pid_c) %>%
  # Plot
  ggplot(data = .) +
  aes(x = rating_positive,
      y = intensity_char) +
  geom_density_ridges2(scale = 1) +
  geom_point(position = position_nudge(y = 0.1),
             shape = 21,
             fill = '#FFFFFF') +
  geom_point(data = central[central$PID %in% pid_c, ],
```

```
            aes(x = value, fill = type),
            shape = 21,
            size = 3,
            alpha = 0.8,
            position = position_nudge(y = 0.6)) +
  scale_fill_viridis_d(name = 'Centrality measure',
                       labels = c('Mean', 'Median',
                                  'Geometric mean',
                                  'Tukey trimean')) +
  scale_x_continuous(limits = c(-5, 105),
                     breaks = seq(from = 0, to = 100, by = 20),
                     expand = c(0,0)) +
  labs(title = 'Participant-level density distribution of SPARS ratings at each stimulus i
       subtitle = 'White points: individual data points | Coloured points: measures of cen
       x = 'SPARS rating [0-100, positive-corrected scale]',
       y = 'Stimulus intensity (J)') +
  facet_wrap(~PID, ncol = 3) +
  theme_bw() +
  theme(legend.position = 'top')
```

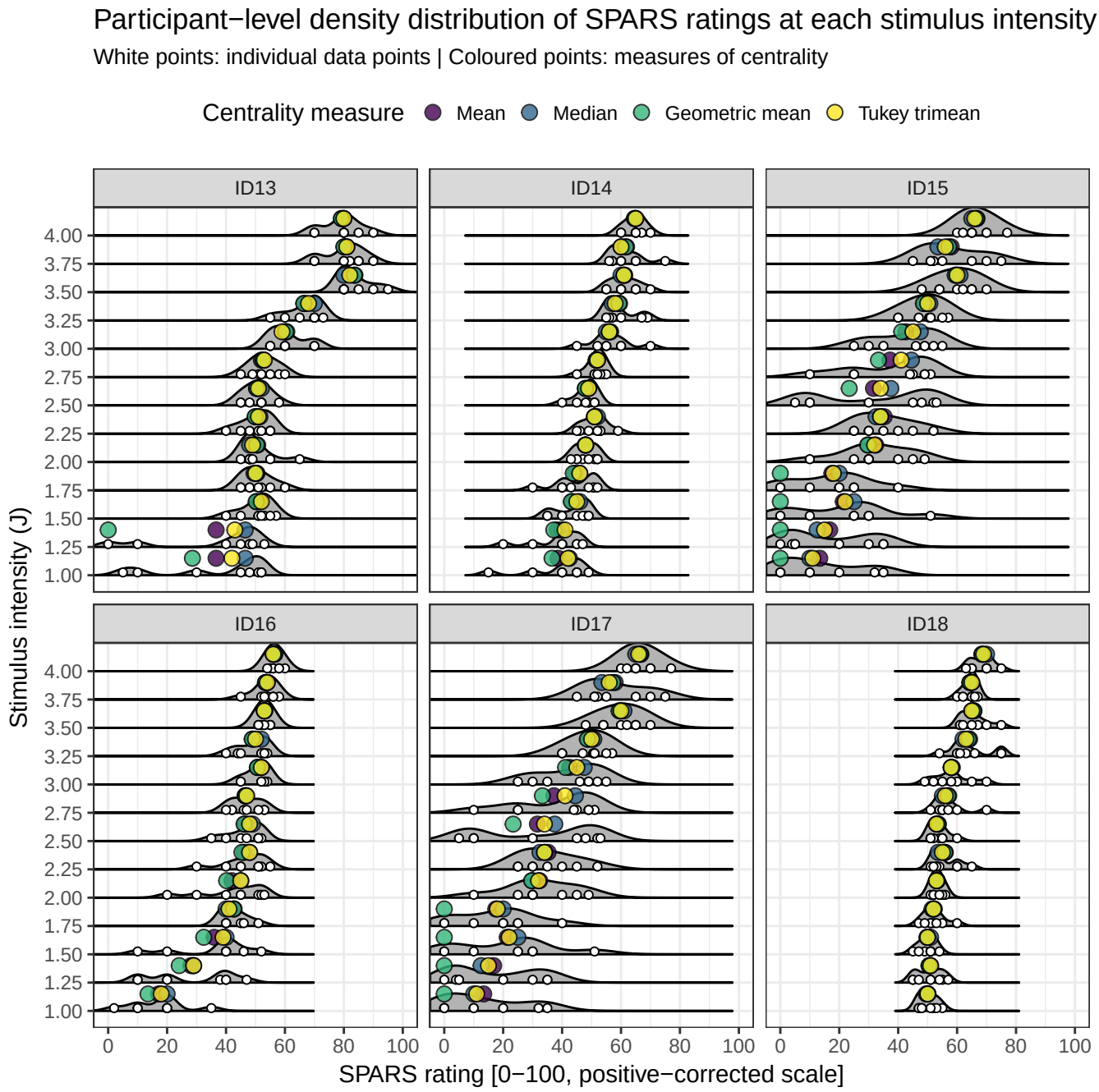
```
# Print plots
p_1to6
```


Participant−level density distribution of SPARS ratings at each stimulus intensity
White points: individual data points | Coloured points: measures of centrality
Centrality measure: Mean Median Geometric mean Tukey trimean
ID01
ID02
ID03
ID04
ID05
ID06
Stimulus intensity (J)
4.00
3.75
3.50
3.25
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0
20
40
60
80
100
SPARS rating [0−100, positive−corrected scale]

```
p_7to12
```


Participant−level density distribution of SPARS ratings at each stimulus intensity
White points: individual data points | Coloured points: measures of centrality
Centrality measure:
Mean
Median
Geometric mean
Tukey trimean
ID07
ID08
ID09
ID10
ID11
ID12
Stimulus intensity (J)
4.00
3.75
3.50
3.25
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0
20
40
60
80
100
SPARS rating [0−100, positive−corrected scale]

```
p_13to18
```



Conclusion

There was significant heterogeneity in the grouping of ratings within and between individuals, with a tendency for data to be left skewed (a few very low values). The heterogeneity makes selecting a measure of centrality difficult. Of the measures of centrality, the *Tukey trimean* and *median* showed the best stability across all stimulus intensities (as expected). The *geometric mean* performed very poorly when there was a strong left skew in the data. The *arithmetic mean* was similarly affected by skewed data. When there was a strong skew, the *Tukey mean* was pulled slightly away from the *median* and towards the affected tail, but otherwise the two measures yielded similar values. Given that the *Tukey trimean* incorporates some of the information in the distribution of the data, but is not overly affected by extreme values, we believe that the *Tukey trimean* should be used for 'averaging' repeated measurements within an individual, at a given stimulus intensity.

Session information

```
sessionInfo()
```

```
## R version 3.5.0 (2018-04-23)
## Platform: x86_64-apple-darwin15.6.0 (64-bit)
## Running under: macOS High Sierra 10.13.5
##
## Matrix products: default
## BLAS: /Library/Frameworks/R.framework/Versions/3.5/Resources/lib/libRblas.0.dylib
## LAPACK: /Library/Frameworks/R.framework/Versions/3.5/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_GB.UTF-8/en_GB.UTF-8/en_GB.UTF-8/C/en_GB.UTF-8/en_GB.UTF-8
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
##  [1] bindrcpp_0.2.2     ggridges_0.5.0     forcats_0.3.0
##  [4] stringr_1.3.1      dplyr_0.7.5        purrr_0.2.5
##  [7] readr_1.1.1        tidyr_0.8.1        tibble_1.4.2
## [10] ggplot2_2.2.1.9000 tidyverse_1.2.1    magrittr_1.5
##
## loaded via a namespace (and not attached):
##  [1] tidyselect_0.2.4  reshape2_1.4.3    haven_1.1.1
##  [4] lattice_0.20-35   colorspace_1.3-2  htmltools_0.3.6
##  [7] viridisLite_0.3.0 yaml_2.1.19       rlang_0.2.1
## [10] pillar_1.2.3      foreign_0.8-70    glue_1.2.0
## [13] withr_2.1.2       modelr_0.1.2      readxl_1.1.0
## [16] bindr_0.1.1       plyr_1.8.4        munsell_0.4.3
## [19] gtable_0.2.0      cellranger_1.1.0  rvest_0.3.2
## [22] psych_1.8.4       evaluate_0.10.1   knitr_1.20
## [25] parallel_3.5.0    broom_0.4.4       Rcpp_0.12.17
## [28] scales_0.5.0.9000 backports_1.1.2   jsonlite_1.5
## [31] mnormt_1.5-5      hms_0.4.2         digest_0.6.15
## [34] stringi_1.2.2     grid_3.5.0        rprojroot_1.3-2
## [37] cli_1.0.0         tools_3.5.0       lazyeval_0.2.1
## [40] crayon_1.3.4      pkgconfig_2.0.1   xml2_1.2.0
## [43] lubridate_1.7.4   assertthat_0.2.0  rmarkdown_1.9
## [46] httr_1.3.1        rstudioapi_0.7    R6_2.2.2
## [49] nlme_3.1-137      compiler_3.5.0
```