Regression to the mean modeling

Regression curves: mean pain rating of 6.2 (SD: 1.7)

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Contents

Premise

Unless the correlation between two sequential measurements is 1, there should be a "flattening" of the relationship between the two, and greater "scatter". This "flattening" and greater "scatter" of the relationship has important implications for the use of cut-off criteria for study inclusion.

This script demonstrates the "flattening" of, and increase in "scatter" in, the relationship between two sequential pain measurements, both with mean = 6.2 and SD = 1.7 (typical baseline values for placebo groups in neuropathic pain RCTs without a cut-off inclusion citerion), over a range of correlational structures: r = 1.0, 0.8, 0.5, and 0.2.

Generate 2x2 covariance matrices

Generate covariance matrices using an SD of 1.7 and correlation of 1.0, 0.8, 0.5, and 0.2.

```
# Correlation matrices
cor_10 <- matrix(c(1, 1, 1, 1), ncol = 2)
cor_08 <- matrix(c(1, 0.8, 0.8, 1), ncol = 2)
cor_05 <- matrix(c(1, 0.5, 0.5, 1), ncol = 2)
cor_02 <- matrix(c(1, 0.2, 0.2, 1), ncol = 2)

# Standard deviation
std <- c(1.7, 1.7)

# Covariance matrices
cov_10 <- cor2cov(cor.mat = cor_10,
                  sd = std)
cov_10
```

```
##      [,1] [,2]
## [1,] 2.89 2.89
## [2,] 2.89 2.89
```

```
cov_08 <- cor2cov(cor.mat = cor_08,
                  sd = std)
cov_08
```

```
##       [,1]  [,2]
## [1,] 2.890 2.312
## [2,] 2.312 2.890
```

```
cov_05 <- cor2cov(cor.mat = cor_05,
                  sd = std)
cov_05
```

```
##       [,1]  [,2]
## [1,] 2.890 1.445
## [2,] 1.445 2.890
```

```
cov_02 <- cor2cov(cor.mat = cor_02,
                  sd = std)
cov_02
```

```
##       [,1]  [,2]
## [1,] 2.890 0.578
## [2,] 0.578 2.890
```

Mean = 6.2, SD = 1.7, r = 1.0

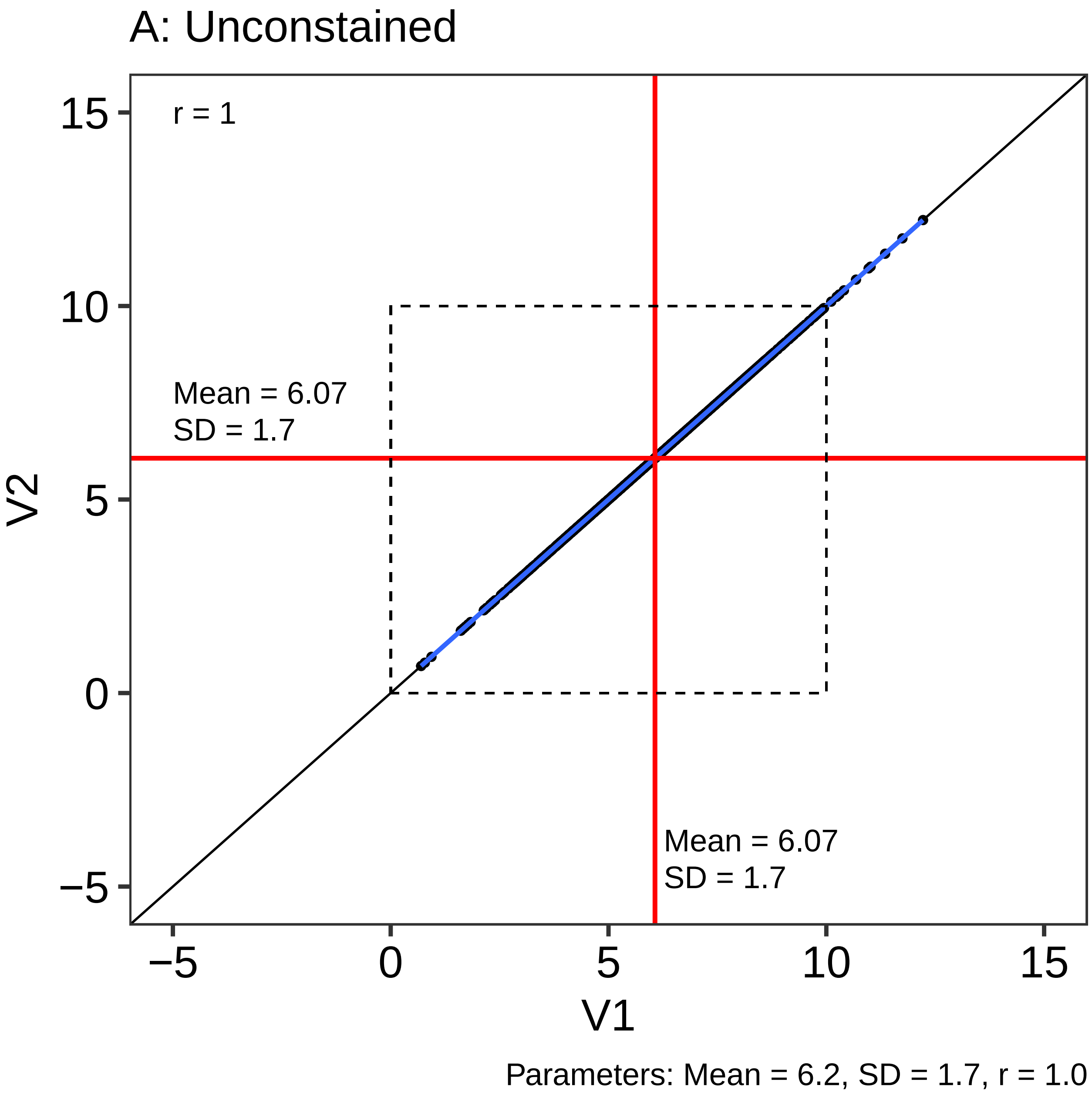
Generate and summarise data

Base data

```r
# Set the random seed for reproducibility
set.seed(2019)

# Generate the data (1000 pairs)
cor_10.base <- as.data.frame(mvrnorm(n = 1000, mu = c(6.2, 6.2), Sigma = cov_10))

# Plot base data
ggplot(data = cor_10.base) +
    aes(x = V1, y = V2) +
    geom_point() +
    geom_abline(slope = 1, intercept = 0) +
    geom_smooth(method = 'lm',
                se = FALSE) +
    geom_hline(yintercept = mean(cor_10.base$V2),
               colour = 'red', size = 1) +
    geom_vline(xintercept = mean(cor_10.base$V1),
               colour = 'red', size = 1) +
    geom_rect(ymin = 0, ymax = 10,
              xmin = 0, xmax = 10,
              colour = '#000000',
              alpha = 0,
              linetype = 2) +
    annotate(geom = 'text', x = -5, y = 15, hjust = 0, size = 5,
             label = str_glue("r = {round(cor(cor_10.base$V1,
                              cor_10.base$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_10.base$V2) + 1.7,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_10.base$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_10.base$V2) + 0.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_10.base$V2),2)}")) +
    annotate(geom = 'text', x = mean(cor_10.base$V1) + 0.2, y = -3.8,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_10.base$V1), 2)}")) +
    annotate(geom = 'text', x = mean(cor_10.base$V1) + 0.2, y = -4.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_10.base$V1), 2)}")) +
    labs(title = 'A: Unconstained',
         caption = 'Parameters: Mean = 6.2, SD = 1.7, r = 1.0') +
    scale_y_continuous(limits = c(-5, 15)) +
    scale_x_continuous(limits = c(-5, 15)) +
    theme(plot.caption = element_text(size = 14))
```



```r
# Linear regression
summary(lm(V2 ~ V1, data = cor_10.base))
```

```
## Warning in summary.lm(lm(V2 ~ V1, data = cor_10.base)): essentially perfect
## fit: summary may be unreliable

##
## Call:
## lm(formula = V2 ~ V1, data = cor_10.base)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -5.232e-14  3.000e-18  4.900e-17  9.800e-17  1.548e-15
##
## Coefficients:
```

```
##                Estimate Std. Error    t value Pr(>|t|)    
## (Intercept) -5.393e-15  1.945e-16 -2.773e+01   <2e-16 ***
## V1           1.000e+00  3.087e-17  3.240e+16   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 1.662e-15 on 998 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1 
## F-statistic: 1.05e+33 on 1 and 998 DF,  p-value: < 2.2e-16
```

```
Confint(lm(V2 ~ V1, data = cor_10.base))
```

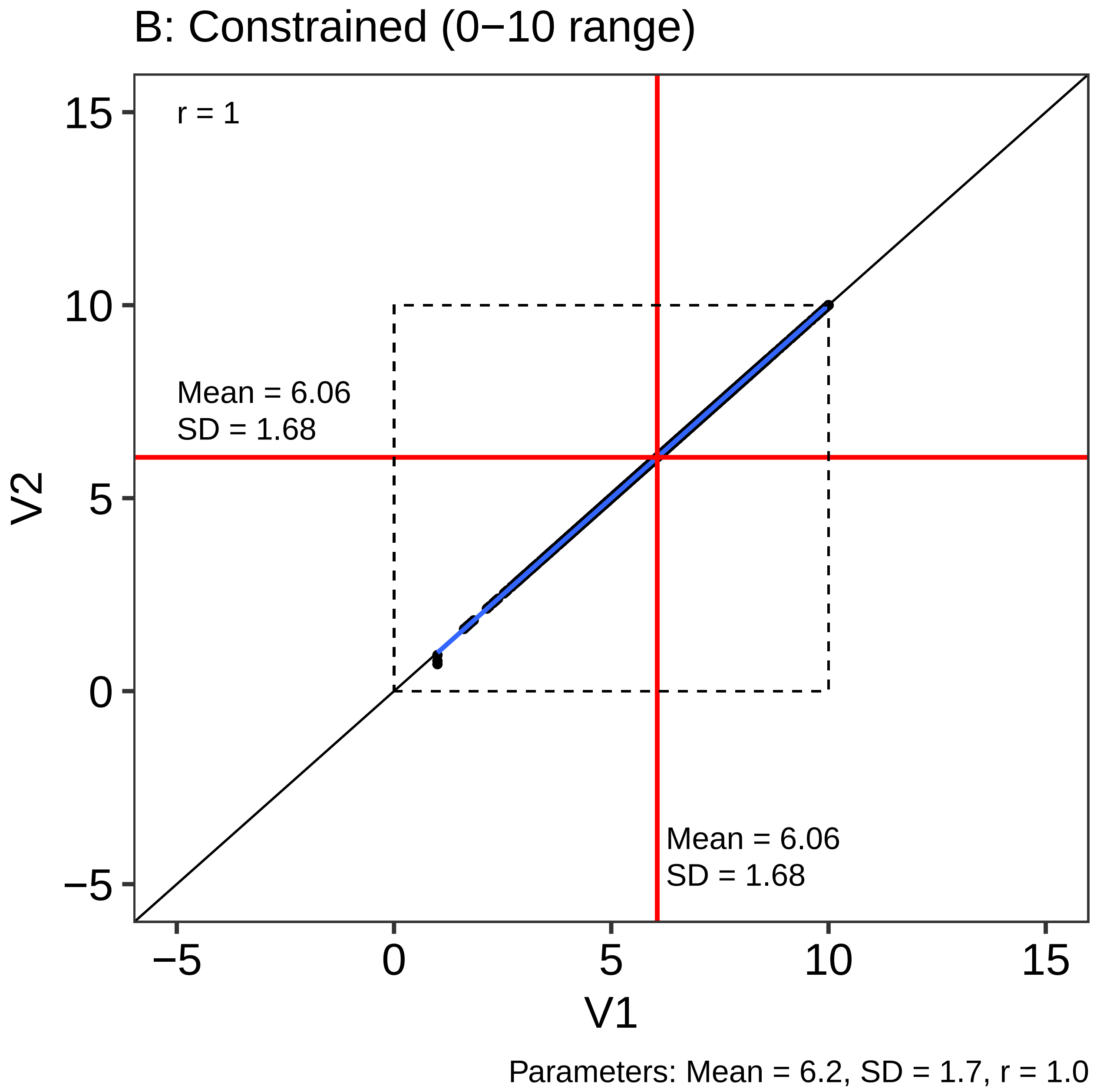
```
## Warning in summary.lm(object, ...): essentially perfect fit: summary may be
## unreliable
```

```
##                  Estimate         2.5 %        97.5 %
## (Intercept) -5.39264e-15 -5.774318e-15 -5.010962e-15
## V1           1.00000e+00  1.000000e+00  1.000000e+00
```

Constrain values to 0-10 range

```
# Process data
cor_10.constrained <- cor_10.base %>%
    mutate(V1 = case_when(
                V1 < 1 ~ 1,
                V1 > 10 ~ 10,
                TRUE ~ V1)) %>%
    mutate(V2 = case_when(
                V2 < 0 ~ 0,
                V2 > 10 ~ 10,
                TRUE ~ V2)) %>%
    mutate(group = 'No threshold')

# Plot processed data
ggplot(data = cor_10.constrained) +
    aes(x = V1, y = V2) +
    geom_point() +
    geom_abline(slope = 1, intercept = 0) +
    geom_smooth(method = 'lm',
                se = FALSE) +
    geom_hline(yintercept = mean(cor_10.constrained$V2),
               colour = 'red', size = 1) +
    geom_vline(xintercept = mean(cor_10.constrained$V1),
               colour = 'red', size = 1) +
    geom_rect(ymin = 0, ymax = 10,
              xmin = 0, xmax = 10,
              colour = '#000000',
              alpha = 0,
              linetype = 2) +
    annotate(geom = 'text', x = -5, y = 15, hjust = 0, size = 5,
             label = str_glue("r = {round(cor(cor_10.constrained$V1,
                                         cor_10.constrained$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_10.constrained$V2) + 1.7,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_10.constrained$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_10.constrained$V2) + 0.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_10.constrained$V2),2)}")) +
    annotate(geom = 'text', x = mean(cor_10.constrained$V1) + 0.2, y = -3.8,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_10.constrained$V1), 2)}")) +
    annotate(geom = 'text', x = mean(cor_10.constrained$V1) + 0.2, y = -4.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_10.constrained$V1), 2)}")) +
    labs(title = 'B: Constrained (0-10 range)',
         caption = 'Parameters: Mean = 6.2, SD = 1.7, r = 1.0') +
    scale_y_continuous(limits = c(-5, 15)) +
    scale_x_continuous(limits = c(-5, 15)) +
    theme(plot.caption = element_text(size = 14))
```



```r
# Linear regression
summary(lm(V2 ~ V1, data = cor_10.constrained))
```

```
##
## Call:
## lm(formula = V2 ~ V1, data = cor_10.constrained)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.295454 -0.000643  0.000662  0.001717  0.005245
##
## Coefficients:
##                Estimate Std. Error  t value Pr(>|t|)
## (Intercept) -0.0069328  0.0013969   -4.963 8.16e-07 ***
## V1           1.0010482  0.0002222 4504.746  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01178 on 998 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 2.029e+07 on 1 and 998 DF,  p-value: < 2.2e-16
```

```
Confint(lm(V2 ~ V1, data = cor_10.constrained))
```

```
##                 Estimate        2.5 %       97.5 %
## (Intercept) -0.006932785 -0.009674038 -0.004191532
## V1           1.001048195  1.000612122  1.001484269
```

Mean = 6.2, SD = 1.7, r = 0.8

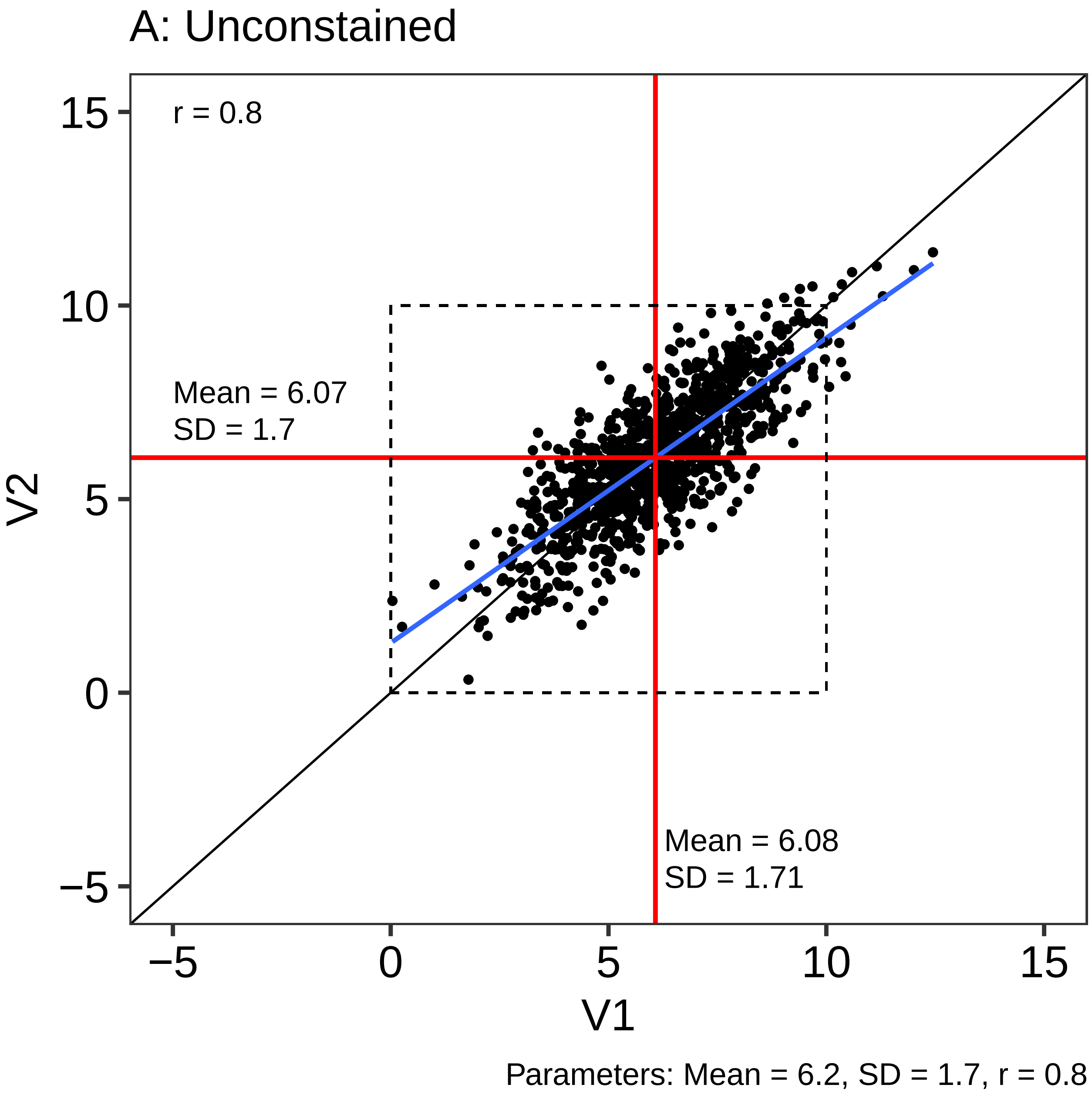
Generate and summarise data

Base data

```
# Set the random seed for reproducibility
set.seed(2019)

# Generate the data (1000 pairs)
cor_08.base <- as.data.frame(mvrnorm(n = 1000, mu = c(6.2, 6.2), Sigma = cov_08))

# Plot base data
ggplot(data = cor_08.base) +
    aes(x = V1, y = V2) +
    geom_point() +
    geom_abline(slope = 1, intercept = 0) +
    geom_smooth(method = 'lm',
                se = FALSE) +
    geom_hline(yintercept = mean(cor_08.base$V2),
               colour = 'red', size = 1) +
    geom_vline(xintercept = mean(cor_08.base$V1),
               colour = 'red', size = 1) +
    geom_rect(ymin = 0, ymax = 10,
              xmin = 0, xmax = 10,
              colour = '#000000',
              alpha = 0,
              linetype = 2) +
    annotate(geom = 'text', x = -5, y = 15, hjust = 0, size = 5,
             label = str_glue("r = {round(cor(cor_08.base$V1,
                              cor_08.base$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_08.base$V2) + 1.7,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_08.base$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_08.base$V2) + 0.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_08.base$V2),2)}")) +
    annotate(geom = 'text', x = mean(cor_08.base$V1) + 0.2, y = -3.8,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_08.base$V1), 2)}")) +
    annotate(geom = 'text', x = mean(cor_08.base$V1) + 0.2, y = -4.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_08.base$V1), 2)}")) +
    labs(title = 'A: Unconstained',
         caption = 'Parameters: Mean = 6.2, SD = 1.7, r = 0.8') +
    scale_y_continuous(limits = c(-5, 15)) +
    scale_x_continuous(limits = c(-5, 15)) +
    theme(plot.caption = element_text(size = 14))
```



```
# Linear regression
summary(lm(V2 ~ V1, data = cor_08.base))
```

```
## 
## Call:
## lm(formula = V2 ~ V1, data = cor_08.base)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -2.9824 -0.6632  0.0201  0.6799  3.3484 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  1.28292    0.12006   10.69   <2e-16 ***
## V1           0.78780    0.01901   41.43   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 1.03 on 998 degrees of freedom
## Multiple R-squared:  0.6323, Adjusted R-squared:  0.632 
## F-statistic:  1716 on 1 and 998 DF,  p-value: < 2.2e-16
```

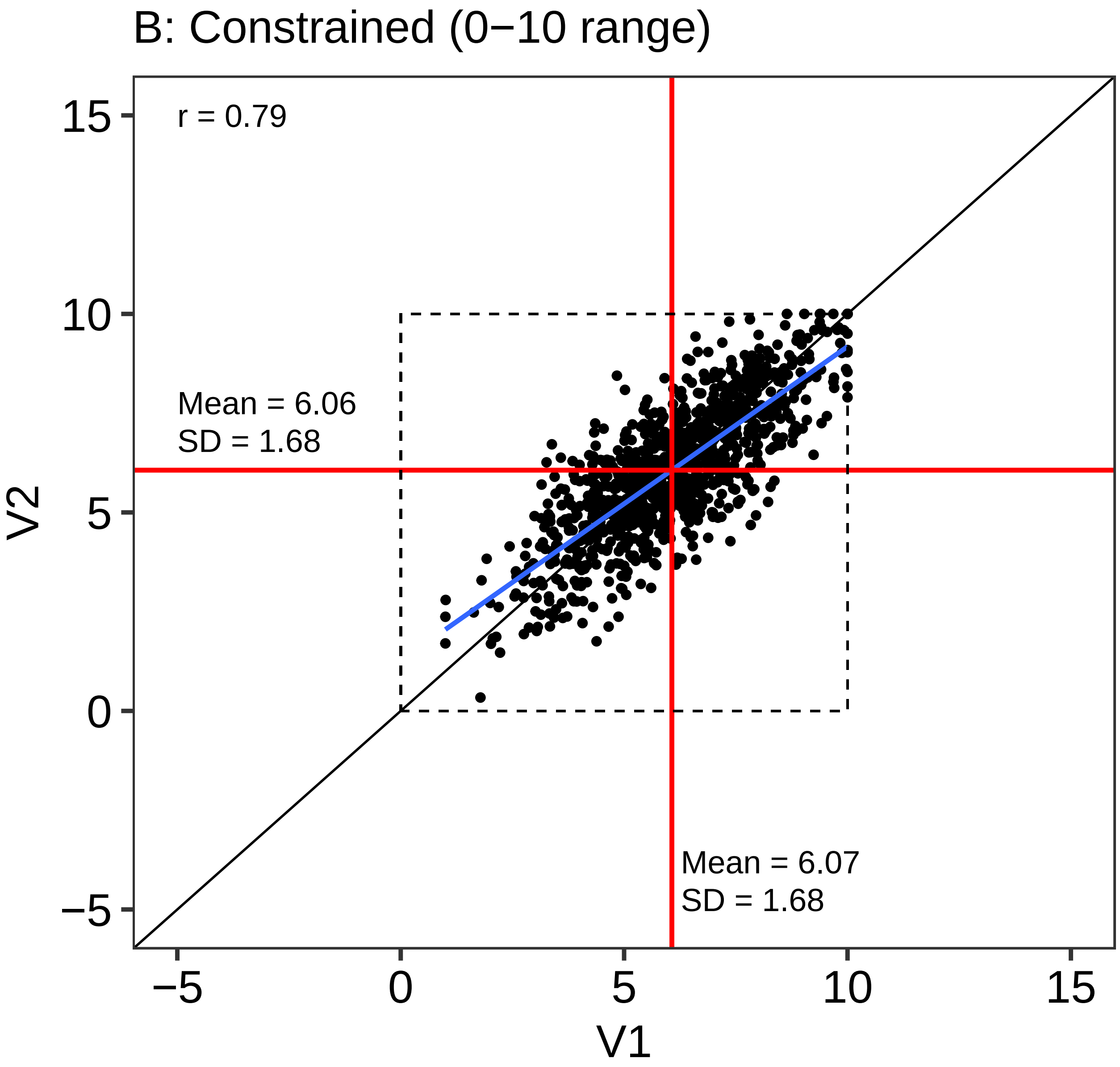
```r
Confint(lm(V2 ~ V1, data = cor_08.base))
```

```
##              Estimate     2.5 %    97.5 %
## (Intercept) 1.2829201 1.0473188 1.5185214
## V1          0.7878005 0.7504867 0.8251143
```

Constrain values to 0-10 range

```r
# Process data
cor_08.constrained <- cor_08.base %>%
    mutate(V1 = case_when(
               V1 < 1 ~ 1,
               V1 > 10 ~ 10,
               TRUE ~ V1)) %>%
    mutate(V2 = case_when(
               V2 < 0 ~ 0,
               V2 > 10 ~ 10,
               TRUE ~ V2)) %>%
    mutate(group = 'No threshold')

# Plot processed data
ggplot(data = cor_08.constrained) +
    aes(x = V1, y = V2) +
    geom_point() +
    geom_abline(slope = 1, intercept = 0) +
    geom_smooth(method = 'lm',
                se = FALSE) +
    geom_hline(yintercept = mean(cor_08.constrained$V2),
               colour = 'red', size = 1) +
    geom_vline(xintercept = mean(cor_08.constrained$V1),
               colour = 'red', size = 1) +
    geom_rect(ymin = 0, ymax = 10,
              xmin = 0, xmax = 10,
              colour = '#000000',
              alpha = 0,
              linetype = 2) +
    annotate(geom = 'text', x = -5, y = 15, hjust = 0, size = 5,
             label = str_glue("r = {round(cor(cor_08.constrained$V1,
                              cor_08.constrained$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_08.constrained$V2) + 1.7,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_08.constrained$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_08.constrained$V2) + 0.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_08.constrained$V2),2)}")) +
    annotate(geom = 'text', x = mean(cor_08.constrained$V1) + 0.2, y = -3.8,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_08.constrained$V1), 2)}")) +
    annotate(geom = 'text', x = mean(cor_08.constrained$V1) + 0.2, y = -4.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_08.constrained$V1), 2)}")) +
    labs(title = 'B: Constrained (0-10 range)',
         caption = 'Parameters: Mean = 6.2, SD = 1.7, r = 0.8') +
    scale_y_continuous(limits = c(-5, 15)) +
    scale_x_continuous(limits = c(-5, 15)) +
    theme(plot.caption = element_text(size = 14))
```


B: Constrained (0−10 range)

Parameters: Mean = 6.2, SD = 1.7, r = 0.8

```r
# Linear regression
summary(lm(V2 ~ V1, data = cor_08.constrained))
```

```
##
## Call:
## lm(formula = V2 ~ V1, data = cor_08.constrained)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9756 -0.6595  0.0179  0.6843  3.3534
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.25863    0.12171   10.34   <2e-16 ***
## V1           0.79179    0.01933   40.97   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.027 on 998 degrees of freedom
## Multiple R-squared:  0.6271, Adjusted R-squared:  0.6267
## F-statistic:  1678 on 1 and 998 DF,  p-value: < 2.2e-16
```

```r
Confint(lm(V2 ~ V1, data = cor_08.constrained))
```

```
##              Estimate     2.5 %    97.5 %
## (Intercept) 1.2586257 1.0197797 1.4974718
## V1          0.7917882 0.7538616 0.8297147
```

```r
## Slope different to 1
```

Mean = 6.2, SD = 1.7, r = 0.5

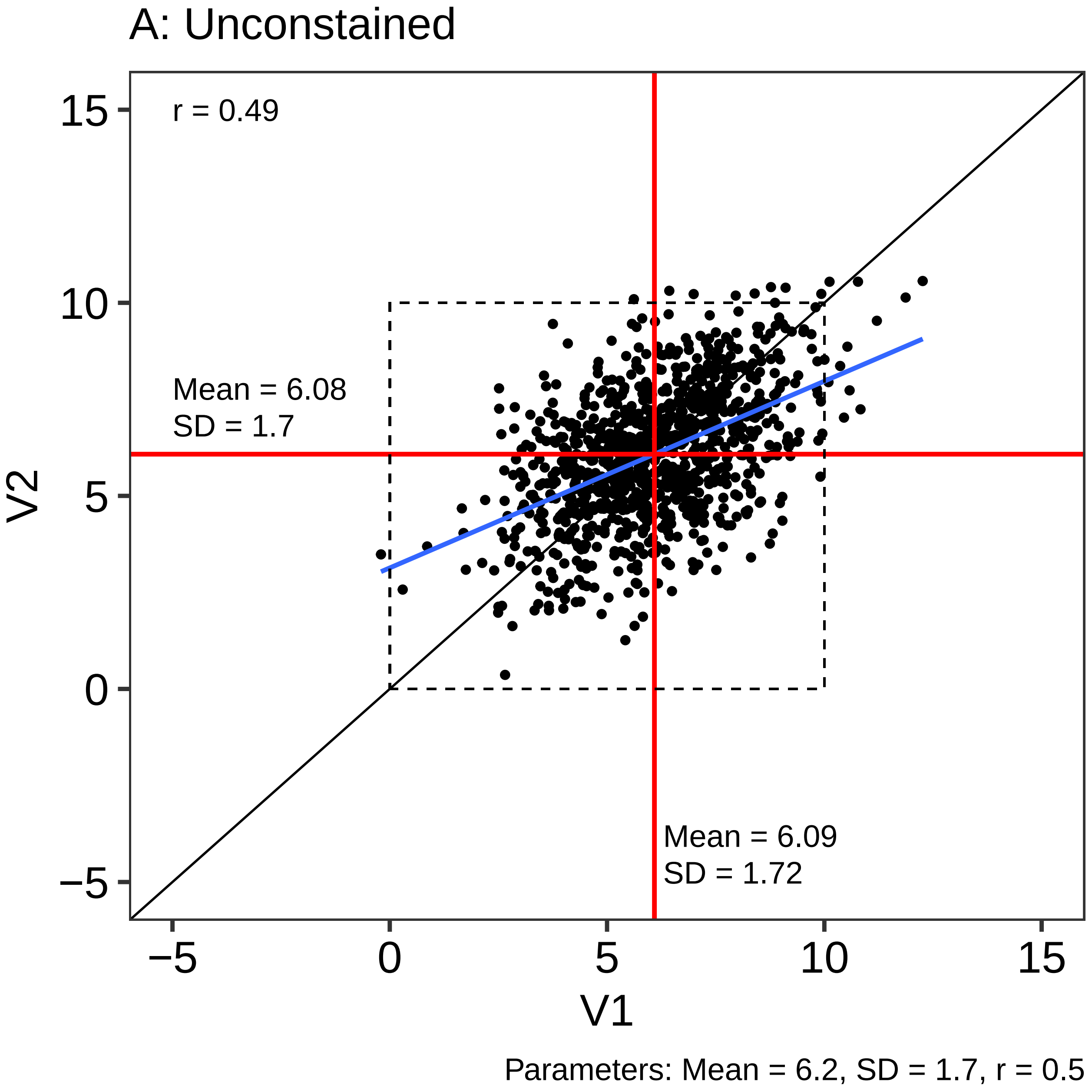
Generate and summarise data

Base data

```r
# Set the random seed for reproducibility
set.seed(2019)

# Generate the data (1000 pairs)
cor_05.base <- as.data.frame(mvrnorm(n = 1000, mu = c(6.2, 6.2), Sigma = cov_05))

# Plot base data
ggplot(data = cor_05.base) +
    aes(x = V1, y = V2) +
    geom_point() +
    geom_abline(slope = 1, intercept = 0) +
    geom_smooth(method = 'lm',
                se = FALSE) +
    geom_hline(yintercept = mean(cor_05.base$V2),
               colour = 'red', size = 1) +
    geom_vline(xintercept = mean(cor_05.base$V1),
               colour = 'red', size = 1) +
    geom_rect(ymin = 0, ymax = 10,
              xmin = 0, xmax = 10,
              colour = '#000000',
              alpha = 0,
              linetype = 2) +
    annotate(geom = 'text', x = -5, y = 15, hjust = 0, size = 5,
             label = str_glue("r = {round(cor(cor_05.base$V1,
                                 cor_05.base$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_05.base$V2) + 1.7,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_05.base$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_05.base$V2) + 0.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_05.base$V2),2)}")) +
    annotate(geom = 'text', x = mean(cor_05.base$V1) + 0.2, y = -3.8,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_05.base$V1), 2)}")) +
    annotate(geom = 'text', x = mean(cor_05.base$V1) + 0.2, y = -4.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_05.base$V1), 2)}")) +
    labs(title = 'A: Unconstained',
         caption = 'Parameters: Mean = 6.2, SD = 1.7, r = 0.5') +
    scale_y_continuous(limits = c(-5, 15)) +
    scale_x_continuous(limits = c(-5, 15)) +
    theme(plot.caption = element_text(size = 14))
```



```
# Linear regression
summary(lm(V2 ~ V1, data = cor_05.base))
```

```
##
## Call:
## lm(formula = V2 ~ V1, data = cor_05.base)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.4932 -0.9854  0.0613  0.9882  4.5026
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.13520    0.17229   18.20   <2e-16 ***
## V1           0.48347    0.02723   17.76   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.481 on 998 degrees of freedom
## Multiple R-squared:  0.2401, Adjusted R-squared:  0.2394
## F-statistic: 315.4 on 1 and 998 DF,  p-value: < 2.2e-16
```

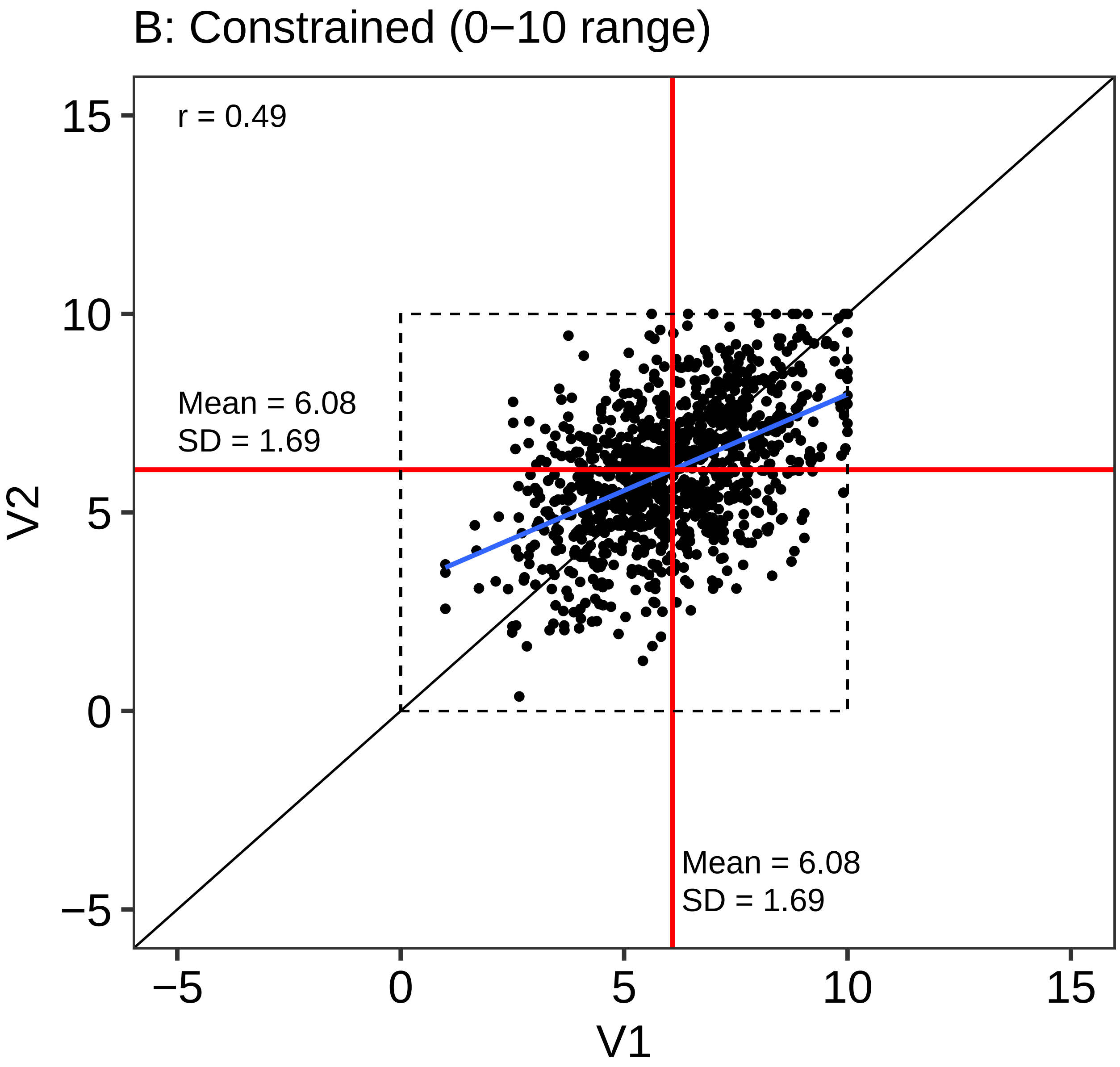
```
Confint(lm(V2 ~ V1, data = cor_05.base))
```

```
##              Estimate     2.5 %    97.5 %
## (Intercept) 3.1352016 2.7971072 3.4732960
## V1          0.4834716 0.4300464 0.5368969
```

Constrain values to 0-10 range

```
# Process data
cor_05.constrained <- cor_05.base %>%
    mutate(V1 = case_when(
               V1 < 1 ~ 1,
               V1 > 10 ~ 10,
               TRUE ~ V1)) %>%
    mutate(V2 = case_when(
               V2 < 0 ~ 0,
               V2 > 10 ~ 10,
               TRUE ~ V2)) %>%
    mutate(group = 'No threshold')

# Plot processed data
ggplot(data = cor_05.constrained) +
    aes(x = V1, y = V2) +
    geom_point() +
    geom_abline(slope = 1, intercept = 0) +
    geom_smooth(method = 'lm',
                se = FALSE) +
    geom_hline(yintercept = mean(cor_05.constrained$V2),
               colour = 'red', size = 1) +
    geom_vline(xintercept = mean(cor_05.constrained$V1),
               colour = 'red', size = 1) +
    geom_rect(ymin = 0, ymax = 10,
              xmin = 0, xmax = 10,
              colour = '#000000',
              alpha = 0,
              linetype = 2) +
    annotate(geom = 'text', x = -5, y = 15, hjust = 0, size = 5,
             label = str_glue("r = {round(cor(cor_05.constrained$V1,
                              cor_05.constrained$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_05.constrained$V2) + 1.7,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_05.constrained$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_05.constrained$V2) + 0.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_05.constrained$V2),2)}")) +
    annotate(geom = 'text', x = mean(cor_05.constrained$V1) + 0.2, y = -3.8,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_05.constrained$V1), 2)}")) +
    annotate(geom = 'text', x = mean(cor_05.constrained$V1) + 0.2, y = -4.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_05.constrained$V1), 2)}")) +
    labs(title = 'B: Constrained (0-10 range)',
         caption = 'Parameters: Mean = 6.2, SD = 1.7, r = 0.5') +
    scale_y_continuous(limits = c(-5, 15)) +
    scale_x_continuous(limits = c(-5, 15)) +
    theme(plot.caption = element_text(size = 14))
```



B: Constrained (0−10 range)

Parameters: Mean = 6.2, SD = 1.7, r = 0.5

```
# Linear regression
summary(lm(V2 ~ V1, data = cor_05.constrained))
```

```
##
## Call:
## lm(formula = V2 ~ V1, data = cor_05.constrained)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.4919 -0.9759  0.0602  0.9929  4.5058
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.12758    0.17463   17.91   <2e-16 ***
## V1           0.48465    0.02766   17.52   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.477 on 998 degrees of freedom
## Multiple R-squared:  0.2352, Adjusted R-squared:  0.2345
## F-statistic:   307 on 1 and 998 DF,  p-value: < 2.2e-16
```

```r
Confint(lm(V2 ~ V1, data = cor_05.constrained))
```

```
##              Estimate     2.5 %    97.5 %
## (Intercept) 3.1275846 2.7848912 3.4702781
## V1          0.4846463 0.4303639 0.5389286
```

Mean = 6.2, SD = 1.7, r = 0.2

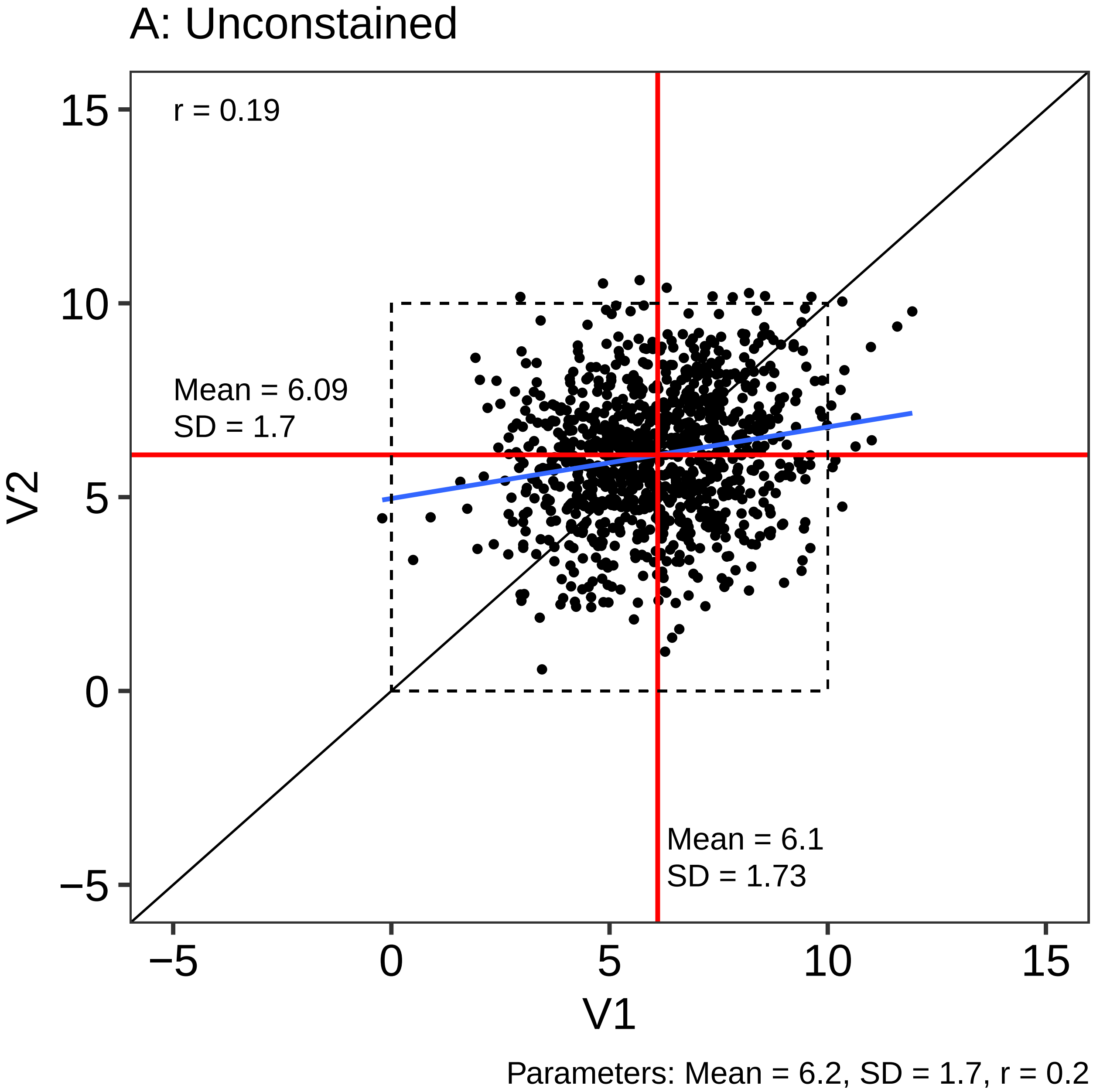
Generate and summarise data

Base data

```r
# Set the random seed for reproducibility
set.seed(2019)

# Generate the data
cor_02.base <- as.data.frame(mvrnorm(n = 1000, mu = c(6.2, 6.2), Sigma = cov_02))

# Plot base data
ggplot(data = cor_02.base) +
    aes(x = V1, y = V2) +
    geom_point() +
    geom_abline(slope = 1, intercept = 0) +
    geom_smooth(method = 'lm',
                se = FALSE) +
    geom_hline(yintercept = mean(cor_02.base$V2),
               colour = 'red', size = 1) +
    geom_vline(xintercept = mean(cor_02.base$V1),
               colour = 'red', size = 1) +
    geom_rect(ymin = 0, ymax = 10,
              xmin = 0, xmax = 10,
              colour = '#000000',
              alpha = 0,
              linetype = 2) +
    annotate(geom = 'text', x = -5, y = 15, hjust = 0, size = 5,
             label = str_glue("r = {round(cor(cor_02.base$V1,
                              cor_02.base$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_02.base$V2) + 1.7,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_02.base$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_02.base$V2) + 0.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_02.base$V2),2)}")) +
    annotate(geom = 'text', x = mean(cor_02.base$V1) + 0.2, y = -3.8,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_02.base$V1), 2)}")) +
    annotate(geom = 'text', x = mean(cor_02.base$V1) + 0.2, y = -4.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_02.base$V1), 2)}")) +
    labs(title = 'A: Unconstained',
         caption = 'Parameters: Mean = 6.2, SD = 1.7, r = 0.2') +
    scale_y_continuous(limits = c(-5, 15)) +
    scale_x_continuous(limits = c(-5, 15)) +
    theme(plot.caption = element_text(size = 14))
```



```
# Linear regression
summary(lm(V2 ~ V1, data = cor_02.base))
```

```
##
## Call:
## lm(formula = V2 ~ V1, data = cor_02.base)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.1078 -1.0744  0.0558  1.1482  4.6550
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.96474    0.19426  25.557  < 2e-16 ***
## V1           0.18442    0.03063   6.021 2.43e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.67 on 998 degrees of freedom
## Multiple R-squared:  0.03506,    Adjusted R-squared:  0.03409
## F-statistic: 36.26 on 1 and 998 DF,  p-value: 2.427e-09
```

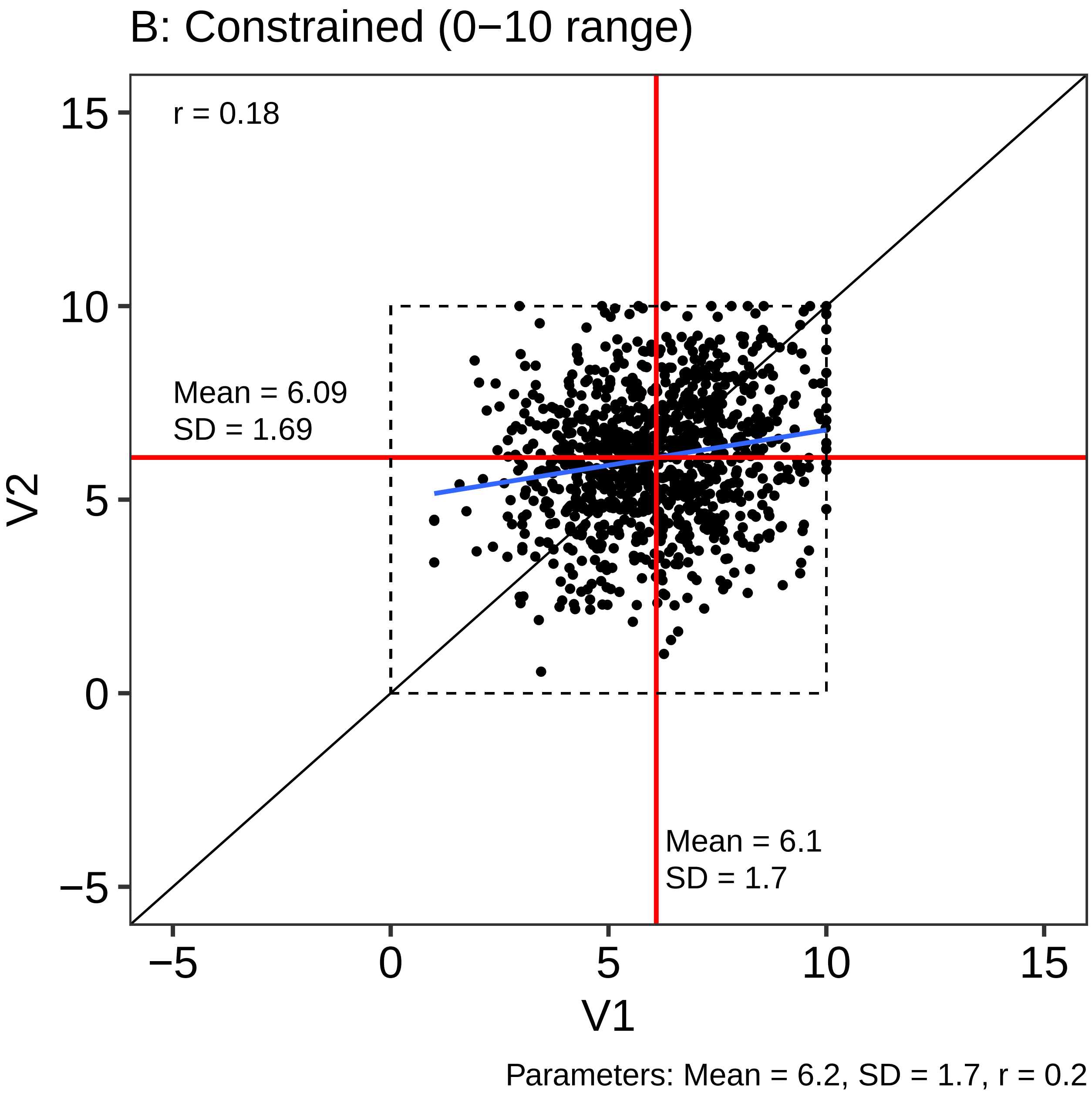
```
Confint(lm(V2 ~ V1, data = cor_02.base))
```

```
##              Estimate     2.5 %    97.5 %
## (Intercept) 4.9647355 4.5835301 5.3459409
## V1          0.1844196 0.1243185 0.2445207
```

Constrain values to 0-10 range

```
# Process data
cor_02.constrained <- cor_02.base %>%
    mutate(V1 = case_when(
                V1 < 1 ~ 1,
                V1 > 10 ~ 10,
                TRUE ~ V1)) %>%
    mutate(V2 = case_when(
                V2 < 0 ~ 0,
                V2 > 10 ~ 10,
                TRUE ~ V2)) %>%
    mutate(group = 'No threshold')

# Plot processed data
ggplot(data = cor_02.constrained) +
    aes(x = V1, y = V2) +
    geom_point() +
    geom_abline(slope = 1, intercept = 0) +
    geom_smooth(method = 'lm',
                 se = FALSE) +
    geom_hline(yintercept = mean(cor_02.constrained$V2),
                colour = 'red', size = 1) +
    geom_vline(xintercept = mean(cor_02.constrained$V1),
                colour = 'red', size = 1) +
    geom_rect(ymin = 0, ymax = 10,
              xmin = 0, xmax = 10,
              colour = '#000000',
              alpha = 0,
              linetype = 2) +
    annotate(geom = 'text', x = -5, y = 15, hjust = 0, size = 5,
             label = str_glue("r = {round(cor(cor_02.constrained$V1,
                              cor_02.constrained$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_02.constrained$V2) + 1.7,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_02.constrained$V2), 2)}")) +
    annotate(geom = 'text', x = -5, y = mean(cor_02.constrained$V2) + 0.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_02.constrained$V2),2)}")) +
    annotate(geom = 'text', x = mean(cor_02.constrained$V1) + 0.2, y = -3.8,
             hjust = 0, size = 5,
             label = str_glue("Mean = {round(mean(cor_02.constrained$V1), 2)}")) +
    annotate(geom = 'text', x = mean(cor_02.constrained$V1) + 0.2, y = -4.75,
             hjust = 0, size = 5,
             label = str_glue("SD = {round(sd(cor_02.constrained$V1), 2)}")) +
    labs(title = 'B: Constrained (0-10 range)',
         caption = 'Parameters: Mean = 6.2, SD = 1.7, r = 0.2') +
    scale_y_continuous(limits = c(-5, 15)) +
    scale_x_continuous(limits = c(-5, 15)) +
    theme(plot.caption = element_text(size = 14))
```


B: Constrained (0−10 range)

Parameters: Mean = 6.2, SD = 1.7, r = 0.2

```r
# Linear regression
summary(lm(V2 ~ V1, data = cor_02.constrained))
```

```
##
## Call:
## lm(formula = V2 ~ V1, data = cor_02.constrained)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.1061 -1.0728  0.0554  1.1495  4.4858
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.97445    0.19646  25.321  < 2e-16 ***
## V1           0.18259    0.03104   5.882 5.52e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.665 on 998 degrees of freedom
## Multiple R-squared:  0.03351,    Adjusted R-squared:  0.03254
## F-statistic:  34.6 on 1 and 998 DF,  p-value: 5.524e-09
```

```
Confint(lm(V2 ~ V1, data = cor_02.constrained))
```

```
##              Estimate     2.5 %    97.5 %
## (Intercept) 4.9744475 4.5889347 5.3599602
## V1          0.1825921 0.1216767 0.2435075
```

Session information

```
sessionInfo()
```

```
## R version 3.6.1 (2019-07-05)
## Platform: x86_64-apple-darwin15.6.0 (64-bit)
## Running under: macOS Mojave 10.14.6
##
## Matrix products: default
## BLAS:   /Library/Frameworks/R.framework/Versions/3.6/Resources/lib/libRblas.0.dylib
## LAPACK: /Library/Frameworks/R.framework/Versions/3.6/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
##  [1] smatr_3.4-8     car_3.0-3       carData_3.0-2   MBESS_4.6.0
##  [5] MASS_7.3-51.4   magrittr_1.5    forcats_0.4.0   stringr_1.4.0
##  [9] dplyr_0.8.3     purrr_0.3.2     readr_1.3.1     tidyr_1.0.0
## [13] tibble_2.1.3    ggplot2_3.2.1   tidyverse_1.2.1
##
## loaded via a namespace (and not attached):
##  [1] tidyselect_0.2.5  xfun_0.10         haven_2.1.1
##  [4] lattice_0.20-38   colorspace_1.4-1  vctrs_0.2.0
##  [7] generics_0.0.2    htmltools_0.4.0   yaml_2.2.0
## [10] rlang_0.4.0       pillar_1.4.2      foreign_0.8-72
## [13] glue_1.3.1        withr_2.1.2       modelr_0.1.5
## [16] readxl_1.3.1      lifecycle_0.1.0   munsell_0.5.0
## [19] gtable_0.3.0      cellranger_1.1.0  zip_2.0.4
## [22] rvest_0.3.4       evaluate_0.14     labeling_0.3
## [25] rio_0.5.16        knitr_1.25        curl_4.2
## [28] broom_0.5.2       Rcpp_1.0.2        scales_1.0.0
## [31] backports_1.1.5   jsonlite_1.6      abind_1.4-5
## [34] hms_0.5.1         digest_0.6.21     openxlsx_4.1.0.1
## [37] stringi_1.4.3     grid_3.6.1        cli_1.1.0
## [40] tools_3.6.1       lazyeval_0.2.2    crayon_1.3.4
## [43] pkgconfig_2.0.3   zeallot_0.1.0     data.table_1.12.2
## [46] xml2_1.2.2        lubridate_1.7.4   assertthat_0.2.1
## [49] rmarkdown_1.16    httr_1.4.1        rstudioapi_0.10
## [52] R6_2.4.0          nlme_3.1-141      compiler_3.6.1
```