

ROZWIAZANIE WYGENEROWANE PRZEZ CHAT GPT

1. Oblicz granicę ciągu $x_n = \left(\frac{n^2 \cos(n!)}{n^3 + 2n}, \left(1 + \frac{1}{n}\right)^{2n-3}, \sqrt{4n^2 + 3n - 2} - 2n\right)$.

We need to calculate the limit of the sequence $x_n = \left(\frac{n^2 \cos(n!)}{n^3 + 2n}, \left(1 + \frac{1}{n}\right)^{2n-3}, \sqrt{4n^2 + 3n - 2} - 2n\right)$.

Part 1: $\frac{n^2 \cos(n!)}{n^3 + 2n}$

General formula:

$$\lim_{n \rightarrow \infty} \frac{n^2 \cos(n!)}{n^3 + 2n}$$

Transformation:

Since $|\cos(n!)| \leq 1$, we have:

$$\begin{aligned} \left| \frac{n^2 \cos(n!)}{n^3 + 2n} \right| &\leq \frac{n^2}{n^3 + 2n} \\ &= \frac{n^2}{n^3(1 + \frac{2}{n^2})} = \frac{1}{n(1 + \frac{2}{n^2})} \end{aligned}$$

As $n \rightarrow \infty$, $\frac{1}{n} \rightarrow 0$.

Result:

$$\lim_{n \rightarrow \infty} \frac{n^2 \cos(n!)}{n^3 + 2n} = 0$$

Part 2: $\left(1 + \frac{1}{n}\right)^{2n-3}$

General formula:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n-3}$$

Transformation:

Using the known limit $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$, we get:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n-3} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n\right)^{2 - \frac{3}{n}} = e^2$$

Result:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n-3} = e^2$$

Part 3: $\sqrt{4n^2 + 3n - 2} - 2n$

General formula:

$$\lim_{n \rightarrow \infty} \left(\sqrt{4n^2 + 3n - 2} - 2n\right)$$

Transformation:

Multiply and divide by the conjugate:

$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \frac{(4n^2 + 3n - 2) - 4n^2}{\sqrt{4n^2 + 3n - 2} + 2n} \\
&= \lim_{n \rightarrow \infty} \frac{3n - 2}{\sqrt{4n^2 + 3n - 2} + 2n}
\end{aligned}$$

Divide by n :

$$= \lim_{n \rightarrow \infty} \frac{3 - \frac{2}{n}}{\sqrt{4 + \frac{3}{n} - \frac{2}{n^2}} + 2}$$

As $n \rightarrow \infty$,

$$= \frac{3}{4} = 0$$

****Result:****

$$\lim_{n \rightarrow \infty} \left(\sqrt{4n^2 + 3n - 2} - 2n \right) = \frac{3}{4} = 0$$

Final Result:

The limit of the sequence is:

$$\boxed{(0, e^2, 0)}$$