RĪGAS TEHNISKĀ UNIVERSITĀTE
Datorzinātnes un informācijas tehnoloģijas fakultāte
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Mākslīgā intelekta un sistēmu inženierijas katedra

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SALIDZINOŠĀ ANALĪZE DATU KOPU FORMĀTIEM PYTORCH ATTĒLU KLASIFIKĀCIJAS UZDEVUMIEM

Atskaite par bakalaura darbu

Zinātniskais vadītājs
Mg.sc.ing, Pētnieks
ĒVALDS URTĀNS

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.. vel kādas

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Python introduction

My power function

```
from decimal import Decimal
from functools import reduce
from itertools import repeat
from operator import mul
from typing import Union


def my_pow(number: Union[int, float, Decimal], power: int):
    if power > 0:
        return reduce(mul, repeat(number, power))
    elif power == 0:
        return 1

    return 1 / reduce(mul, repeat(number, abs(power)))
```

Matrix dot product:

```
import numpy as np


def dot(a, b):
    try:
        a_height, a_width = np.shape(a)
    except ValueError:
        a_height, a_width = np.shape(a)[0], 1

    try:
        b_height, b_width = np.shape(b)
    except ValueError:
        b_height, b_width = np.shape(b)[0], 1

    if a_width != b_height:
        raise ValueError(f'Wrong shape of matrix: {np.shape(a)=} {np.shape(b)=}')

    c = np.array(
        tuple(
            sum(x * y for x, y in zip(a[row_idx], b[:, col_idx]))
            for row_idx in range(a_height)
            for col_idx in range(b_width)
        )
    )

    return c.reshape((a_height, b_width))
```

Asteroids game

Implemented helper functions:

```
def rotation_mat(degrees: float):
    """
    Rotating around Z axis
    :param degrees:
    :return:
    """
    theta = np.radians(degrees)
    c = np.cos(theta)
    s = np.sin(theta)

    return np.array([
        [c, -s, 0.0],
        [s, c, 0.0],
        [0.0, 0.0, 1.0],
    ])


def translation_mat(dx: float, dy: float):
    return np.array([
        [1.0, 0.0, dx],
        [0.0, 1.0, dy],
        [0.0, 0.0, 1.0],
    ])


def scale_mat(sx: float, sy: float):
    return np.array([
        [sx, 0.0, 0.0],
        [0.0, sy, 0.0],
        [0.0, 0.0, 1.0],
    ])


def dot(a, b):
    try:
        a_height, a_width = np.shape(a)
    except ValueError:
        a_height, a_width = np.shape(a)[0], 1

    try:
        b_height, b_width = np.shape(b)
    except ValueError:
        b_height, b_width = np.shape(b)[0], 1

    if a_width != b_height:
        raise ValueError(f'Wrong shape of matrix: {np.shape(a)=} {np.shape(b)=}')

    c = np.array(
        tuple(
            sum(x * y for x, y in zip(a[row_idx], b[:, col_idx]))
            for row_idx in range(a_height)
```

```python
            for col_idx in range(b_width)
        )
    )

    return c.reshape((a_height, b_width))


def vec2d_to_vec3d(vec2d):
    i = np.array((
        (1.0, 0.0),
        (0.0, 1.0),
        (0.0, 0.0),
    ))

    return dot(i, vec2d[:, None]).transpose()[0] + np.array([0.0, 0.0, 1.0])


def vec3d_to_vec2d(vec3d):
    i = np.array((
        (1.0, 0.0, 0.0),
        (0.0, 1.0, 0.0),
    ))

    return dot(i, vec3d[:, None])
```

Robot arm

1) Implement 3-segment robot arm.
I’ve implemented a dynamic N-segment robot arm by slightly prettifying initial code.

```
def rotation(theta: float):
    """
    :param theta: in radians
    :return:
    """
    c = np.cos(theta)
    s = np.sin(theta)

    return np.array((
        (c, -s),
        (s, c),
    ))


def d_rotation(theta: float):
    """
    :param theta: in radians
    :return:
    """
    c = np.cos(theta)
    s = np.sin(theta)

    return np.array((
        (-s, -c),
        (c, -s),
    ))
```

2) Implemented multi-segment robot arm:

```
prev_r = None

for segment_idx in range(SEGMENT_COUNT):
    # getting rotation value
    theta = thetas[segment_idx]
    # getting rotation matrix
    r = rotation(theta)
    dr_theta_1 = d_rotation(theta)
    # calculating current segment vector by adding rotated segment template to the tip of the previous segment
    np_joints[segment_idx+1] = np.dot(r, segment) + np_joints[segment_idx]

    # STILL BLACK MAGIC FOR ME
    x = dr_theta_1 @ segment

    if segment_idx:
        x = prev_r @ x

    # is this somehow related to derivative of the loss function?
    d_theta_1 = np.sum(x * -2 * (TARGET_POINT - np_joints[-1]))
```

```
    # END OF BLACK MAGIC

    # updating and storing new rotation value for the current segment
    thetas[segment_idx] -= d_theta_1 * LEARNING_RATE

    prev_r = r

loss = np.sum((TARGET_POINT - np_joints[-1]) ** 2)
plt.title(f'loss: {loss:.4f} thetas: {tuple(round(np.rad2deg(theta)) for theta in thetas)}')
```

3) Loss of MSE from tip of robot arm (last vector) to target point:

```
loss = np.sum((TARGET_POINT - np_joints[-1]) ** 2)
```

House prices

1) Implement by hand equations of derivative using LaTeX (I will use similar format in LibreOffice/ODF, sorry)

$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$f_x = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

2) Writing equations for housings:

$$dw\,linear: \frac{\partial}{\partial W}[W \cdot x + b] = x$$

$$dx\,linear: \frac{\partial}{\partial x}[W \cdot x + b] = W$$

$$db\,linear: \frac{\partial}{\partial b}[W \cdot x + b] = 1$$

3) Cost function

We will use mean squared error cost function:

$$J(\theta_0, \theta_1) = L_{MSE} = \frac{1}{N} \cdot \sum_{i=0}^{N} (h_\theta(x_i) - y_i)^2$$

where $h_\theta(x_i) = \theta_0 + \theta_1 \cdot x_i$ (is our model)

We need to minimize cost function’s result.

4) Gradient descent

The goal of this is to update θ_0, b and θ_1, W to minimize cost function result.

$$\theta_0 := \theta_0 - \alpha \cdot \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

$$b := b - \alpha \cdot \frac{\partial}{\partial b} J(b, W)$$

$$\theta_1 := \theta_1 - \alpha \cdot \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

$$W := W - \alpha \cdot \frac{\partial}{\partial W} J(b, W)$$

but to simultaneously update both variables we need to assign them to temporary variables first, so b doesn’t affect calculation of the W.

α - is a learning rate, which defines how fast we are going to change b and W.

By intuition what is going to happen – partial derivative of the cost function will be a positive slope (will return a positive number) if the mse will on the right side of the local minima, as a result we will subtract a slope value multiplied by learning rate from W (or b). Otherwise we will add it. In ideal situation once MSE is 0 – we don’t move anymore.

So let’s try to figure out partial derivatives:

For θ_0 or b :

$$\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_0} \cdot \frac{1}{N} \cdot \sum_{i=0}^{N} (h_\theta(x_i) - y_i)^2 = \frac{\partial}{\partial \theta_0} \cdot \frac{1}{N} \cdot \sum_{i=0}^{N} (\theta_0 + \theta_1 x_i - y_i)^2 = \frac{2}{N} \sum_{i=0}^{N} (\theta_0 + \theta_1 x_i - y_i) \cdot 1$$

equals to:

$$\frac{2}{N}\sum_{i=0}^{N}(\theta_0+\theta_1 x_i-y_i)\cdot 1=\frac{2}{N}\sum_{i=0}^{N}(b+W\cdot x_i-y_i)\cdot 1$$

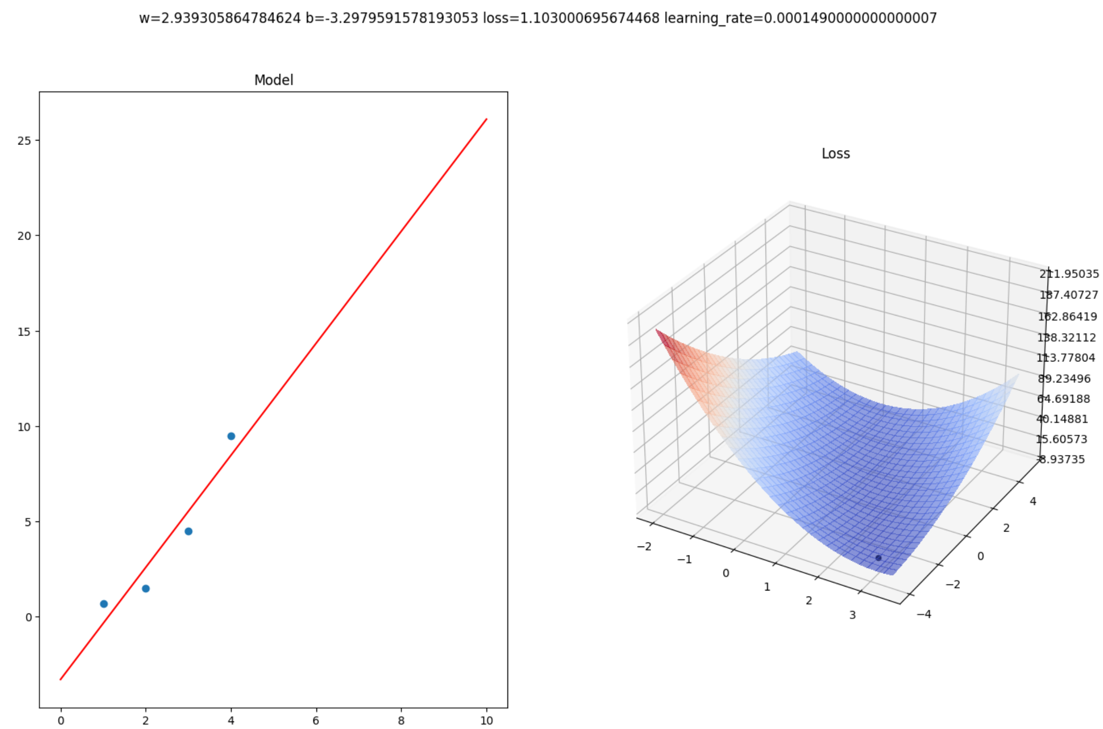
and for θ_1 or W :

$$\frac{\partial}{\partial\theta_1}J(\theta_0,\theta_1)=\frac{\partial}{\partial\theta_1}\cdot\frac{1}{N}\cdot\sum_{i=0}^{N}(h_\theta(x_i)-y_i)^2=\frac{\partial}{\partial\theta_1}\cdot\frac{1}{N}\cdot\sum_{i=0}^{N}(\theta_0+\theta_1 x_i-y_i)^2=\frac{2}{N}\sum_{i=0}^{N}(\theta_0+\theta_1 x_i-y_i)\cdot x_i$$

equals to:

$$\frac{2}{N}\sum_{i=0}^{N}(\theta_0+\theta_1 x_i-y_i)\cdot x_i=\frac{2}{N}\sum_{i=0}^{N}(b+W\cdot x_i-y_i)\cdot x_i$$

For linear model:



For sigmoid model:

$$\frac{1}{1+e^{-x}}$$

We need to find a gradient descent:

$$\theta_0:=\theta_0-\alpha\cdot\frac{\partial}{\partial\theta_0}J(\theta_0,\theta_1)$$

$$b:=b-\alpha\cdot\frac{\partial}{\partial b}J(b,W)$$

$$\theta_1:=\theta_1-\alpha\cdot\frac{\partial}{\partial\theta_1}J(\theta_0,\theta_1)$$

$$W:=W-\alpha\cdot\frac{\partial}{\partial W}J(b,W)$$

$$\frac{\partial}{\partial\theta_0}J(\theta_0,\theta_1)=\frac{\partial}{\partial\theta_0}\cdot\frac{1}{N}\cdot\sum_{i=0}^{N}(h_\theta(x_i)-y_i)^2$$

where $h_\theta(x) = \frac{1}{1+e^{-(\theta_0+\theta_1 \cdot x)}} = \frac{1}{1+e^{-(b+W \cdot x)}}$

let's substitute $a = b+W \cdot x$ so $h_\theta(x) = \frac{1}{1+e^{-a}}$

$$\frac{\partial}{\partial a}\frac{1}{1+e^{-a}} = \frac{\partial}{\partial a}(1+e^{-a})^{-1} = -(1+e^{-a})^{-2} \cdot \frac{\partial}{\partial a}(1+e^{-a}) = -(1+e^{-a})^{-2} \cdot (\frac{\partial}{\partial a}1 + \frac{\partial}{\partial a}e^{-a}) =$$

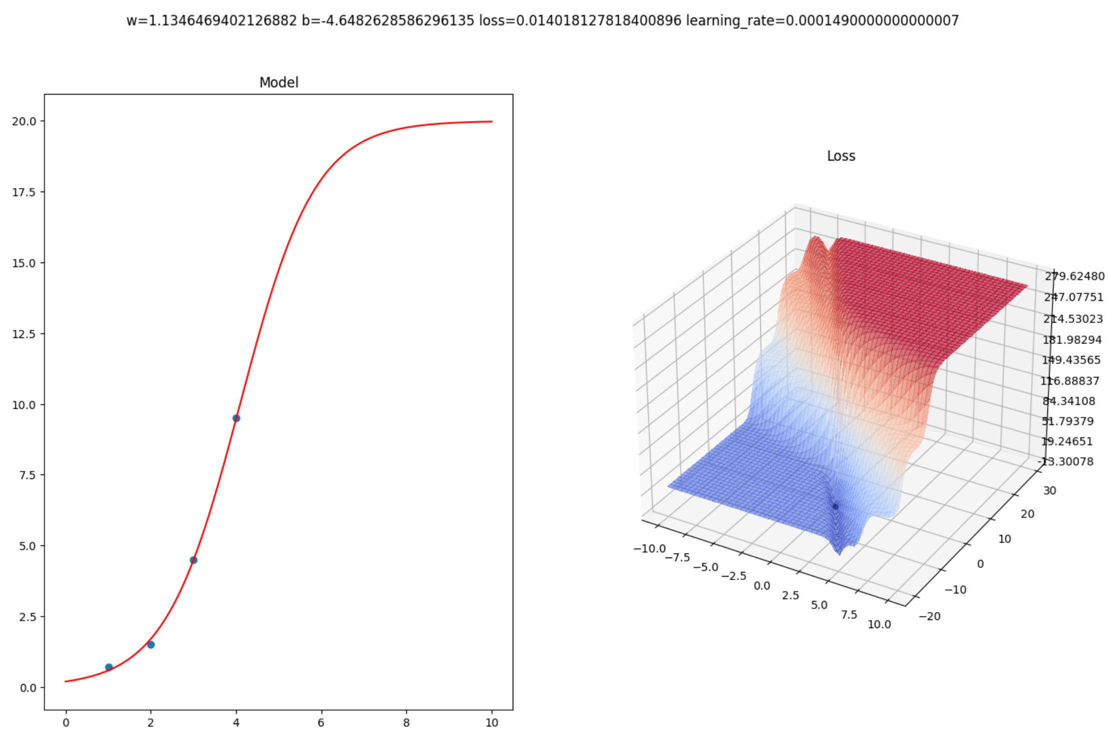
$$= -(1+e^{-a})^{-2} \cdot (0+e^{-a} \cdot \frac{\partial}{\partial a}[-a]) = -(1+e^{-a})^{-2} \cdot (e^{-a} \cdot -1) = \frac{e^{-a}}{(1+e^{-a})^2} =$$

$$= \frac{e^{-a}}{(1+e^{-a}) \cdot (1+e^{-a})} = \frac{1 \cdot e^{-a}}{(1+e^{-a}) \cdot (1+e^{-a})} = \frac{1}{1+e^{-a}} \cdot \frac{e^{-a}}{1+e^{-a}} =$$

$$= \frac{1}{1+e^{-a}} \cdot \frac{e^{-a}+1-1}{1+e^{-a}} = \frac{1}{1+e^{-a}} \cdot (\frac{1+e^{-a}}{1+e^{-a}} - \frac{1}{1+e^{-a}}) = \frac{1}{1+e^{-a}} \cdot (1 - \frac{1}{1+e^{-a}})$$

$$\frac{\partial}{\partial \theta_0}\frac{1}{1+e^{-a}} = \frac{\partial}{\partial b}\frac{1}{1+e^{-a}} = \frac{\partial}{\partial a} \cdot \frac{\partial}{\partial b} = \frac{e^{-a}}{(1+e^{-a})^2} \cdot 1 = \frac{1}{1+e^{-a}} \cdot (1 - \frac{1}{1+e^{-a}}) \cdot 1$$

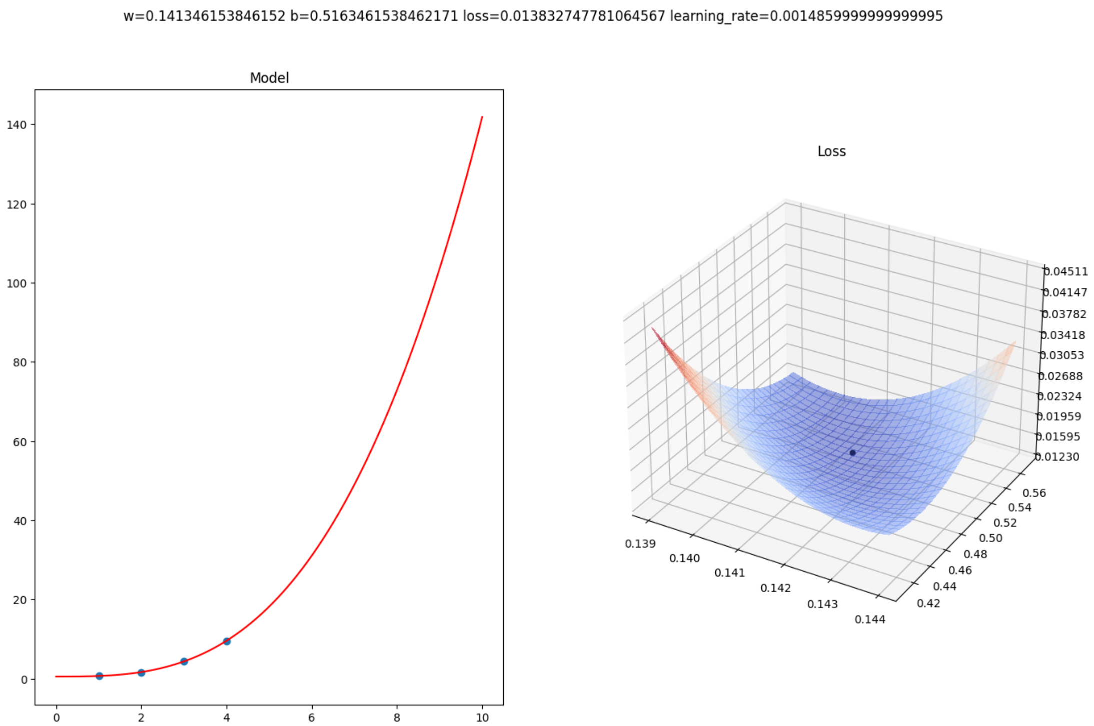
$$\frac{\partial}{\partial \theta_1}\frac{1}{1+e^{-a}} = \frac{\partial}{\partial W}\frac{1}{1+e^{-a}} = \frac{\partial}{\partial a} \cdot \frac{\partial}{\partial W} = \frac{e^{-a}}{(1+e^{-a})^2} \cdot x = \frac{1}{1+e^{-a}} \cdot (1 - \frac{1}{1+e^{-a}}) \cdot x$$



Animated version: https://www.youtube.com/watch?v=4hFCo9tbU34
It looks cool but I don't find sigmoid logical here because there is no chance that 10 floor building may cost same as 100 floor. What about cubic $b+W \cdot x^3$? Where b would shift the line vertically and

W would regulate its width. That was my intuition on how to fit the line. The nice part here is that loss function derivative dW, db is the same as for linear. And dx is:

$$dx\ cubic: \frac{\partial}{\partial x}[W \cdot x^3 + b] = W \cdot 3 \cdot x^2$$



And it worked. That’s much better.

Backpropagation

$$y' = M(x) = Linear(W_1, b_1, W_2, b_2, x) = Linear(W_2, b_2, ReLU(Linear(W_1, b_1, x)))$$
$$Linear(W_i, b_i, x_i) = W_i \cdot x_i + b_i$$
$$ReLU(x_i) = \begin{cases} x_i, x_i \geq 0 \\ 0, x_i < 0 \end{cases}$$
$$MAE(y', y) = \frac{1}{N} \sum_{i=1}^{N} (y_i - y_i')$$

SGD:

$$W_i' = W_i - \alpha \cdot \frac{MAE(y, W_1, b_1, W_2, b_2, x)}{\partial W_i}$$
$$b_i' = b_i - \alpha \cdot \frac{MAE(y, W_1, b_1, W_2, b_2, x)}{\partial b_i}$$

$$\frac{MAE(y, M(x))}{\partial W_i} = \frac{|y - M(x)|}{\partial W_i}$$

Let's assume: $a = y - M(x)$

Then: $\frac{|a|}{\partial a} = \frac{\sqrt{a}}{\partial a} = \frac{(a^2)^{\frac{1}{2}}}{\partial a} = \frac{1}{2} \cdot (a^2)^{-\frac{1}{2}} \cdot \frac{a^2}{\partial a} = \frac{1}{2} \cdot (a^2)^{-\frac{1}{2}} \cdot 2a = a \cdot (a^2)^{-\frac{1}{2}} = a \cdot \frac{1}{|a|} = \frac{a}{|a|} = \frac{y - M(x)}{|y - M(x)|}$

$$\frac{Linear(W_i, b_i, x)}{\partial W_i} = \frac{W_i \cdot x + b_i}{\partial W_i} = x$$

$$\frac{Linear(W_i, b_i, x)}{\partial b_i} = \frac{W_i \cdot x + b_i}{\partial b_i} = 1$$

$$\frac{MAE(y, W_1, b_1, W_2, b_2, x)}{\partial W_2} = \frac{|a|}{\partial a} \cdot \frac{M(x)}{\partial W_2} = \frac{y - M(x)}{|y - M(x)|} \cdot \frac{Linear(W_2, b_2, ReLU(Linear(W_1, b_1, x)))}{\partial W_2} =$$
$$= \frac{y - M(x)}{|y - M(x)|} \cdot ReLU(Linear(W_1, b_1, x))$$
$$\frac{MAE(y, W_1, b_1, W_2, b_2, x)}{\partial b_2} = \frac{|a|}{\partial a} \cdot \frac{M(x)}{\partial b_2} = \frac{y - M(x)}{|y - M(x)|} \cdot \frac{Linear(W_2, b_2, ReLU(Linear(W_1, b_1, x)))}{\partial b_2} =$$
$$= \frac{y - M(x)}{|y - M(x)|} \cdot 1$$

$$M(x) = Linear(W_2, b_2, ReLU(Linear(W_1, b_1, x)))$$
$$z = ReLU(q)$$
$$q = Linear(W_1, b_1, x)$$
$$M(x) = Linear(W_2, b_2, z) = W_2 \cdot z + b_2$$

$$\frac{MAE(y,W_1,b_1,W_2,b_2,x)}{\partial W_1}=\frac{|a|}{\partial a}\cdot\frac{M(x)}{\partial W_1}=\frac{|a|}{\partial a}\cdot\frac{Linear(W_2,b_2,z)}{\partial W_1}=\frac{|a|}{\partial a}\cdot\frac{Linear(W_2,b_2,z)}{\partial z}\cdot\frac{z}{\partial W_1}=$$

$$=\frac{|a|}{\partial a}\cdot\frac{W_2\cdot z+b_2}{\partial z}\cdot\frac{z}{\partial W_1}=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(Linear(W_1,b_1,x))}{\partial W_1}=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}\cdot\frac{q}{\partial W_1}=$$

$$=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}\cdot\frac{Linear(W_1,b_1,x)}{\partial W_1}=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}\cdot\frac{W_1\cdot x+b_1}{\partial W_1}=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}\cdot x=$$

$$=\frac{y-M(x)}{|y-M(x)|}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}\cdot x$$

$$\frac{ReLU(q)}{\partial q}=\begin{cases}1, q\geq 0\\ 0, q<0\end{cases}$$

$$\frac{ReLU(W_1\cdot x+b_1)}{\partial[W_1\cdot x+b_1]}=\begin{cases}1, W_1\cdot x+b_1\geq 0\\ 0, W_1\cdot x+b_1<0\end{cases}$$

$$\frac{MAE(y,W_1,b_1,W_2,b_2,x)}{\partial b_1}=\frac{|a|}{\partial a}\cdot\frac{M(x)}{\partial b_1}=\frac{|a|}{\partial a}\cdot\frac{Linear(W_2,b_2,z)}{\partial b_1}=\frac{|a|}{\partial a}\cdot\frac{Linear(W_2,b_2,z)}{\partial z}\cdot\frac{z}{\partial b_1}=$$

$$=\frac{|a|}{\partial a}\cdot\frac{W_2\cdot z+b_2}{\partial z}\cdot\frac{z}{\partial b_1}=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(Linear(W_1,b_1,x))}{\partial b_1}=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}\cdot\frac{q}{\partial b_1}=$$

$$=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}\cdot\frac{Linear(W_1,b_1,x)}{\partial b_1}=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}\cdot\frac{W_1\cdot x+b_1}{\partial b_1}=\frac{|a|}{\partial a}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}\cdot 1=$$

$$=\frac{y-M(x)}{|y-M(x)|}\cdot W_2\cdot\frac{ReLU(q)}{\partial q}$$

LeakyReLu task

Model: $y' = M(x) = LeakyReLU(Linear(\tanh(Linear(W \cdot x + b))))$

$$y' = M(x) = LeakyReLu(Linear(W_1, b_1, W_2, b_2, x), \alpha) =$$
$$= LeakyReLu(Linear(W_2, b_2, \tanh(Linear(W_1, b_1, x))), \alpha)$$

Where:

$$Linear(x) = W \cdot x + b$$

$$dw\, linear: \frac{\partial}{\partial W}[W \cdot x + b] = x$$
$$dx\, linear: \frac{\partial}{\partial x}[W \cdot x + b] = W$$
$$db\, linear: \frac{\partial}{\partial b}[W \cdot x + b] = 1$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{\tanh(x)}{\delta x} = \delta x \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{(e^x - e^{-x}) \cdot (e^x + e^{-x})^{-1}}{\delta x} = \frac{(e^x - e^{-x})}{\delta x} \cdot (e^x + e^{-x})^{-1} + (e^x - e^{-x}) \cdot \frac{(e^x + e^{-x})^{-1}}{\delta x} =$$
$$= \frac{(e^x - e^{-x})}{\partial (e^x - e^{-x})} \cdot \frac{(e^x - e^{-x})}{\partial x} \cdot (e^x + e^{-x})^{-1} + (e^x - e^{-x}) \cdot \frac{(e^x + e^{-x})^{-1}}{\partial (e^x + e^{-x})} \cdot \frac{(e^x + e^{-x})^{-1}}{\partial x} =$$
$$= (e^x + e^{-x}) \cdot (e^x + e^{-x})^{-1} - (e^x - e^{-x}) \cdot (e^x + e^{-x})^{-2} \cdot (e^x - e^{-x}) = 1 - (e^x - e^{-x})^2 \cdot (e^x + e^{-x})^{-2}$$

$LeakyReLU(x) = \begin{cases} x, x > 0 \\ \alpha \cdot x, x \le 0 \end{cases}$ here α is a slope, not the learning rate

$$\frac{LeakyReLU(x)}{\partial x} = \begin{cases} 1, x > 0 \\ \alpha, x \le 0 \end{cases}$$

MAE loss function:

$$MAE(y', y) = \frac{1}{N} \sum_{i=1}^{N} (y_i - y_i')$$

SGD:

$$W_i' = W_i - \alpha \cdot \frac{MAE(y, W_1, b_1, W_2, b_2, x)}{\partial W_i}$$
$$b_i' = b_i - \alpha \cdot \frac{MAE(y, W_1, b_1, W_2, b_2, x)}{\partial b_i}$$

$$y' = M(x) = LeakyReLu(Linear(W_1, b_1, W_2, b_2, x), \alpha) =$$
$$= LeakyReLu(Linear(W_2, b_2, \tanh(Linear(W_1, b_1, x))), \alpha)$$

$$m = Linear(W_1, b_1, x)$$
$$k = Linear(W_2, b_2, \tanh(Linear(W_1, b_1, x)))$$
$$l = \tanh(Linear(W_1, b_1, x))$$

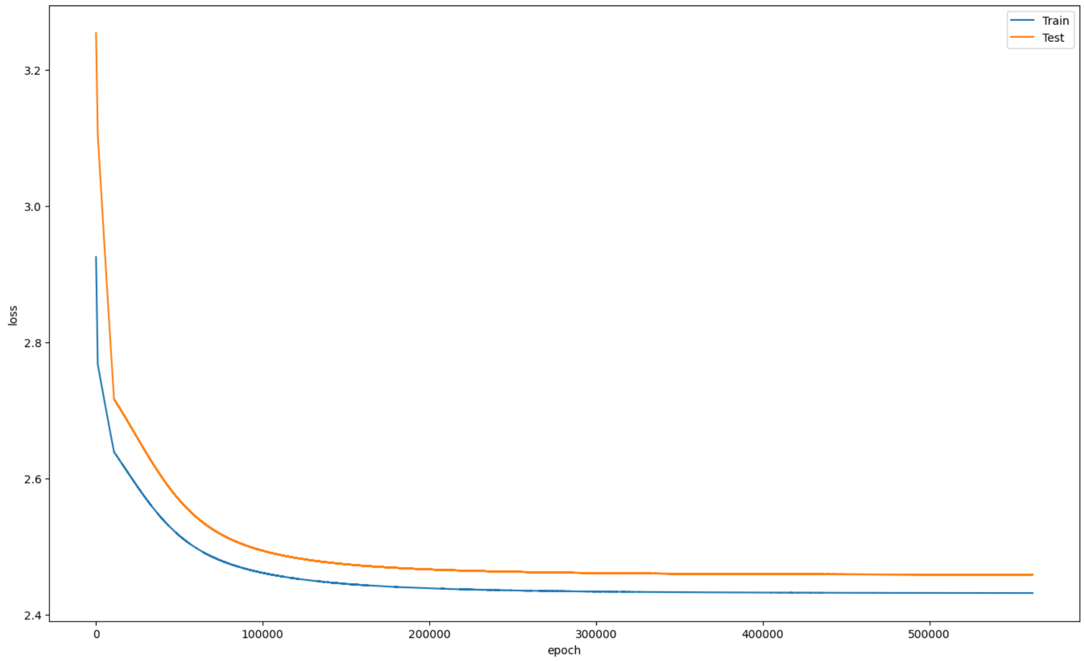
$$\frac{y'}{\partial W_2} = \frac{LeakyReLu(k)}{\partial k} \cdot \frac{k}{\partial W_2} =$$
$$= \frac{LeakyReLu(k)}{\partial k} \cdot \frac{Linear(W_2, b_2, l)}{\partial W_2} =$$
$$= \frac{LeakyReLu(k)}{\partial k} \cdot l = \frac{LeakyReLu(k)}{\partial k} \cdot \tanh(Linear(W_1, b_1, x))$$

$$\frac{y'}{\partial b_2} = \frac{LeakyReLu(k)}{\partial k} \cdot \frac{k}{\partial b_2} = \frac{LeakyReLu(k)}{\partial k} \cdot \frac{Linear(W_2, b_2, l)}{\partial b_2} = \frac{LeakyReLu(k)}{\partial k} \cdot 1$$

$$\frac{y'}{\partial W_1} = \frac{LeakyReLu(k)}{\partial k} \cdot \frac{k}{\partial W_1} =$$
$$= \frac{LeakyReLu(k)}{\partial k} \cdot \frac{Linear(W_2, b_2, l)}{\partial W_1} =$$
$$= \frac{LeakyReLu(k)}{\partial k} \cdot \frac{Linear(W_2, b_2, l)}{\partial l} \cdot \frac{l}{\partial W_1} =$$
$$= \frac{LeakyReLu(k)}{\partial k} \cdot \frac{Linear(W_2, b_2, l)}{\partial l} \cdot \frac{\tanh(Linear(W_1, b_1, x))}{\partial W_1} =$$
$$= \frac{LeakyReLu(k)}{\partial k} \cdot \frac{Linear(W_2, b_2, l)}{\partial l} \cdot \frac{\tanh(m)}{\partial m} \cdot \frac{m}{\partial W_1} =$$
$$= \frac{LeakyReLu(k)}{\partial k} \cdot \frac{Linear(W_2, b_2, l)}{\partial l} \cdot \frac{\tanh(m)}{\partial m} \cdot \frac{Linear(W_1, b_1, x)}{\partial W_1} =$$
$$= \frac{LeakyReLu(k)}{\partial k} \cdot \frac{Linear(W_2, b_2, l)}{\partial l} \cdot \frac{\tanh(m)}{\partial m} \cdot x$$

$$\frac{y'}{\partial b_1} = \frac{LeakyReLu(k)}{\partial k} \cdot \frac{k}{\partial b_1} =$$
$$\ldots same\ as\ for\ W_1 \ldots$$
$$= \frac{LeakyReLu(k)}{\partial k} \cdot \frac{Linear(W_2, b_2, l)}{\partial l} \cdot \frac{\tanh(m)}{\partial m} \cdot 1$$

Running this model produces following result:


Train
Test
3.2
3.0
2.8
2.6
2.4
loss
0
100000
200000
300000
400000
500000
epoch

NumPy + OOP version

1) Implement dataset normalization to get X and Y features in range from -1..1.

$$X_i = 2 \cdot \left(\frac{X_i - min(X_i)}{max(X_i) - min(X_i)} - 0.5 \right)$$

Need also function to convert Y back to real values.
Solution: To convert values back – we need to remember min and max values of initial dataset, otherwise we don't know according to what values we have -1 and 1 boundaries.

```
def normalize(values: np.ndarray) -> Tuple[np.ndarray, np.ndarray, np.ndarray]:
    max_values = np.max(values, axis=0)
    min_values = np.min(values, axis=0)

    return 2.0 * ((values - min_values) / (max_values - min_values) - 0.5), min_values, max_values



def denormalize(values: np.ndarray, min_values: np.ndarray, max_values: np.ndarray) -> np.ndarray:
    return (values / 2.0 + 0.5) * (max_values - min_values) + min_values
```

2) Implement model with new functions

- Use code from 4. (?) task
- Add classes LossMSE (Mean square error loss function), LayerSigmoid

```
class SigmoidLayer:
    def __init__(self):
        self.x = None
        self.output = None

    def forward(self, x: Variable) -> Variable:
        self.x = x
        self.output = Variable(
            1.0 / (1.0 + np.exp(-x.value))
        )
        return self.output

    def backward(self):
        self.x.grad = -1.0 / (1.0 + np.exp(-self.x.value)) ** 2 * self.output.grad
```

```
class MSELoss:
    def __init__(self):
        self.y: Optional[Variable] = None
        self.y_prim: Optional[Variable] = None

    def forward(self, y: Variable, y_prim: Variable) -> float:
        self.y = y
        self.y_prim = y_prim
        return np.mean((y.value - y_prim.value) ** 2)

    def backward(self):
        self.y_prim.grad = 2.0 * (self.y_prim.value - self.y.value)
```

- Replace ReLU with Sigmoid in Model

We will use same Sigmoid formulas:

$$\frac{1}{1+e^{-x}}$$

$$\frac{\partial}{\partial x}\frac{1}{1+e^{-x}}=\frac{1}{1+e^{-x}}\cdot\left(1-\frac{1}{1+e^{-x}}\right)$$

- Train with LossMSE

And same MSE cost function:

$$L_{MSE}=\frac{1}{N}\cdot\sum_{i=0}^{N}\left(h_{\theta}\left(x_i\right)-y_i\right)^2$$

$$\frac{\partial}{\partial L_{MSE}}=\frac{2}{N}\sum_{i=0}^{N}\left(b+W\cdot x_i-y_i\right)$$

- Fine tune Hyper parameters so you can get lowest error in 300 epochs

Resulting model layers are:

```
self.layers = [
    LinearLayer(in_features=8, out_features=4),
    SigmoidLayer(),
    LinearLayer(in_features=4, out_features=4),
    SigmoidLayer(),
    LinearLayer(in_features=4, out_features=1)
]
```