Systemy Inteligentne 2

Pamięć asocjacyjna

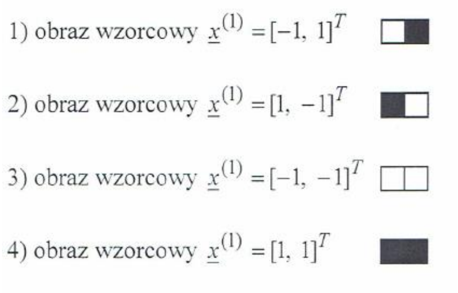
Polecenie:

a) Narysować strukturę pamięci asocjacyjnej (neuronowej, rekurencyjnej sieci Hopfielda), w której można zapisać obraz wzorcowy przedstawiony poniżej.

b) Stosując regułę Hebba zapisu do pamięci, określić macierz wag pamięci zawierającej przedstawiony poniżej obraz wzorcowy.

c) Zbudować graf przejść uzyskanej pamięci dla odczytu asynchronicznego.

d) Zbudować graf przejść uzyskanej pamięci dla odczytu synchronicznego.



1) obraz wzorcowy $\underline{x}^{(1)} = [-1,\ 1]^T$

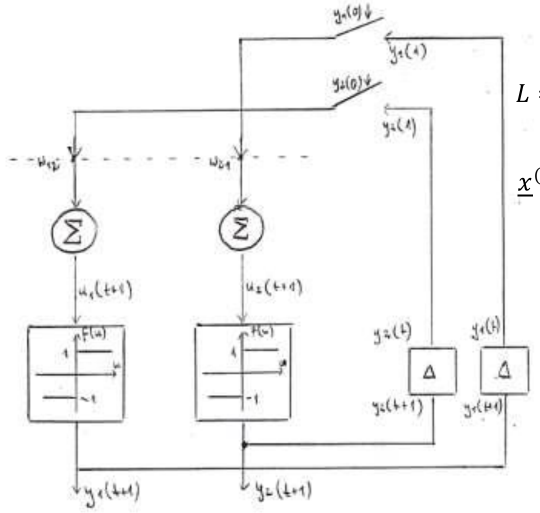
2) obraz wzorcowy $\underline{x}^{(1)} = [1,\ -1]^T$

3) obraz wzorcowy $\underline{x}^{(1)} = [-1,\ -1]^T$

4) obraz wzorcowy $\underline{x}^{(1)} = [1,\ 1]^T$

Przykład 1.

a)



$L = \{\underline{x}^{(k)}\}_{k=1}^{K} = \left\{\left[x_1^{(k)}, x_2^{(k)}\right]^T\right\}_{k=1}^{K=1} \qquad n = 2$

$\underline{x}^{(1)} = [-1,1]^T$

b)

$$\underline{W} = \sum_{k=1}^{K} \underline{x}^{(k)} * \underline{x}^{(k)^T} - KI\,, gdzie\ I - macierz\ jednostkowa$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{W} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} * [-1 \quad 1] - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$\underline{W} = \begin{bmatrix} 0 & w_{12} \\ w_{21} & 0 \end{bmatrix} = \begin{bmatrix} \underline{w_1} \\ \underline{w_2} \end{bmatrix}^T$$

$\underline{w_1} = [0\ w_{12}]^T \quad \underline{w_2} = [w_{21}\ 0]^T$

$\underline{y}(t) = [y_1(t)\ y_2(t)]^T$

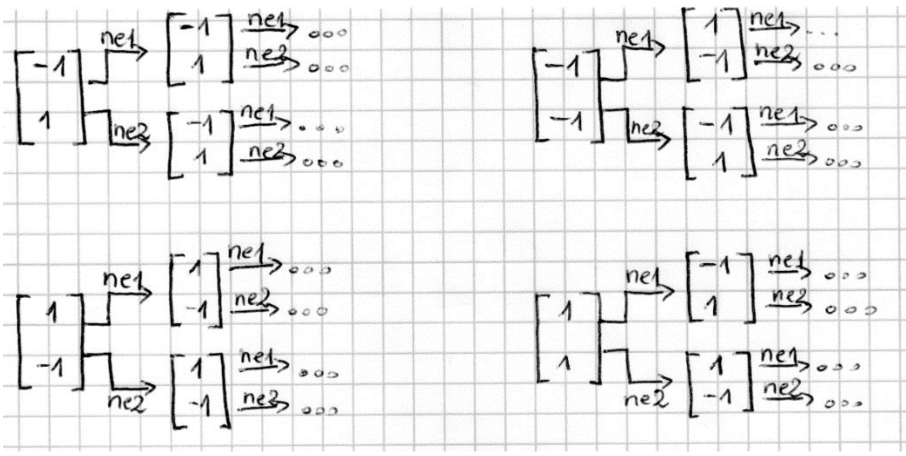
Energia:

$$E(y) = -\frac{1}{2}\underline{y}^T * \underline{W} * \underline{y}$$

$$E(y) = -\frac{1}{2} * [y_1 \quad y_2] * \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = -\frac{1}{2} * \begin{bmatrix} -y_2 \\ -y_1 \end{bmatrix} * \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = -\frac{1}{2} * \begin{bmatrix} -y_1 y_2 \\ -y_1 y_2 \end{bmatrix} = \frac{1}{2} y_1 y_2 + \frac{1}{2} y_1 y_2 = y_1 y_2$$

$E([-1 \quad -1]^T) = 1$
$E([1 \quad 1]^T) = 1$
$E([-1 \quad 1]^T) = -1$
$E([1 \quad -1]^T) = -1$

c)



$$sgn(x) = \begin{cases} 1, x > 0 \\ stan\ poprzedni, x = 0 \\ -1, x < 0 \end{cases}$$

$sgn\left([0 \quad -1] \begin{bmatrix} -1 \\ 1 \end{bmatrix}\right) = sgn(-1) = -1$

$sgn\left([-1 \quad 0] \begin{bmatrix} -1 \\ 1 \end{bmatrix}\right) = sgn(1) = 1$

$sgn\left([0 \quad -1] \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = sgn(1) = 1$

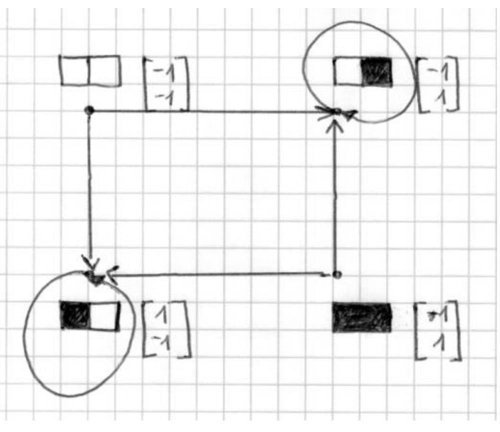
$sgn\left([-1 \quad 0] \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = sgn(-1) = -1$

$sgn\left([0 \quad -1] \begin{bmatrix} -1 \\ -1 \end{bmatrix}\right) = sgn(1) = 1$

$sgn\left([-1 \quad 0] \begin{bmatrix} -1 \\ -1 \end{bmatrix}\right) = sgn(1) = 1$

$sgn\left([0 \quad -1] \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = sgn(-1) = -1$

$sgn\left([-1 \quad 0] \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = sgn(-1) = -1$



d)

$y_0 = [-1 \quad 1]^T$

$sgn\left(\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right) = sgn\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$y_0 = [1 \quad -1]^T$

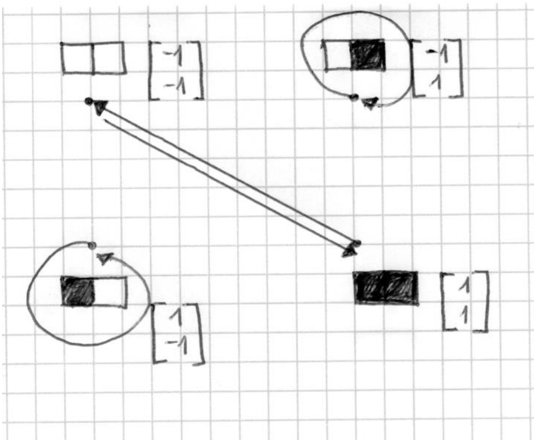
$sgn\left(\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = sgn\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$y_0 = [1 \;\; 1]^T$

$sgn\left(\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = sgn\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

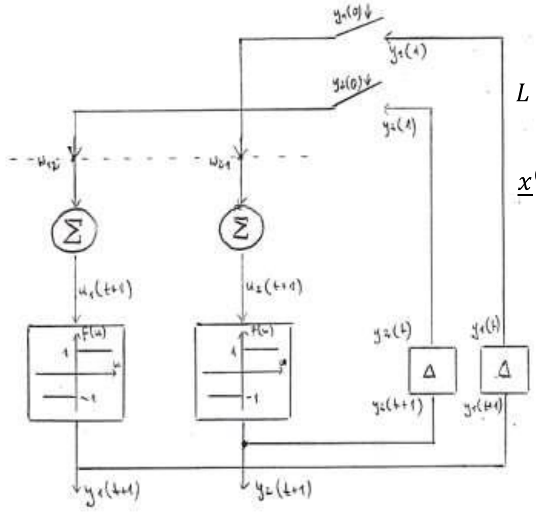
$y_0 = [-1 \quad -1]^T$

$sgn\left(\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix}\right) = sgn\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



Przykład 3.

a)



$L = \{\underline{x}^{(k)}\}_{k=1}^{K} = \left\{\left[x_1^{(k)}, x_2^{(k)}\right]^T\right\}_{k=1}^{K=1} \qquad n = 2$

$\underline{x}^{(1)} = [-1, -1]^T$

b)

$$\underline{W} = \sum_{k=1}^{K} \underline{x}^{(k)} * \underline{x}^{(k)^T} - KI \text{ , } gdzie\ I - macierz\ jednostkowa$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{W} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} * \begin{bmatrix} -1 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\underline{W} = \begin{bmatrix} 0 & w_{12} \\ w_{21} & 0 \end{bmatrix} = \begin{bmatrix} \underline{w_1} \\ \underline{w_2} \end{bmatrix}^T$$

$\underline{w_1} = [0\ w_{12}]^T \quad \underline{w_2} = [w_{21}\ 0]^T$

$\underline{y}(t) = [y_1(t)\ y_2(t)]^T$

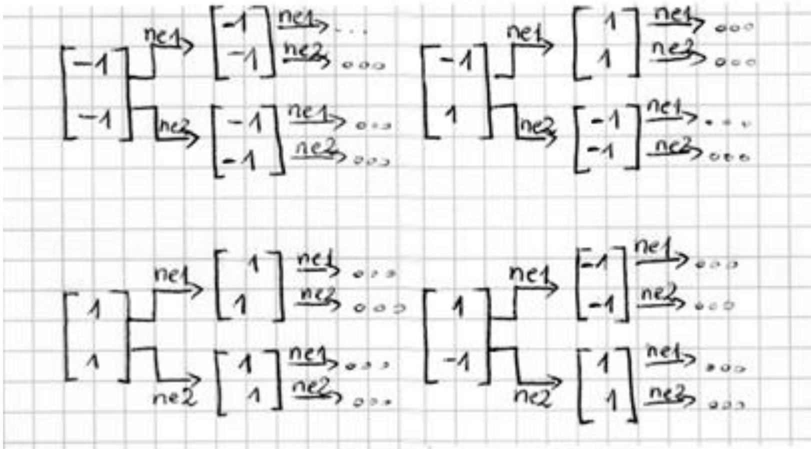
Energia:

$$E(y) = -\frac{1}{2}\underline{y}^T * \underline{W} * \underline{y}$$

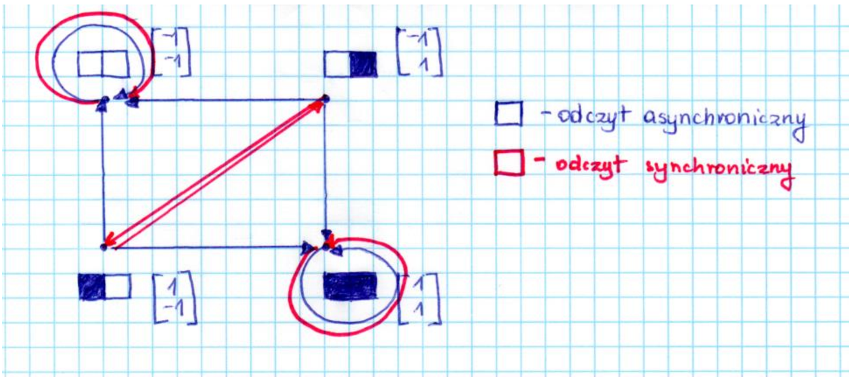
$$E(y) = -\frac{1}{2} * \begin{bmatrix} y_1 & y_2 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = -\frac{1}{2} * \begin{bmatrix} y_2 \\ y_1 \end{bmatrix} * \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = -\frac{1}{2} * \begin{bmatrix} y_1 y_2 \\ y_1 y_2 \end{bmatrix} = -\frac{1}{2} y_1 y_2 - \frac{1}{2} y_1 y_2 = -y_1 y_2$$

$E([-1 \quad -1]^T) = -1$
$E([1 \quad 1]^T) = -1$
$E([-1 \quad 1]^T) = 1$
$E([1 \quad -1]^T) = 1$

c)



$$sgn\left(\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}-1\\-1\end{bmatrix}\right) = sgn(-1) = -1$$
$$sgn\left(\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}-1\\-1\end{bmatrix}\right) = sgn(-1) = -1$$
$$sgn\left(\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\right) = sgn(1) = 1$$
$$sgn\left(\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\right) = sgn(1) = 1$$
$$sgn\left(\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}\right) = sgn(1) = 1$$
$$sgn\left(\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}\right) = sgn(-1) = -1$$
$$sgn\left(\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}\right) = sgn(-1) = -1$$
$$sgn\left(\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}\right) = sgn(1) = 1$$



d)

$y_0 = [-1 \;\; 1]^T$

$$sgn\left(\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}\right) = sgn\left(\begin{bmatrix}1\\-1\end{bmatrix}\right) = \begin{bmatrix}1\\-1\end{bmatrix}$$

$y_0 = [1 \;\; -1]^T$

$$sgn\left(\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}\right) = sgn\left(\begin{bmatrix}-1\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\1\end{bmatrix}$$

$y_0 = [1 \;\; 1]^T$

$$sgn\left(\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\right) = sgn\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix}$$

$y_0 = [-1 \;\; -1]^T$

$$sgn\left(\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}-1\\-1\end{bmatrix}\right) = sgn\left(\begin{bmatrix}-1\\-1\end{bmatrix}\right) = \begin{bmatrix}-1\\-1\end{bmatrix}$$

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