HW4 DSCI552

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1 ISLR, 4.8.3

$$Q_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}\left(x-\mu_k\right)^2\right)}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma_l} \exp\left(-\frac{1}{2\sigma_l^2}\left(x-\mu_l\right)^2\right)} \tag{1}$$

$$\begin{aligned}\log\left(p_k(x)\right)\log\left(\sum_{l=1}^{K}\pi_l \frac{1}{\sqrt{2\pi}\sigma_l}\exp\left(-\frac{1}{2\sigma_l^2}\left(x-\mu_l\right)^2\right)\right) \\ = \log\left(\pi_k\right) + \log\left(\frac{1}{\sqrt{2\pi}\sigma_k}\right) + -\frac{1}{2\sigma_k^2}\left(x-\mu_k\right)^2\end{aligned} \tag{2}$$

$$\delta_k(x) = \log\left(\pi_k\right) + \log\left(\frac{1}{\sqrt{2\pi}\sigma_k}\right) + -\frac{1}{2\sigma_k^2}\left(x-\mu_k\right)^2 \tag{3}$$

This is equivalent to assigning the observation to the class for which $\delta_k(x)$ is largest. As you can see, $\delta_k(x)$ is a quadratic function of x.

2 ISLR, 4.8.7

$$p_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_k\right)^2\right)}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_l\right)^2\right)}$$

$$\begin{aligned}
p_{\text{yes}}(x) =& \frac{\pi_{\text{yes}} \exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_{\text{yes}}\right)^2\right)}{\sum_{l=1}^{K} \pi_l \exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_l\right)^2\right)} \\
=& \frac{\pi_{yes} \exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_{\text{yes}}\right)^2\right)}{\pi_{\text{yes}} \exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_{\text{yes}}\right)^2\right) + \pi_{no} \exp\left(-\frac{1}{2\sigma^2}\left(x-\mu_{no}\right)^2\right)} \\
=& \frac{0.80 \exp\left(-\frac{1}{2*36}(x-10)^2\right)}{0.80 \exp\left(-\frac{1}{2*36}(x-10)^2\right) + 0.20 \exp\left(-\frac{1}{2*36}x^2\right)}
\end{aligned}$$

$$p_{yes}(4) = \frac{0.80 \exp\left(-\frac{1}{2*36}(4-10)^2\right)}{0.80 \exp\left(-\frac{1}{2*36}(4-10)^2\right) + 0.20 \exp\left(-\frac{1}{2*36}4^2\right)} = 75.2\% \tag{4}$$