algorithmica
re-engineering it talent

Top-20 Training Program
(DP Thinking)

Apply the recursive thinking discussed in class and check whether we need memory(dynamic programming) technique to solve the following problems.

Problem1: Computing Pascal Coefficients

Write an efficient function to calculate n c r which is recursively defined as follows:

$$\binom{n}{r} = \begin{cases} 1 & r = 0 \text{ or } r = n \\ \binom{n-1}{r-1} + \binom{n-1}{r} & otherwise \end{cases}$$

What are the time and space complexities of your solution?

Problem2: Longest Nesting of Boxes

A d-dimensional box with dimensions (x1, x2,..., xd) nests within another box with dimensions (y1, y2,..., yd) if there exists a permutation π on {1, 2,..., d} such that xπ(1) < y1, xπ(2) < y2,..., xπ(d)< yd. Given a set of n 3-dimensional boxes, write an efficient function to determine the length of the longest sequence of boxes b1, b2, ..,bk such that each box bi nests in box bi+1($1 \le k < n$). What are the time and space complexities of your solution?

Problem3: Maximum Profitable Job Execution

There are n jobs each of which takes a unit time to execute. Each job say i has a deadline d_i and a profit p_i associated to it. The job earns the profit only if it is scheduled before its deadline otherwise the profit earned is zero. At any instant only **one** job can be executed.An optimal schedule is a schedule that maximizes the overall profit earned from the execution of the jobs. Give a polynomial time algorithm that takes the number of jobs, the profit matrix and the deadline matrix and returns the profit earned by an optimal schedule of the jobs.

Function Prototype:

```
int schedule(int n, int []profit, int []deadline);
```

Example:

Suppose n = 4; profit = [50, 10, 15, 30]; deadline = [2, 1, 2, 1].
The optimal schedule is: 4, 1
The maximum profit is: 80

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Problem4: Maximum Contiguous Squared Area of Ones

Given an N*N array A of zeroes and ones, write an efficient function to find the size(or area) of the largest contiguous all-ones square.