Star Scheduling

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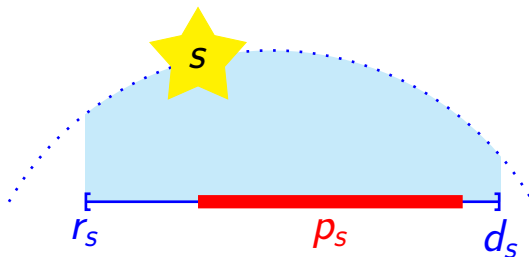
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Looking at the stars

- $[r_s; d_s)$ is the visibility interval
- p_s is the required duration of observation
- w_s is the interest

scheduling the observation of star s means observing s for a continuous duration p_s within the visibility interval $[r_s; d_s)$, rewarding w_s

Star scheduling (one night)

Instance: a set $\mathcal{S}$ of stars; each star $s \in \mathcal{S}$ has an interest w_s, an observation duration p_s and a visibility window $[r_s; d_s)$
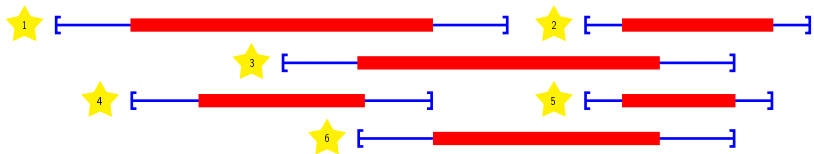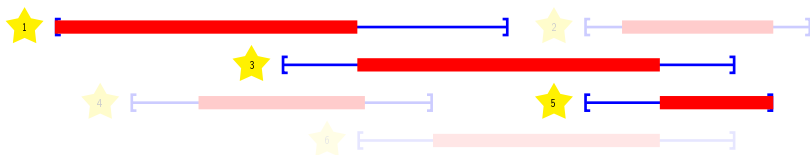
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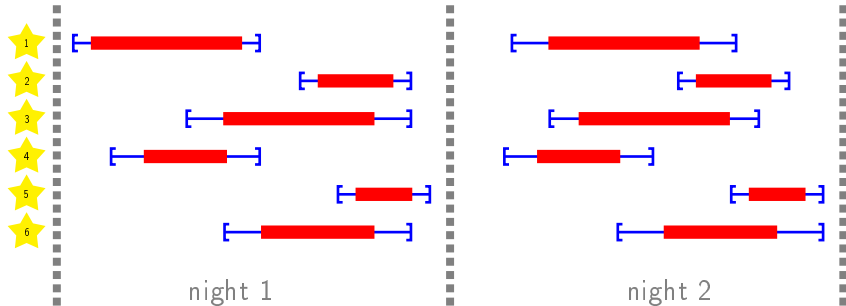

Question: find $\mathcal{S}' \subset \mathcal{S}$ and starting times $t_s, \forall s \in \mathcal{S}'$ such that

- for each $s \in \mathcal{S}'$: $[t_s; t_s + p_s) \subset [r_s; d_s)$
- for each $(s_1, s_2) \in \mathcal{S}'^2$: $[t_{s_1}; t_{s_1} + p_{s_1}) \cap [t_{s_2}; t_{s_2} + p_{s_2}) = \emptyset$
- $\sum_{s \in \mathcal{S}'} w_s$ is maximized

Star scheduling (several nights)

Instance: a set $\mathcal{N}$ of nights, a set $\mathcal{S}$ of stars; each star $s \in \mathcal{S}$ has an interest w_s, an observation duration p_s^n and a visibility window $[r_s^n; d_s^n)$, depending on the night n of the observation

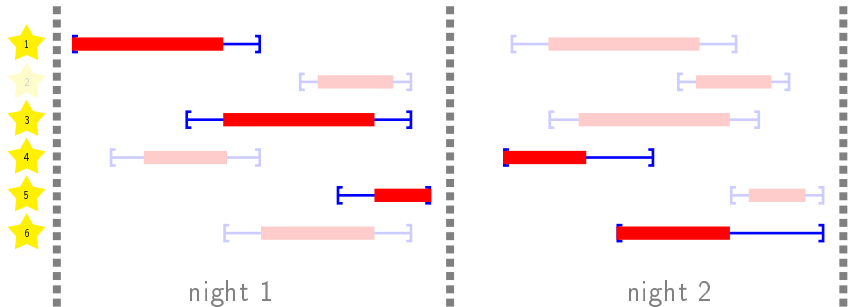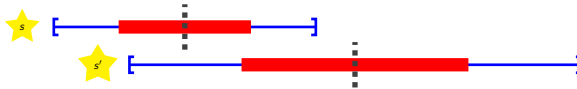night 1   night 2

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night 1   night 2

The order is known!

The meridian instant ($m_s = \frac{r_s + d_s}{2}$) is a **mandatory** instant of observation, that is: for every star s, $p_s^n \geq \frac{d_s^n - r_s^n}{2}$

Property

If for all star s: $p_s^n \geq \frac{d_s^n - r_s^n}{2}$, then observations must be scheduled by non-decreasing meridian time

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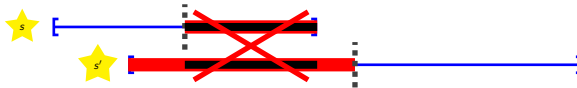

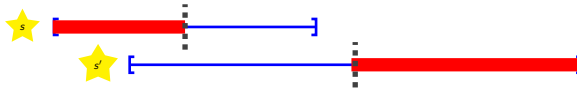## Property

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A MIP model

$$\max \sum_{s \in \mathcal{S}} w_s \boxed{z_s} \longrightarrow = 1 \text{ iff } s \text{ observed}$$

$$s.c. \sum_{n \in \mathcal{N}} \boxed{z_s^n} = z_s \longrightarrow = 1 \text{ iff } s \text{ observed on night } n$$

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visibility window
of night n
$$\begin{cases} r_s^n z_s^n \leq \boxed{t_s} \longrightarrow \text{ starting time of observation } s \\ \\ t_s + p_s^n z_s^n \leq d_s^n z_s^n + M(1 - z_s^n) \end{cases}$$

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$s \prec s'$ if observed the same night

$$z_s^n + z_{s'}^n - 1 \leq \boxed{y_{ss'}} \longrightarrow = 1 \text{ iff } s \text{ and } s' \text{ observed the same night}$$

$$t_s + p_s^n \leq t_{s'} + M(1 - z_{ss'})$$

Introduction

Complexity

Solving star scheduling

Conclusions and perspectives

Scheduling one night

Instance: a set $\mathcal{S}$ of stars; each star $s \in \mathcal{S}$ has an interest w_s, an observation duration p_s and a visibility window $[r_s; d_s)$ such that $p_s \geq (d_s - r_s)/2$; a bound W

Question: find a subset of stars so that the total interest is at least W, visibility windows are respected and observations do not overlap

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Complexity of the one night case

Star scheduling of one night is NP-Hard (even if $w_s = 1$)

Scheduling of one night is NP-hard

- Variant of Partition : $2n$ pairs (a_{2i-1}, a_{2i}) so that $\sum_i a_i = 2B$. Can a total of B be made with one item from each pair?

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- For each pair of items (a_{2i}, a_{2i+1}) create a pair of incompatible stars (s_{2i-1}, s_{2i}): same visibility window and $p_s \geq (d_s - r_s)/2$

$$p_{2i-1} = p_{2i} = 2B + a_i$$

G-SCOP

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- Set visibility windows so that scheduling a star from a pair does not prevent scheduling a star from another pair and each pair of stars have dedicated instants

$$r_{2i-1} = r_{2i} = 2(i-1)B + \sum_{j=1}^{i-1} \min(a_{2i-1}, a_{2i})$$
$$d_{2i-1} = d_{2i} = 2iB + \sum_{j=1}^{i} \max(a_{2i-1}, a_{2i})$$
$$d_{2n-1} = d_{2n} = (2n+1)B$$

$$p_{2i-1} = p_{2i} = 2B + a_i$$
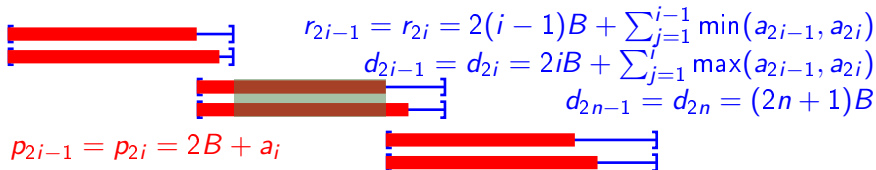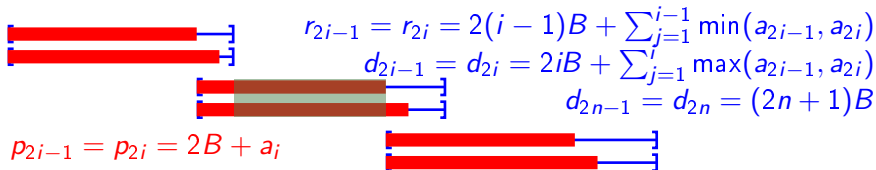
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- "yes" to Partition $\iff$ non-idling schedule of length $(2n+1)B$

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$$p_{2i-1} = p_{2i} = 2B + a_i$$

A pseudo-polynomial algorithm

$f(i, t)$: maximum interest with stars 1 to i, and such that s_i ends before time t

$$f(i, t) =$$

$$\begin{cases} \min(f(i-1, t), f(i-1, t - p_i) + w_i) & \forall i \in [1, m], t \in [r_i + p_i, T] \\ f(i-1, t) & \forall i \in [1, m], t \in [0, r_i + p_i[\\ -\infty & \forall i \in [1, m], t < 0 \\ 0 & i = 0, \forall t \in [0, T] \end{cases}$$

We are looking for $f(m, T)$ which can be computed in $O(mT)$

Scheduling several nights

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Complexity of the several nights case

Star scheduling of several nights is unary NP-Hard (even if $w_s = 1$ and all nights are identical)

Logic-based Bender decomposition

Master problem:
assignment of stars to nights

$$\max \sum_{s \in \mathcal{S}} w_s z_s$$

$$s.c. \sum_{n \in \mathcal{N}} z_s^n = z_s$$

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Master problem:
assignment of stars to nights

Slave problem:
scheduling of each nights

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Night 1: ok?

Night 2: ok?

...

G-SCOP

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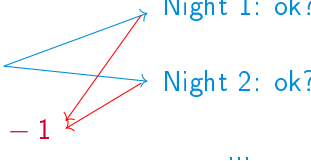$$\sum_{s \in C_k^n} z_s^n \leq |C_k^n| - 1$$

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G-SCOP
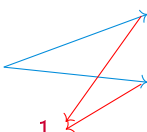
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...

- efficient MIP
- upper bound at each iteration

- n independent problems
- linear complexity

Column generation

Night patterns: Ω_n, set of all possible schedules for night n

Pattern k for night n: $p_{n1}^k \ldots p_{n|\mathcal{S}|}^k$, where $p_{ns}^k = 1$ iff star s belongs to the k-th pattern of night n; weight $w_n^k = \sum_{s \in \mathcal{S}} w_s p_{ns}^k$

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$$\max \sum_{n \in \mathcal{N}} \sum_{k \in \Omega_n} w_n^k \boxed{\rho_n^k} \longrightarrow = 1 \text{ iff pattern } k \text{ used, night } n$$

$$\sum_{k \in \Omega_n} \rho_n^k = 1 \quad \forall n \in \mathcal{N}$$

$$\sum_{n \in \mathcal{N}} \sum_{k \in \Omega_n} p_{ns}^k \rho_n^k \leq 1 \quad \forall s \in \mathcal{S}$$

$$\rho_n^k \in \{0, 1\} \quad \forall n \in \mathcal{N}, \forall k \in \Omega_n$$

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$\alpha_n \qquad \sum_{k \in \Omega_n} \rho_n^k = 1 \qquad \forall n \in \mathcal{N}$

$\beta_s \qquad \sum_{n \in \mathcal{N}} \sum_{k \in \Omega_n} p_{ns}^k \rho_n^k \leq 1 \qquad \forall s \in \mathcal{S}$

$\qquad \qquad \rho_n^k \in \{0, 1\} \qquad \forall n \in \mathcal{N}, \forall k \in \Omega_n$

Reduced cost of ρ_n^k: $w_n^k - \alpha_n - \sum_{s \in \mathcal{S}} p_{ns}^k \beta_s$

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α_n

β_s

$$\sum_{k \in \Omega_n} \rho_n^k = 1 \quad \forall n \in \mathcal{N}$$

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Reduced cost of ρ_n^k: $w_n^k - \alpha_n - \sum_{s \in \mathcal{S}} p_{ns}^k \beta_s$

Pattern of max reduced cost? Single Night Case with costs $w_s - \beta_s$

Local search

Classical local-search procedure

Neighborhoods:

- moving a star from one night to another
- inserting an unobserved star
- exchanging two stars

An optimal schedule for each night is computed systematically

Solutions

instance	$|\mathcal{S}|$	$|\mathcal{N}|$	BD (OPT/*UB*)		CG (UB)		LS (LB)	
			val	cpu(s)	val	cpu(s)	val	cpu(s)
pb1	200	32	***5200***	900	**5200**	1.47	**5200**	0.28
pb2	200	32	***3310***	900	**3310**	0.99	**3310**	0.33
pb3	200	69	**7800**	100	**7800**	1.59	**7800**	0.17
pb4	200	69	-	-	**4870**	1.63	**4870**	0.11
pb5	400	69	*12660*	900	**11910**	5.11	**11910**	12.39
pb6	400	69	*9250*	900	9099.9	19.57	9070	773.95
pb7	400	142	-	-	**13680**	11.85	**13680**	0.15
pb8	400	142	***9760***	900	**9760**	13.71	**9760**	0.21
real	800	142	*18930*	900	18620	92.41	18510	689.60
							18480	306.07

Example solution

solution

Star scheduling

- a particular interval-scheduling problem with known order
- NP-hardness is proven for both one night and general cases
- several solution methods are proposed and tested

Work in progress...

- Instances
  - we need more instances!
  - study the structure of the real instance
    (e.g., similarity between nights)

- Solution methods
  - enrich numerical experiments
  - embed the CG into a branch & bound to get optimal solutions
  - analyze (and improve) LS behavior
  - get advantage of nights' similarities

- Complexity
  - draw precise complexity frontiers
  - study approximability