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SCIENCES POUR LA CONCEPTION, L'OPTIMISATION ET LA PRODUCTION

Star Scheduling

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june 2015

UMR 5272

Laboratoire des Sciences pour la Conception, l'Optimisation et la production

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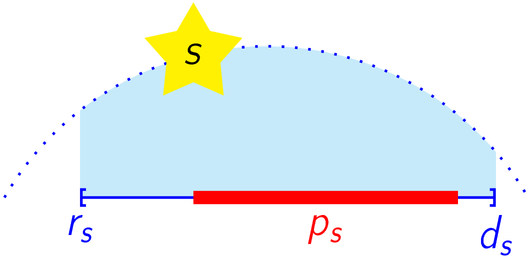
Looking at the stars

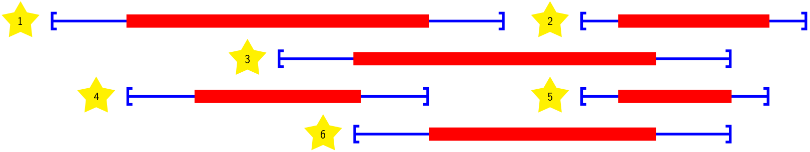


- $[r_s; d_s)$ is the visibility interval
- p_s is the required duration of observation
- w_s is the interest

scheduling the observation of star s means observing s for a continuous duration p_s within the visibility interval $[r_s; d_s)$, rewarding w_s

Star scheduling (one night)

Instance: a set $\mathcal{S}$ of stars; each star $s \in \mathcal{S}$ has an interest w_s, an observation duration p_s and a visibility window $[r_s; d_s)$



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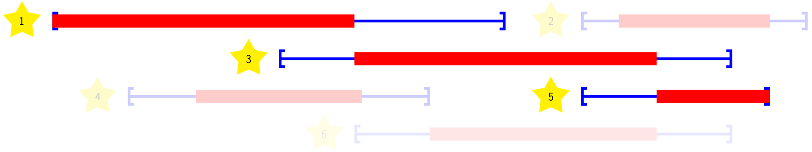
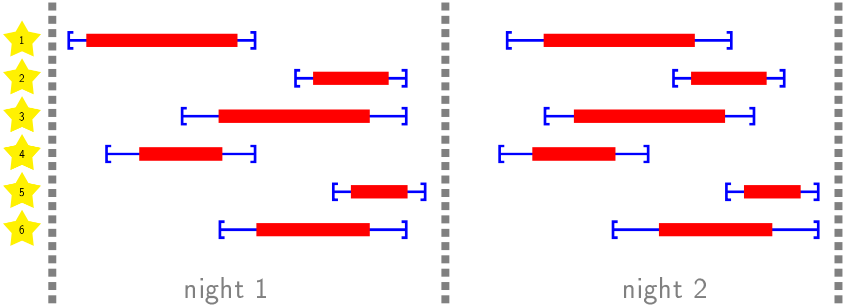


Question: find $\mathcal{S}' \subset \mathcal{S}$ and starting times $t_s, \forall s \in \mathcal{S}'$ such that

- for each $s \in \mathcal{S}'$: $[t_s; t_s + p_s) \subset [r_s; d_s)$
- for each $(s_1, s_2) \in \mathcal{S}'^2$: $[t_{s_1}; t_{s_1} + p_{s_1}) \cap [t_{s_2}; t_{s_2} + p_{s_2}) = \emptyset$
- $\sum_{s \in \mathcal{S}'} w_s$ is maximized

Star scheduling (several nights)

Instance: a set $\mathcal{N}$ of nights, a set $\mathcal{S}$ of stars; each star $s \in \mathcal{S}$ has an interest w_s, an observation duration p_s^n and a visibility window $[r_s^n; d_s^n)$, depending on the night n of the observation



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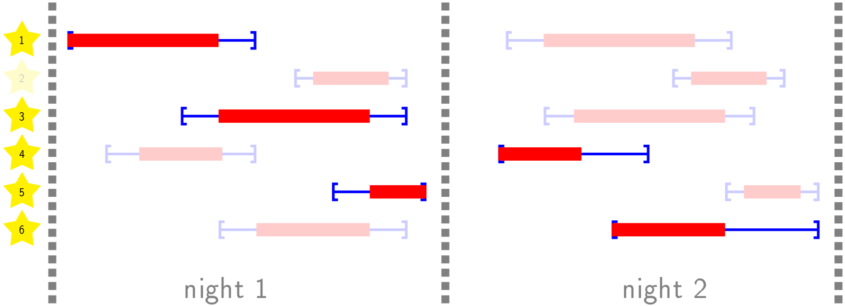

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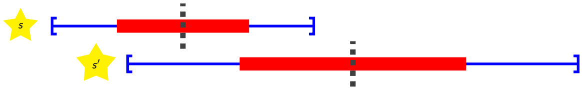
The order is known!

The meridian instant ($m_s = \frac{r_s+d_s}{2}$) is a mandatory instant of observation, that is: for every star s, $p_s^n \geq \frac{d_s^n - r_s^n}{2}$



Property

If for all star s: $p_s^n \geq \frac{d_s^n - r_s^n}{2}$, then observations must be scheduled by non-decreasing meridian time

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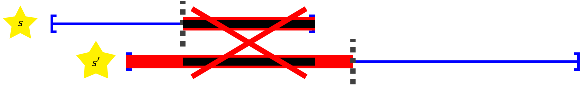

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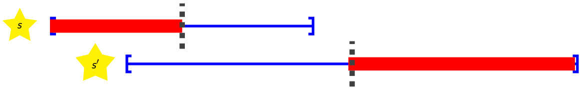

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A MIP model

$$\max \sum_{s \in \mathcal{S}} w_s \boxed{z_s} \longrightarrow = 1 \text{ iff } s \text{ observed}$$

$$s.c. \sum_{n \in \mathcal{N}} \boxed{z_s^n} = z_s \longrightarrow = 1 \text{ iff } s \text{ observed on night } n$$

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visibility window of night n

$$\begin{cases} r_s^n z_s^n \leq \boxed{t_s} \longrightarrow \text{starting time of observation } s \\ t_s + p_s^n z_s^n \leq d_s^n z_s^n + M(1 - z_s^n) \end{cases}$$

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$s \prec s'$ if observed the same night

$$\left\{\begin{array}{l} z_s^n + z_{s'}^n - 1 \le \boxed{y_{ss'}} \longrightarrow = 1 \text{ iff } s \text{ and } s' \text{ observed the same night} \\ t_s + p_s^n \le t_{s'} + M(1 - z_{ss'}) \end{array}\right.$$

Introduction

Complexity

Solving star scheduling

Conclusions and perspectives

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Scheduling one night

Instance: a set $\mathcal{S}$ of stars; each star $s \in \mathcal{S}$ has an interest w_s, an observation duration p_s and a visibility window $[r_s; d_s)$ such that $p_s \geq (d_s - r_s)/2$; a bound W

Question: find a subset of stars so that the total interest is at least W, visibility windows are respected and observations do not overlap

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Complexity of the one night case

Star scheduling of one night is NP-Hard (even if $w_s = 1$)

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Scheduling of one night is NP-hard

- Variant of Partition : $2n$ pairs (a_{2i-1}, a_{2i}) so that $\sum_i a_i = 2B$. Can a total of B be made with one item from each pair?

Scheduling of one night is NP-hard

- Variant of Partition : $2n$ pairs (a_{2i-1}, a_{2i}) so that $\sum_i a_i = 2B$. Can a total of B be made with one item from each pair?
- For each pair of items (a_{2i}, a_{2i+1}) create a pair of incompatible stars (s_{2i-1}, s_{2i}): same visibility window and $p_s \geq (d_s - r_s)/2$

$p_{2i-1} = p_{2i} = 2B + a_i$

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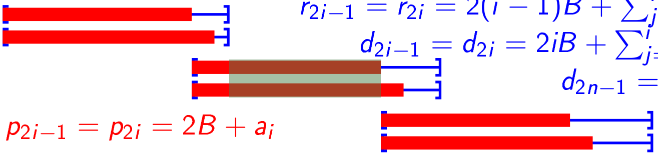
- Set visibility windows so that scheduling a star from a pair does not prevent scheduling a star from another pair and each pair of stars have dedicated instants



$$r_{2i-1} = r_{2i} = 2(i-1)B + \sum_{j=1}^{i-1} \min(a_{2i-1}, a_{2i})$$

$$d_{2i-1} = d_{2i} = 2iB + \sum_{j=1}^{i} \max(a_{2i-1}, a_{2i})$$

$$d_{2n-1} = d_{2n} = (2n+1)B$$

$$p_{2i-1} = p_{2i} = 2B + a_i$$

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- Set visibility windows so that scheduling a star from a pair does not prevent scheduling a star from another pair and each pair of stars have dedicated instants
- "yes" to Partition $\iff$ non-idling schedule of length $(2n+1)B$

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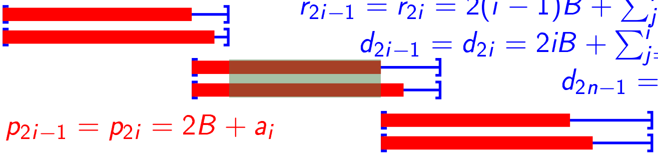

$$p_{2i-1} = p_{2i} = 2B + a_i$$

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A pseudo-polynomial algorithm

$f(i,t)$: maximum interest with stars 1 to i, and such that s_i ends before time t

$$f(i,t) = \begin{cases} \min(f(i-1,t), f(i-1,t-p_i)+w_i) & \forall i \in [1,m], t \in [r_i+p_i, T] \\ f(i-1,t) & \forall i \in [1,m], t \in [0, r_i+p_i[\\ -\infty & \forall i \in [1,m], t < 0 \\ 0 & i = 0, \forall t \in [0,T] \end{cases}$$

We are looking for $f(m,T)$ which can be computed in $O(mT)$

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- $P|r_j|\sum w_j U_j$ (identical nights) is NP-complete

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Complexity of the several nights case

Star scheduling of several nights is unary NP-Hard (even if $w_s = 1$ and all nights are identical)

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Logic-based Bender decomposition

Master problem:
assignment of stars to nights

$$\max \sum_{s \in \mathcal{S}} w_s z_s$$
$$s.c. \sum_{n \in \mathcal{N}} z_s^n = z_s$$

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Slave problem:
scheduling of each nights

Night 1: ok?

Night 2: ok?

...

Logic-based Bender decomposition

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$$\max \sum_{s\in\mathcal{S}} w_s z_s$$

$$s.c. \sum_{n\in\mathcal{N}} z_s^n = z_s$$

$$\sum_{s\in C_k^n} z_s^n \leq |C_k^n| - 1$$

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Logic-based Bender decomposition

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- efficient MIP
- upper bound at each iteration

Slave problem:
scheduling of each nights

Night 1: ok?

Night 2: ok?

...

- n independent problems
- linear complexity

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Column generation

Night patterns: Ω_n, set of all possible schedules for night n

Pattern k for night n: $p^k_{n1} \ldots p^k_{n|\mathcal{S}|}$, where $p^k_{ns} = 1$ iff star s belongs to the k-th pattern of night n; weight $w^k_n = \sum_{s \in \mathcal{S}} w_s p^k_{ns}$

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$$\max \sum_{n \in \mathcal{N}} \sum_{k \in \Omega_n} w^k_n \boxed{\rho^k_n} \longrightarrow = 1 \text{ iff pattern } k \text{ used, night } n$$

$$\begin{array}{rcll} \sum_{k \in \Omega_n} \rho^k_n & = & 1 & \forall n \in \mathcal{N} \\ \sum_{n \in \mathcal{N}} \sum_{k \in \Omega_n} p^k_{ns} \rho^k_n & \leq & 1 & \forall s \in \mathcal{S} \\ \rho^k_n \in \{0, 1\} & & & \forall n \in \mathcal{N}, \forall k \in \Omega_n \end{array}$$

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$$
\begin{array}{lrcll}
 & \max \sum_{n \in \mathcal{N}} \sum_{k \in \Omega_n} w^k_n \boxed{\rho^k_n} & \longrightarrow & = 1 \text{ iff pattern } k \text{ used, night } n & \\
\alpha_n & \sum_{k \in \Omega_n} \rho^k_n & = & 1 & \forall n \in \mathcal{N} \\
\beta_s & \sum_{n \in \mathcal{N}} \sum_{k \in \Omega_n} p^k_{ns} \rho^k_n & \leq & 1 & \forall s \in \mathcal{S} \\
 & \rho^k_n \in \{0,1\} & & & \forall n \in \mathcal{N}, \forall k \in \Omega_n
\end{array}
$$

Reduced cost of ρ^k_n: $w^k_n - \alpha_n - \sum_{s \in \mathcal{S}} p^k_{ns} \beta_s$

Column generation

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\alpha_n & \sum_{k \in \Omega_n} \rho^k_n & = & 1 & \forall n \in \mathcal{N} \\
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 & \rho^k_n \in \{0, 1\} & & & \forall n \in \mathcal{N}, \forall k \in \Omega_n
\end{array}$$

Reduced cost of ρ^k_n: $w^k_n - \alpha_n - \sum_{s \in \mathcal{S}} p^k_{ns} \beta_s$

Pattern of max reduced cost? Single Night Case with costs $w_s - \beta_s$

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Local search

Classical local-search procedure

Neighborhoods:

- moving a star from one night to another
- inserting an unobserved star
- exchanging two stars

An optimal schedule for each night is computed systematically

Solutions

instance	$\|\mathcal{S}\|$	$\|\mathcal{N}\|$	BD (OPT/*UB*) val	BD (OPT/*UB*) cpu(s)	CG (UB) val	CG (UB) cpu(s)	LS (LB) val	LS (LB) cpu(s)
pb1	200	32	***5200***	900	**5200**	1.47	**5200**	0.28
pb2	200	32	***3310***	900	**3310**	0.99	**3310**	0.33
pb3	200	69	**7800**	100	**7800**	1.59	**7800**	0.17
pb4	200	69	-	-	**4870**	1.63	**4870**	0.11
pb5	400	69	*12660*	900	**11910**	5.11	**11910**	12.39
pb6	400	69	*9250*	900	9099.9	19.57	9070	773.95
pb7	400	142	-	-	**13680**	11.85	**13680**	0.15
pb8	400	142	***9760***	900	**9760**	13.71	**9760**	0.21
real	800	142	*18930*	900	18620	92.41	18510	689.60
							18480	306.07

Example solution

solution

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Star scheduling

- a particular interval-scheduling problem with known order
- NP-hardness is proven for both one night and general cases
- several solution methods are proposed and tested

Work in progress...

- Instances
  - we need more instances!
  - study the structure of the real instance (e.g., similarity between nights)
- Solution methods
  - enrich numerical experiments
  - embed the CG into a branch & bound to get optimal solutions
  - analyze (and improve) LS behavior
  - get advantage of nights' similarities
- Complexity
  - draw precise complexity frontiers
  - study approximability