SkaiWD – Laboratorium 4

PCA – przykład

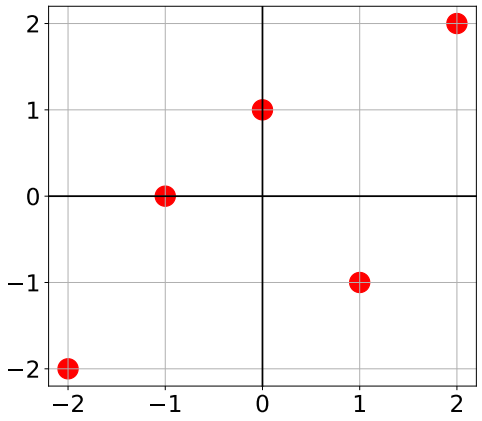
Dane:

	x_1	x_2
	-2	-2
	-1	0
	0	1
	1	-1
	2	2
avg	0	0

Macierz kowariancji:

	x_1	x_2
x_1	2.50	1.75
x_2	1.75	2.50

Wykres:



Miejsce na obliczenia:

Macierz kowariancji X: $Cov_X = \frac{1}{4} X^T X$

Wartości własne Cov_X: $(2.5 - \lambda)^2 - 1.75^2 = 0 \iff \lambda^2 - 5\lambda + \frac{51}{16} = 0 \iff \lambda = 4.25 \vee \lambda = 0.75$

Pierwszy wektor własny k_1: $-1.75x_1 + 1.75x_2 = 0 \wedge 1.75x_1 - 1.75x_2 = 0 \iff x_1 = x_2$

Drugi wektor własny k_2: $1.75x_1 + 1.75x_2 = 0 \wedge 1.75x_1 + 1.75x_2 = 0 \iff x_1 = -x_2$

Posortowane malejąco wartości i wektory własne (znormalizowane):

$\lambda_1 = \underline{4.25}$ $\qquad$ $\lambda_2 = \underline{0.75}$

$$k_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \qquad k_2 = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \qquad K = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

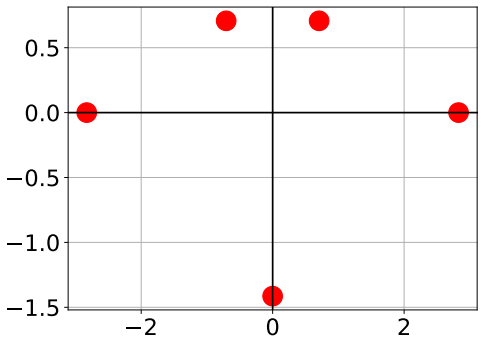
Dane po PCA:

$$Y = XK = \begin{array}{|c|c|} \hline y_1 & y_2 \\ \hline -2\sqrt{2} & 0 \\ \hline -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \hline \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \hline 0 & -\sqrt{2} \\ \hline 2\sqrt{2} & 0 \\ \hline \end{array}$$

Macierz kowariancji:

	y_1	y_2
y_1	4.25	0.00
y_2	0.00	0.75

Wykres:



$y_1(x_1, x_2) = \underline{k_{11}}\ x_1 + \underline{k_{12}}\ x_2$

$y_2(x_1, x_2) = \underline{k_{21}}\ x_1 + \underline{k_{22}}\ x_2$