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UNIVERSITY OF WARSAW

Problem 4

STUCK METALLIC SPHERES

Reporter: Stanisław Rakowski
Authors: Stanisław Rakowski, Kamil Dutkiewicz

Problem statement

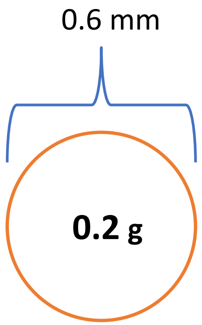
Fill a bottle with small metal/plastic spheres with diameters of the same order of magnitude as the size of the opening. Try to pour the spheres out of the bottle by turning it upside down. Similar to pouring salt from small openings, one can see that after a certain time the spheres become stuck and stop pouring out.

1. **Investigate the phenomenon.**
2. **What is the average time it takes before the system becomes stuck?**
3. **What bottle shapes can prevent the system from getting stuck?**

Relevant parameters

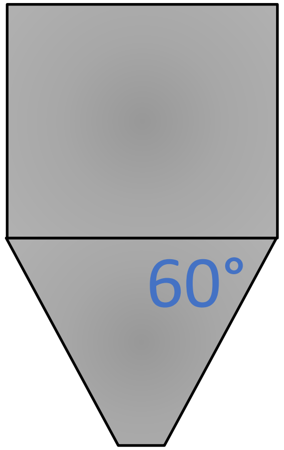
Ball's parameters

- ➔ diameter
- ➔ mass
- ➔ friction/material



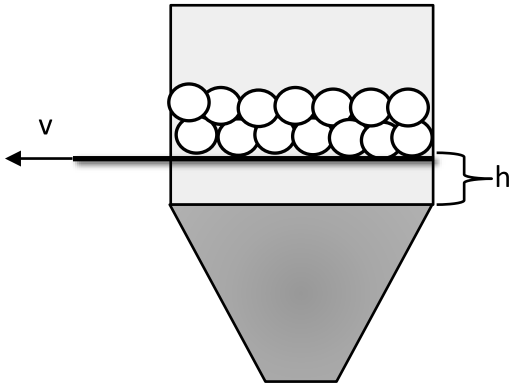
Nozzle parameters

- ➔ orifice diameter
- ➔ Shape/angle



Initial conditions

- ➔ number of balls
- ➔ Initial height
- ➔ Opening method (velocity)



Relevant parameters

Fixed

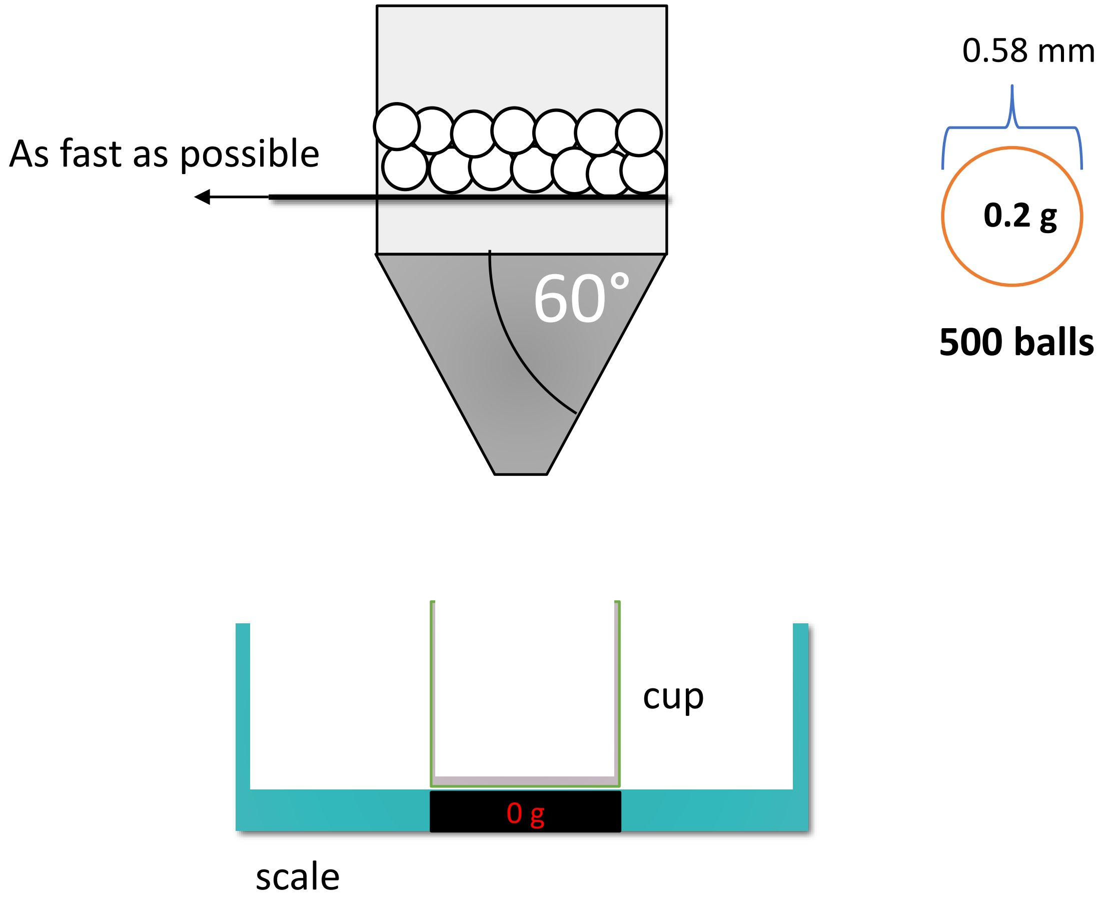
- mass of a ball
- material of a ball
- angle
- number of balls
- initial height
- opening velocity

Variables

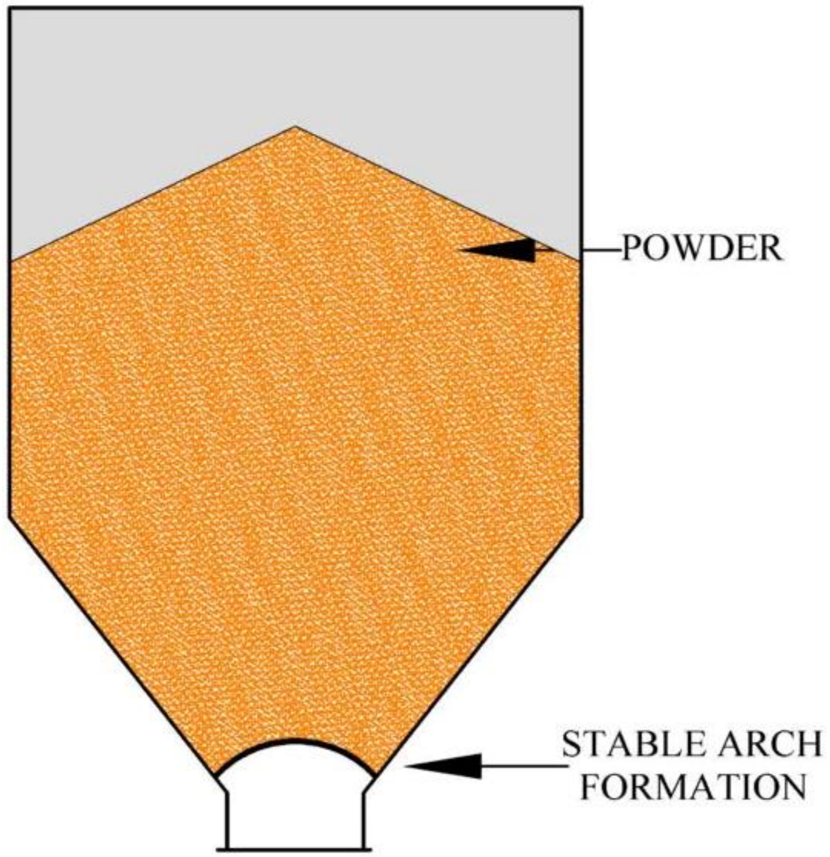
- orifice diameter
- ball's diameter

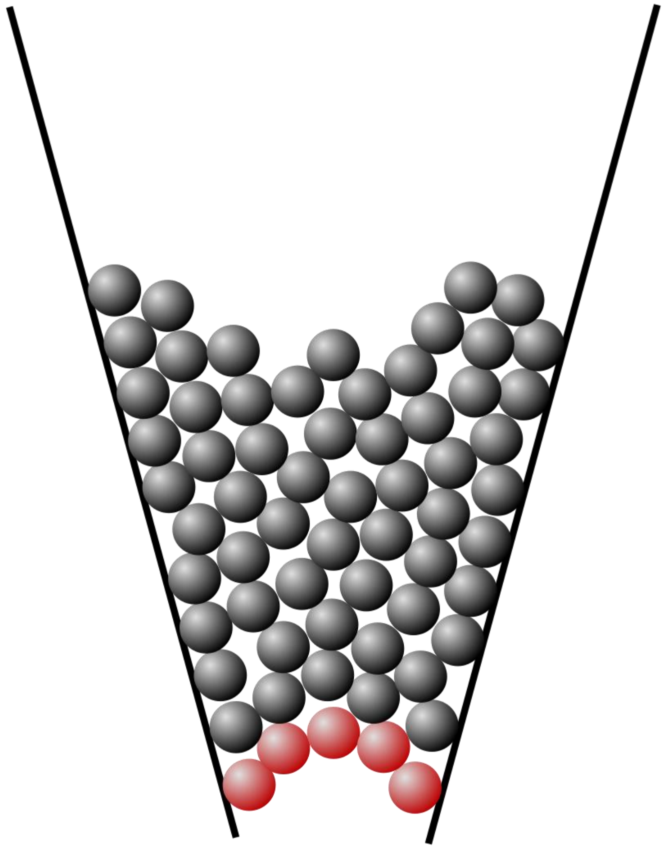
[1] *Jamming of Granular Flow in a Two-Dimensional Hopper*, K. To et al, Phys. Rev. Lett. 86, 71
[2] *Jamming in granular matter*, A. Garcimartin et al, AIP Conference Proceedings 742, 279 (2004)

Experimental setup

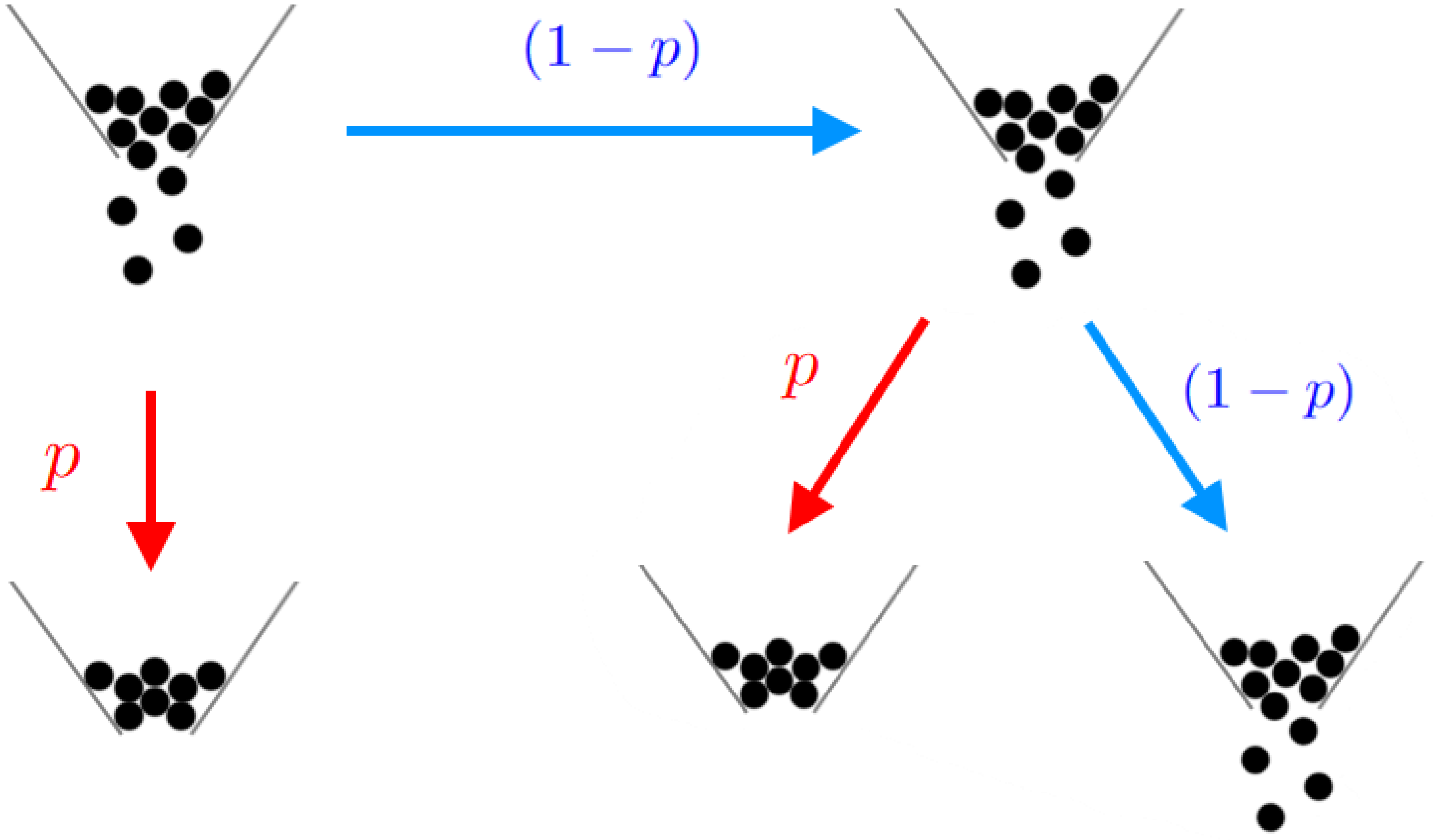


Arch formation





Modeling the flow as a probabilistic process

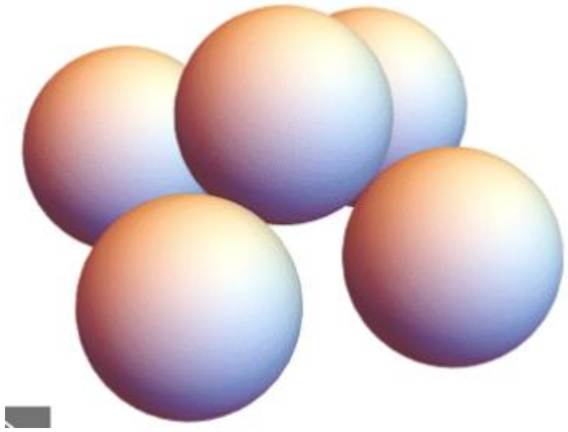


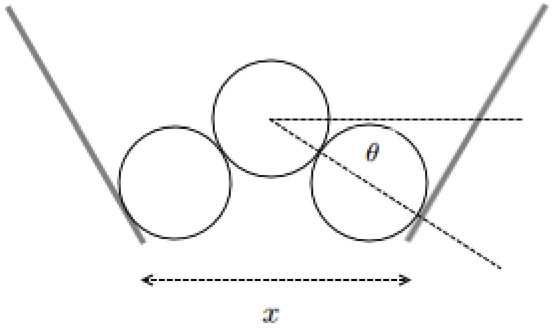
system gets stuck

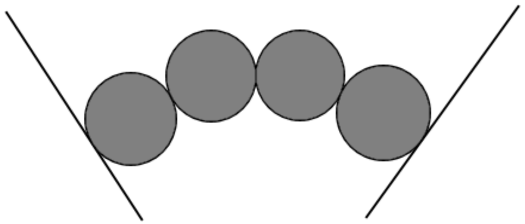
n_0 balls fall down

$$P(n) = (1-p)^{\frac{n}{n_0}} p$$

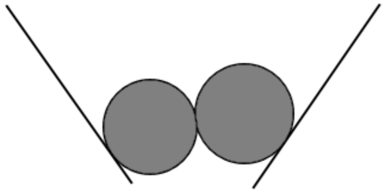
Probability of arch formation model







$p(n) \approx 0$



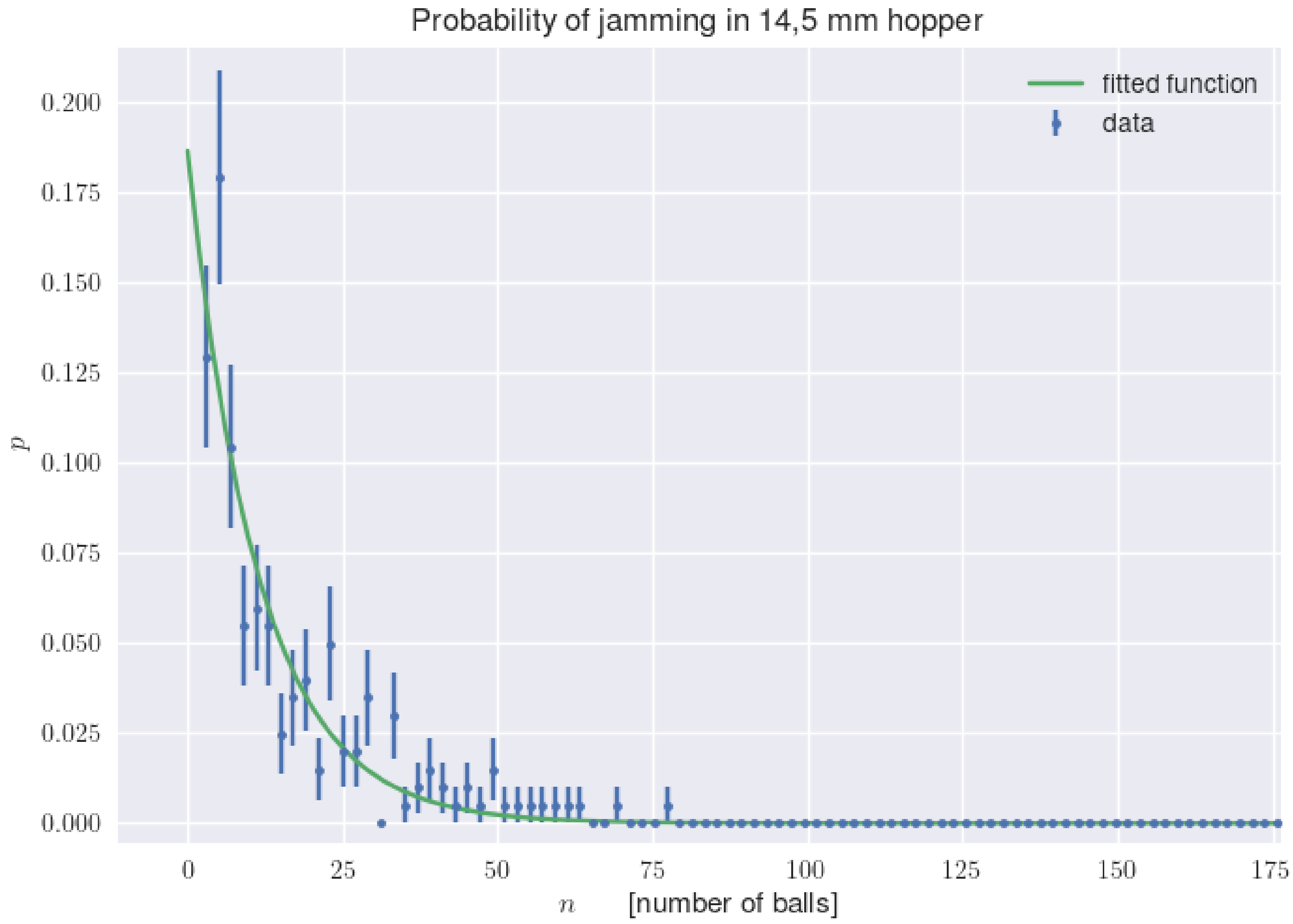
$p(n) \approx 1$

$$P_x(x) = \int_0^{\frac{\pi}{2}} d\theta \; \delta(x - 3\cos(\theta))$$

$$p(d) = \int_d^3 dx \int_0^{\frac{\pi}{2}} d\theta \; \delta(x - 3\cos(\theta))$$

$$p(d) = \frac{\arccos(\frac{d}{3})}{\pi}$$

Measuring probability distribution



$$P(n) = (1-p)^{\frac{n}{n_0}} p$$

$$d = 15 \text{ mm}$$

$$p = 17,8\%$$

$$n_0 = 2.7$$

Measuring probability distribution

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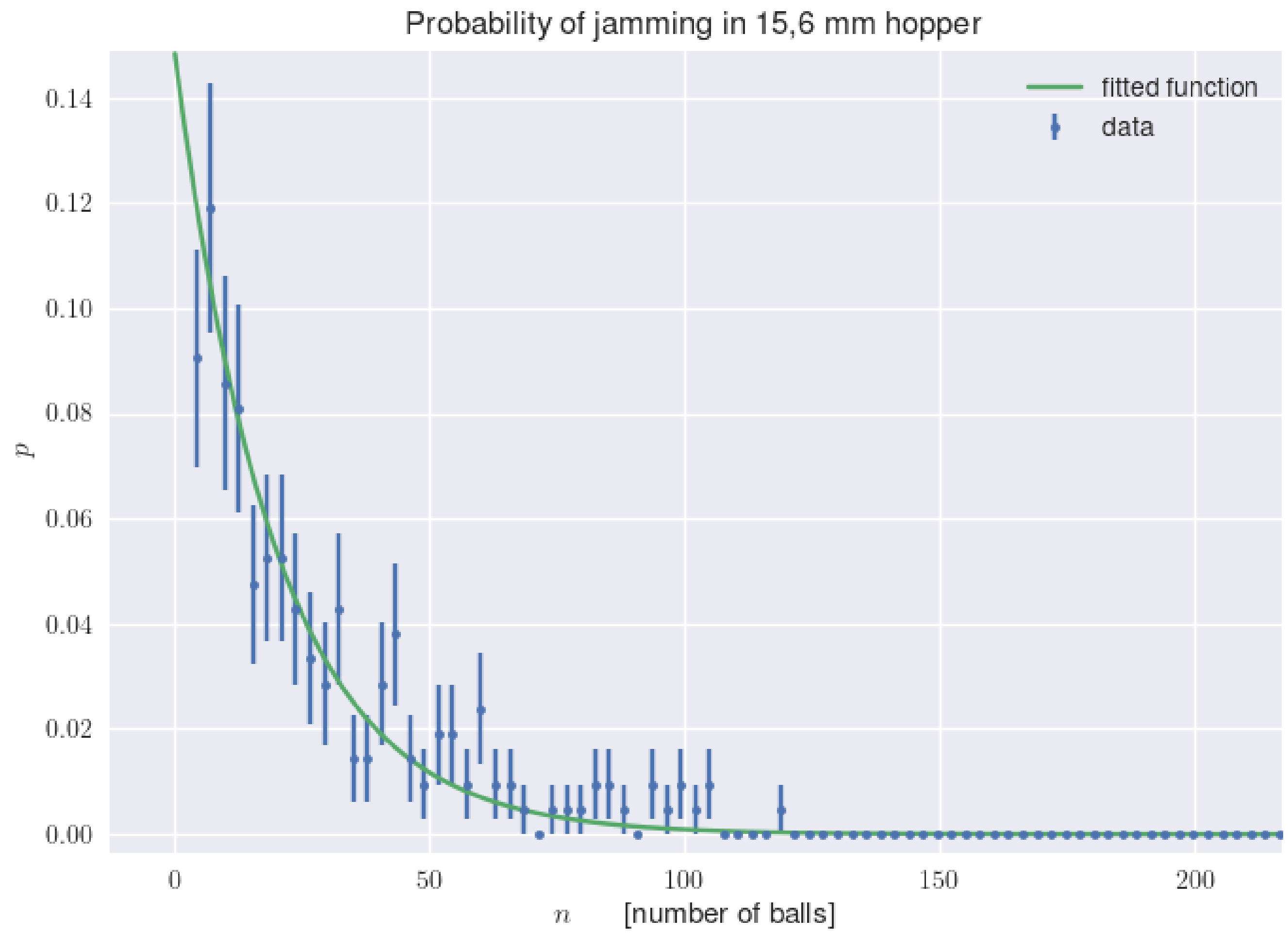


$$P(n) = (1-p)^{\frac{n}{n_0}} p$$

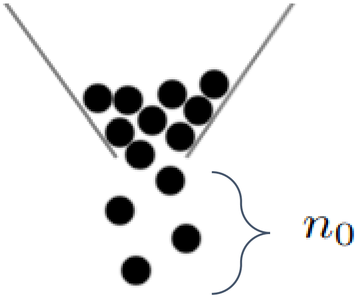
$$d = 15,6 \text{ mm}$$

$$p = 14,8\%$$

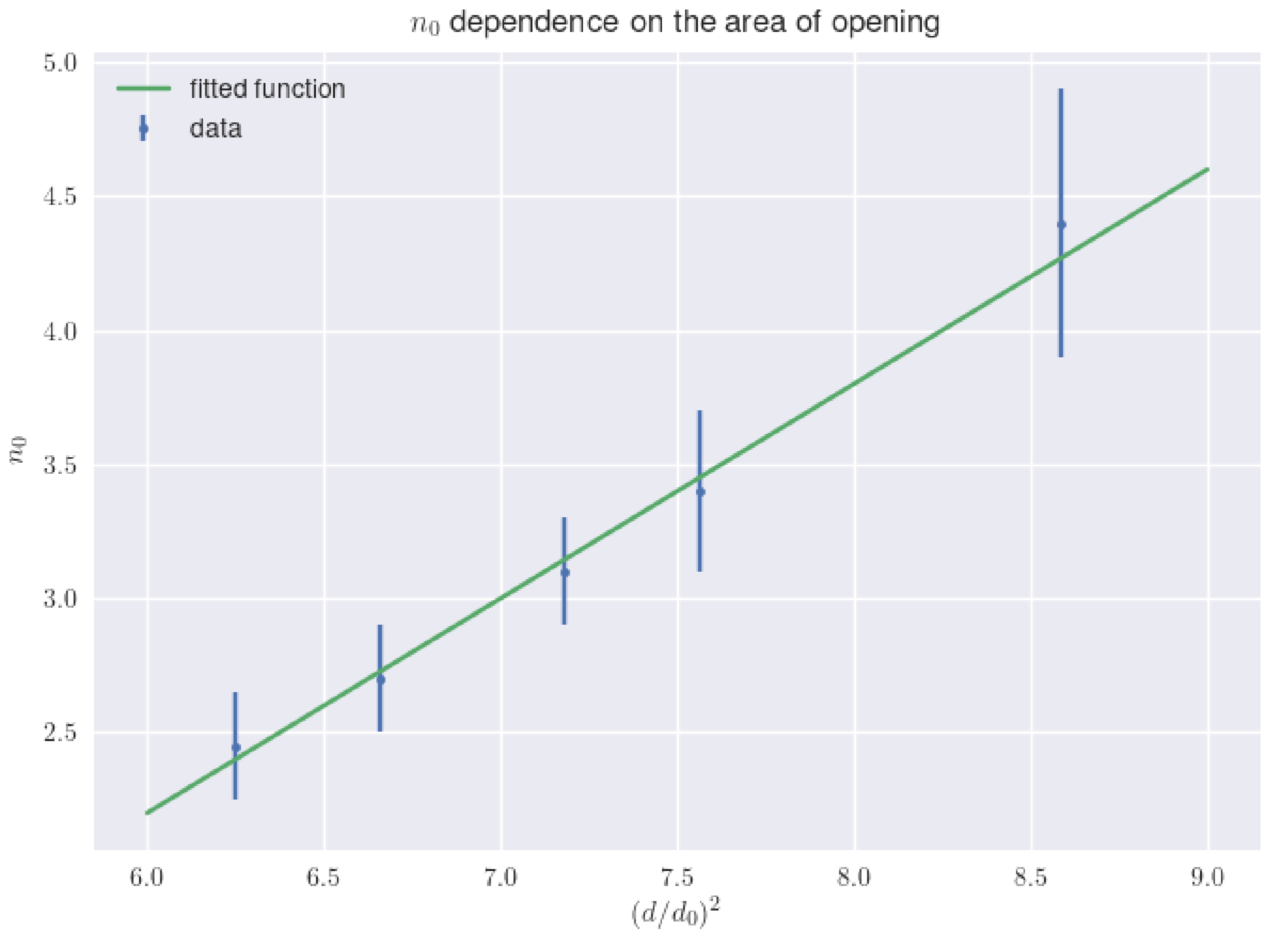
$$n_0 = 3.1$$

How it depends on the diameter of the orifice

Orfice diameter	n_0
14,5 mm	2,45 $\pm$ 0.2
15 mm	2,7 $\pm$ 0.2
15,6 mm	3,1 $\pm$ 0.2
16 mm	3,4 $\pm$ 0.3
17 mm	4,3 $\pm$ 0.5



$$n_0(d) = ad^2 + b$$

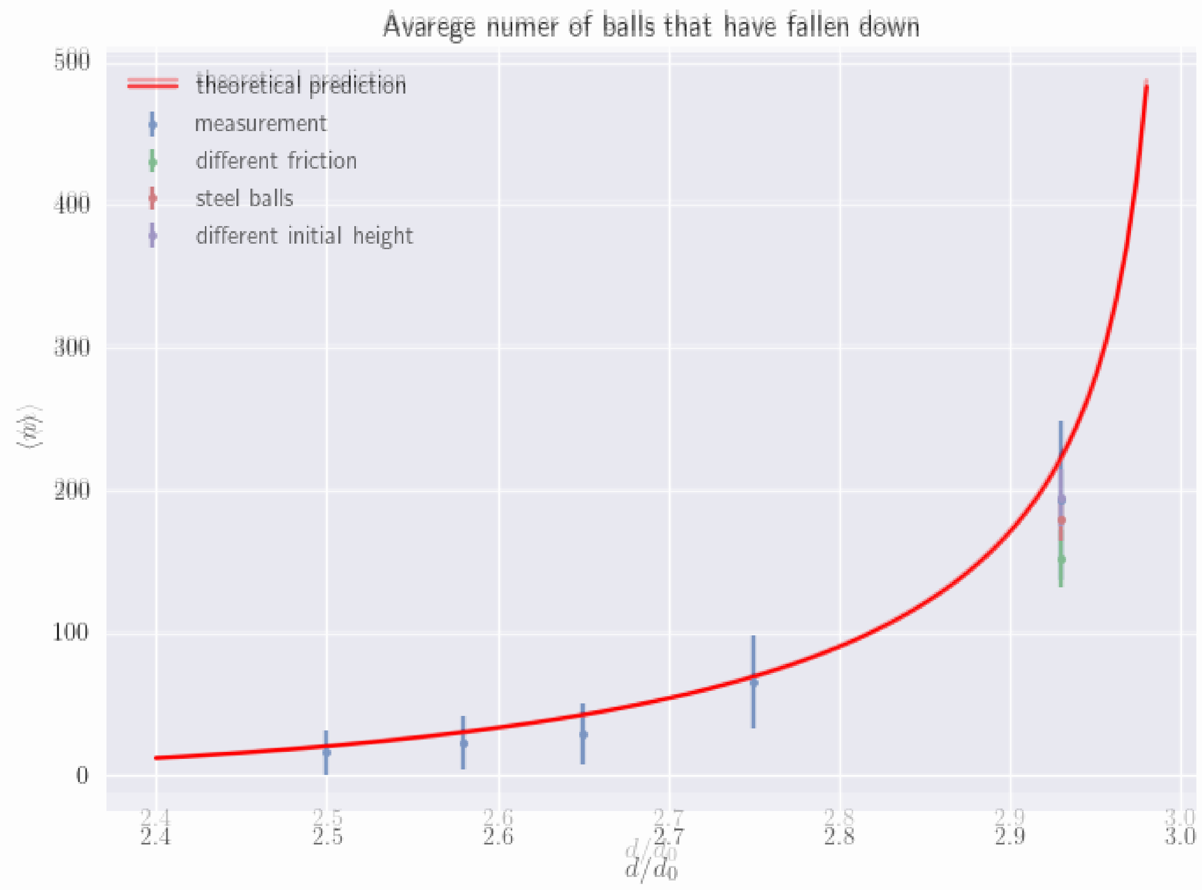


Calculating expected number of balls

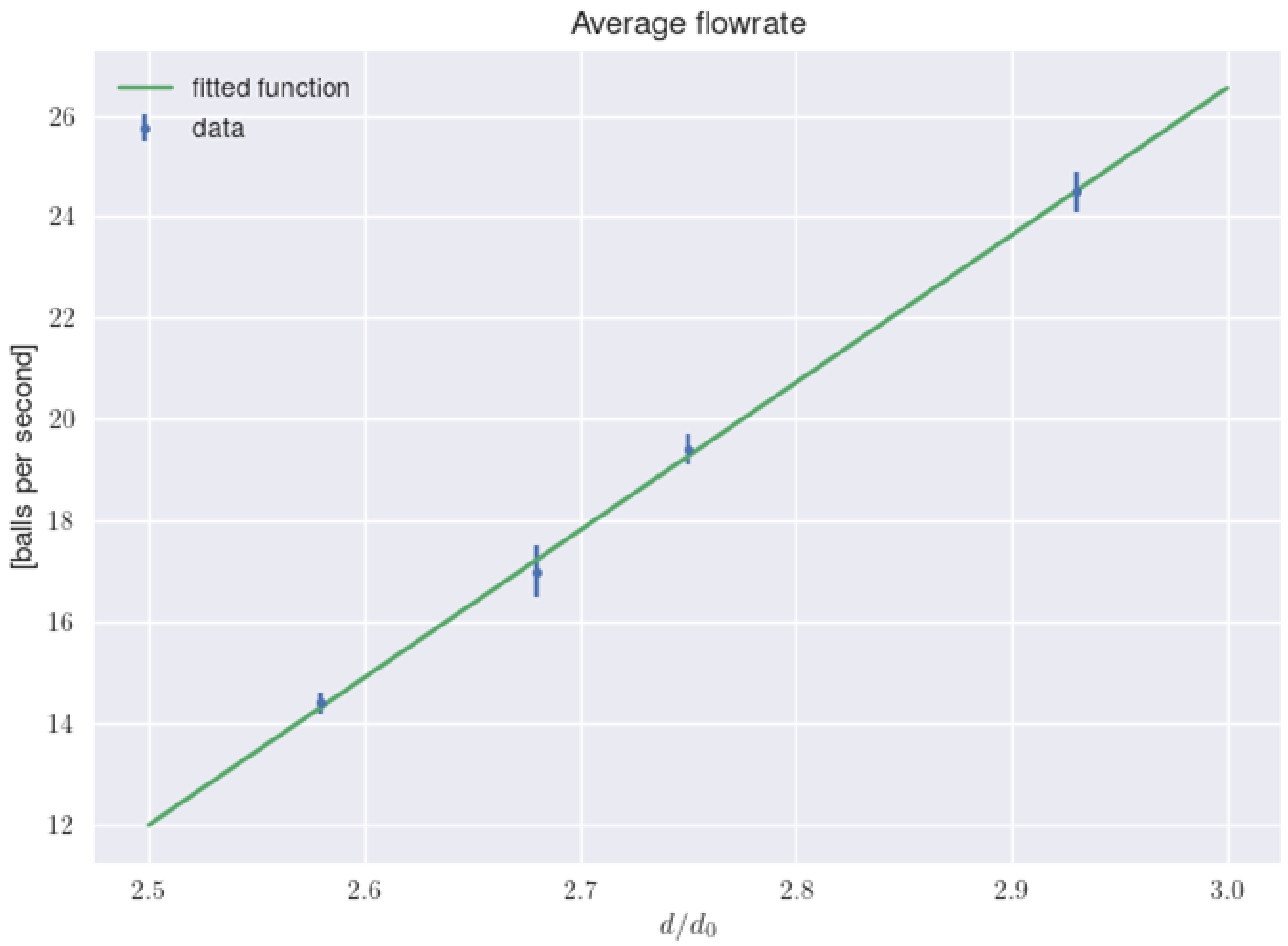
$$\langle n \rangle (d) = \sum_0^\infty nP(n,d) = \frac{p(1-p)^{1/n_0}}{((1-p)^{1/n_0}-1)^2}$$

$$\langle n \rangle (d) = \frac{\frac{\arccos(d/3)}{\pi}\left(1-\frac{\arccos(d/3)}{\pi}\right)^{1/n_0}}{\left(\left(1-\frac{\arccos(d/3)}{\pi}\right)^{1/n_0}-1\right)^2}$$

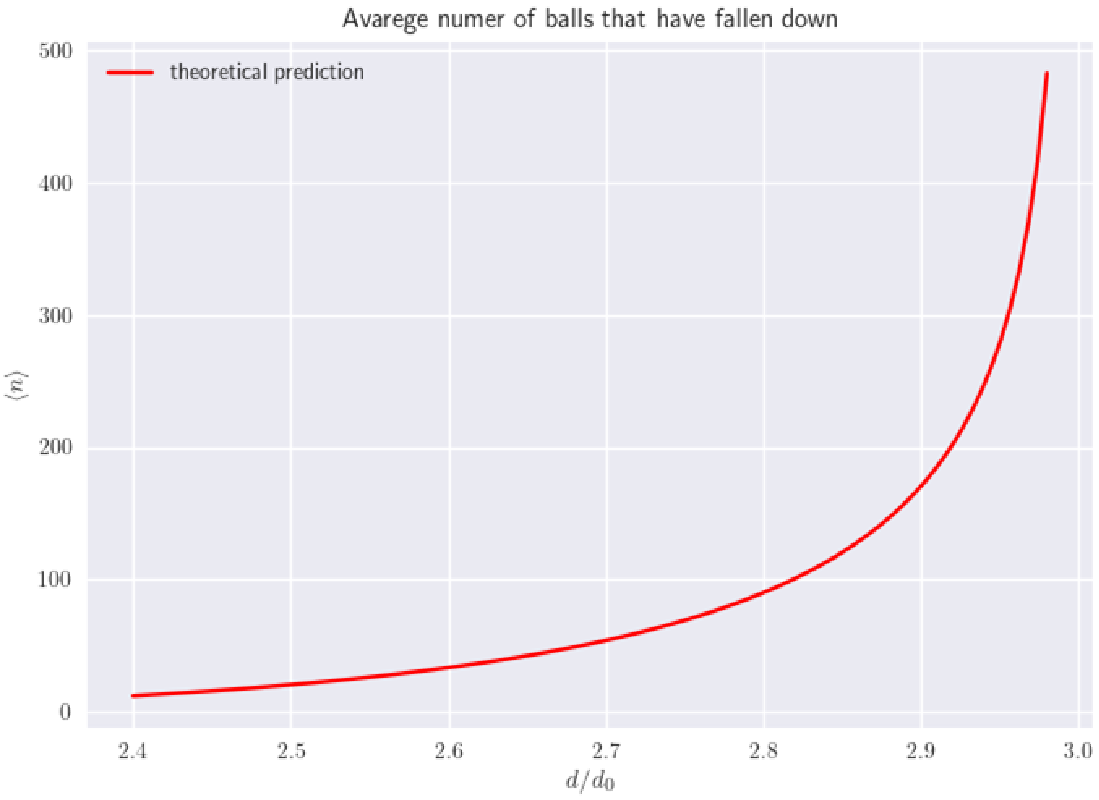
$$n_0(d) = ad^2 + b$$

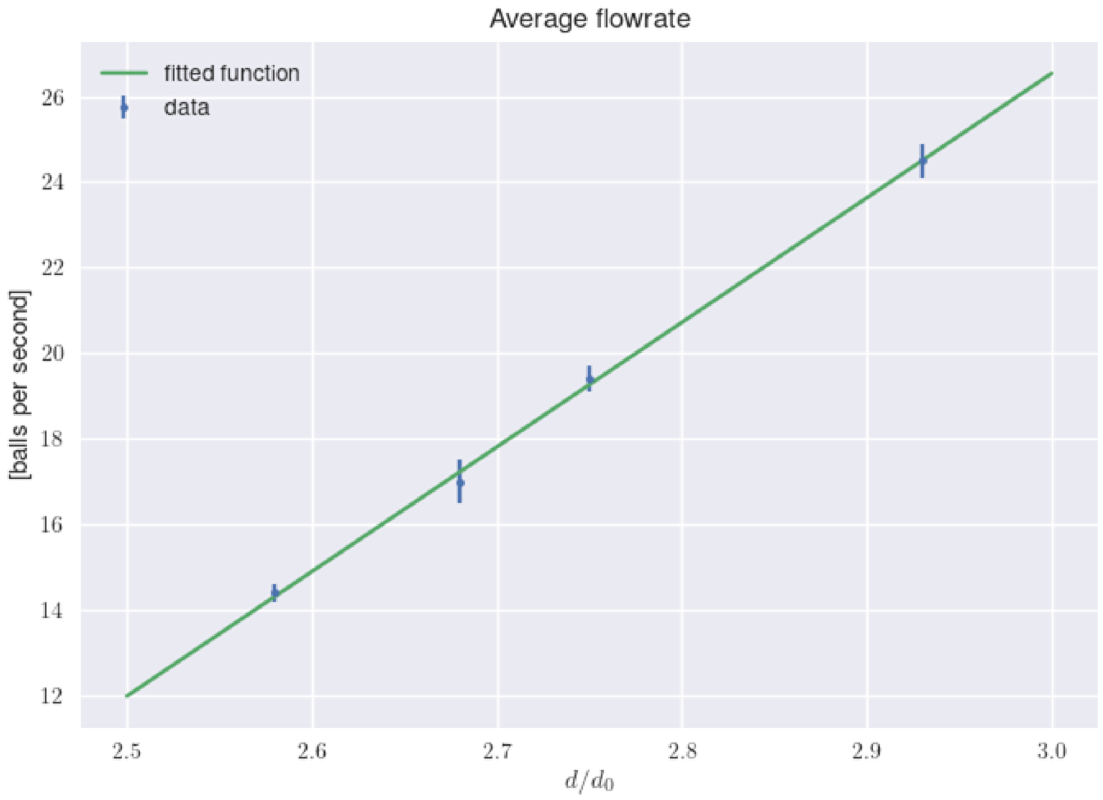


Measuring time



Final solution to n° 2





Designing the optimal nozzle

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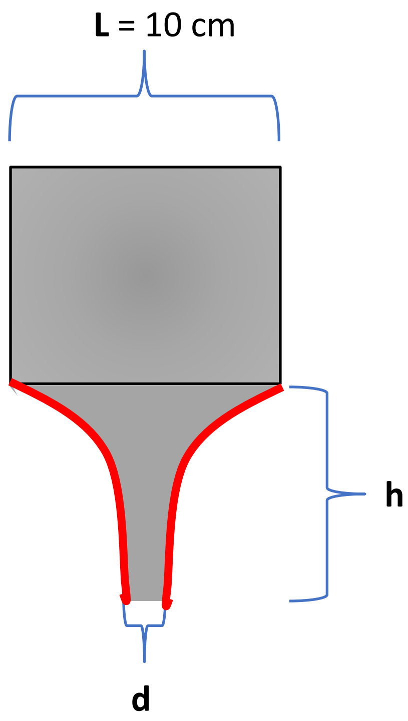
3. What bottle shapes can prevent the system from getting stuck?

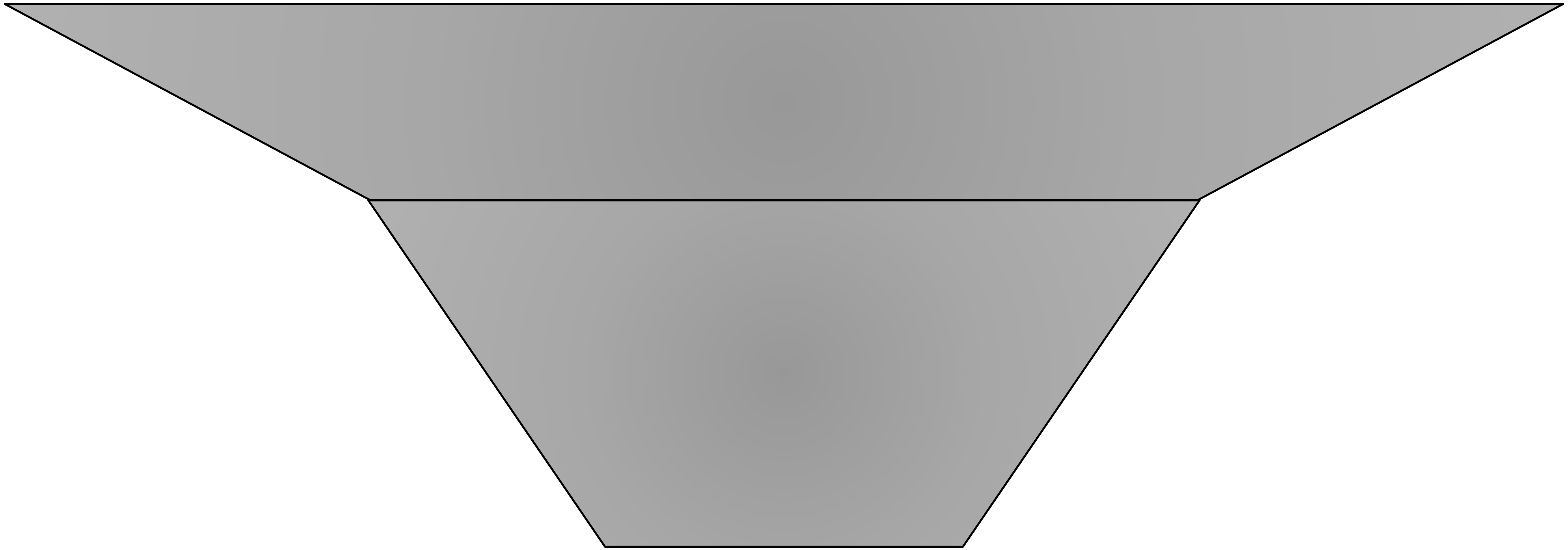
Optimization problem:

Given a cylinder of diameter L and balls of radius r, find the shortest (in terms of h) nozzle, that has an orifice of diameter d and never or almost never gets stuck.

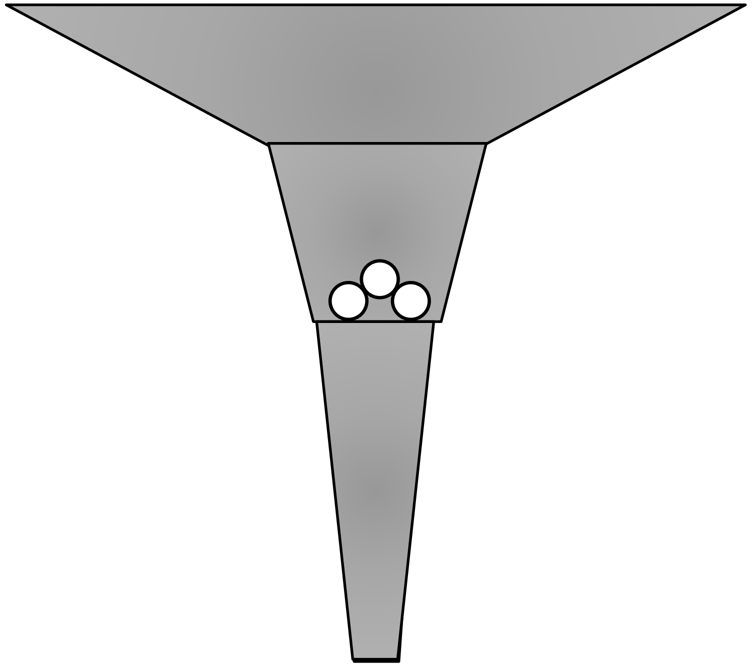
For simplicity, we limit our solutions to cylindrically symmetrical ones.



Sequence of nozzles



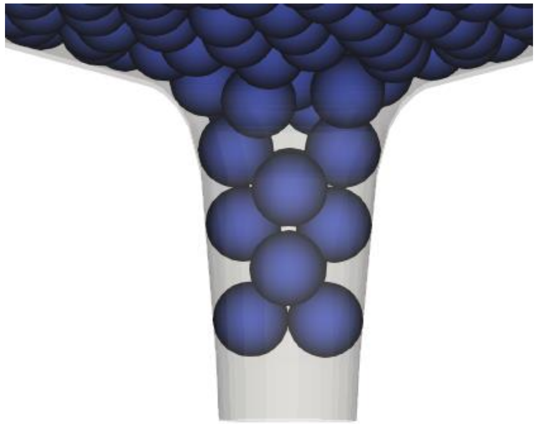
Polishing our nozzle



Simulation

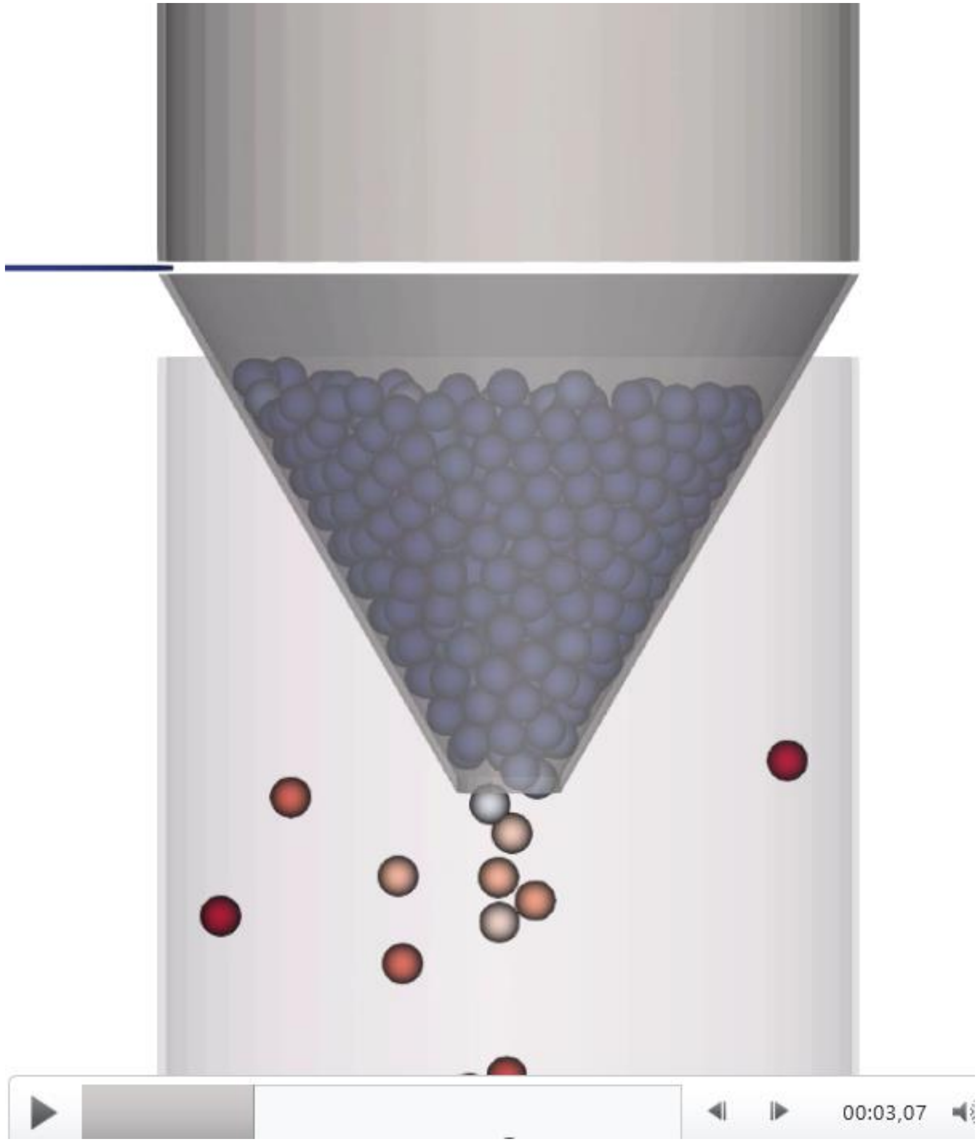
LIGGGHTS – open source discrete element method particle simulation software

https://www.engineerdo.com/2019/10/04/liggghts/

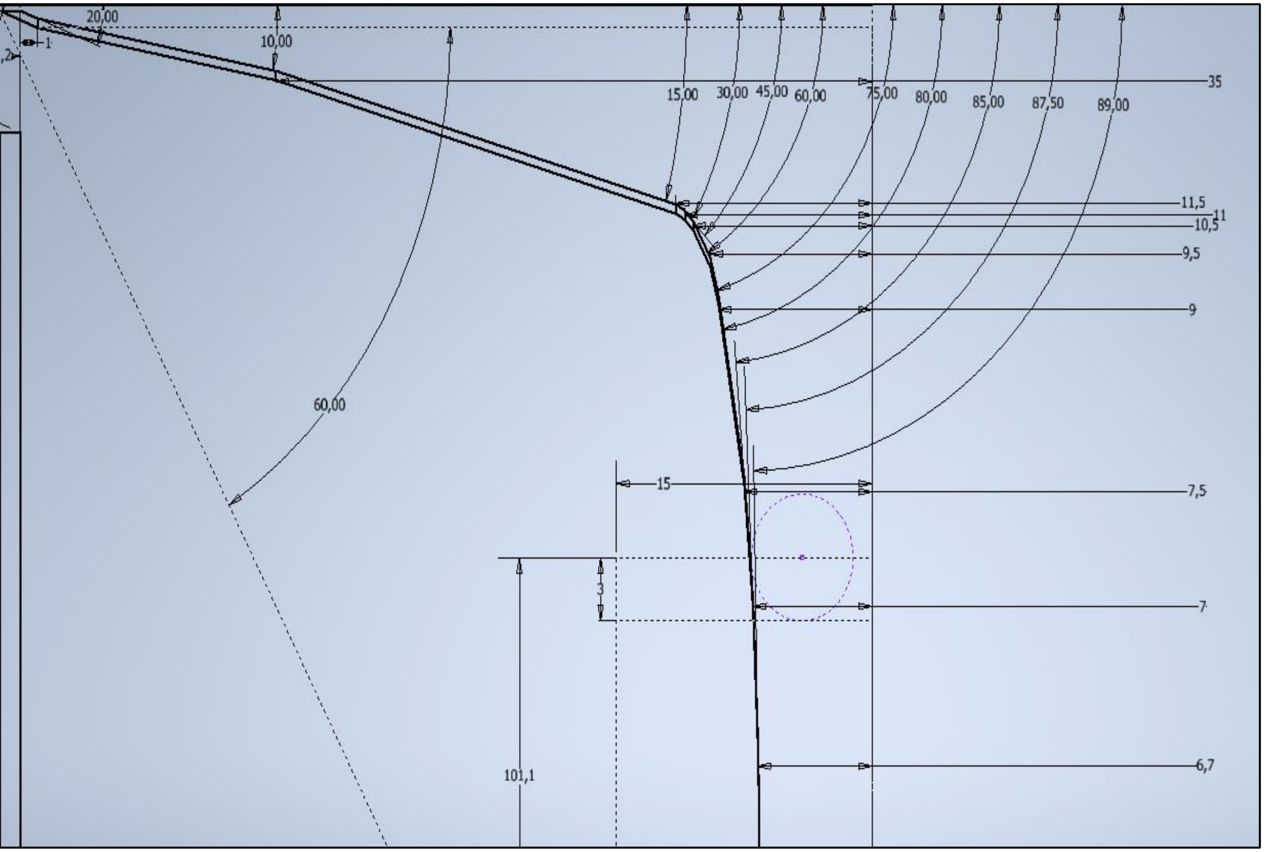


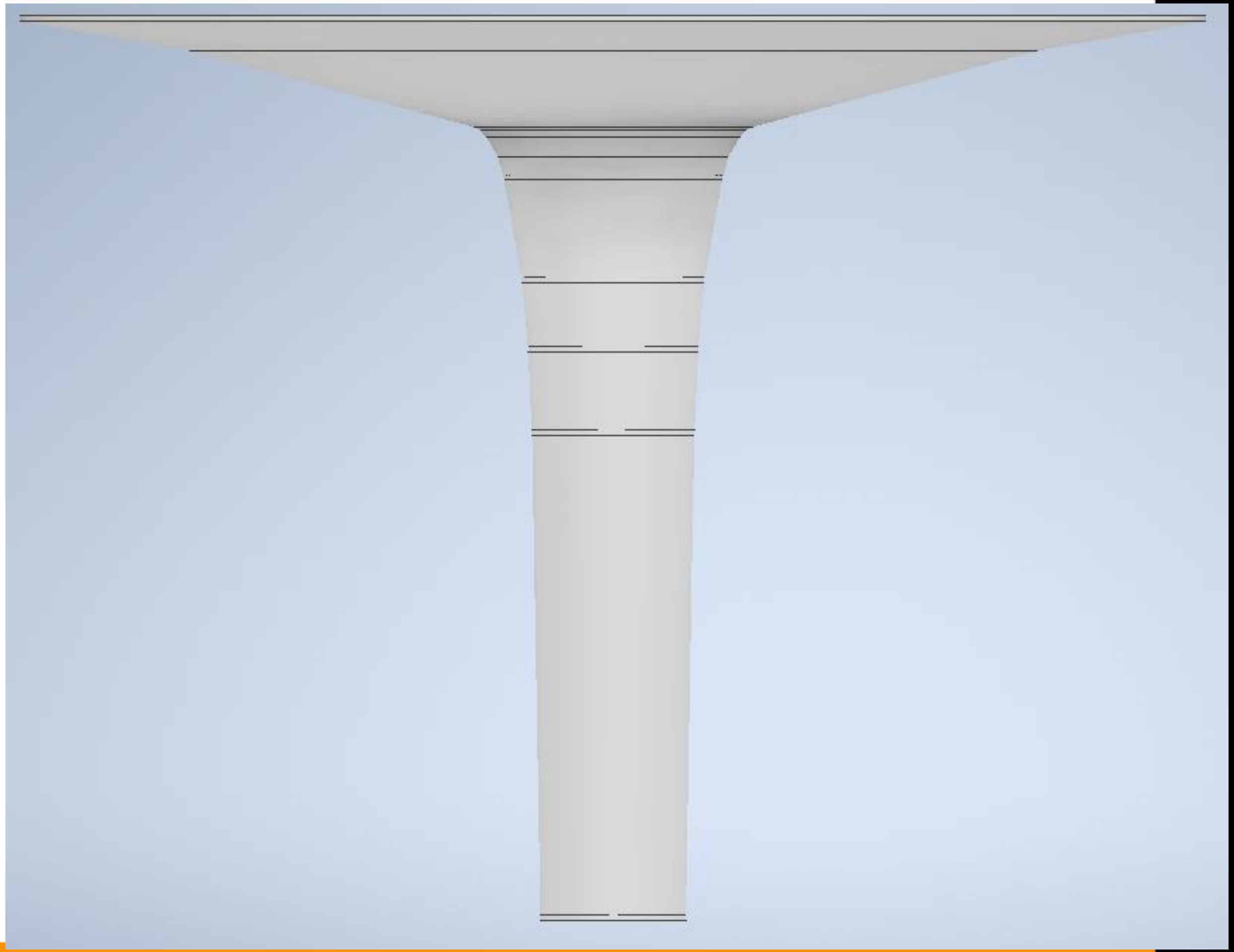
After 16 iterations...

Simulation



Polishing our nozzle





Optimal nozzle comparison

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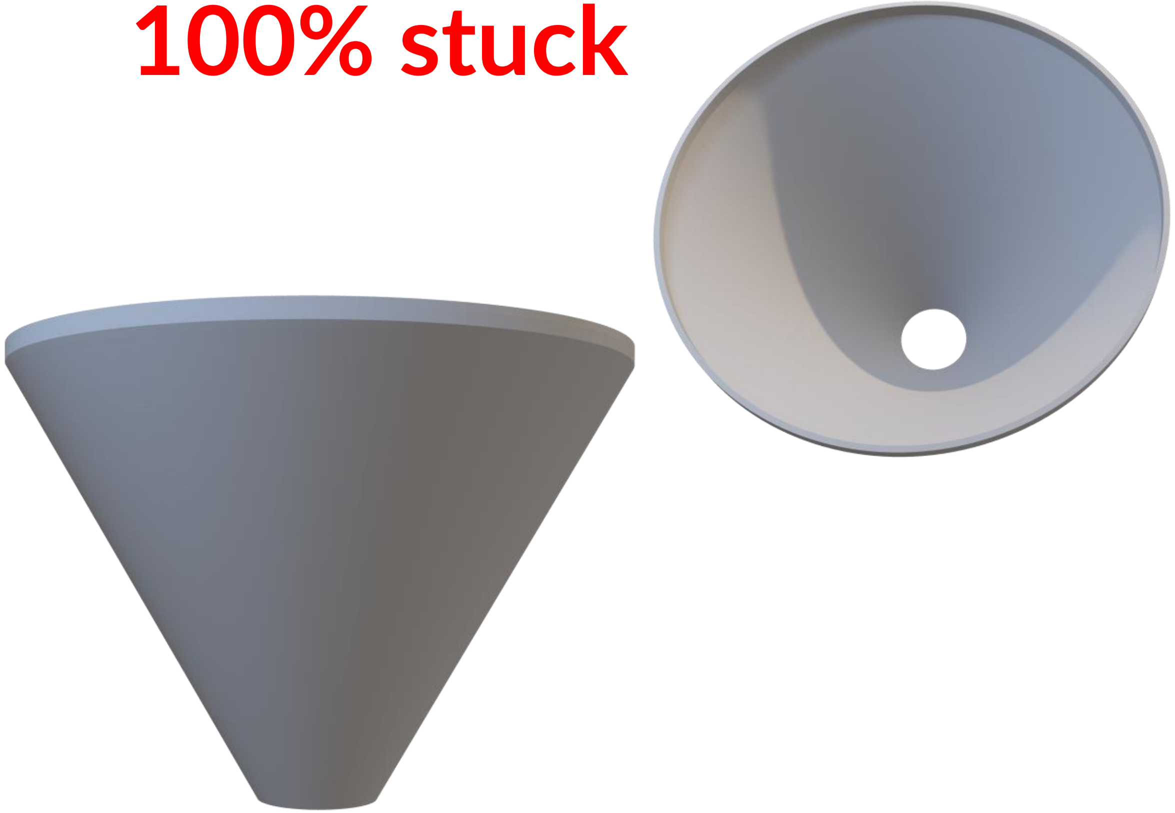
100% stuck



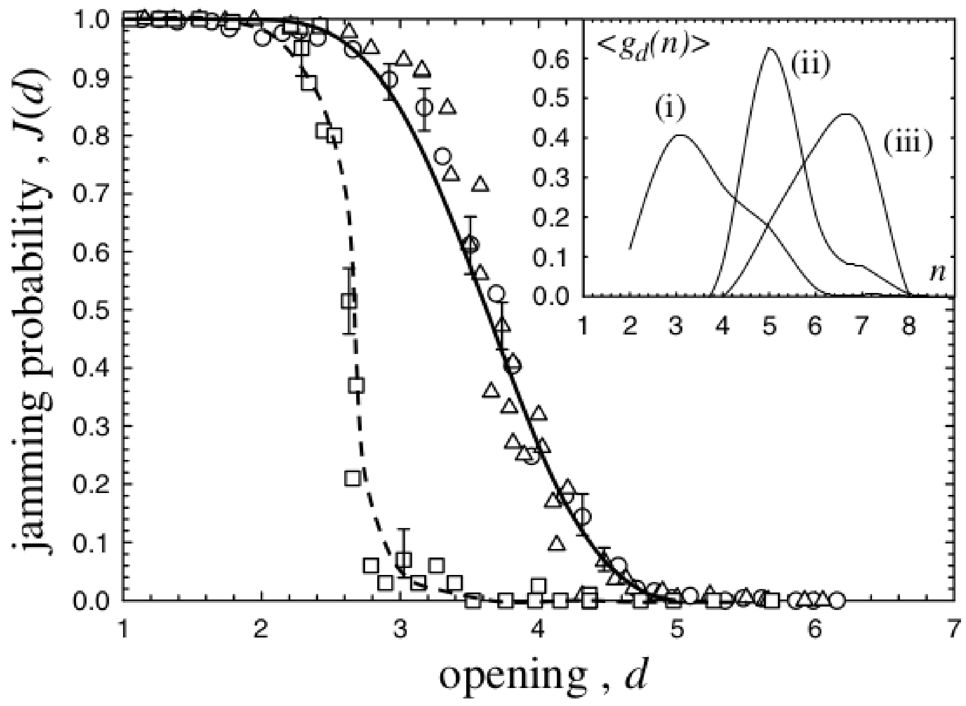
5,5% stuck

Summary

- We stated that spheres in a bottle get stuck because of arch formation
- We developed a theory that explained the distribution of the results and the mechanism behind the phenomenon
- We calculated how much time it take for a system to get stuck
- We designed the shape of a nozzle that prevents the system from jamming in 94.5 % of cases

Angle dependence

between 60° and 75° so that the jamming probability is the same for the two different hoppers ($\phi = 60°$ and 34°) of our experiment. The data $J(d)$ in the experiment with the hopper of $\phi = 75°$ are indeed very different from those of $\phi = 60°$ and 34°. It drops rapidly to zero at $d = 2.5$ im-



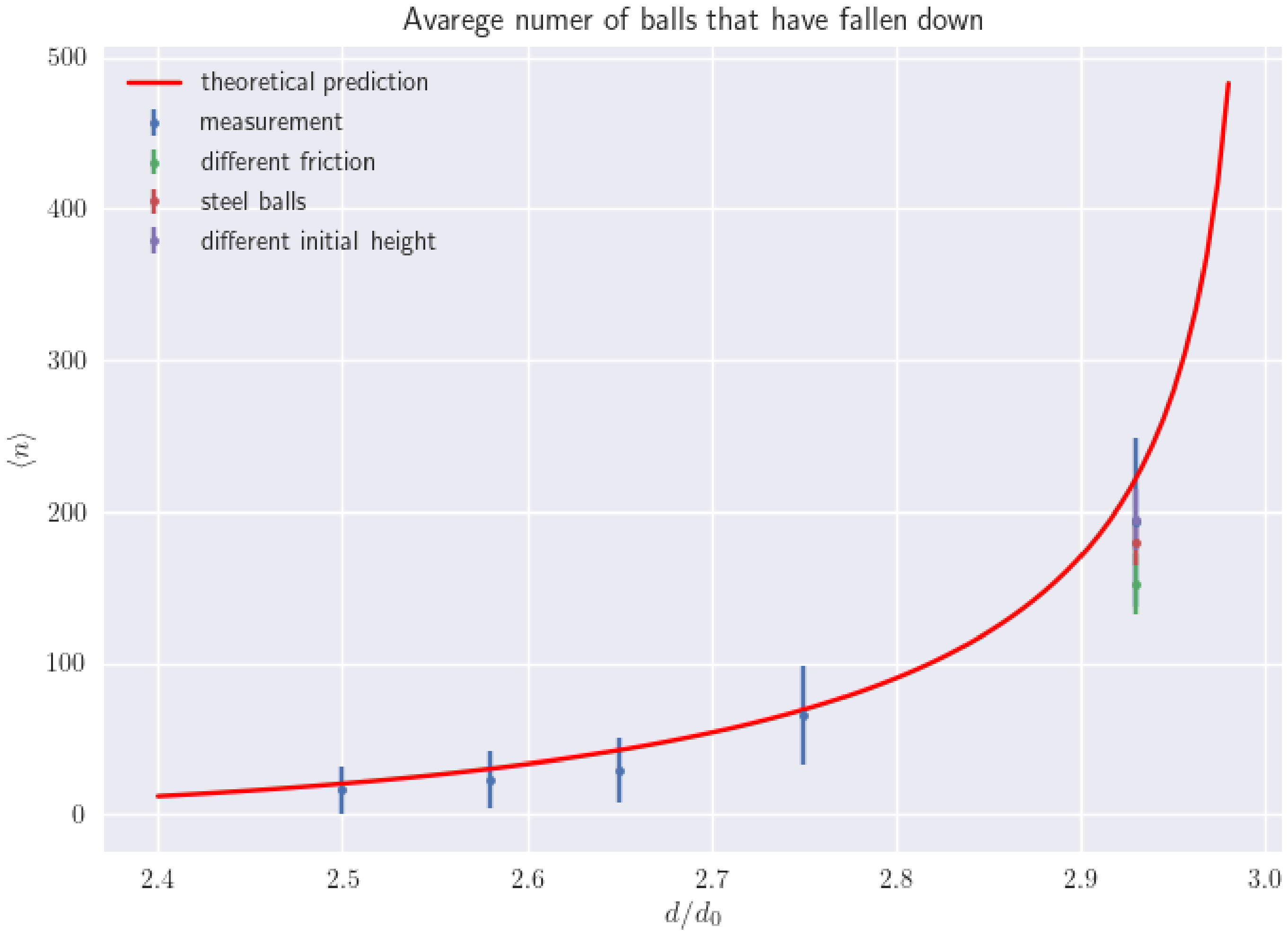
[1]*Jamming of Granular Flow in a Two-Dimensional Hopper*, K. To et al, Phys. Rev. Lett. 86, 71
[2]*Jamming in granular matter*, A. Garcimartin et al, AIP Conference Proceedings 742, 279 (2004)

Friction dependence

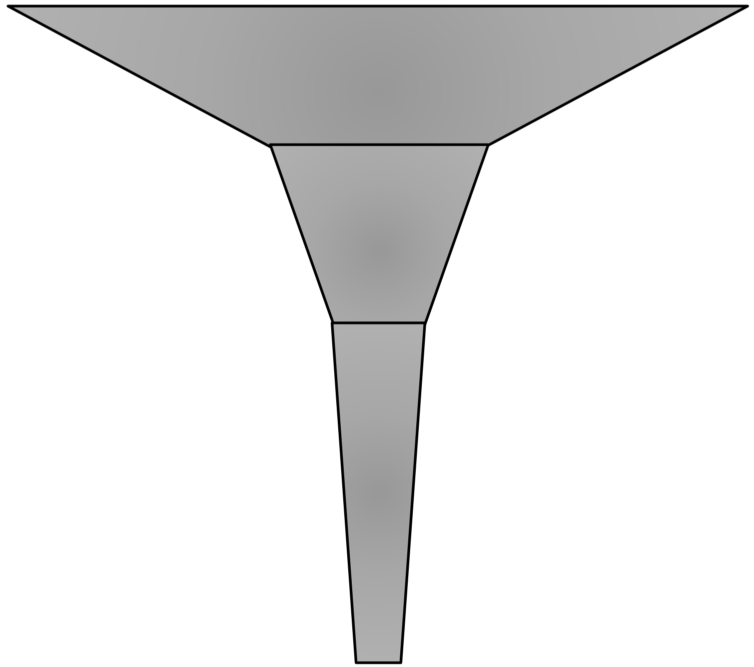
The surprising result is that all these changes do not produce any measurable effect in jamming. Both p (as obtained from the histogram) and the behaviour of $<s>$ (Fig. 5) remain unchanged. This means that jamming is not directly related to the details of friction among grains or the elasticity of the material, nor to the surface properties of the particles.

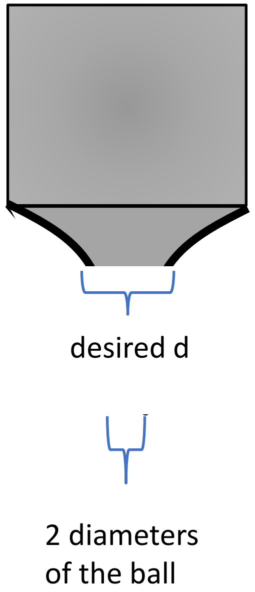
[1]*Jamming of Granular Flow in a Two-Dimensional Hopper*, K. To et al, Phys. Rev. Lett. 86, 71
[2]*Jamming in granular matter*, A. Garcimartin et al, AIP Conference Proceedings 742, 279 (2004)

Initial conditions dependence

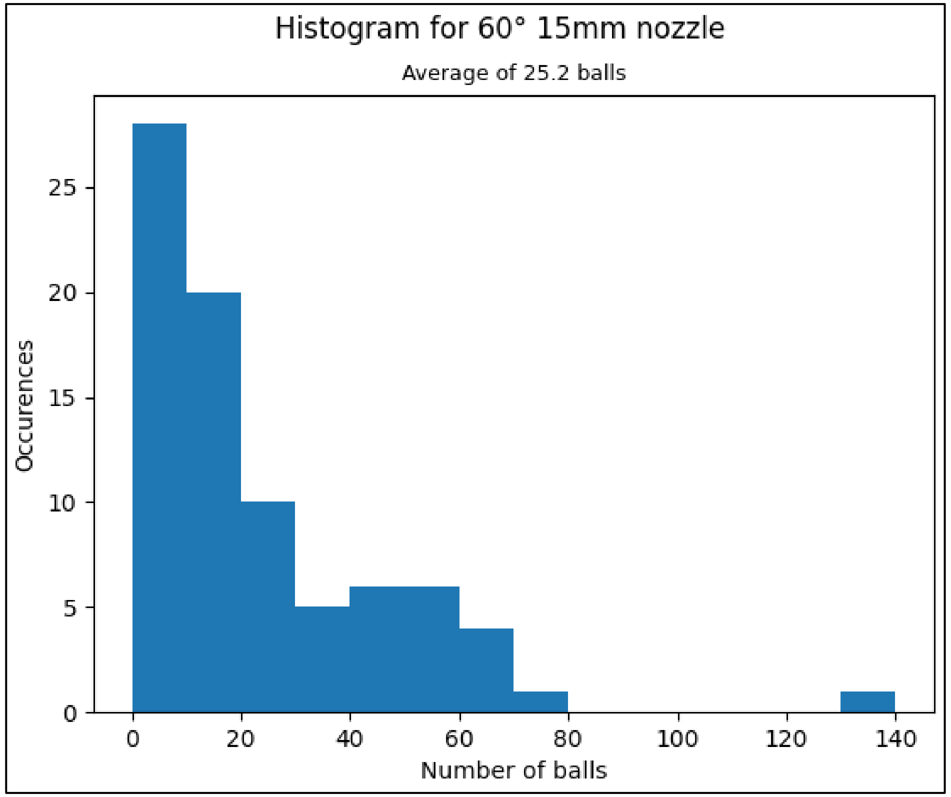


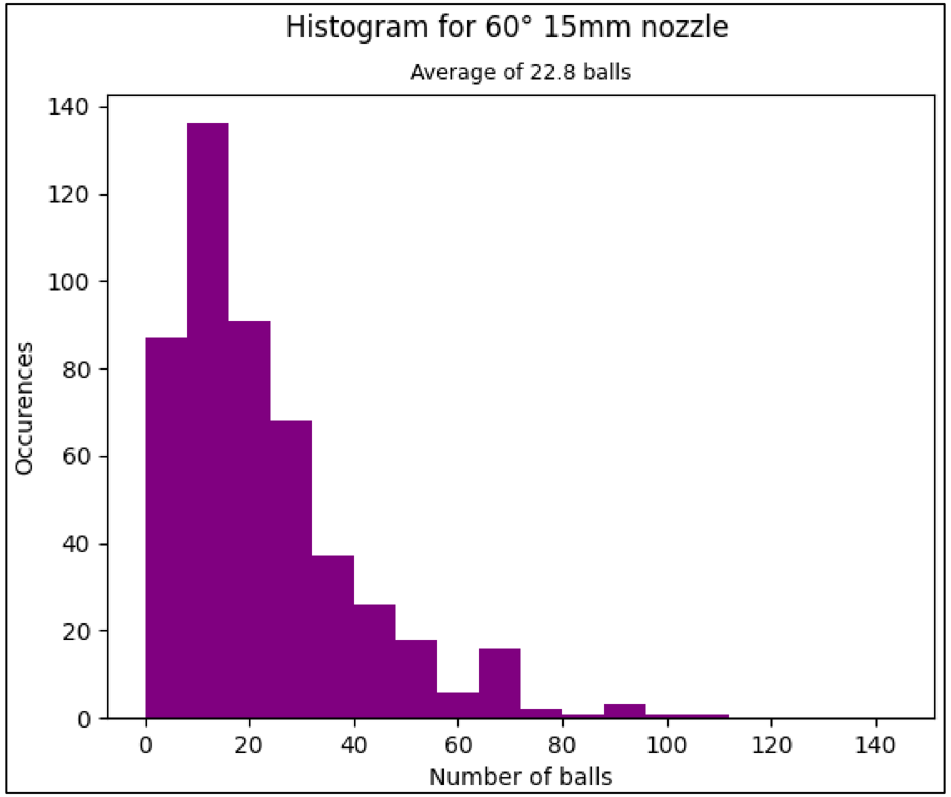
Finding the optimal nozzle



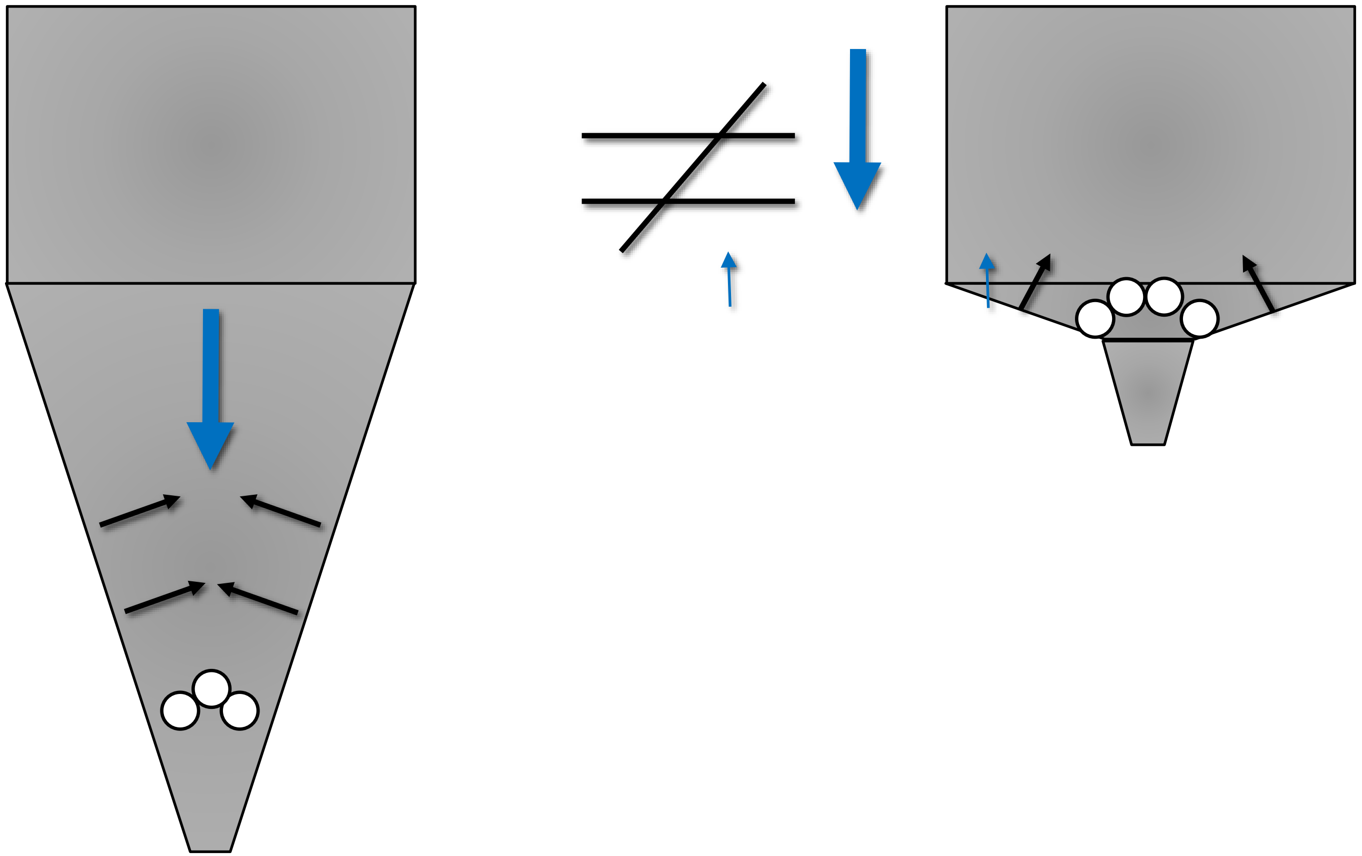


Base experimental and simulated results





But



This is why flat nozzles require more balls to get stuck