Kamieniarz Kacper #14

Numerical Methods (ENUME 2019) – Project
Assignment B: Approximation of functions

1. Make a graph of the function:

$$f(x) = \sqrt{1-x^2}\, e^{x-\frac{1}{3}} \text{ for } x \in [-1,1]$$

and indicate a sequence of its values which will be next used for approximation:

$$\left\{ y_n = f(x_n) \middle| n = 1, 2, ..., N \right\}, \text{ where } x_n = -1 + 2\frac{n-1}{N-1}$$

Repeat this exercise for $N = 10, 20 \text{ and } 30$.

2. Develop a program for the least-squares approximation of the function $f(x)$ on the basis of the data $\left\{ (x_n, y_n) \middle| n = 1, ..., N \right\}$, using the operator of pseudoinversion "\" implemented in MATLAB. Use the Chebyshev polynomials defined by the tripart formula:

$$T_0(x) = 1, \; T_1(x) = x, \; T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x) \text{ for } k = 2, \ldots, K$$

as a basis of linearly independent functions. Check the correctness of the program for several pairs of the values of N and K. Add the results of approximation to the corresponding graphs made according to the instruction provided in Section 1.

3. Carry out a systematic investigation of the dependence of the accuracy of approximation on the values of N and K. Use the following accuracy indicators for this purpose:

$$\delta_2(K, N) = \frac{\left\| \hat{f}(x; K, N) - f(x) \right\|_2}{\left\| f(x) \right\|_2} \quad \text{(the root-mean-square error)}$$

$$\delta_\infty(K, N) = \frac{\left\| \hat{f}(x; K, N) - f(x) \right\|_\infty}{\left\| f(x) \right\|_\infty} \quad \text{(the maximum error)}$$

where $\hat{f}(x; K, N)$ is an approximating function obtained for N and K. Make the three-dimensional graphs of the functions $\delta_2(K, N)$ and $\delta_\infty(K, N)$ for $N \in \{5, ..., 50\}$ and $K < N$.

4. Carry out a systematic investigation of the dependence of the indicators $\delta_2(K, N)$ and $\delta_\infty(K, N)$ on the standard deviation $\sigma_y \in \left[10^{-5}, 10^{-1}\right]$ of random errors the data used for approximation are corrupted with. For this purpose:

- Generate the error-corrupted data according to the formula:

  $$\tilde{y}_n = y_n + \Delta\tilde{y}_n \text{ for } n = 1, ..., N$$

  where $\{\Delta\tilde{y}_n\}$ are pseudorandom numbers following the zero-mean normal distribution with the variance σ_y^2, obtained by means of the MATLAB operator ***randn***.
- For each value of the standard deviation σ_y, determine the values $\breve{N}$ and $\breve{K}$ minimising $\delta_2(K, N)$ and compute $\delta_{2,MIN}(\sigma_y) \equiv \delta_2(\breve{K}, \breve{N})$.
- Approximate the sequence of pairs $\left\langle \sigma_y, \delta_{2,MIN}(\sigma_y) \right\rangle$, determined for several dozen values of σ_y, by means of the MATLAB operator ***polyfit***; present the result of approximation using the logarithmic scale on both axes.