House prices

1) Implement by hand equations of derivative using LaTeX (I will use similar format in LibreOffice/ODF, sorry)

$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

why did I wrote them here?

$$f_x = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

2) Writing equations for housings:

$$dw\, linear: \frac{\delta}{\delta W}[W \cdot x + b] = x$$

$$dx\, linear: \frac{\delta}{\delta x}[W \cdot x + b] = W$$

$$db\, linear: \frac{\delta}{\delta b}[W \cdot x + b] = 1$$

3) Cost function

We will use mean squared error cost function:

$$J(\theta_0, \theta_1) = L_{MSE} = \frac{1}{N} \cdot \sum_{i=0}^{N} (h_\theta(x_i) - y_i)^2$$

where $h_\theta(x_i) = \theta_0 + \theta_1 \cdot x_i$ (is our model)

We need to minimize cost function’s result.

4) Gradient descent

The goal of this is to update θ_0, b and θ_1, W to minimize cost function result.

$$\theta_0 := \theta_0 - \alpha \cdot \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

$$b := b - \alpha \cdot \frac{\partial}{\partial b} J(b, W)$$

$$\theta_1 := \theta_1 - \alpha \cdot \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

$$W := W - \alpha \cdot \frac{\partial}{\partial W} J(b, W)$$

but to simultaneously update both variables we need to assign them to temporary variables first, so b doesn’t affect calculation of the W.

α - is a learning rate, which defines how fast we are going to change b and W.

By intuition what is going to happen – partial derivative of the cost function will be a positive slope (will return a positive number) if the mse will on the right side of the local minima, as a result we will subtract a slope value multiplied by learning rate from W (or b). Otherwise we will add it. In ideal situation once MSE is 0 – we don’t move anymore.

So let’s try to figure out partial derivatives:

For θ_0 or b :

$$\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_0} \cdot \frac{1}{N} \cdot \sum_{i=0}^{N} (h_\theta(x_i) - y_i)^2 = \frac{\partial}{\partial \theta_0} \cdot \frac{1}{N} \cdot \sum_{i=0}^{N} (\theta_0 + \theta_1 x_i - y_i)^2 = \frac{2}{N} \sum_{i=0}^{N} (\theta_0 + \theta_1 x_i - y_i) \cdot 1$$

equals to:

$$\frac{2}{N}\sum_{i=0}^{N}(\theta_0+\theta_1 x_i-y_i)\cdot 1=\frac{2}{N}\sum_{i=0}^{N}(b+W\cdot x_i-y_i)\cdot 1$$

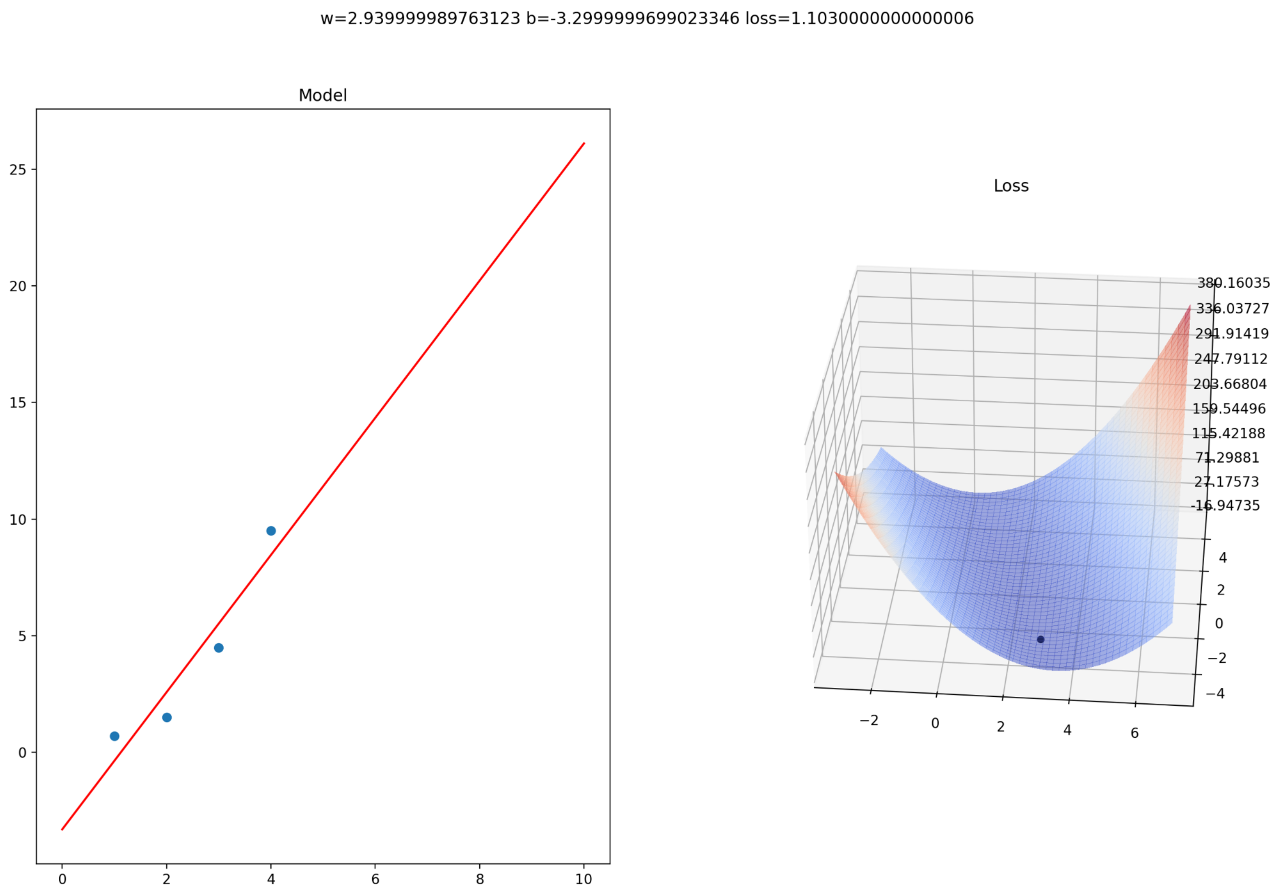
and for θ_1 or W :

$$\frac{\partial}{\partial\theta_1}J(\theta_0,\theta_1)=\frac{\partial}{\partial\theta_1}\cdot\frac{1}{N}\cdot\sum_{i=0}^{N}(h_\theta(x_i)-y_i)^2=\frac{\partial}{\partial\theta_1}\cdot\frac{1}{N}\cdot\sum_{i=0}^{N}(\theta_0+\theta_1 x_i-y_i)^2=\frac{2}{N}\sum_{i=0}^{N}(\theta_0+\theta_1 x_i-y_i)\cdot x_i$$

equals to:

$$\frac{2}{N}\sum_{i=0}^{N}(\theta_0+\theta_1 x_i-y_i)\cdot x_i=\frac{2}{N}\sum_{i=0}^{N}(b+W\cdot x_i-y_i)\cdot x_i$$

For linear model:



For sigmoid model:

$$\frac{1}{1+e^{-x}}$$

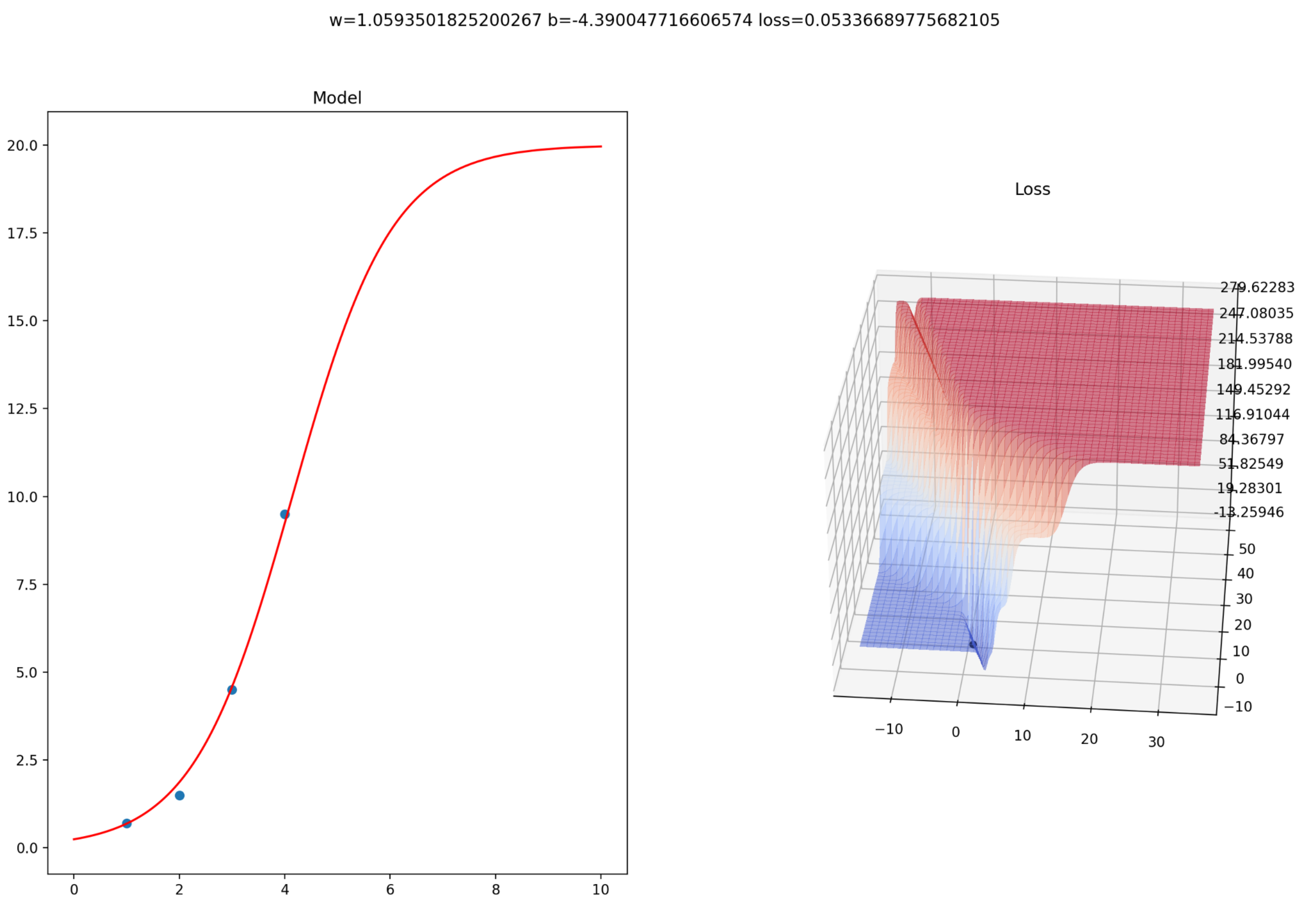
We need to find a gradient descent:

$$\theta_0 := \theta_0 - \alpha \cdot \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$b := b - \alpha \cdot \frac{\partial}{\partial b} J(b, W)$$

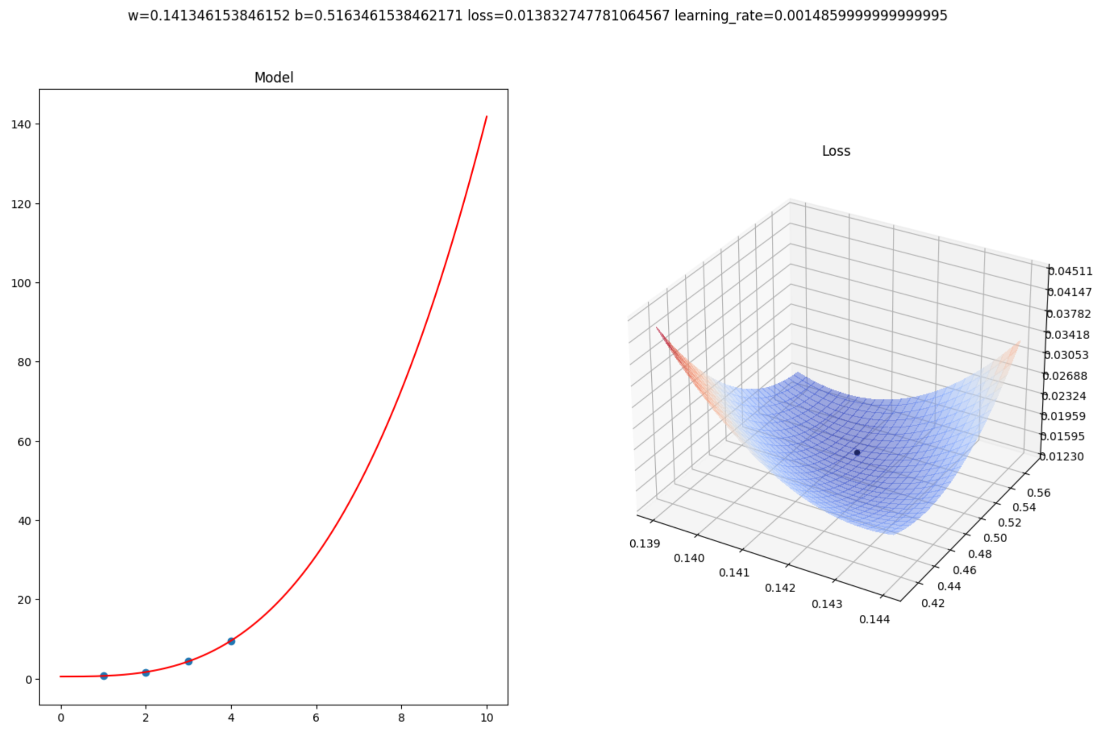
$$\theta_1 := \theta_1 - \alpha \cdot \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$W := W - \alpha \cdot \frac{\partial}{\partial W} J(b, W)$$
$$\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_0} \cdot \frac{1}{N} \cdot \sum_{i=0}^{N} (h_\theta(x_i) - y_i)^2$$

where $h_\theta(x) = \frac{1}{1+e^{-(\theta_0+\theta_1 \cdot x)}} = \frac{1}{1+e^{-(b+W \cdot x)}}$

let’s substitute $a = b + W \cdot x$ so $h_\theta(x) = \frac{1}{1+e^{-a}}$



I don’t find sigmoid logical here because there is no chance that 10 floor building may cost same as 100 floor. What about cubic $b + W \cdot x^3$? Where b would shift line vertically and W would regulate its width. That was my intuition on how to fit the line.



And it worked. That’s much better.