

# Filled Ball

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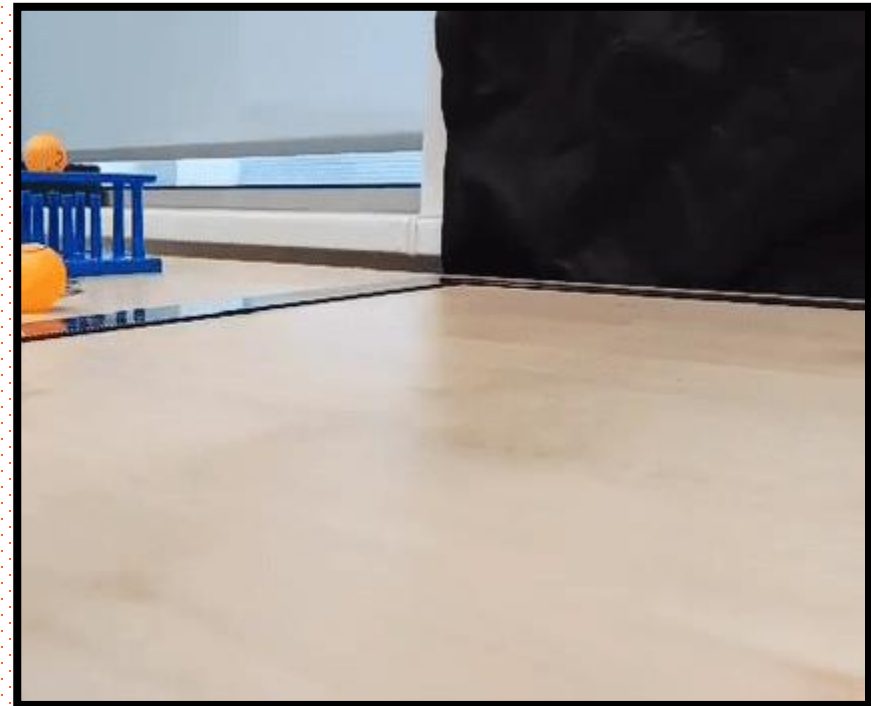
# The problem

*A ping-pong ball that has been partially filled with some fluid or sand will bounce much lower than a filled one. Explain this phenomenon. How does the height of the bounce depend on the relevant parameters?*

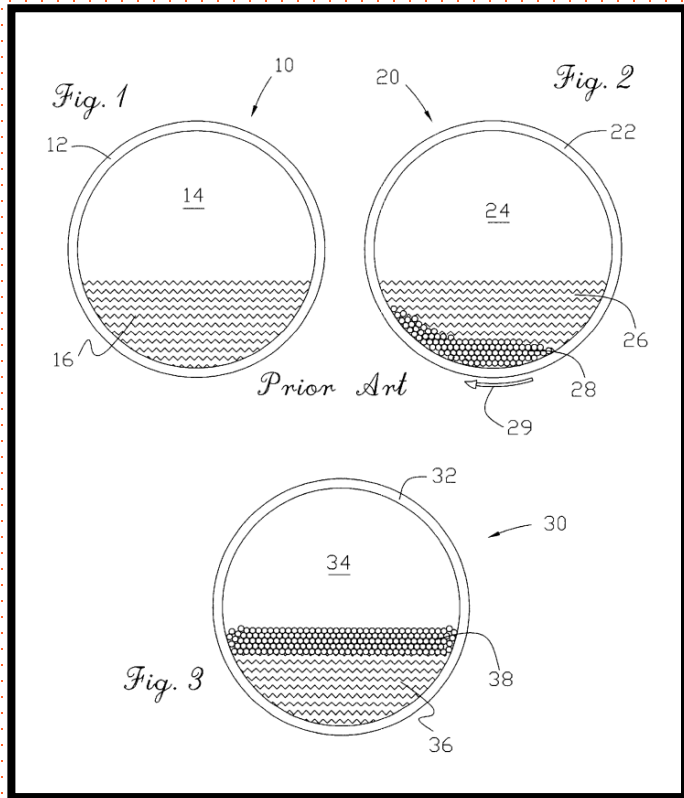
# First impressions

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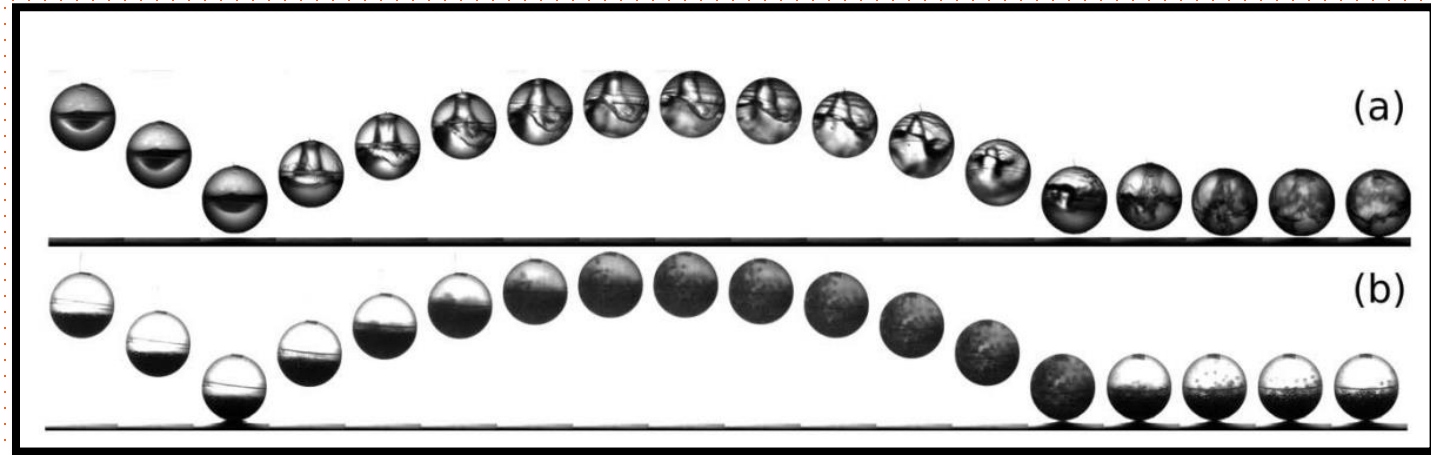
- Bounce height  $h$  varies for different same-height throws
- Bounce height is not proportional to initial height  $h_0$  (damping)
- Lowest  $h$  (for given  $h_0$ ) is for balls filled *around* halfway
- The ball starts to **roll**



# Origin of the problem



[1] – filled street hockey balls

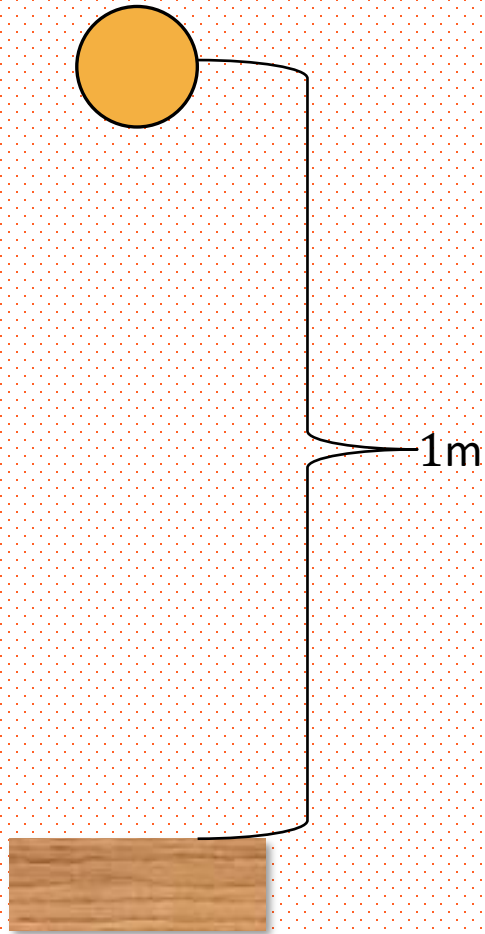


[2] – bouncing sphere partially filled with water (a) and granular material (b)

[1] Street hockey ball, U.S. patent 6,645,098 (11 November 2003)

[2] T. W. Killian, R. A. Klaus, T. T. Truscott, Rebound and jet formation of a fluid-filled sphere

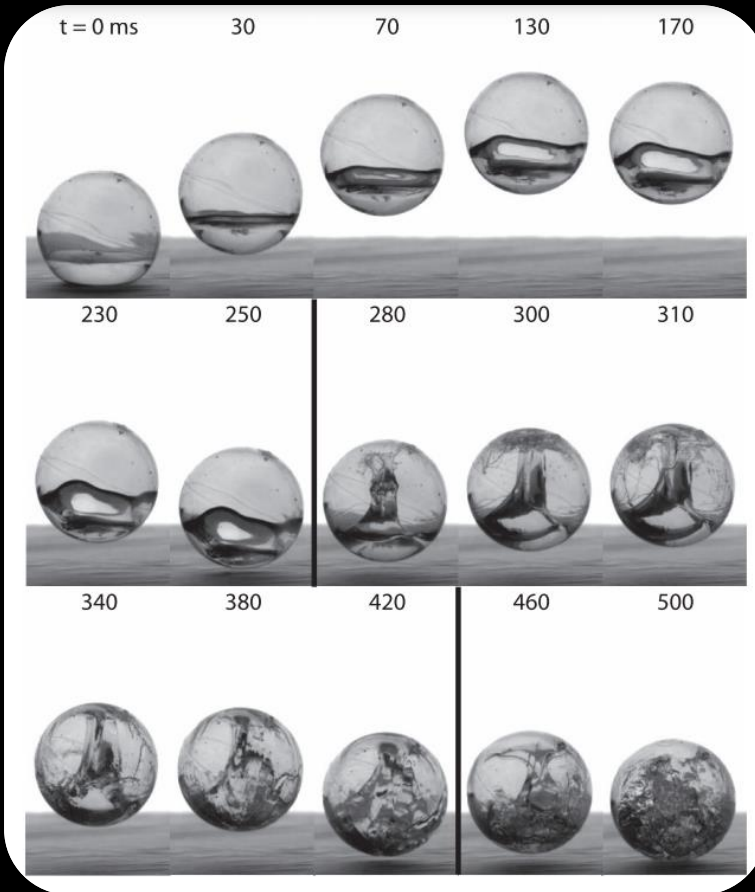
$$m = (2.55 \pm 0.01)\text{g}$$



$$\left\langle \frac{h}{h_0} \right\rangle = 0.587 \pm 0.003$$

$$\alpha = \frac{E}{E_0} = \frac{h}{h_0}$$

60% of energy is conserved



*A ping-pong ball that has  
been partially filled will  
bounce much lower than  
a filled one.*

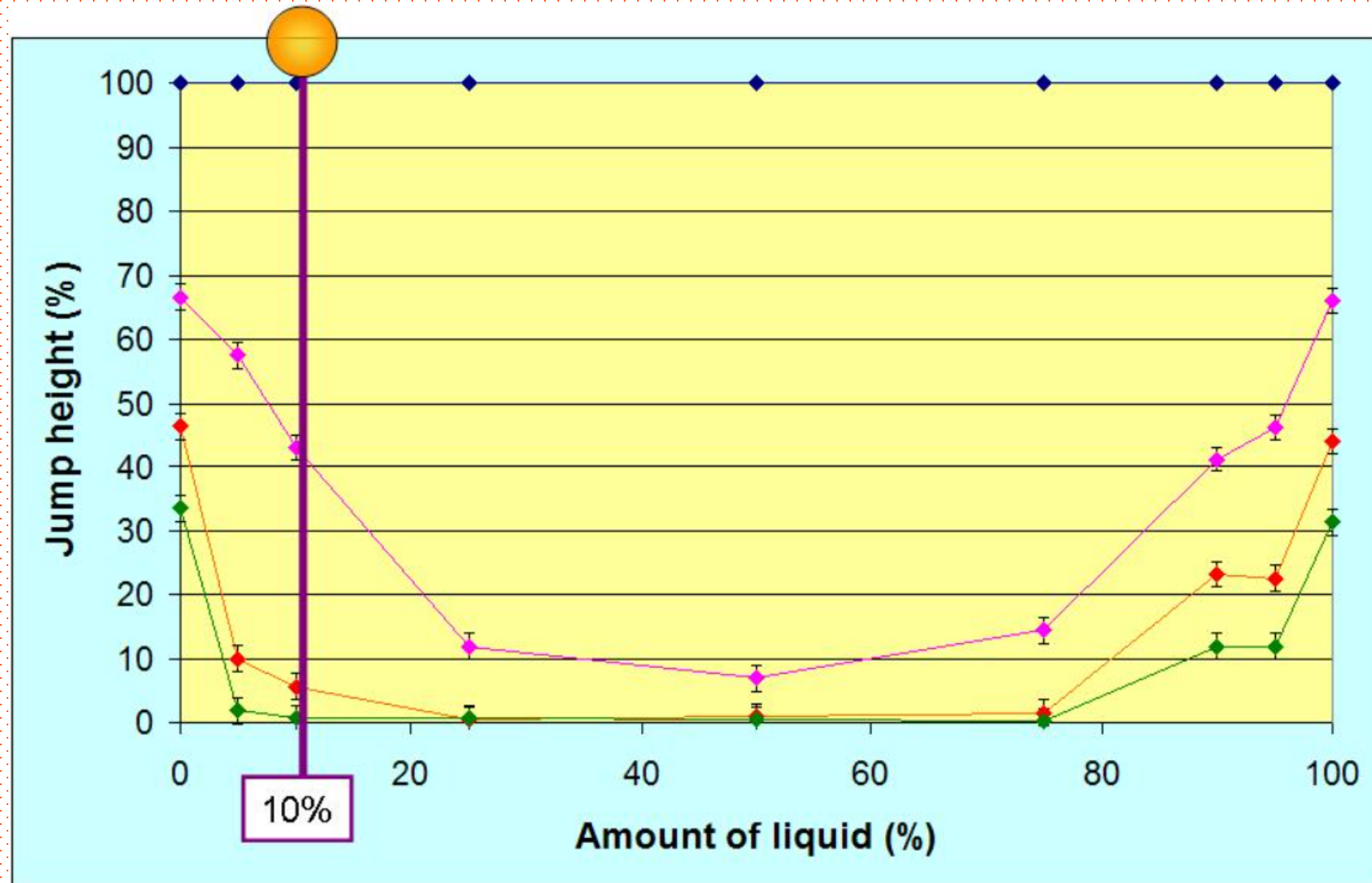
# Previous investigations

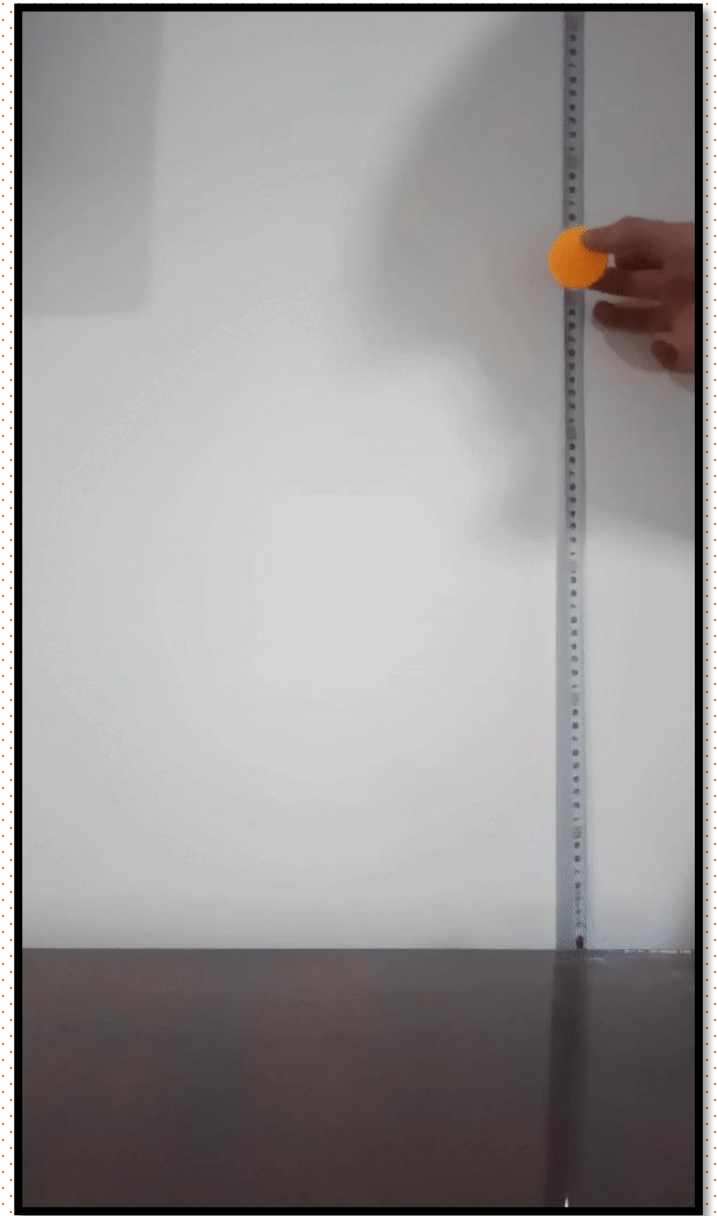
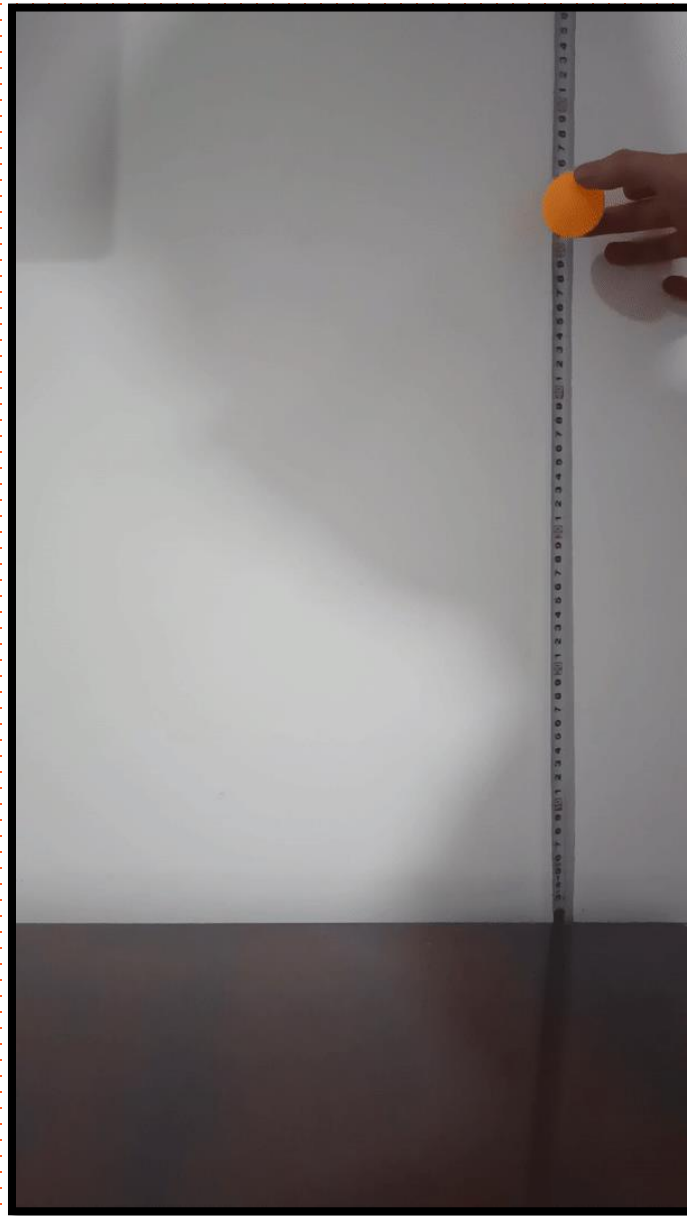
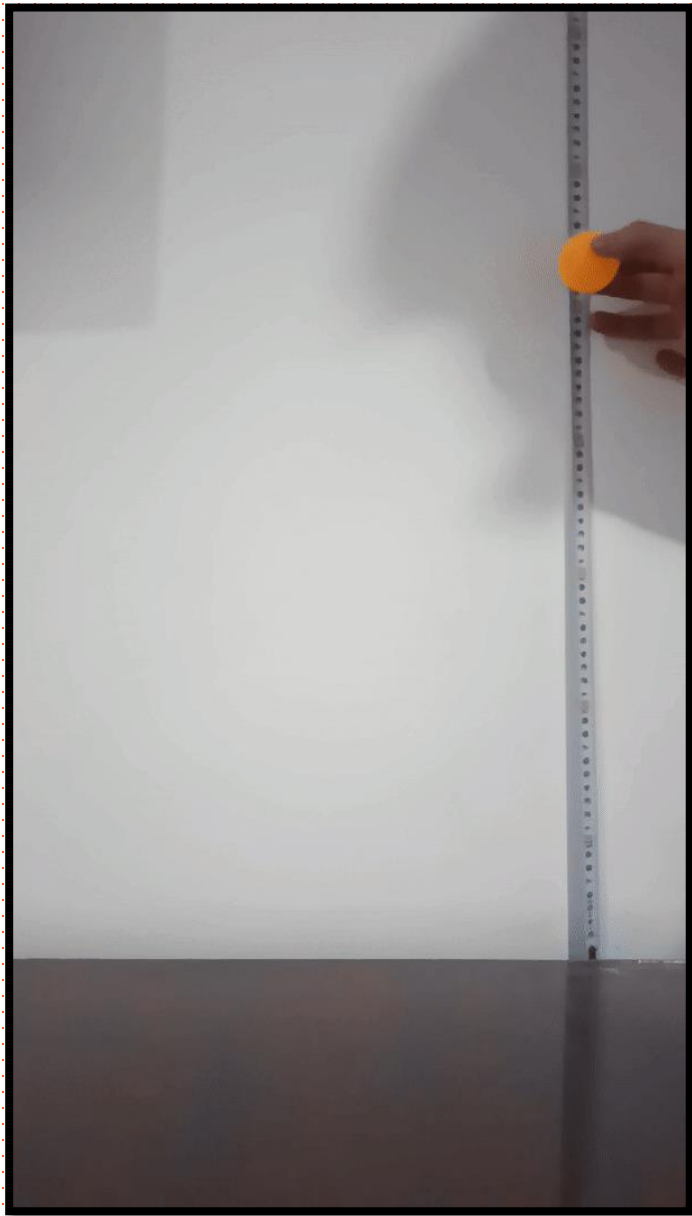
IYPT, 2014

## Russian Team

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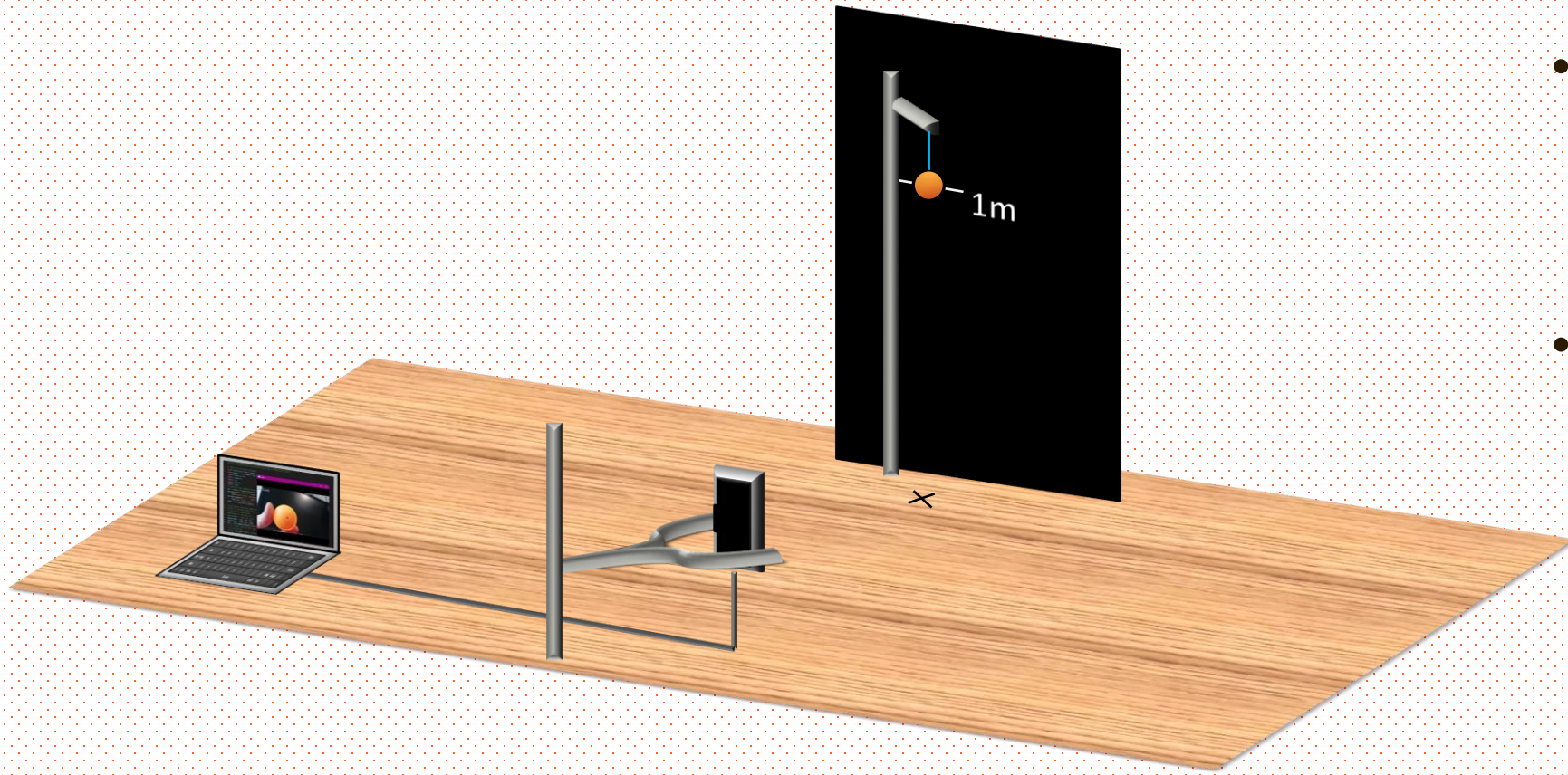
Measurements done from 80cm





A ball with 2ml of water can bounce to either 20, 12 or 4 cm



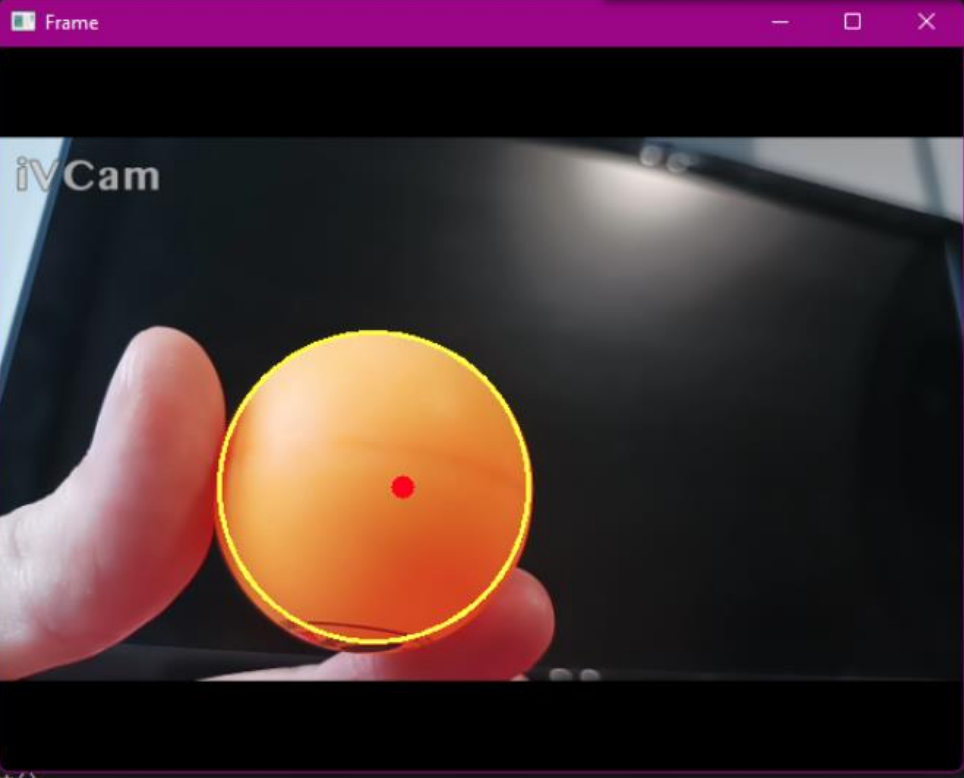


- Motion of the ball established using a Python tracking code
- Ball is hanging on a string and is released by burning the string

# Experimental setup

# Tracker

```
1 https://www.pyimagesearch.com/2015/09/14/ball-tracking-with-opencv/ [4]
2
3 # import the necessary packages
4 from collections import deque
5 from imutils.video import VideoStream
6 import numpy as np
7 import argparse
8 import cv2
9 import imutils
10 import time
11
12 # construct the argument parse and parse the arguments
13 ap = argparse.ArgumentParser()
14 ap.add_argument("-v", "--video",
15                 help="path to the (optional) video file")
16 ap.add_argument("-b", "--buffer",
17                 help="max buffer size")
18 args = vars(ap.parse_args())
19
20 # define the lower and upper boundaries for the color of the
21 # ball in the HSV color space, then initialize a list to
22 # store tracked points
23 greenLower = (29, 86, 6)
24 greenUpper = (64, 255, 255)
25 lower = (15, 120, 60)
26 upper = (45, 232, 230)
27 pts = deque(maxlen=args["buffer"])
28
29 # if a video path was not supplied, grab the webcam feed
30 if not args.get("video", False):
31     vs = VideoStream(src=0).start()
```



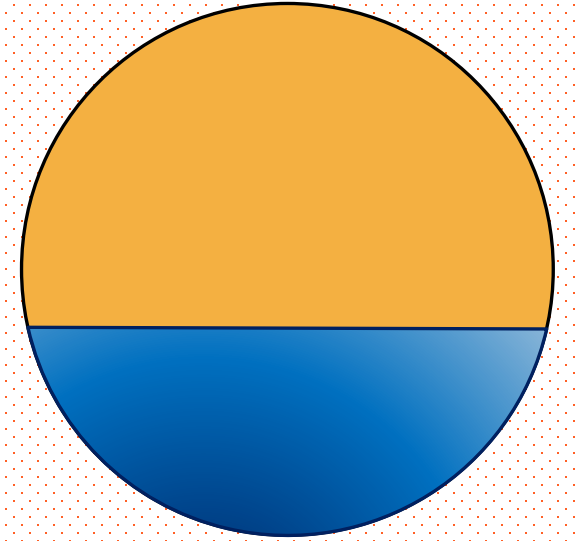


Ball mass around 2.5 g

## Water

Maximum 30 grams

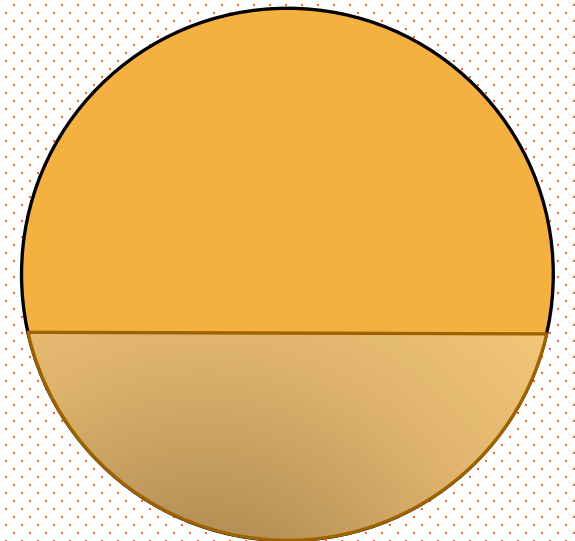
Measurements every 3 grams

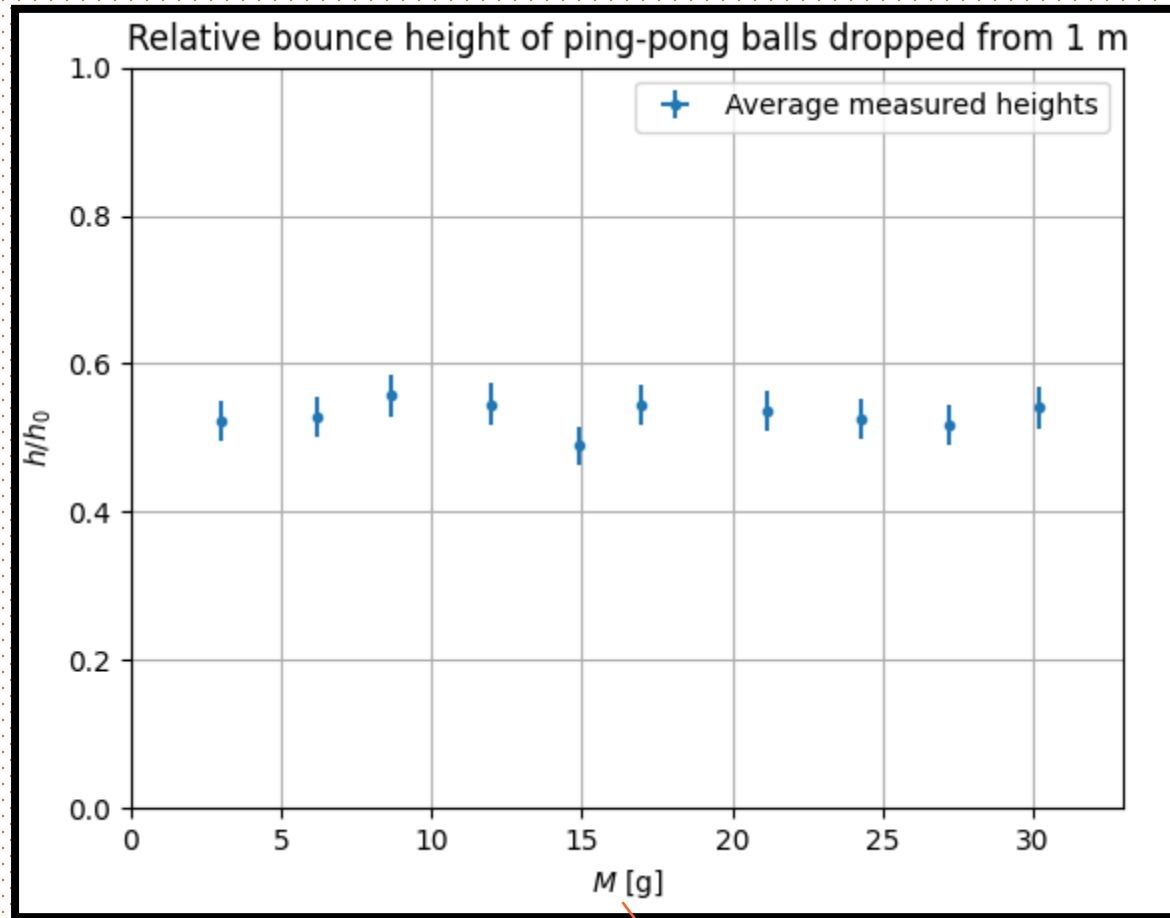


## Sand

Maximum 52 grams

Measurements every 6 grams





Filling mass

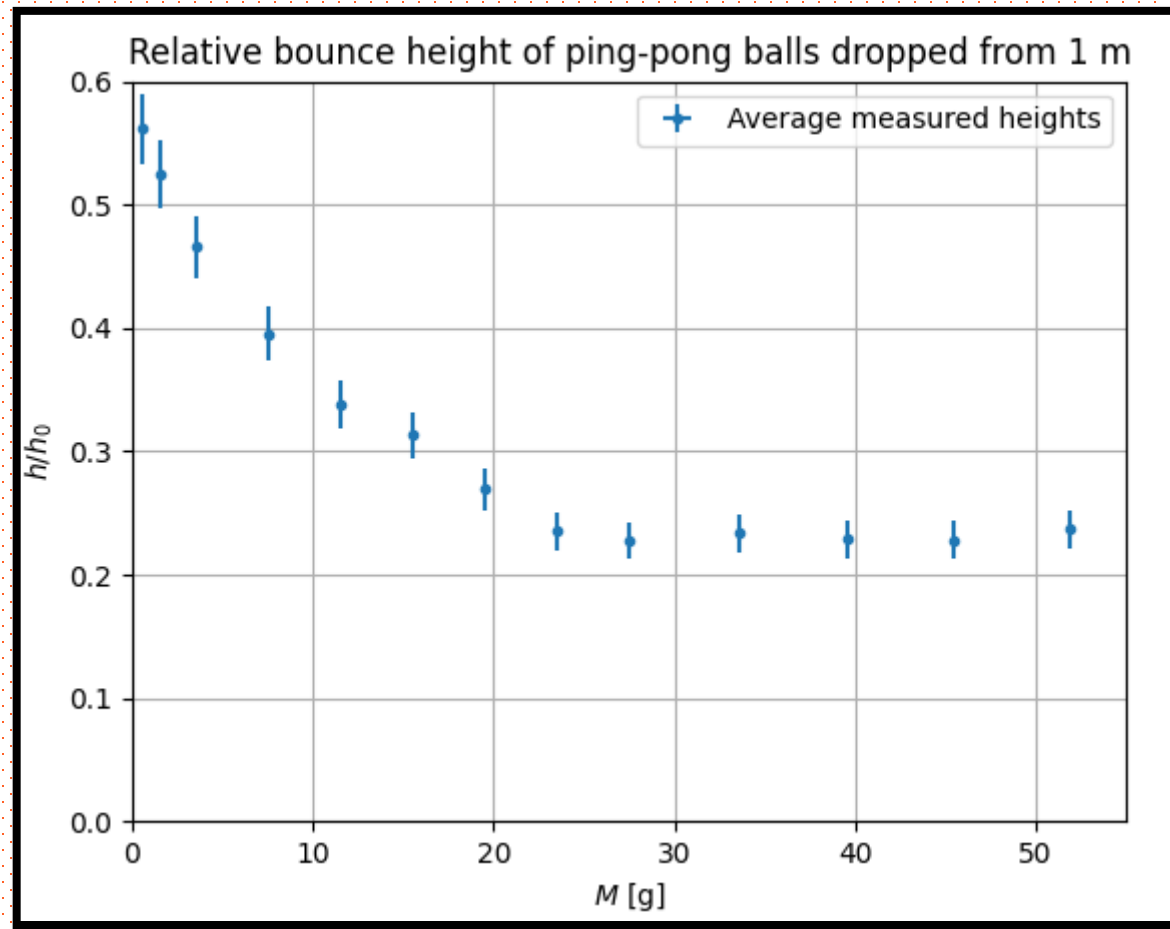
## Results for water

Each data point is an average of three measurements.

# Example

Notice how the ball doesn't rebound the second time  
Ball rebounds from 50 cm to 31 cm  $\rightarrow 0.6$   
15 ml water ball used



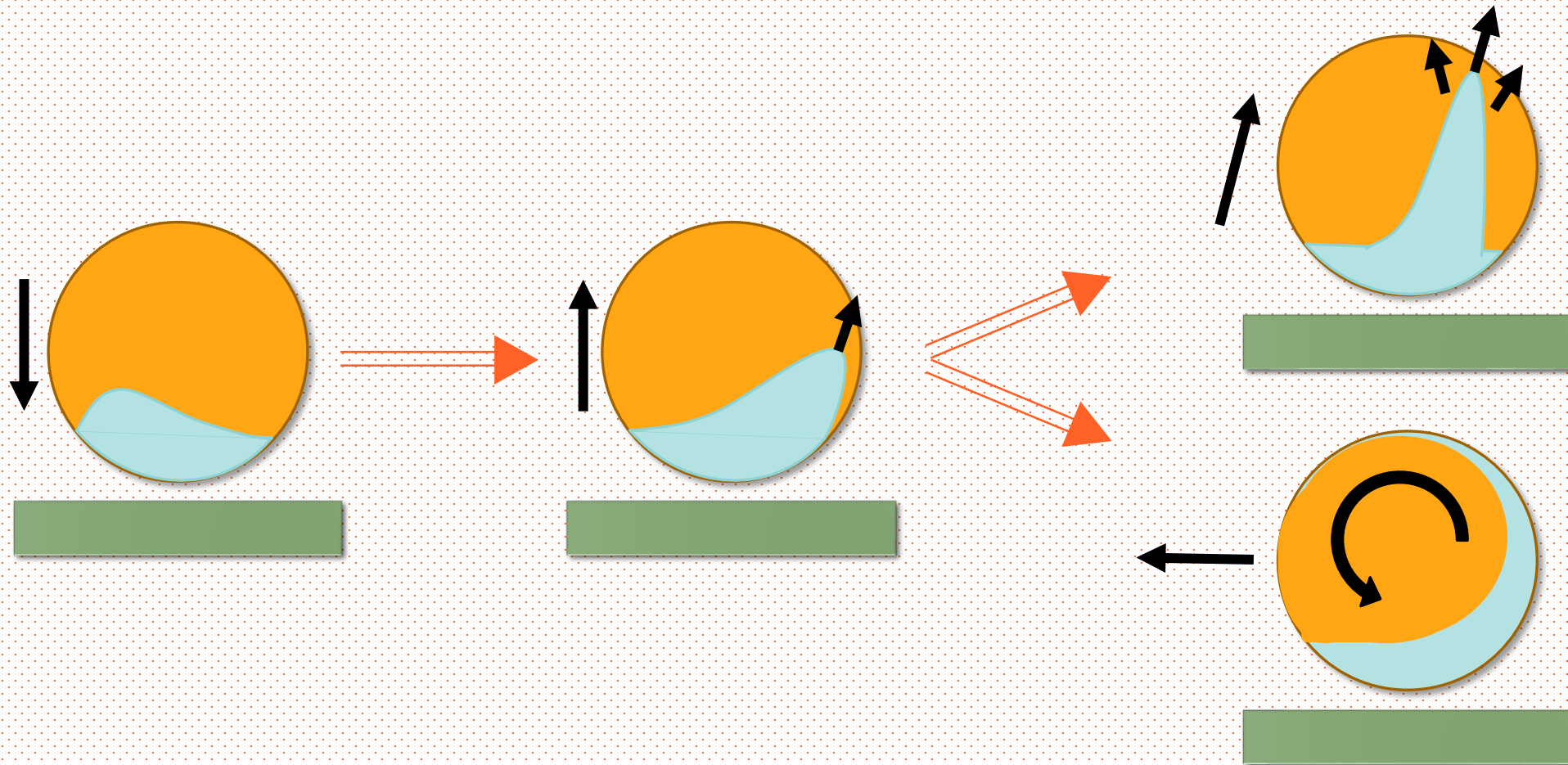


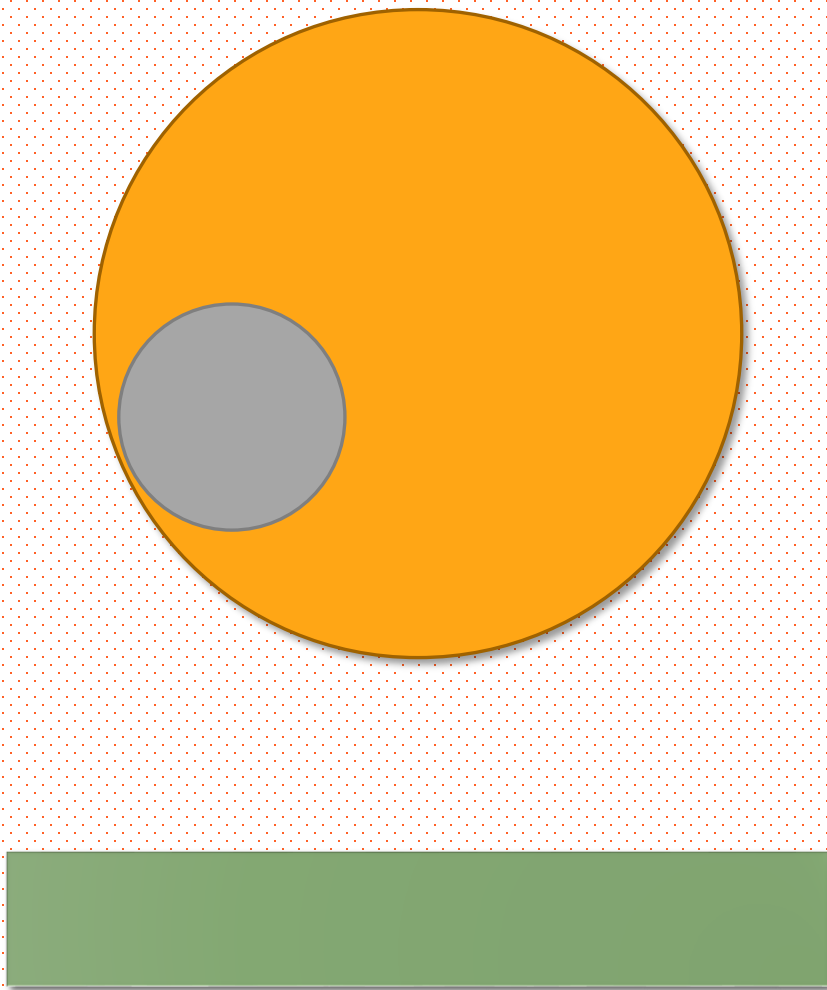
## Results for sand

Each data point is an average of three measurements.

# Energy losses

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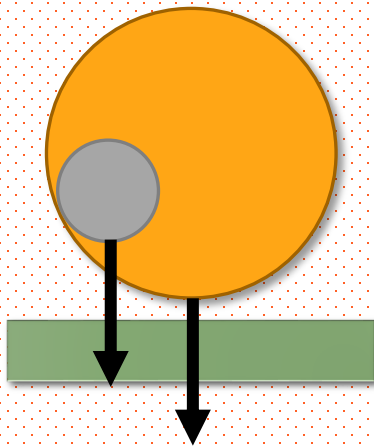


Inner ball analogy

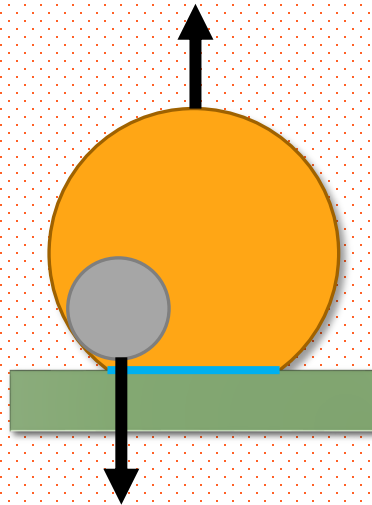


# Stiff ball inside the sphere – rotation

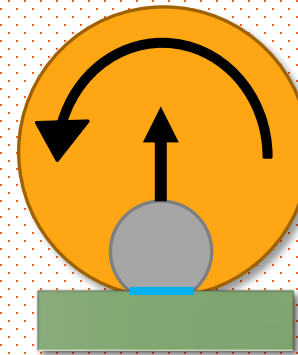
Same initial  
velocities



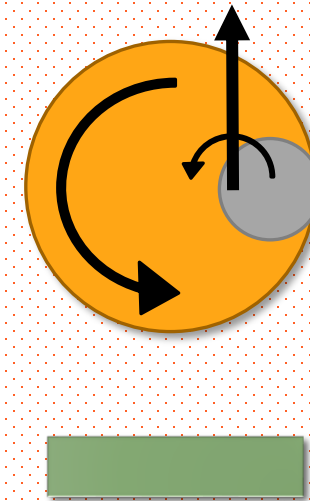
Ping-pong ball  
rebound



Stiff ball  
rebound



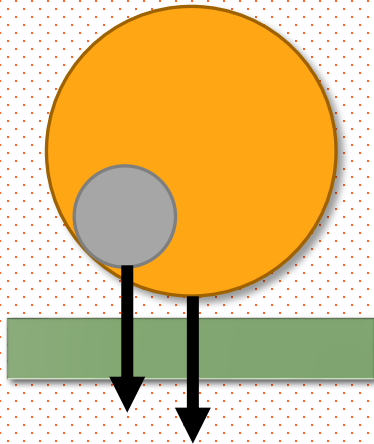
Rotation around  
centre of mass



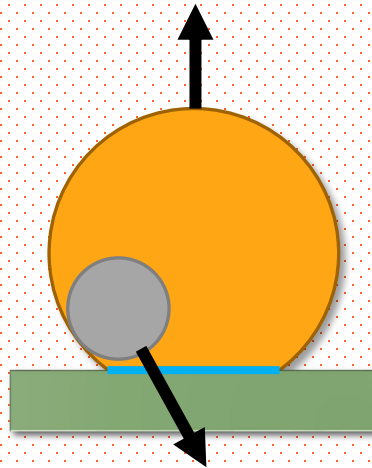
# Stiff ball inside the sphere – 'jets'

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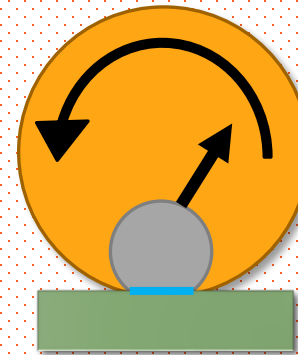
Same initial velocities



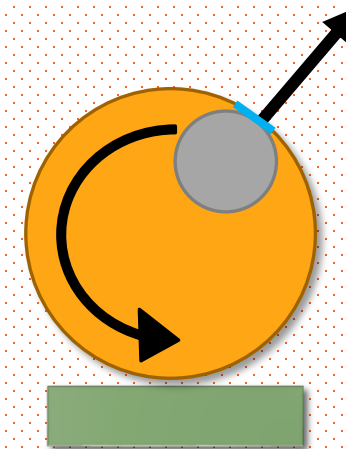
Ping-pong ball rebound



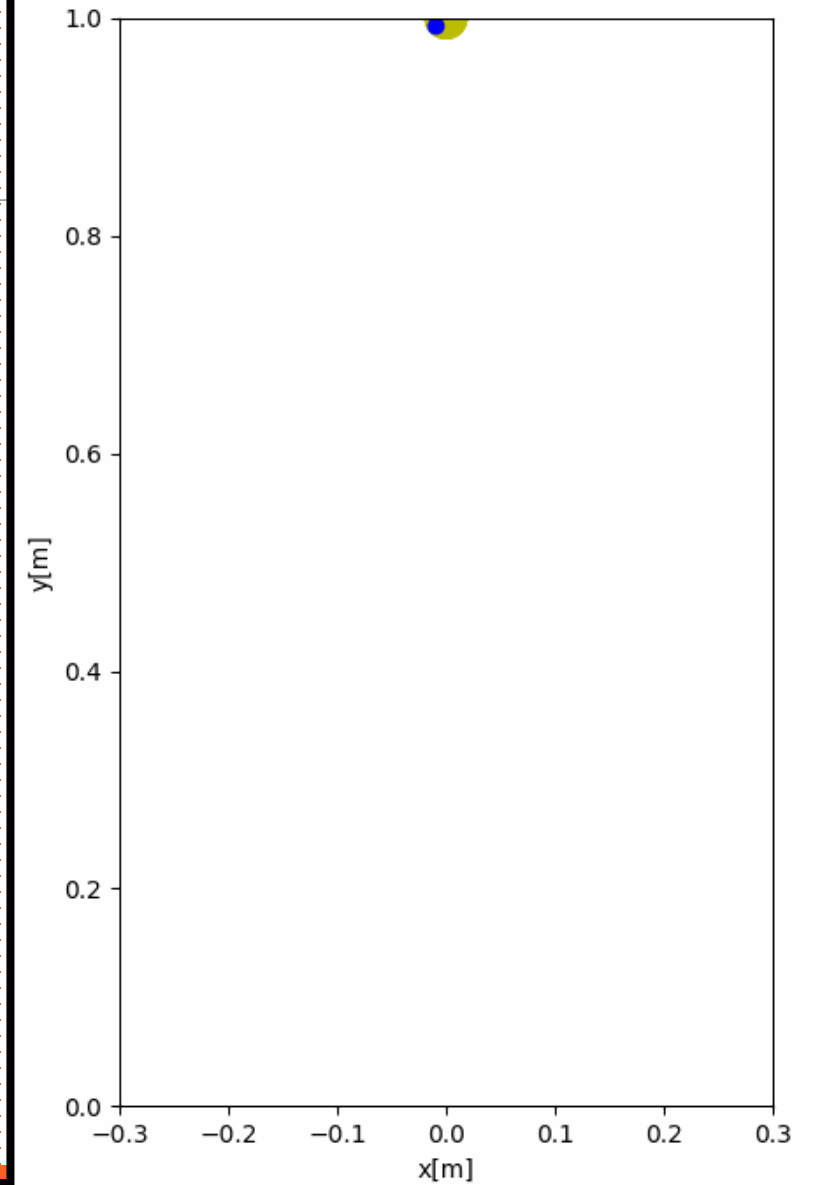
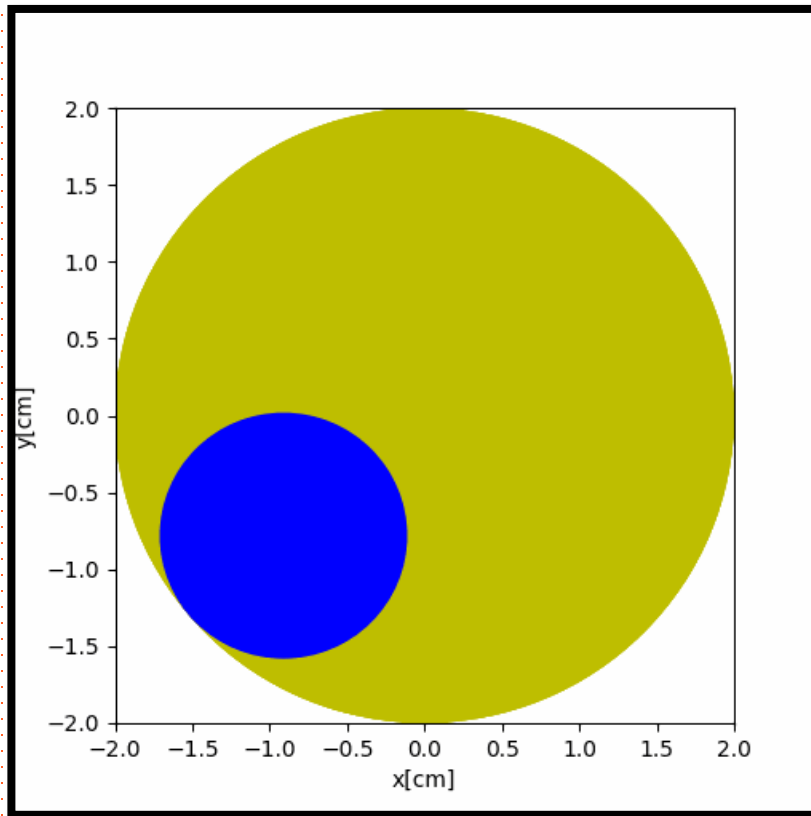
Stiff ball rebound



Collision between the two balls

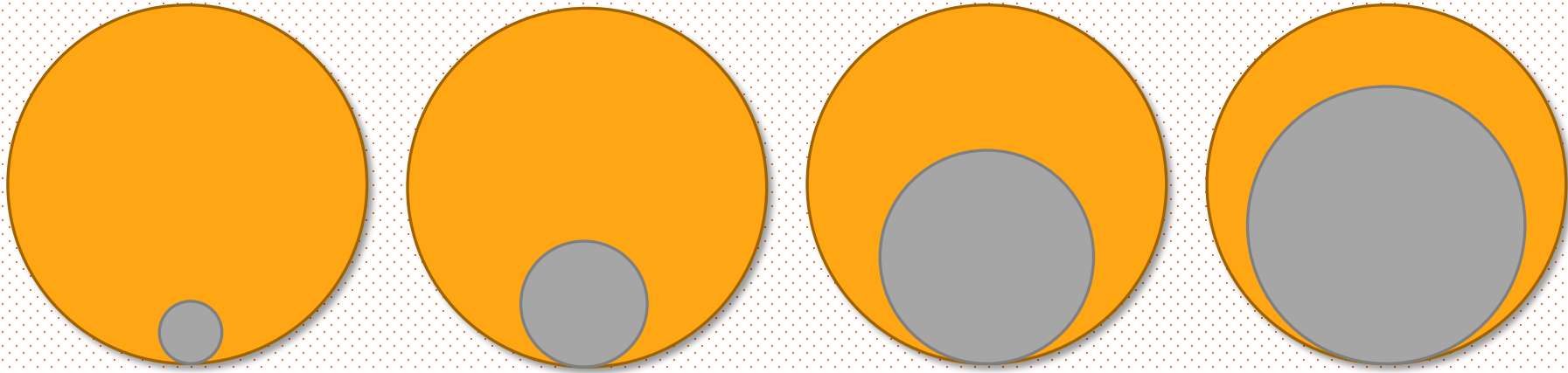


# Numerical simulation

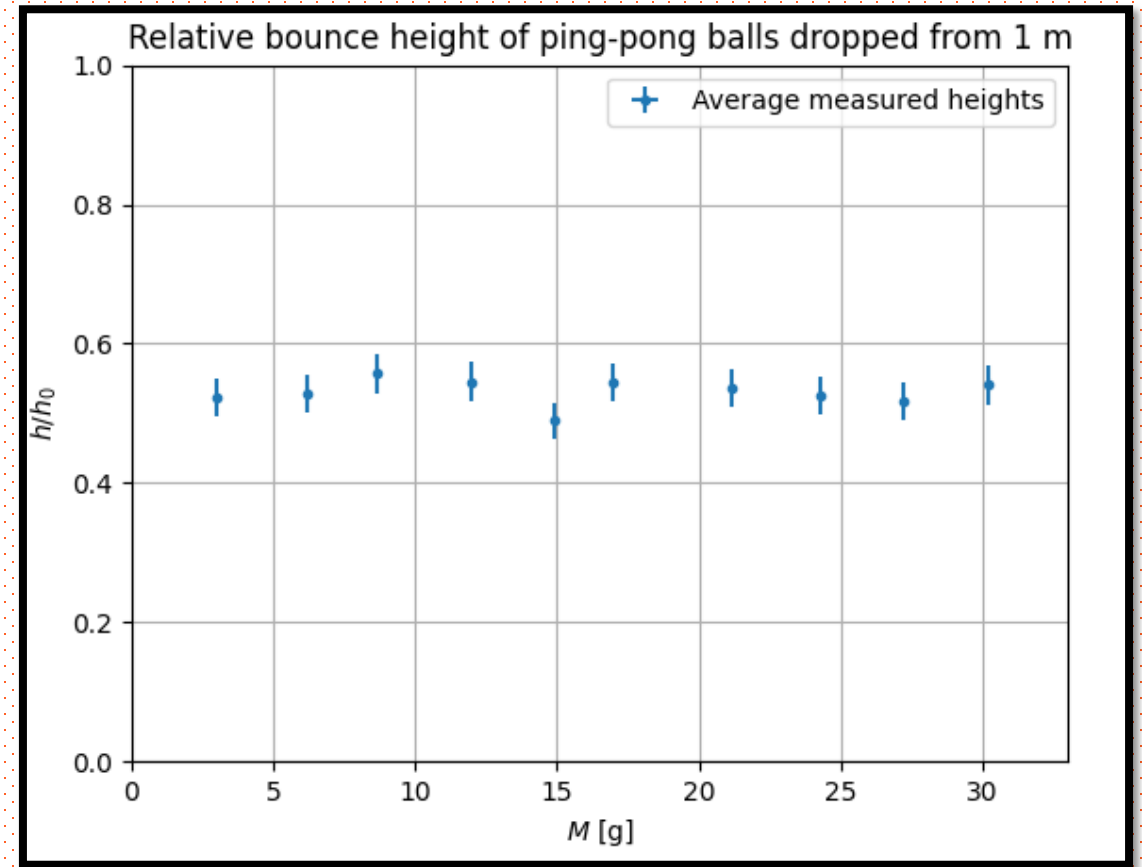
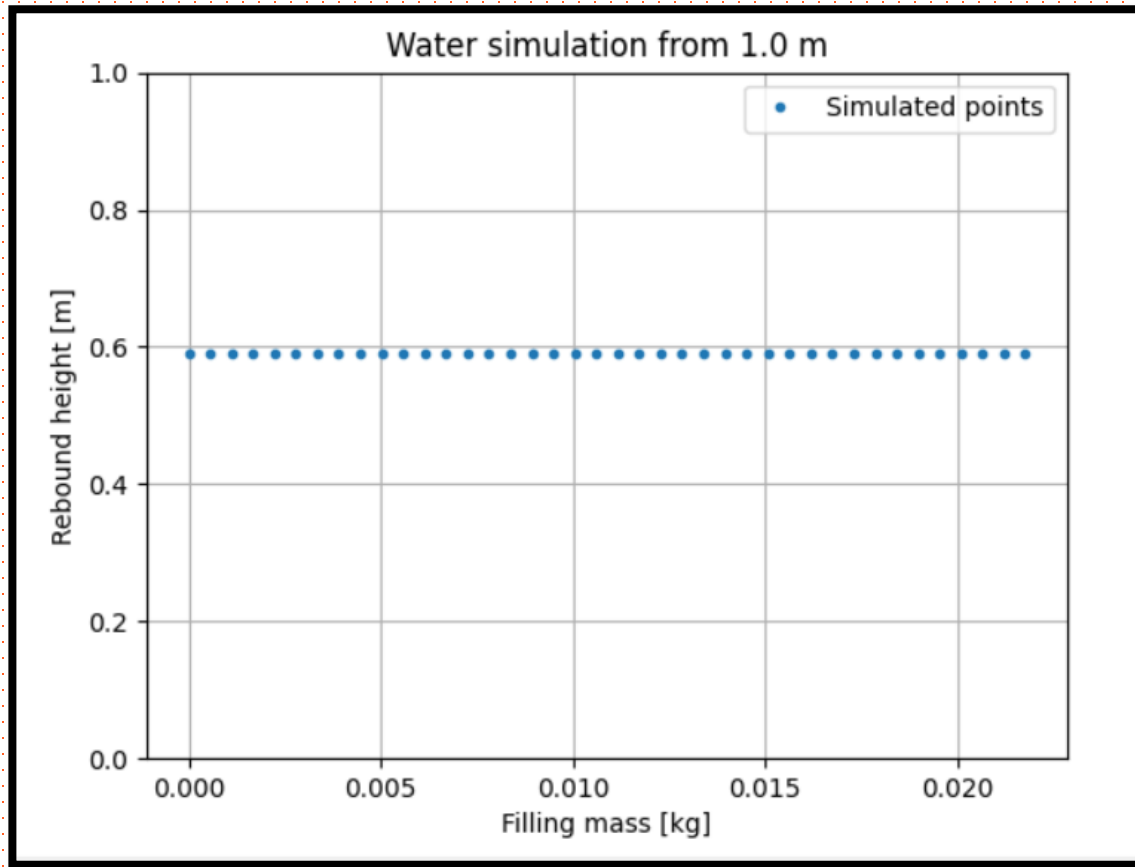


# Filling mass and ball size

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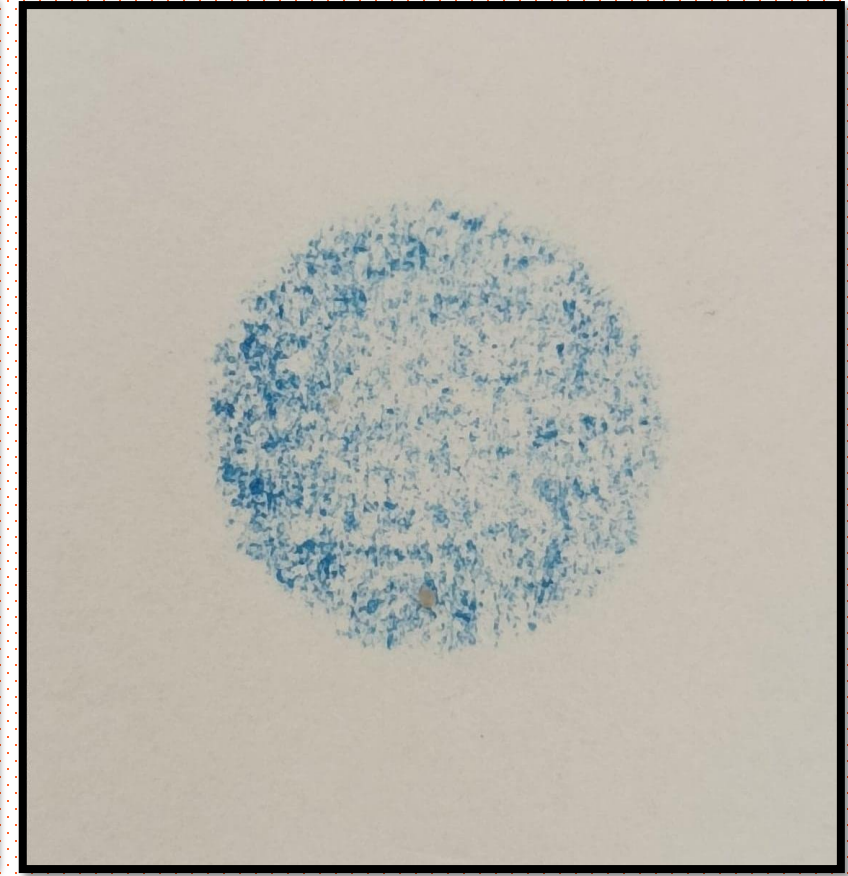
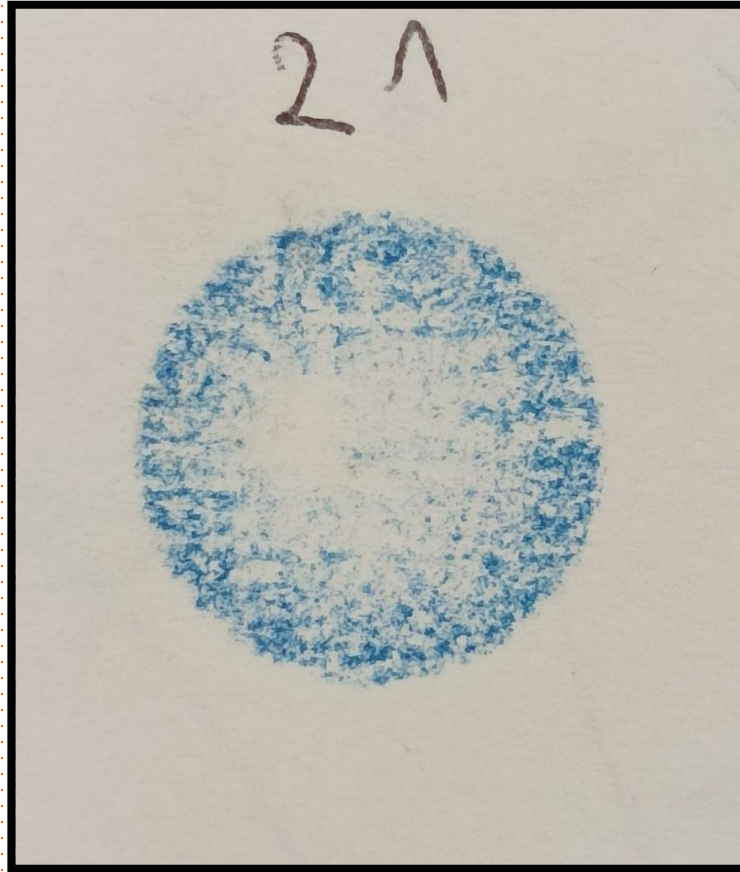


# Simulation for water

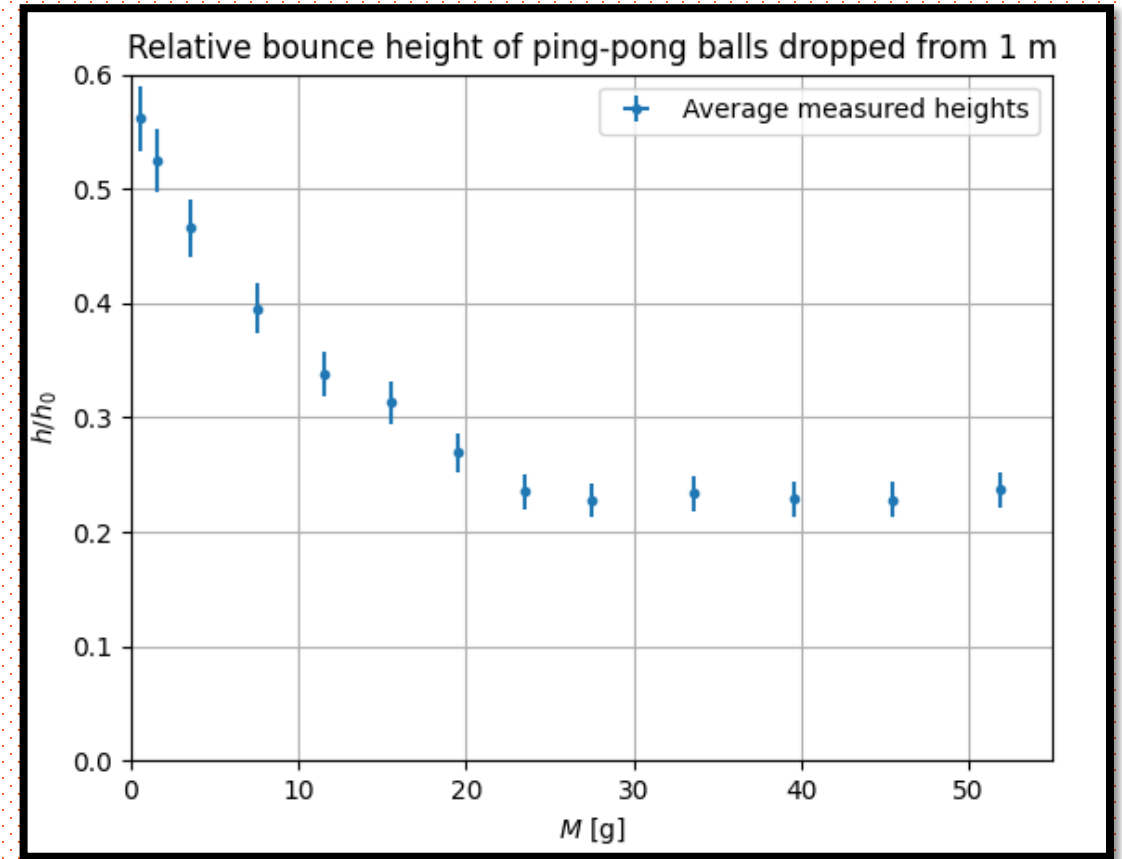
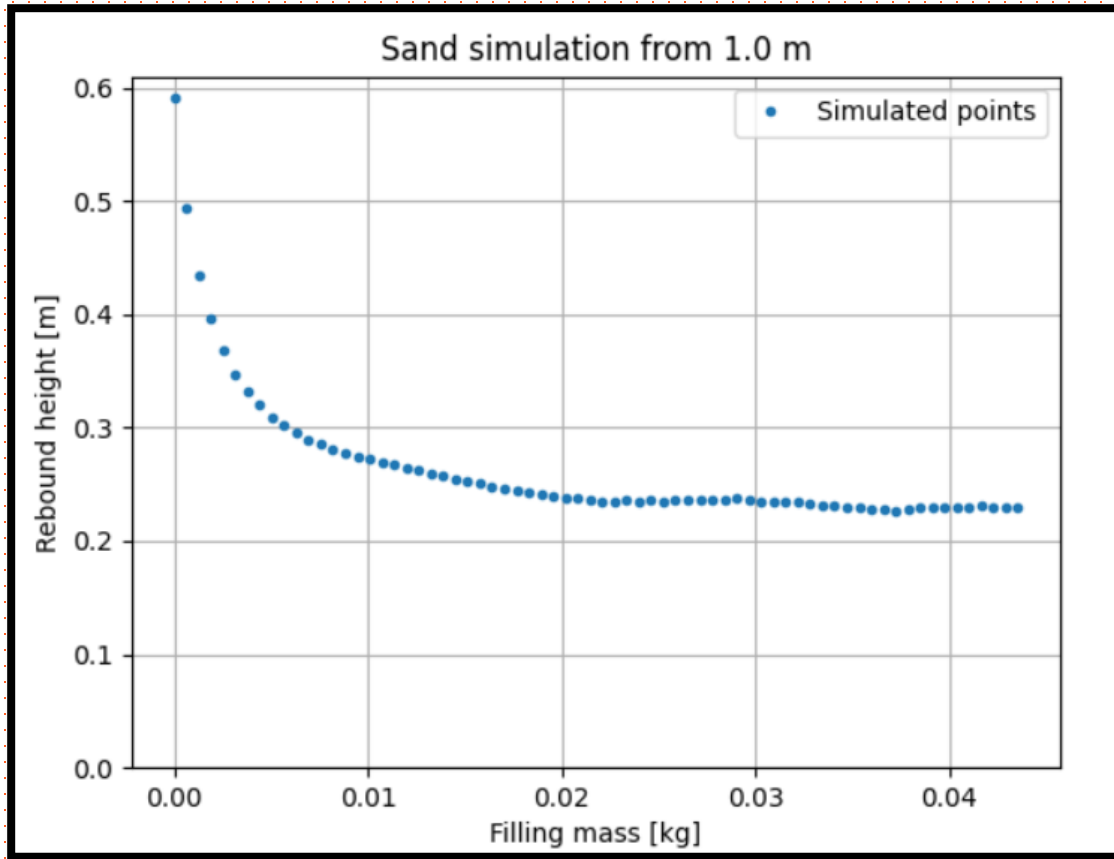


# Differences between water and sand

Ball	Retained Energy
Ping-pong	0.587
Water	0.587
Sand	0.2

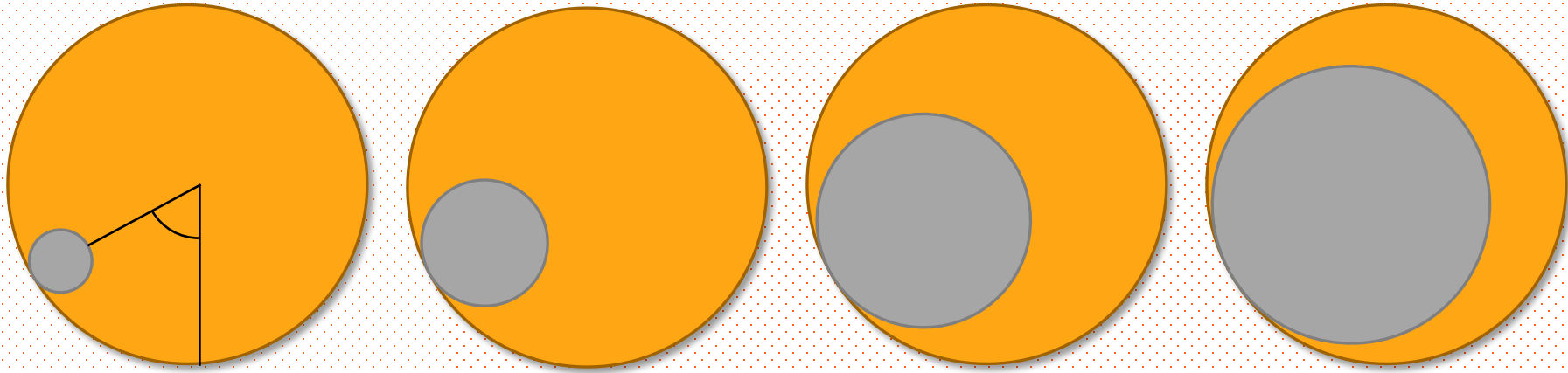


# Simulation for sand



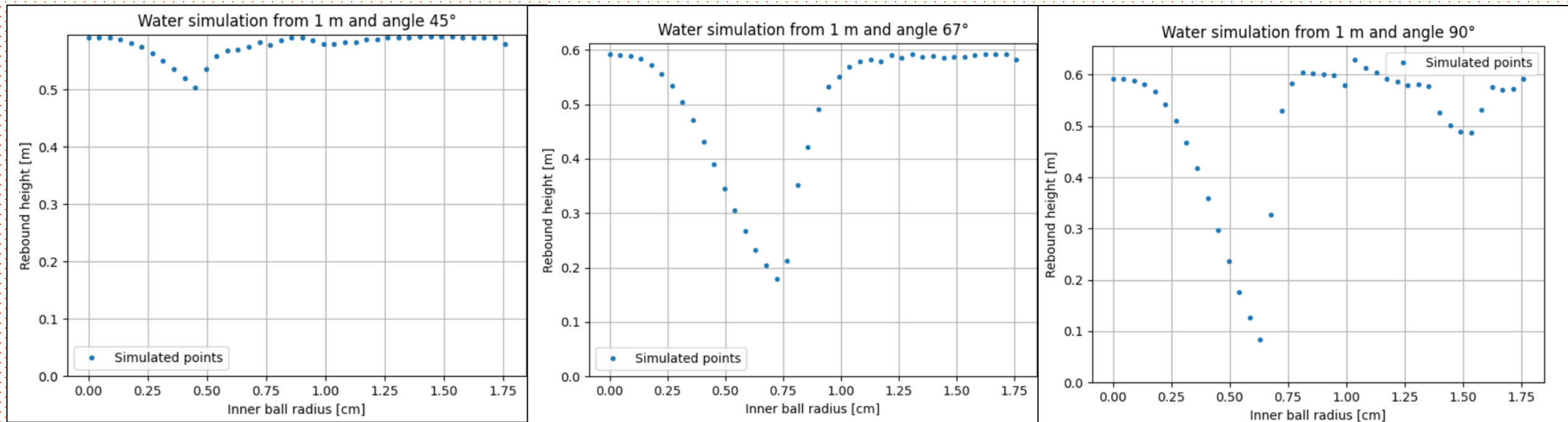
# Initial angle

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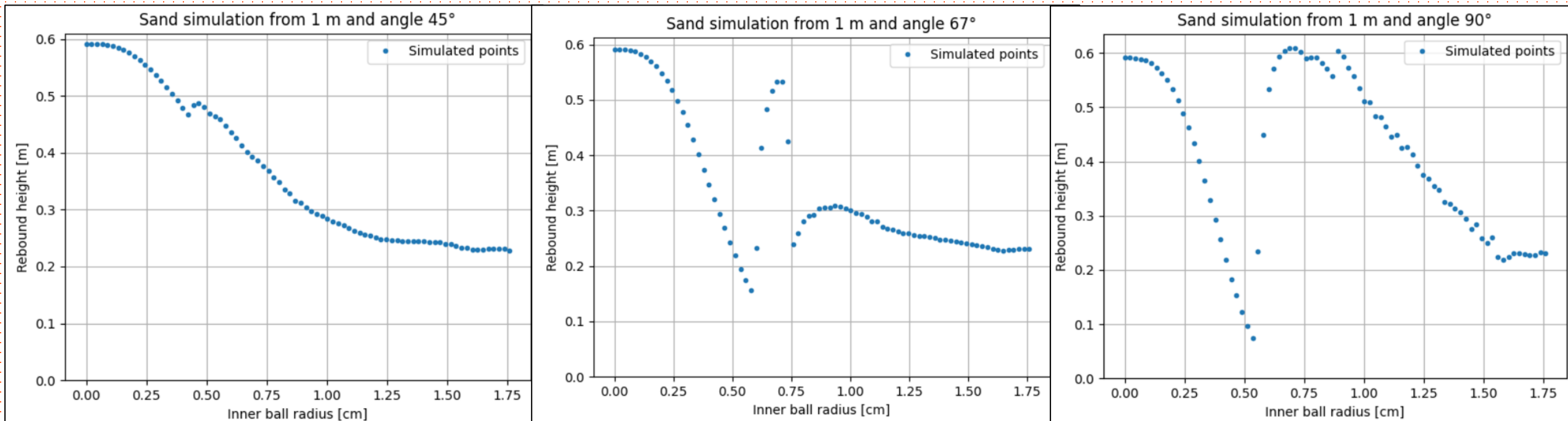




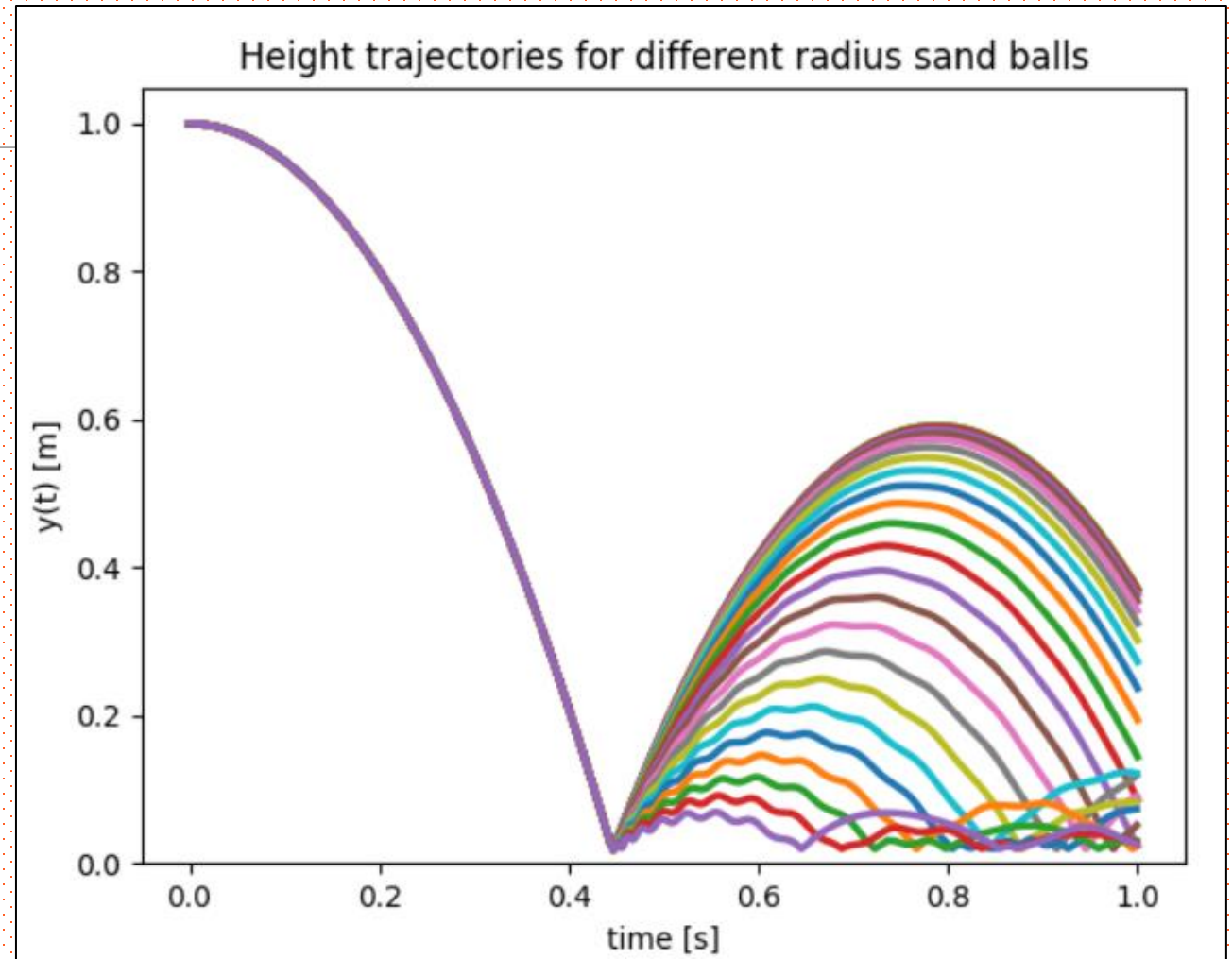
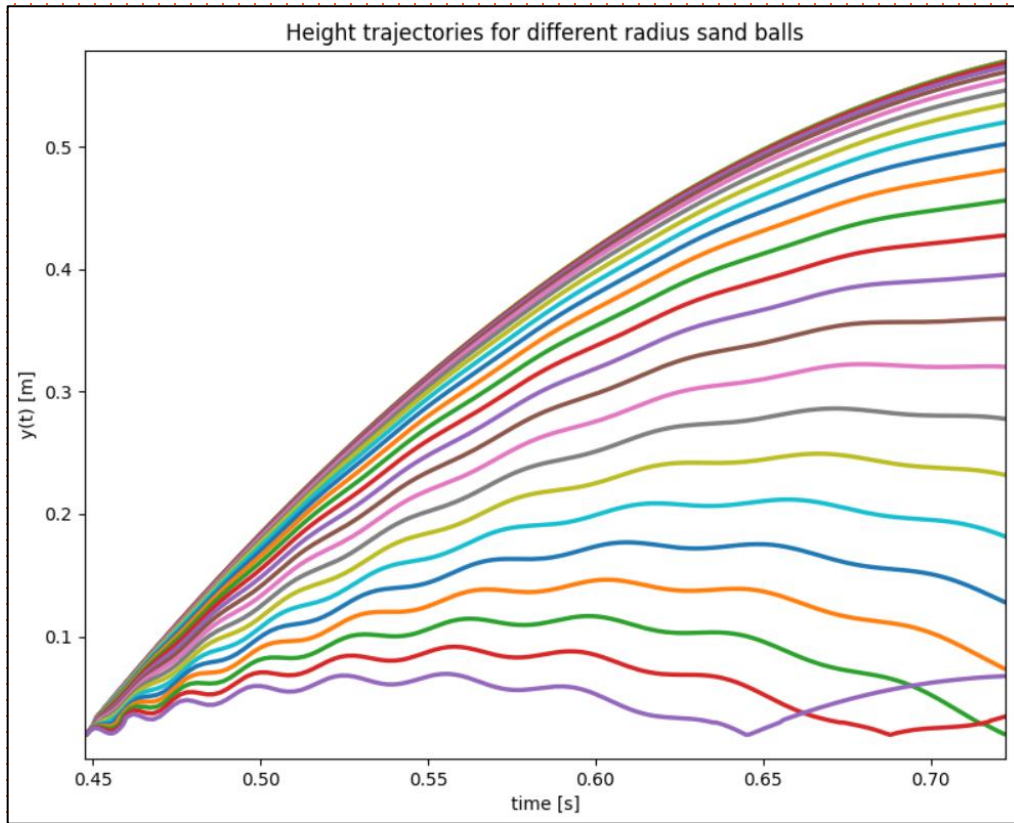
# Different angles - water



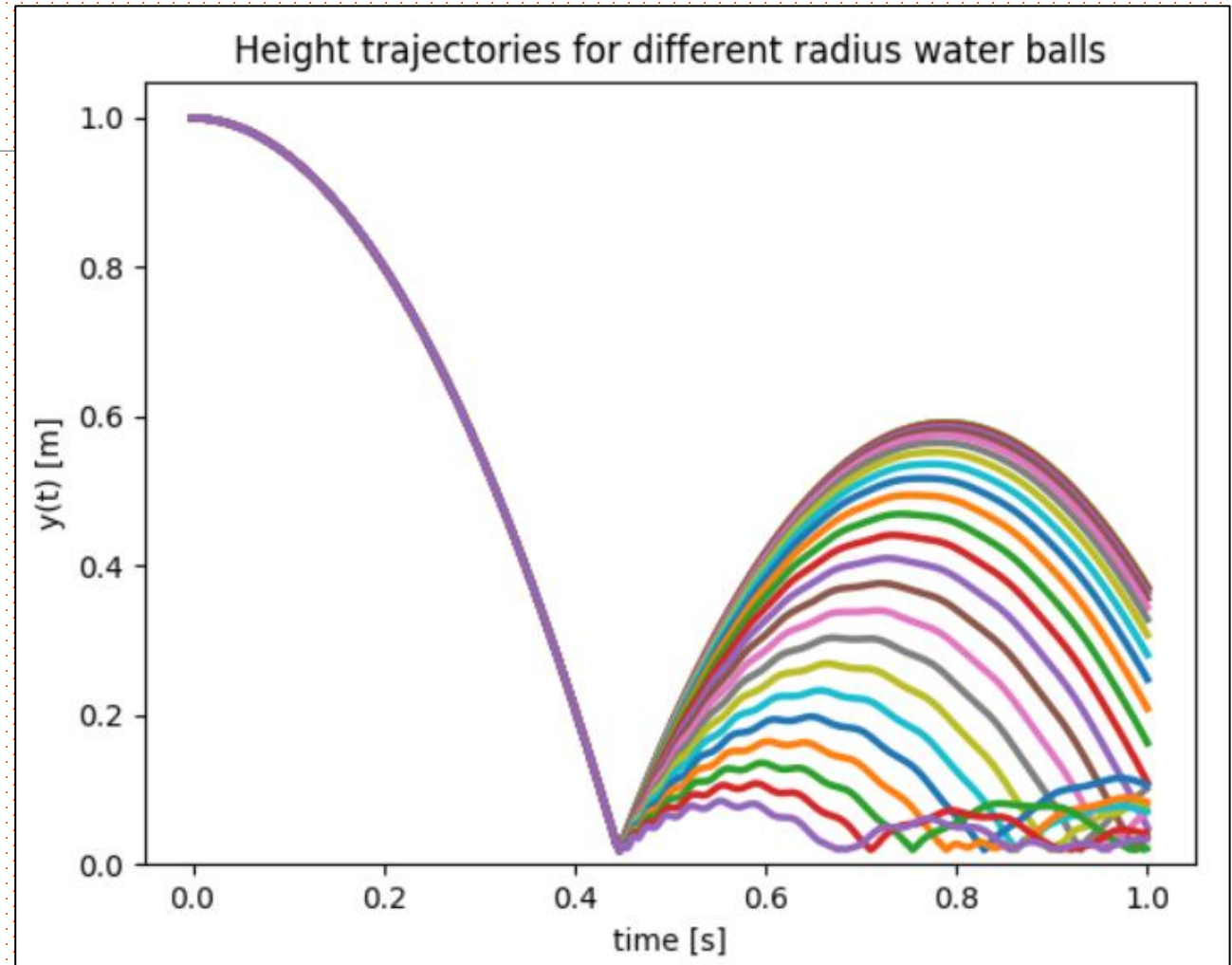
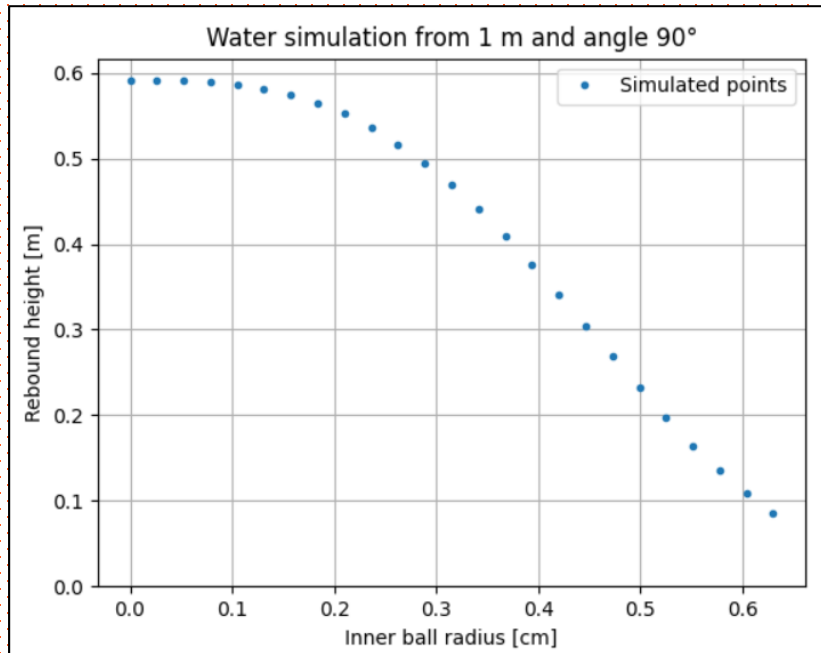
# Different angles - sand



# Sand



# Water



# Conclusions

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- We proved that provided the right setup the height of the bounce can be the same for a half filled ball
- We measured how the height depends on the mass of the filling, the difference between liquid and a granular material
- We proved that the damping is a result of symmetry breakage in initial conditions and explained the movement of filling during impact
- We provided a useful analogy with a stiff ball inside the ping-pong ball and checked how it correlates to the initial problem

# References

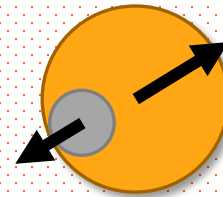
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1. *Street hockey ball*, U.S. patent 6,645,098 (11 November 2003)
2. T. W. Killian, R. A. Klaus, T. T. Truscott, *Rebound and jet formation of a fluid-filled sphere*
3. F. Pacheco-Vázquez , S. Dorbolo, *Rebound of a confined granular material: combination of a bouncing ball and a granular damper*
4. <https://www.pyimagesearch.com/2015/09/14/ball-tracking-with-opencv/>
5. L. Pauchard, S. Rica 1998, *Physics of a ,ping-pong' ball*

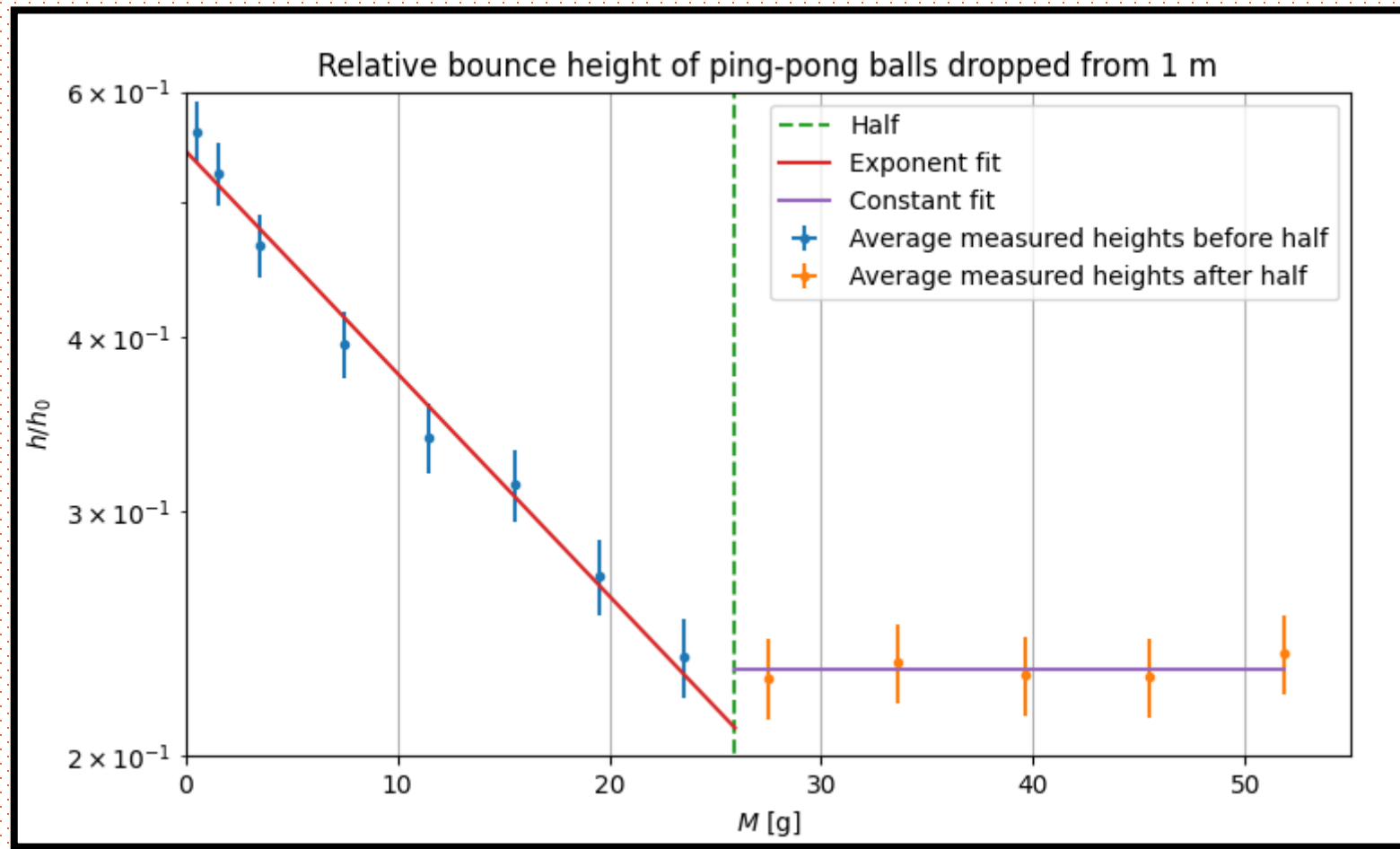
# The Simulation

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- The balls are in constant gravitational field
- Ping-pong ball feels  $k \cdot (\text{radius} - y)$  force (while  $y < \text{radius}$  and  $v < 0$ ), where  $k = 15000$  (to match dynamometer measurement), **but** feels  $0.587 \cdot k \cdot (\text{radius} - y)$  while  $v > 0$
- The inner balls bounce in one time step, losing 0.587 (water) or 0.2 (sand) of its energy
- The balls collision is calculated as one of two bodies with some relative radial velocities, with masses 2.5g for the ping-pong ball and  $\text{volume} \cdot \text{density}$  for the inner ball and the CoR 0.75



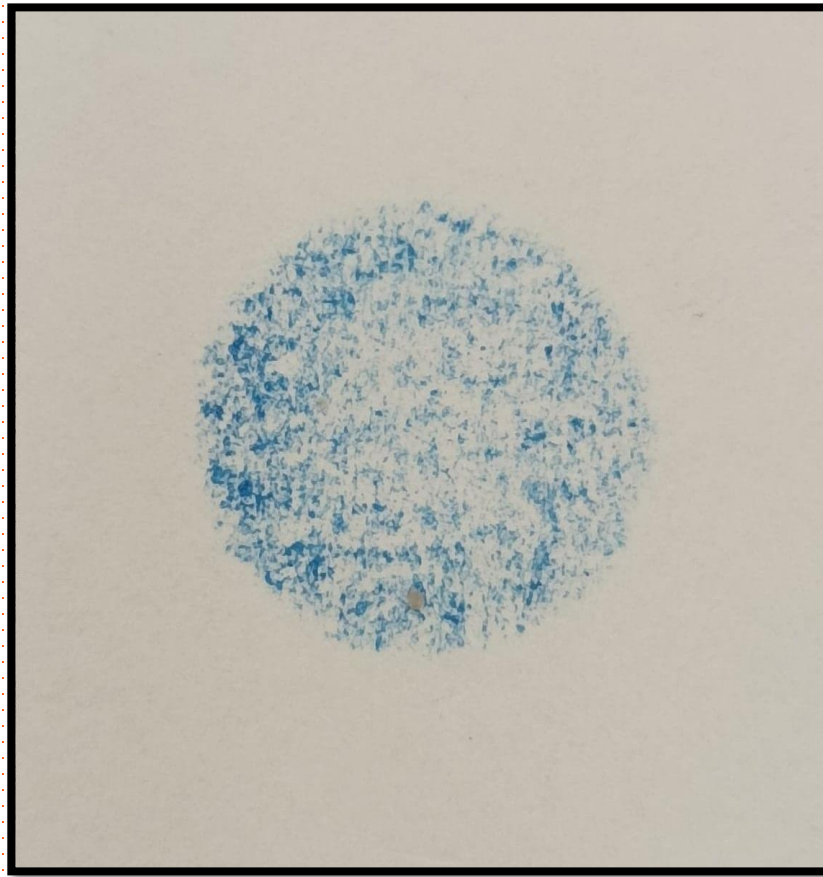
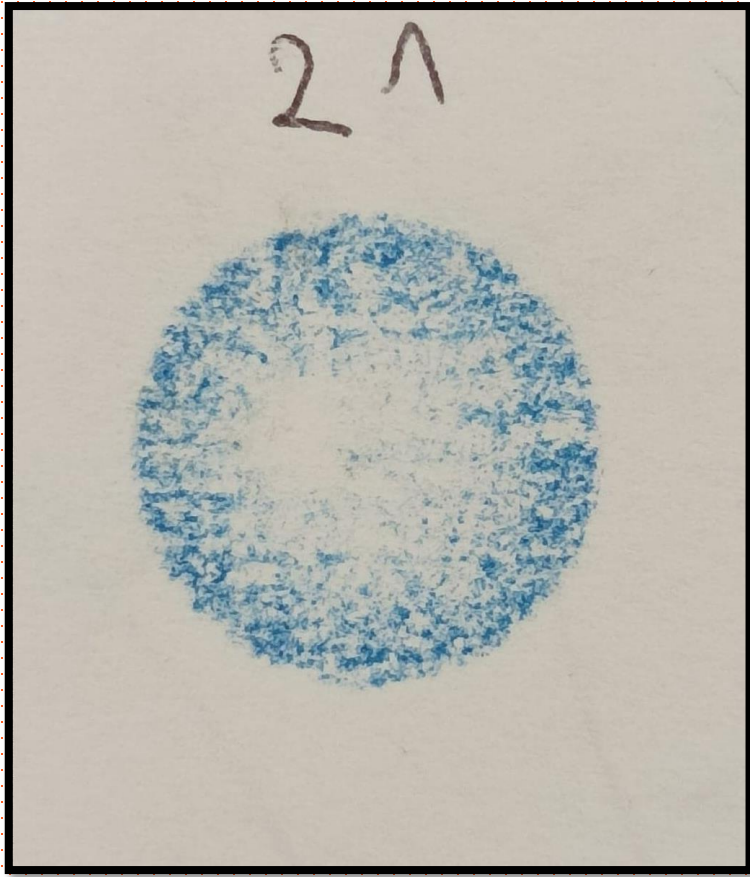
- The model is two dimensional, but mass scales with density times volume



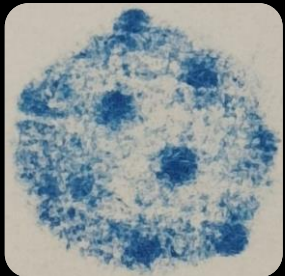
## Results in logarithmic scale

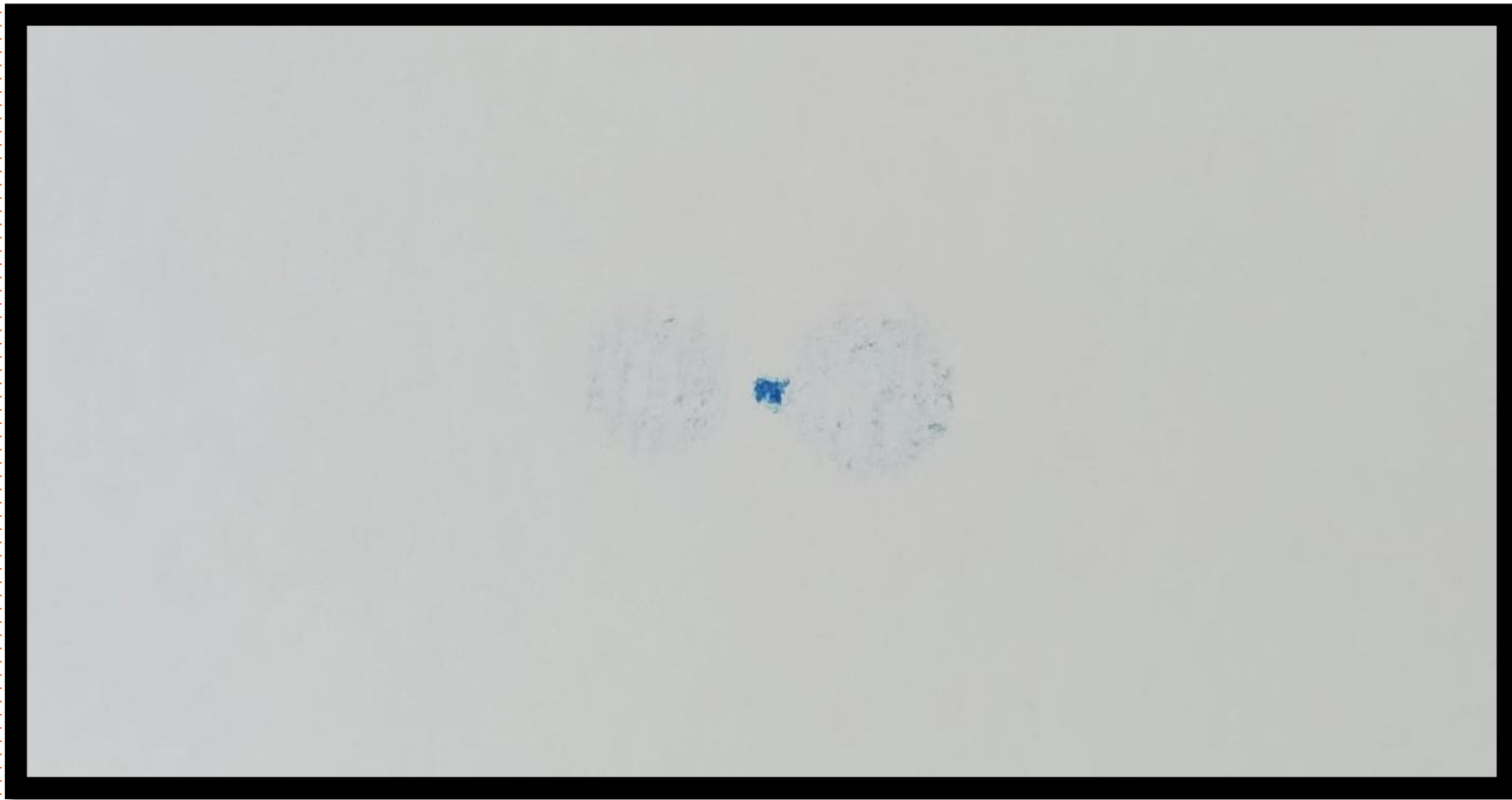
Exponential fit:  $0.587 \cdot \exp(-0.48M)$





Tracing paper

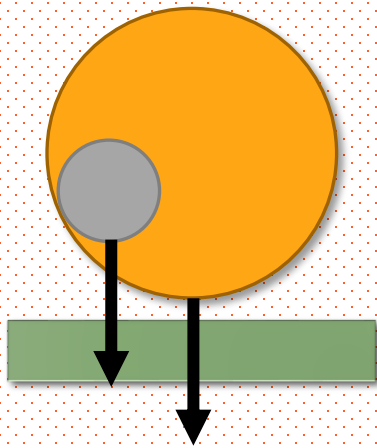




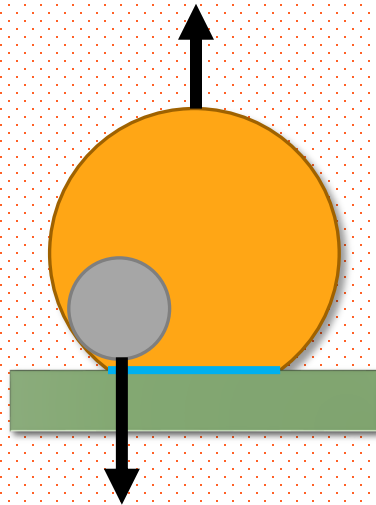
Double rebound

# Steel ball inside the sphere

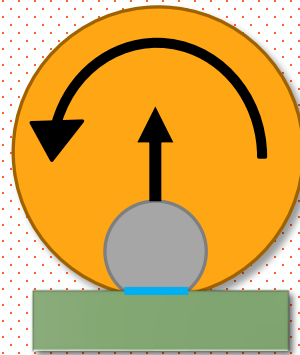
Same initial velocities



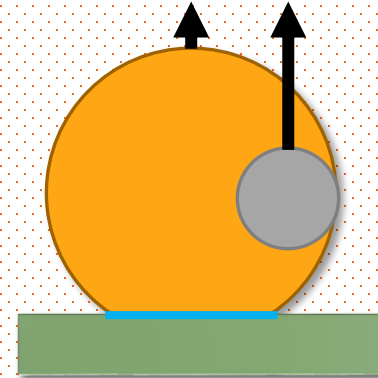
First rebound



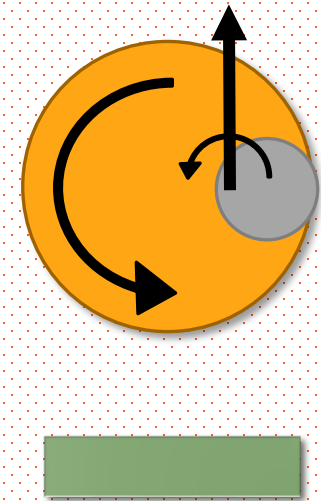
Steel ball rebound



Second rebound

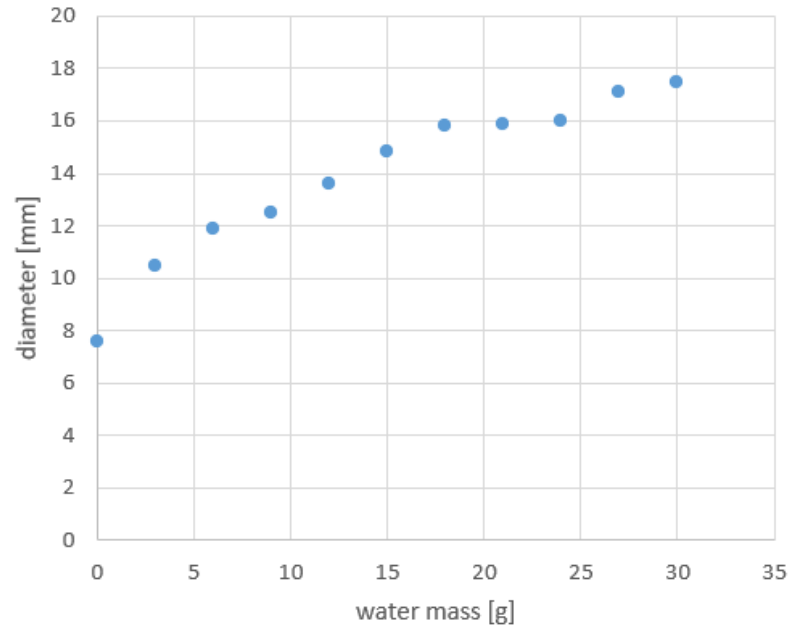


Rotation around centre of mass

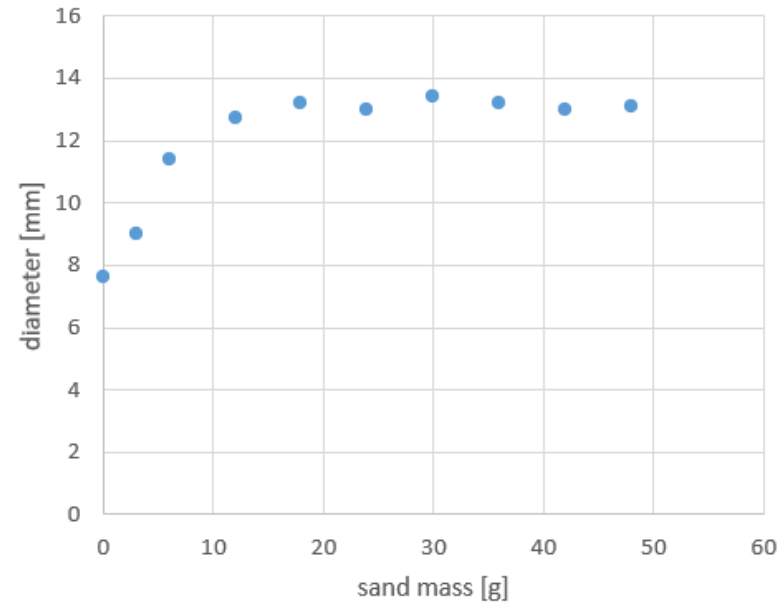


# Trace diameter

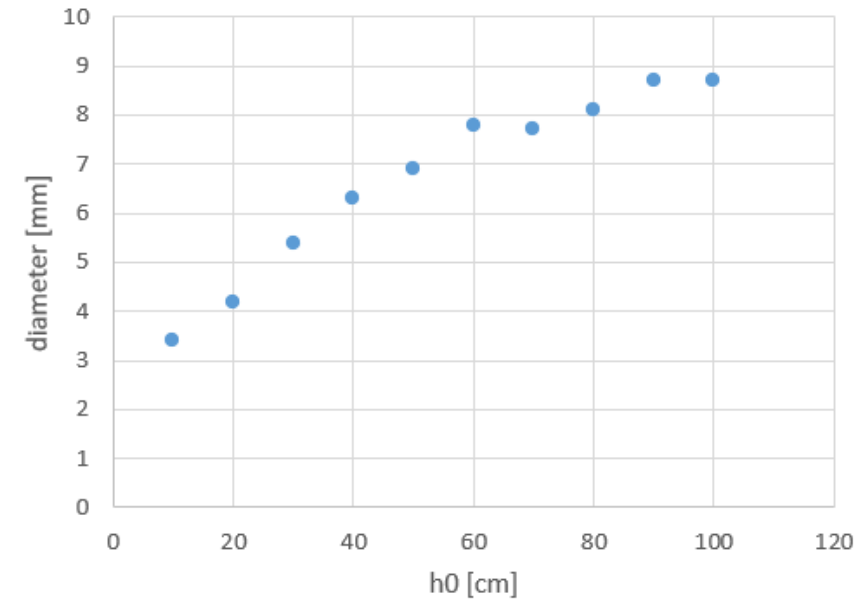
Trace diameter - water from 1m



Trace diameter - sand from 1m



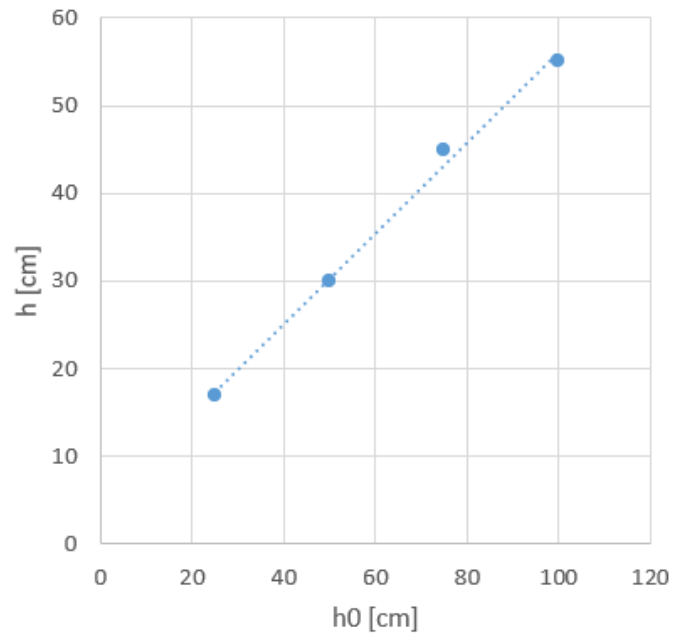
Trace diameter - empty ball



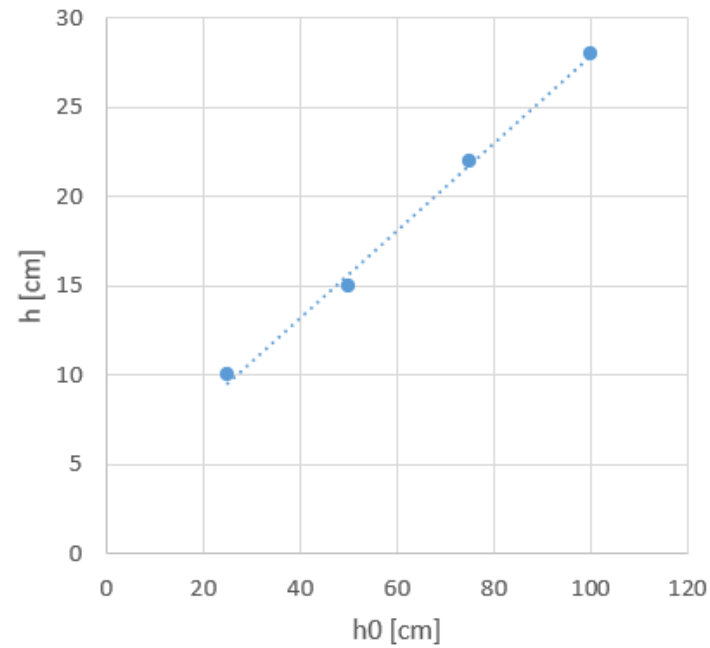
# Initial height

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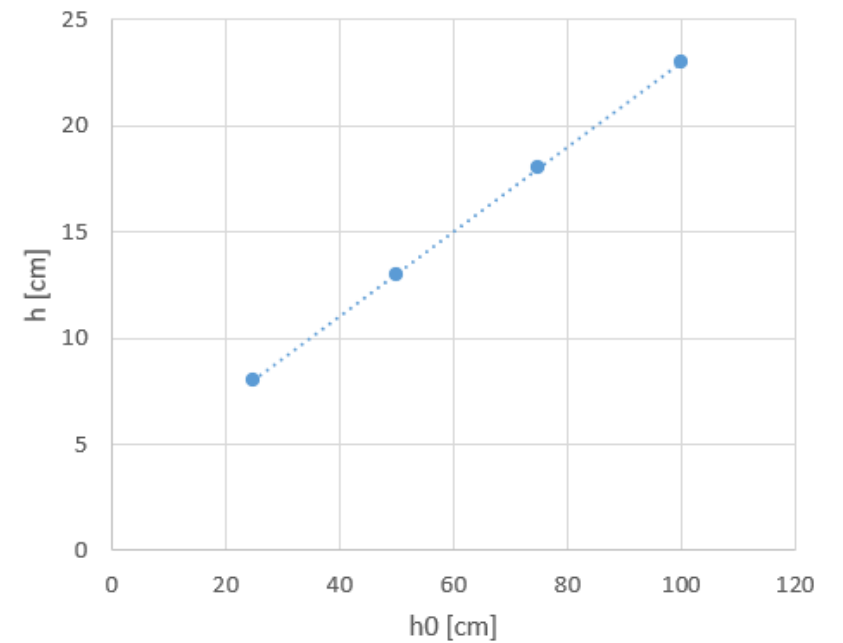
15 ml water



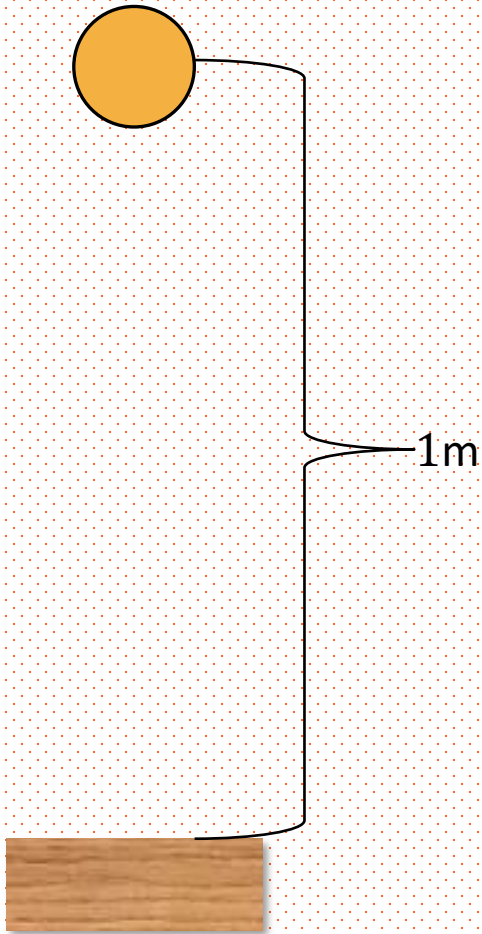
25 g sand



52 g sand



$$m = (2.55 \pm 0.01)\text{g}$$



$$\left\langle \frac{h}{h_0} \right\rangle = 0.587$$

$$\sigma = 0.0126$$

$$\delta h = \frac{\sigma}{\sqrt{n}} = \frac{0.0126}{\sqrt{21}} \approx 0.003$$

$$\alpha = \frac{E}{E_0} = \frac{h}{h_0} = (58.7 \pm 0.3) \cdot 10^{-2}$$

# Limits of our simulation

