Problem 11

THE CHALK TRICK

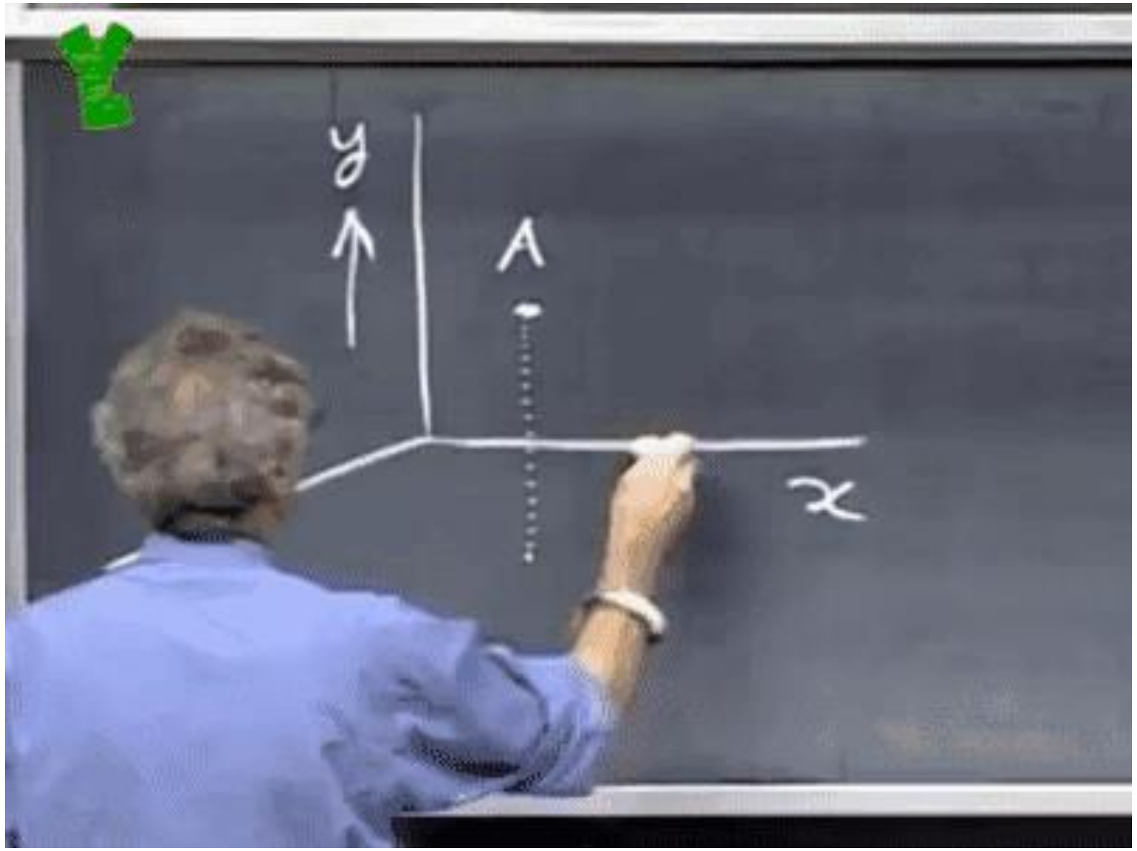
Problem statement

It is possible to draw continuous lines in a blackboard with chalk. However, by changing the angle of contact, the line drawn on the board becomes a dotted line, though the movement is still continuous.

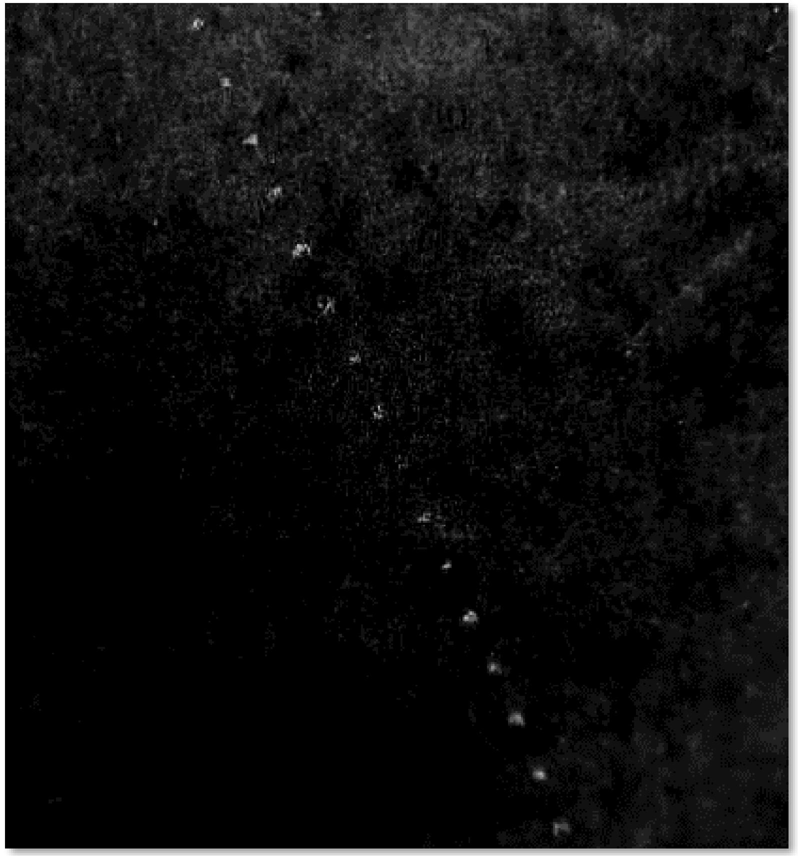
What parameters from the relative movement between the chalk and the board can be inferred from the resulting trace?

Is it possible to infer anything about the dimensions of the chalk?

The chalk trick

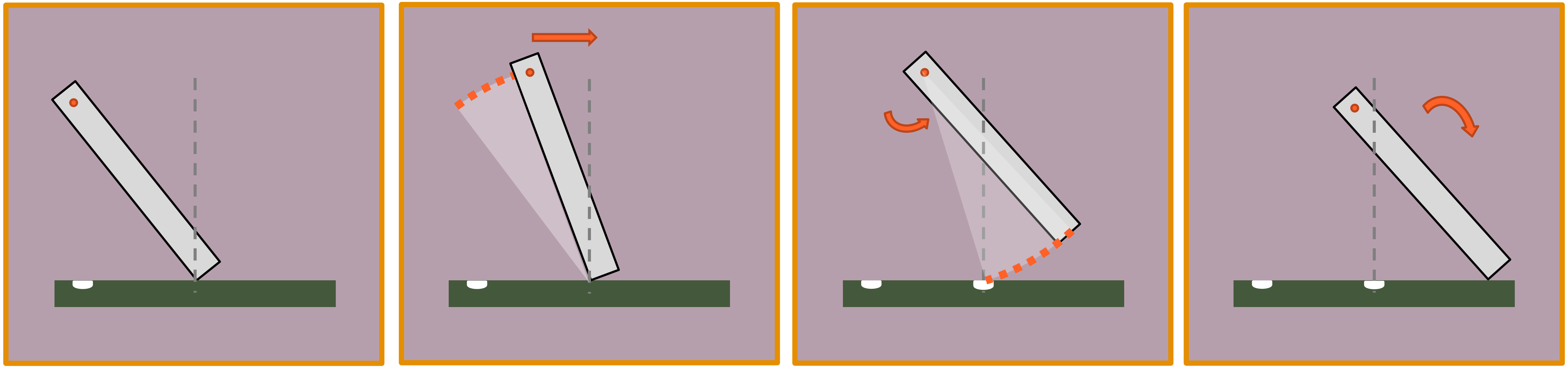


Walter Lewin

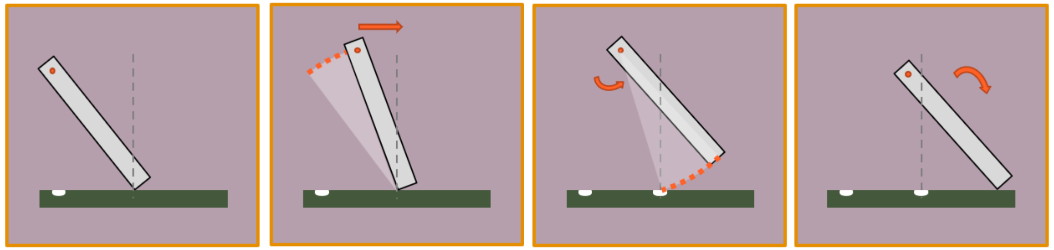


Resulting trace (increased contrast)

What happens?

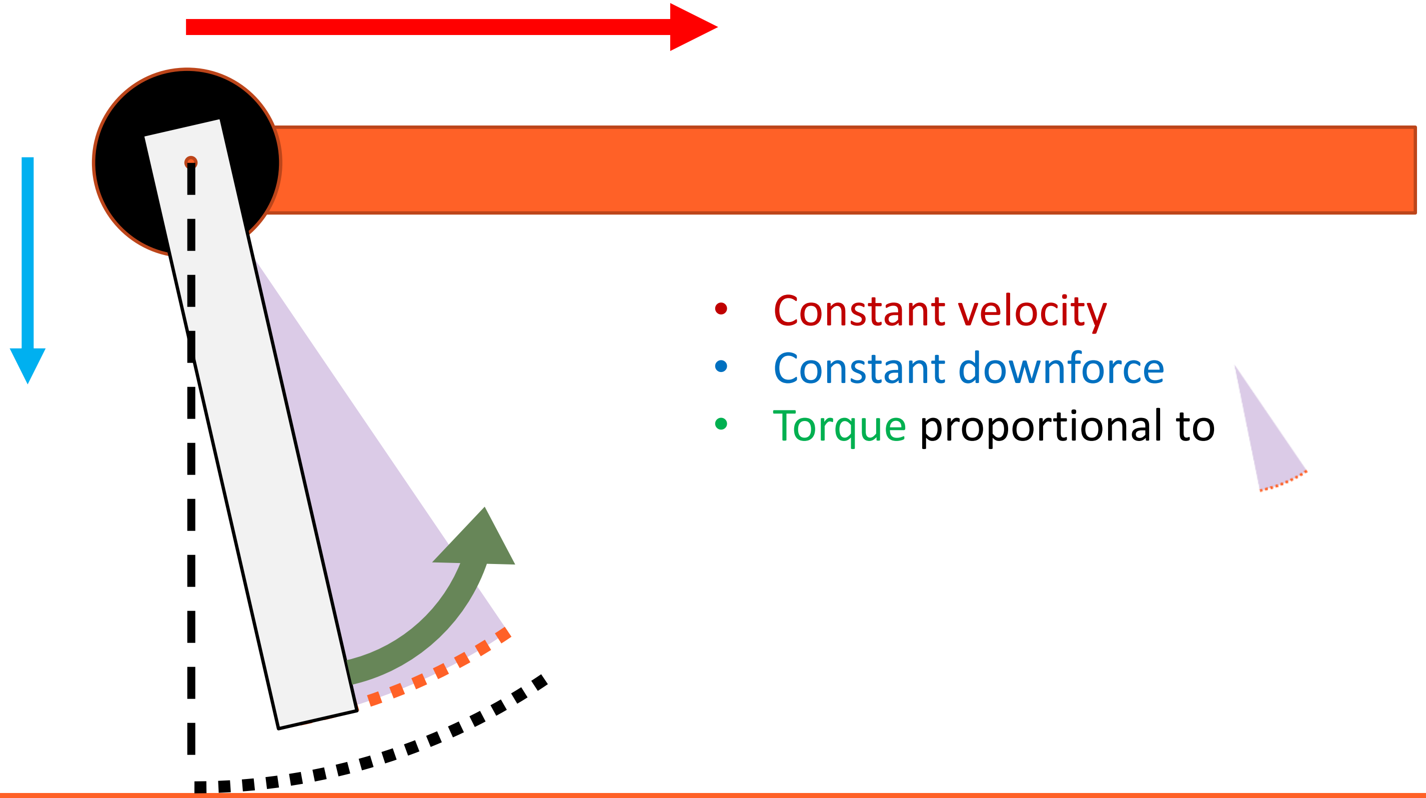


What happens?

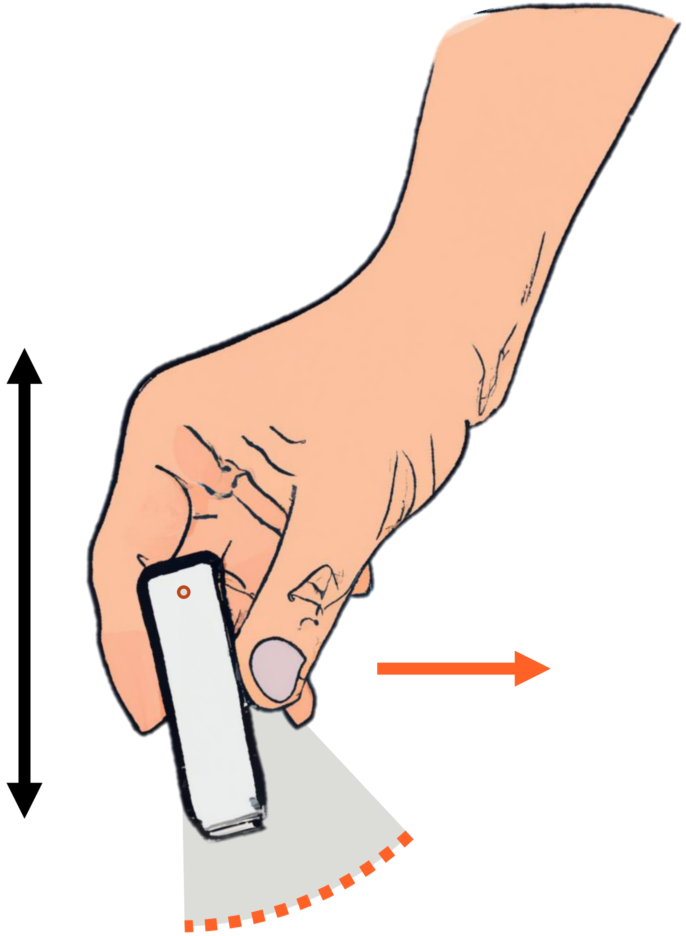


The hand

How can we model the hand?

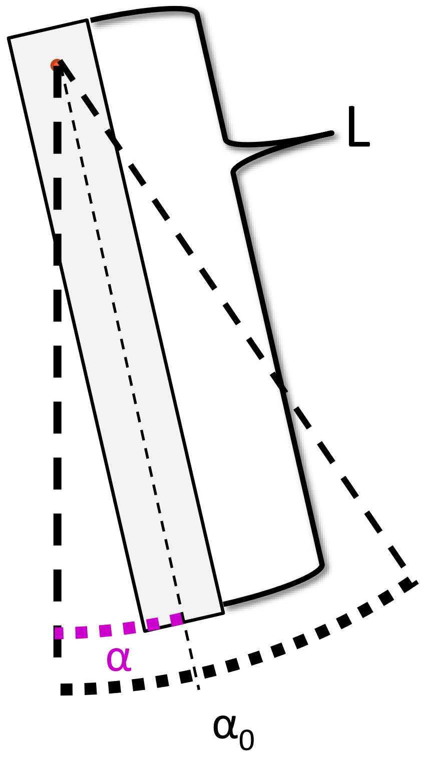


- Constant velocity
- Constant downforce
- Torque proportional to



DALL-E 2

Parameters

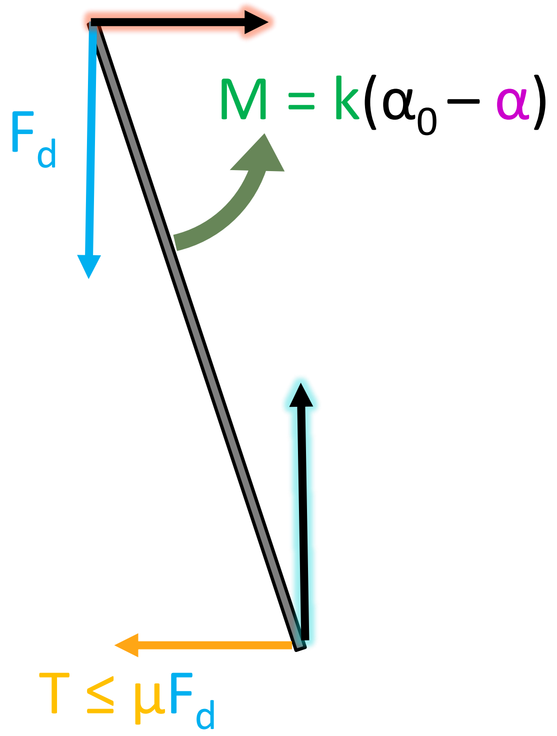


We can manipulate:

- L - length
- F_d – downforce
- k – torque constant
- α_0 – Initial holding angle
- μ (maximum static)

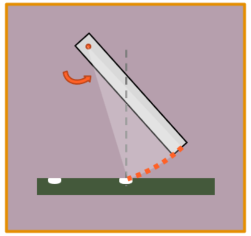
And the velocity v

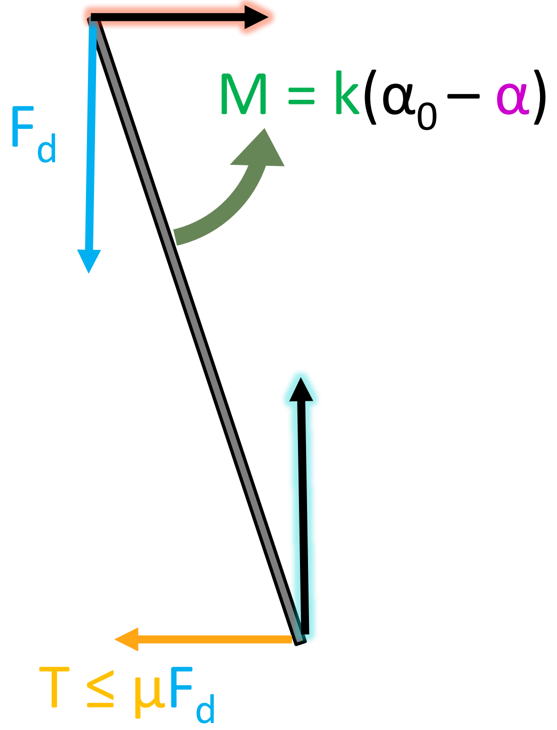
Forces



Theory

Lets calculate α at which $T = \mu F_d$ (the chalk is about to





Balance of the torques:

$$k(\alpha_0 - \alpha^*) + LF_d \sin(\alpha^*) = \mu LF_d \cos(\alpha^*)$$

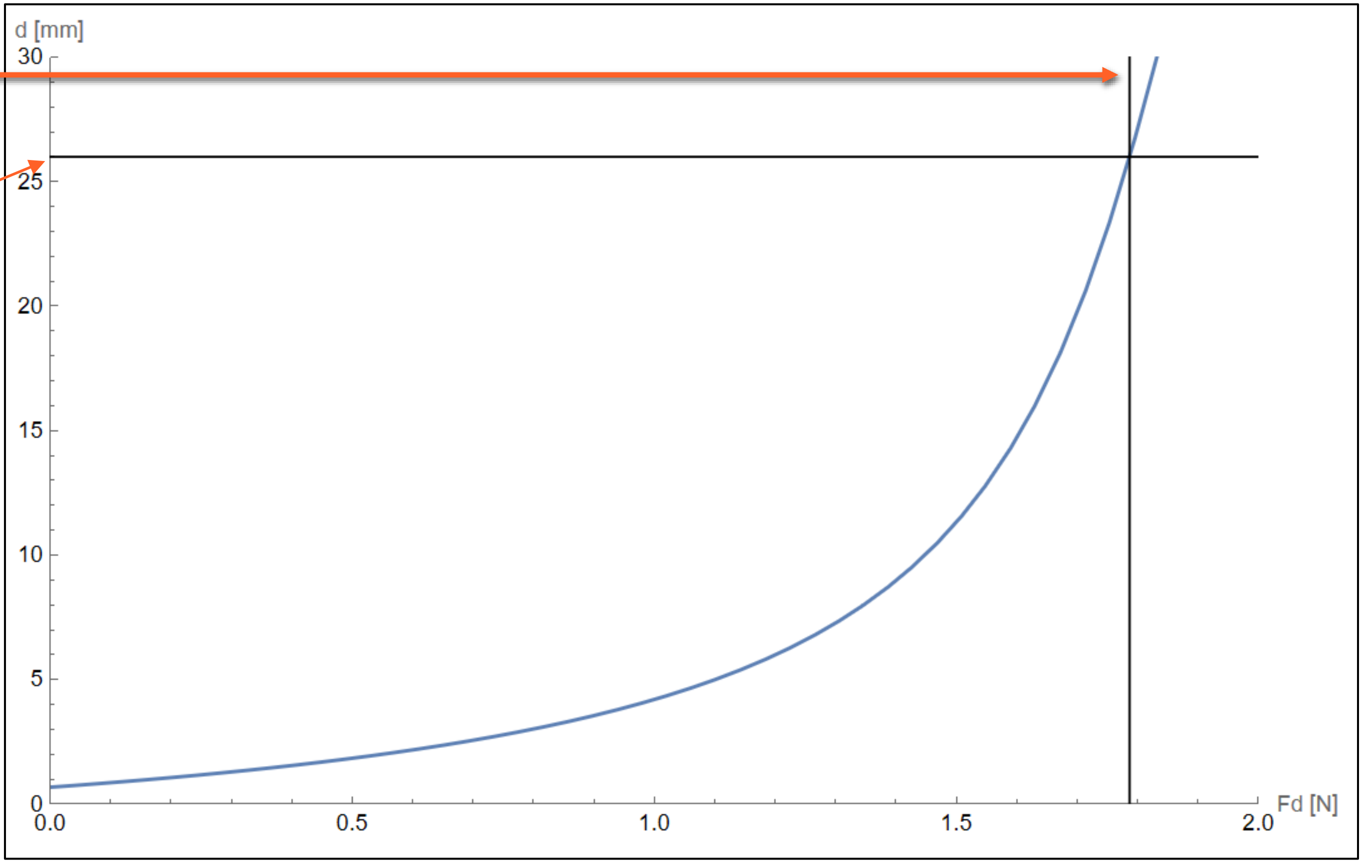
α^* - critical angle

Theory

Then the distance:

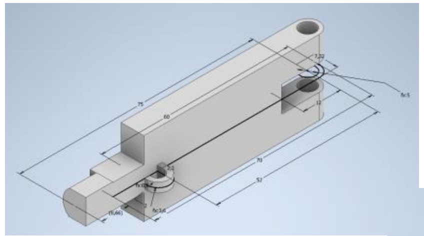
$d = L(\sin[\alpha_0] - \sin[\alpha^*])$

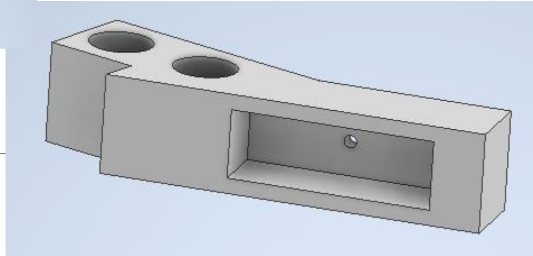
We can also calculate the maximum F_d such that $\alpha^* = 0$ (and from that the max d)

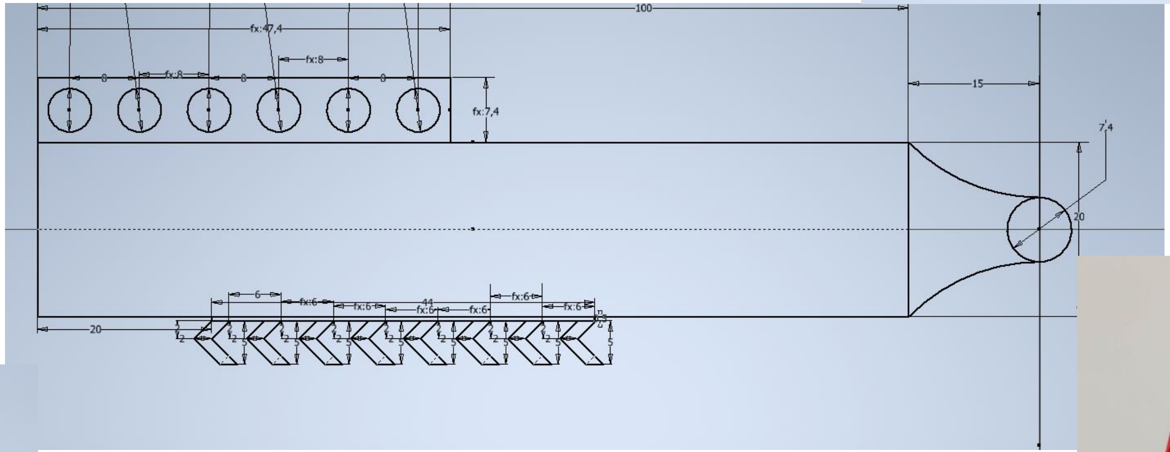


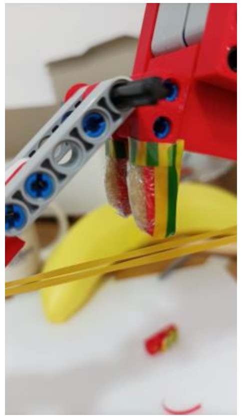
Does it work?

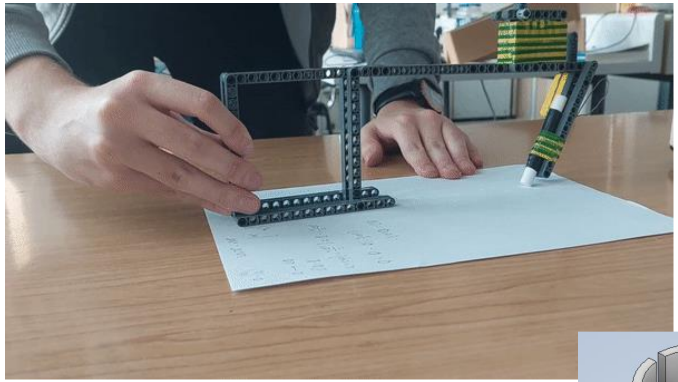
The lengthy process of building a chalking robot

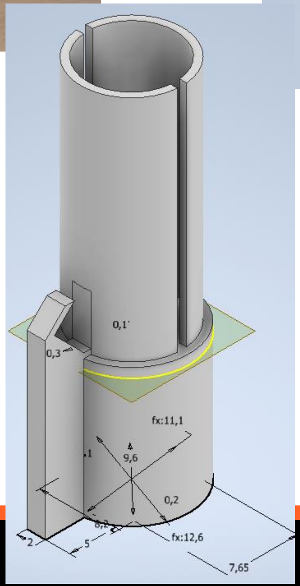


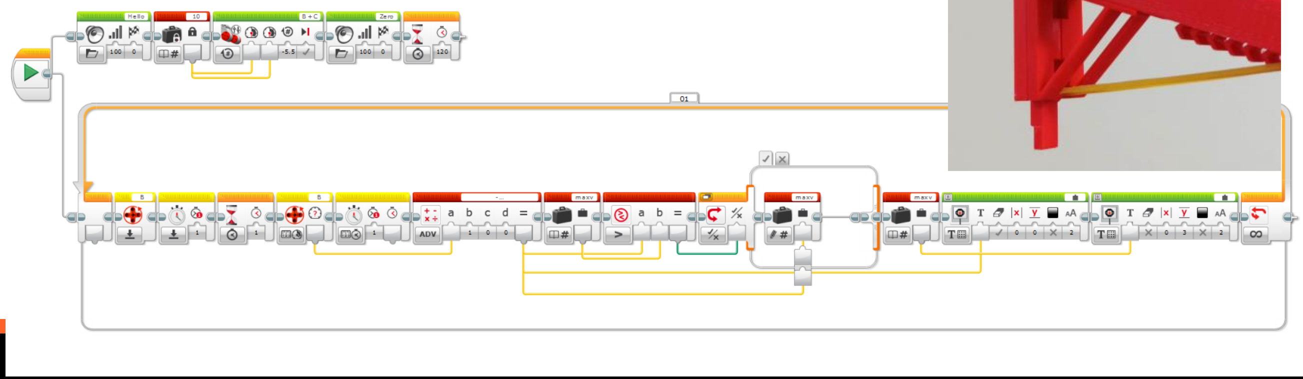


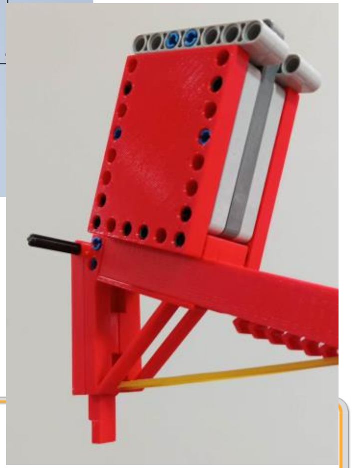


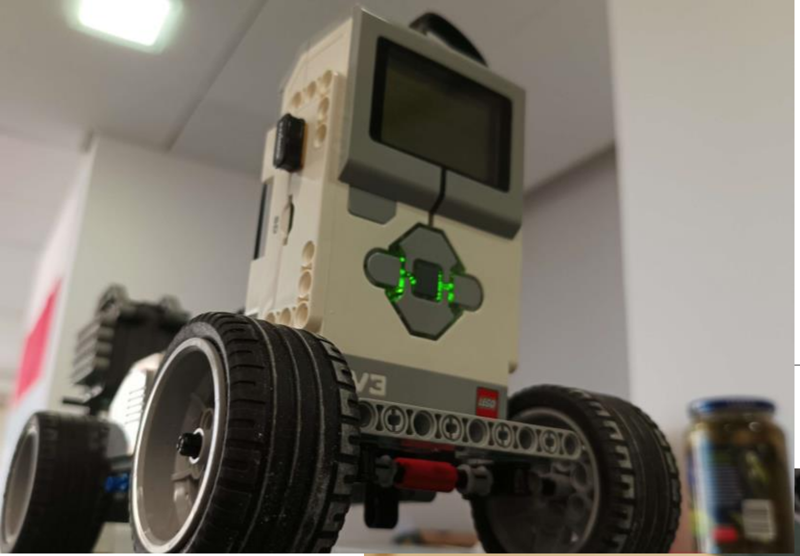




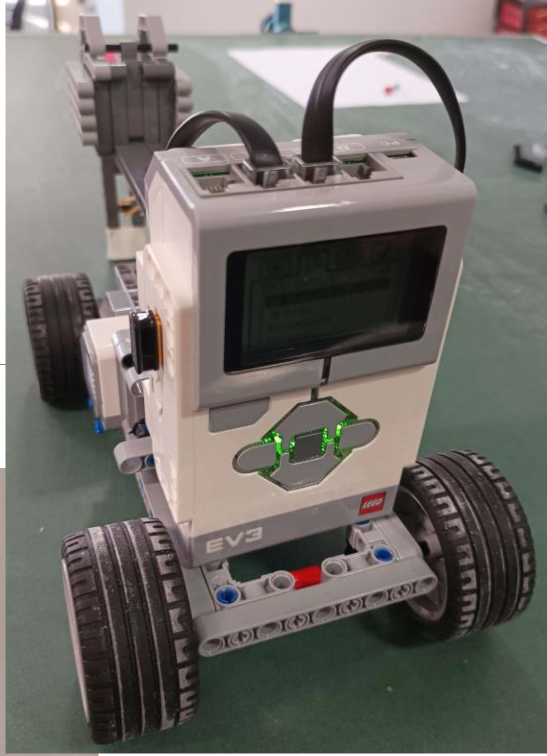




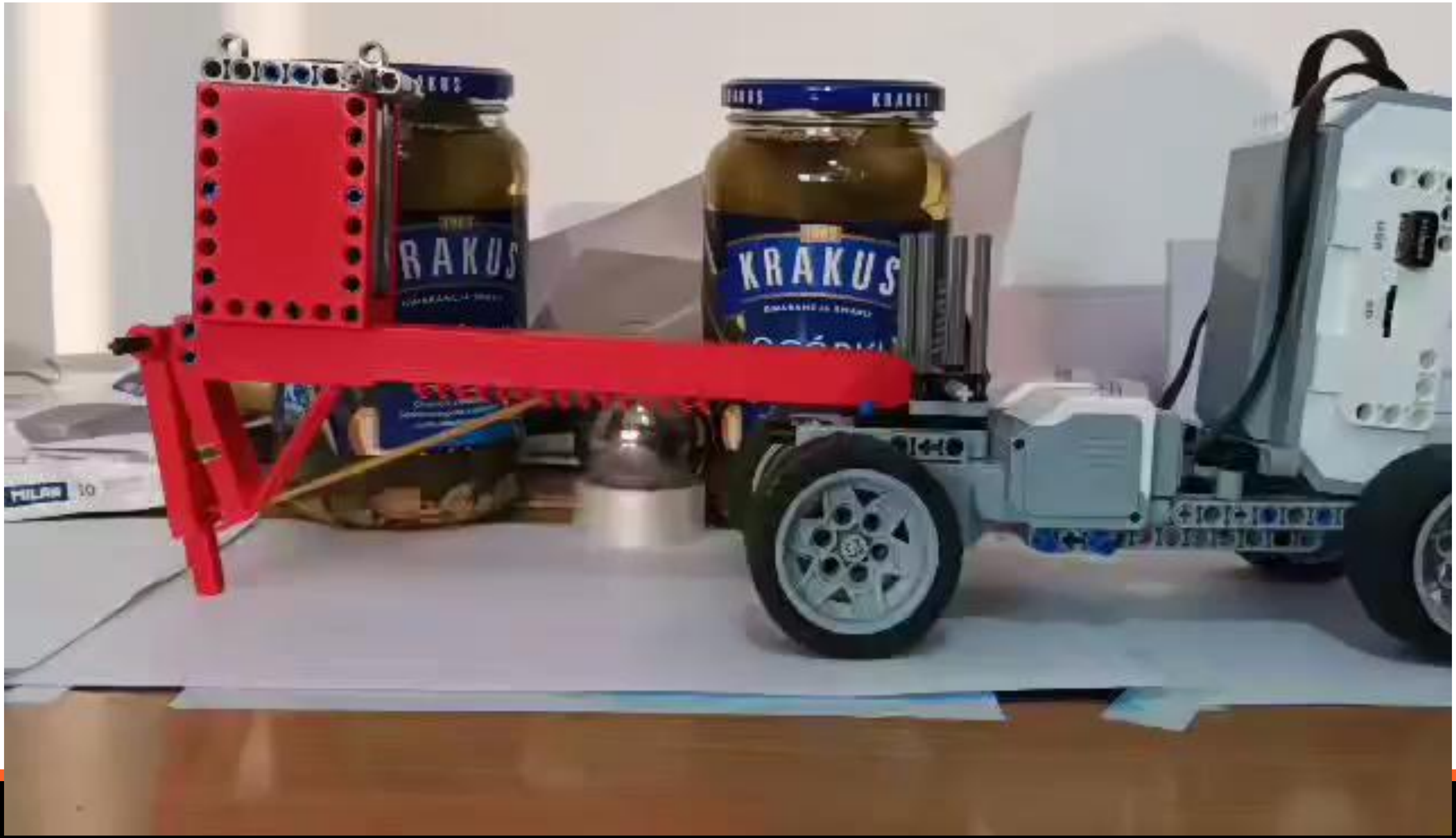




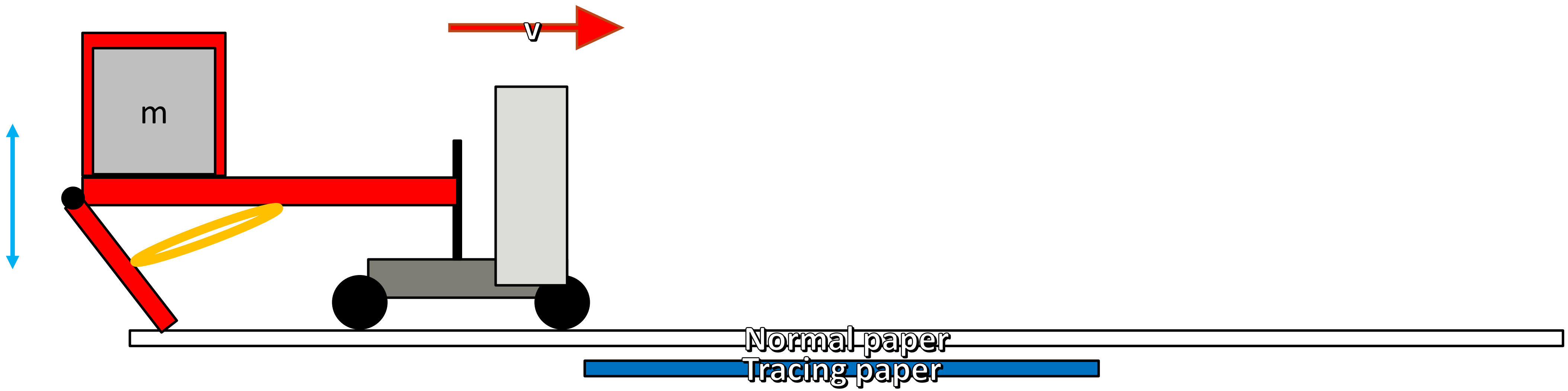
The robot



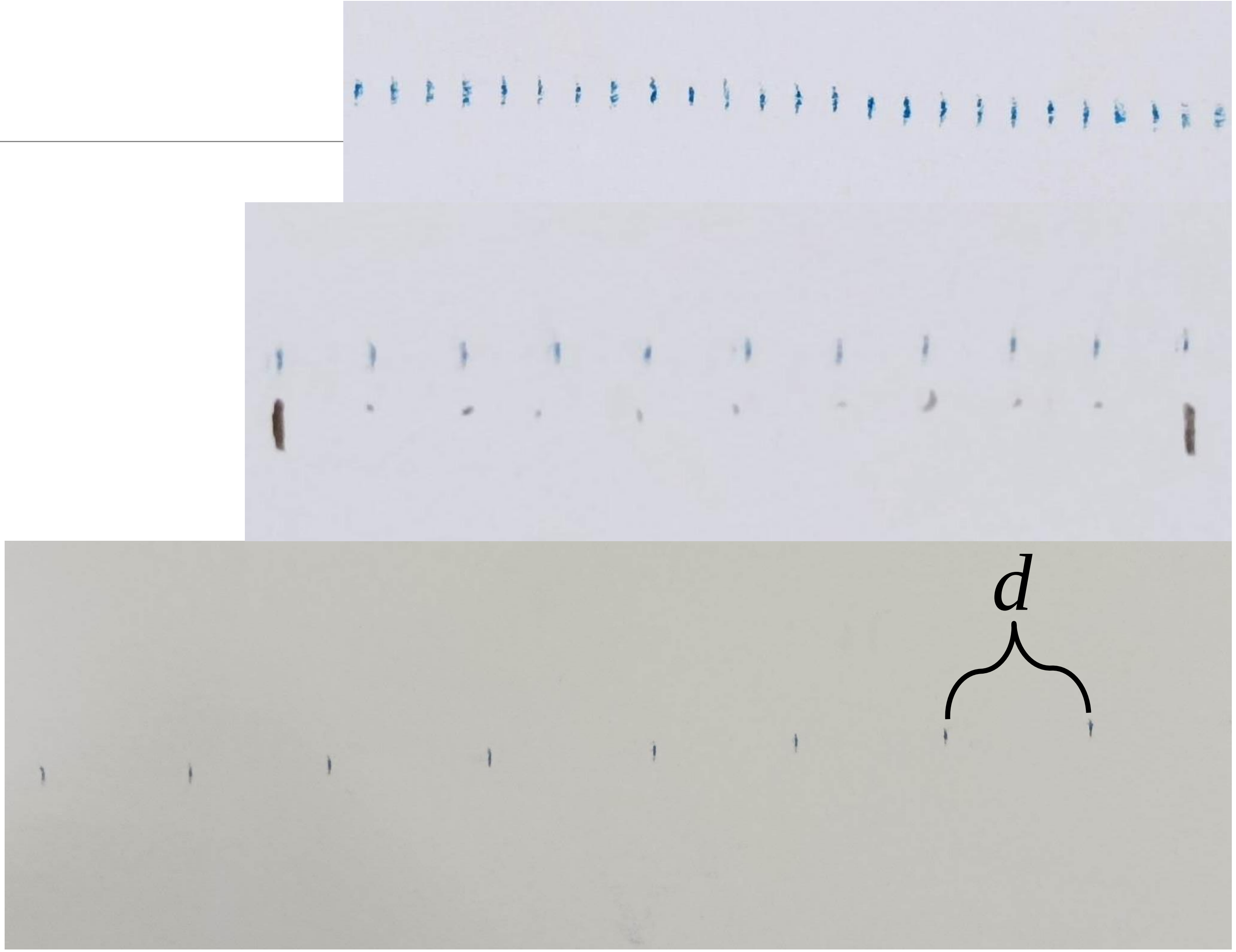
Robot in action



Experiment



Results



Results

k = (0.167 ± 0.01) N*m/[rad]

μ = 0.40 ± 0.03

α_0 = 20° ± 1°

L = (7.5 ± 0.1) cm

(v = 5 cm/s)

k = 0.1717 N*m/[rad]

μ = 0.408

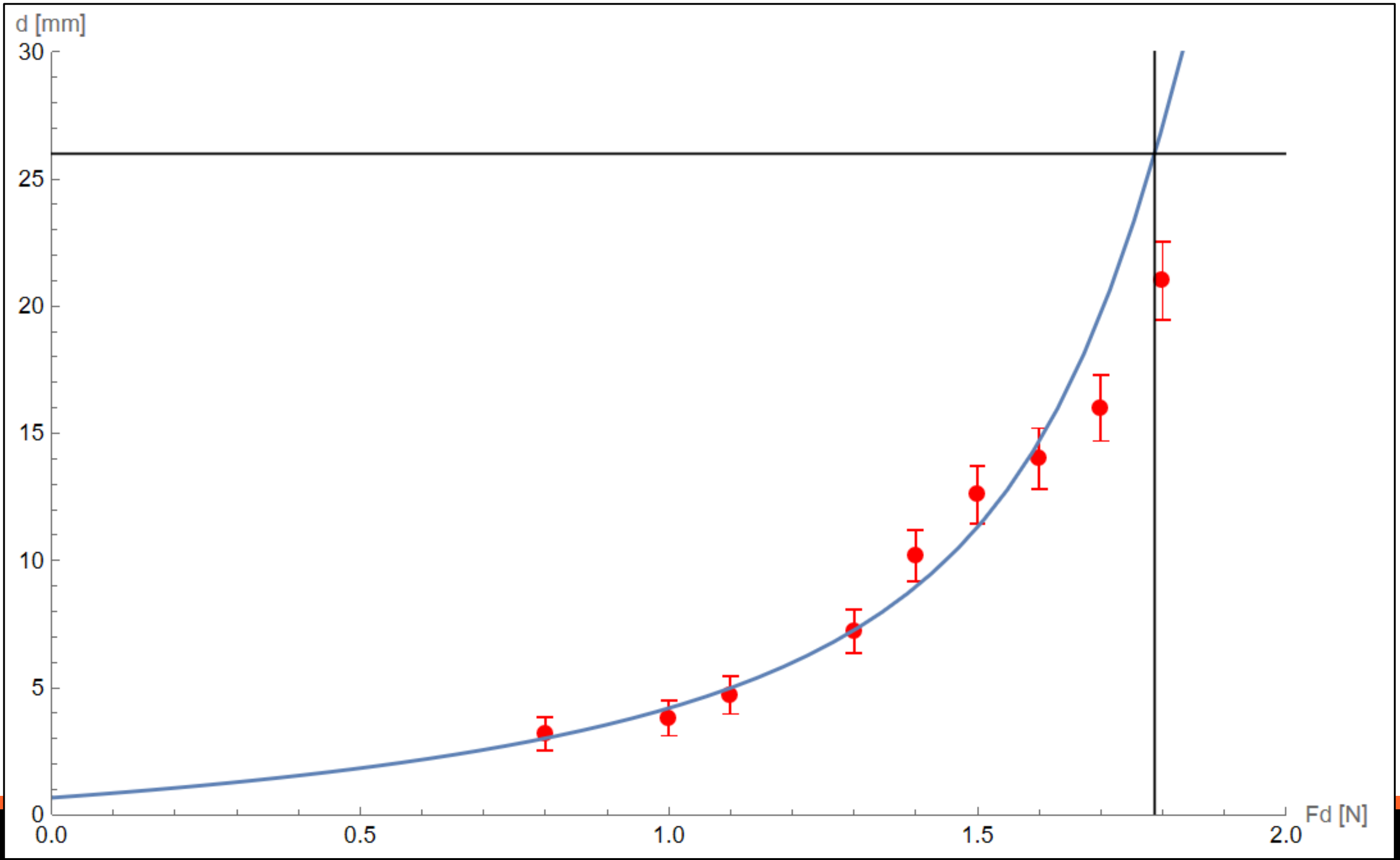
α_0 = 20.57°

Fitted

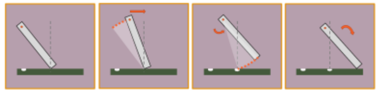
Parameters

Match

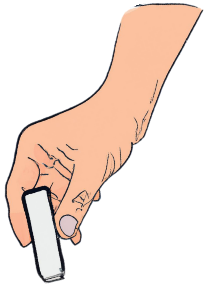
measured



Interpretation

- 
- $k(\alpha_0 - \alpha^*) + LF_d \sin(\alpha^*) = \mu LF_d \cos(\alpha^*)$

- We control Fd, k, α_0 v
- We focus on is v and k - the rest balances out
- But still the above apply



The problem questions

What parameters from the relative movement between the chalk and the board can be inferred from the resulting trace?

With only one dimension we can only say what the relations is between L F_d k α_0 μ (v)

Additional information from dot size?

Is it possible to infer anything about the dimensions of the chalk?

Diststance is linear with L, as long as **k** is increased

$$k(\alpha_0 - \alpha^*) + LF_d \sin(\alpha^*) = \mu LF_d \cos(\alpha^*)$$

$$d = L(\sin[\alpha_0] - \sin[\alpha^*])$$

Velocity

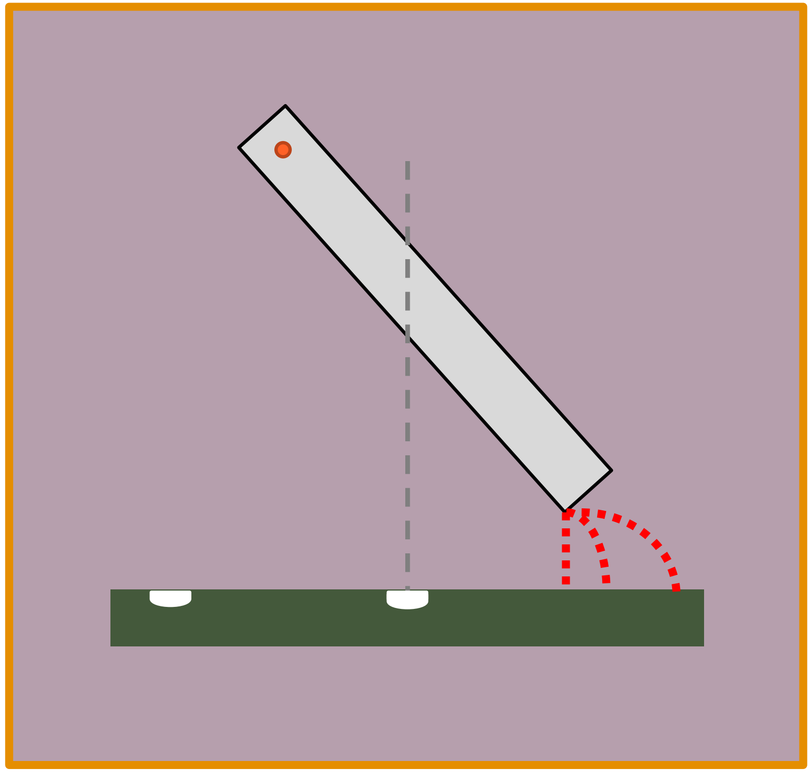
If the chalk moves along a parabola, the distance calculated would be

$d' = d + v\Delta t$

Where Δt is the free fall time

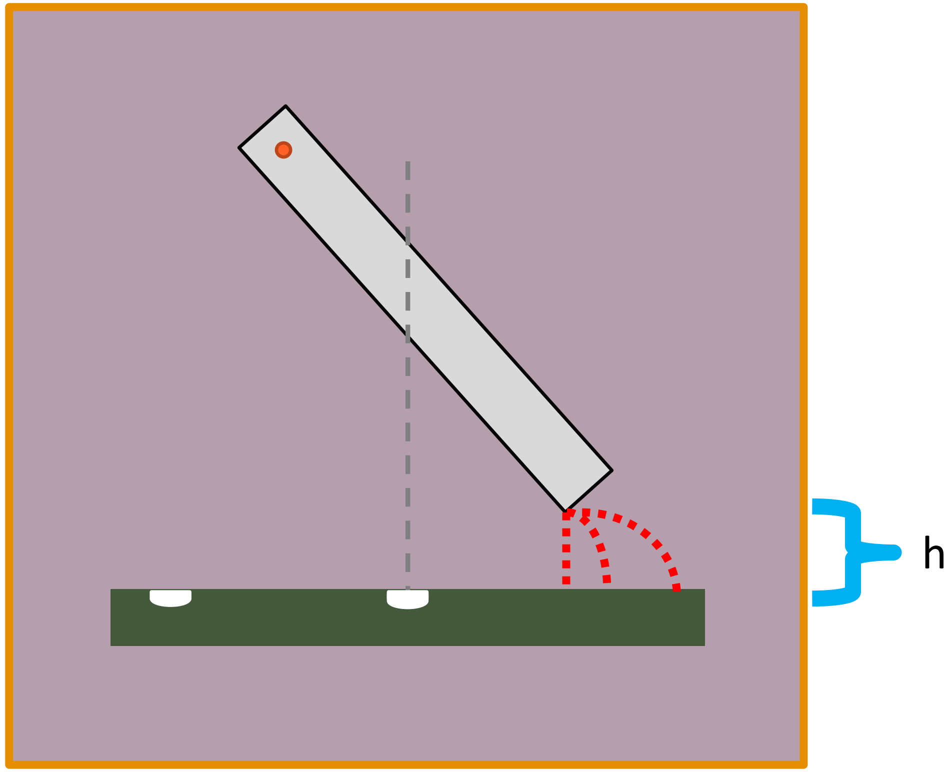
For velocities under 5cm/s

$v\Delta t < 1.5$ mm



Velocity

$$\Delta t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2L(\cos[\alpha*] - \cos[\alpha_0])}{g}}$$



Summary

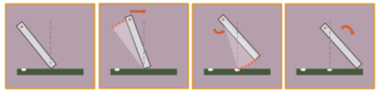
1. We developed a step by step model what happens in the real case

2. We simplified the hand to an easier model

3. We calculated the expected distance based on all the parameters

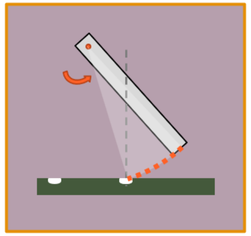
4. We verified agreement between experiment and theory
(and built a cool robot)

Assumptions

- The chalk is stiff, light and thin
- The velocity is small
- The chalk does exactly this: 
- In the last step it falls straight down
- We will assume α is between 0 and α_0

Theory

Lets calculate α at which $T = \mu F_d$ (the chalk is about to

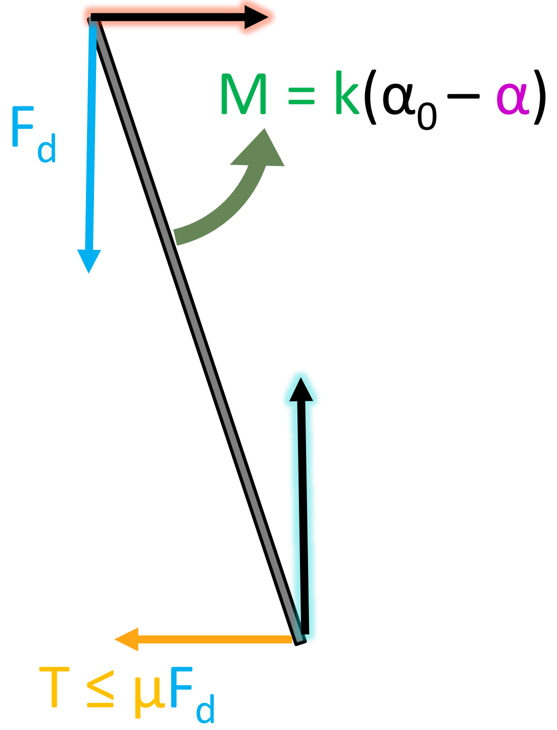


Balance of the torques:

$$k(\alpha_0 - \alpha^*) + LF_d \sin(\alpha^*) = \mu LF_d \cos(\alpha^*)$$

We can solve this numerically or by Taylor expansion. 1st order:

$$\alpha^* = (\alpha_0 - \mu LF_d/k)/(1 - LF_d/k)$$



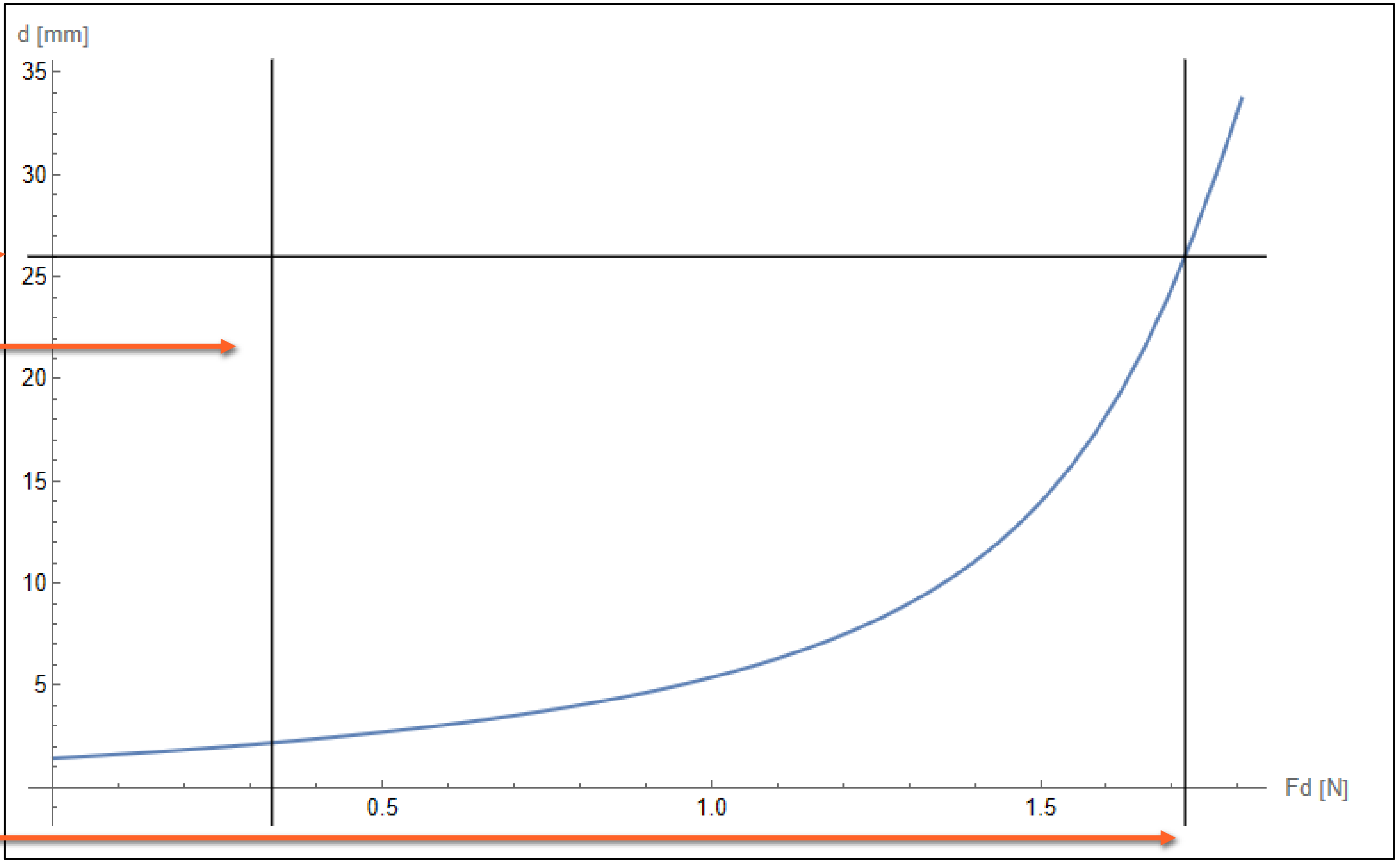
Theory

Then the distance:

$d = L(\sin[\alpha_0] - \sin[\alpha^*])$

We can also calculate the maximum F_d such that $\alpha^* = 0$ (and from that the max d)

We can estimate the minimum F_d because in reality M is not 0 at α_0



Changing μ

sandpaper

The problems

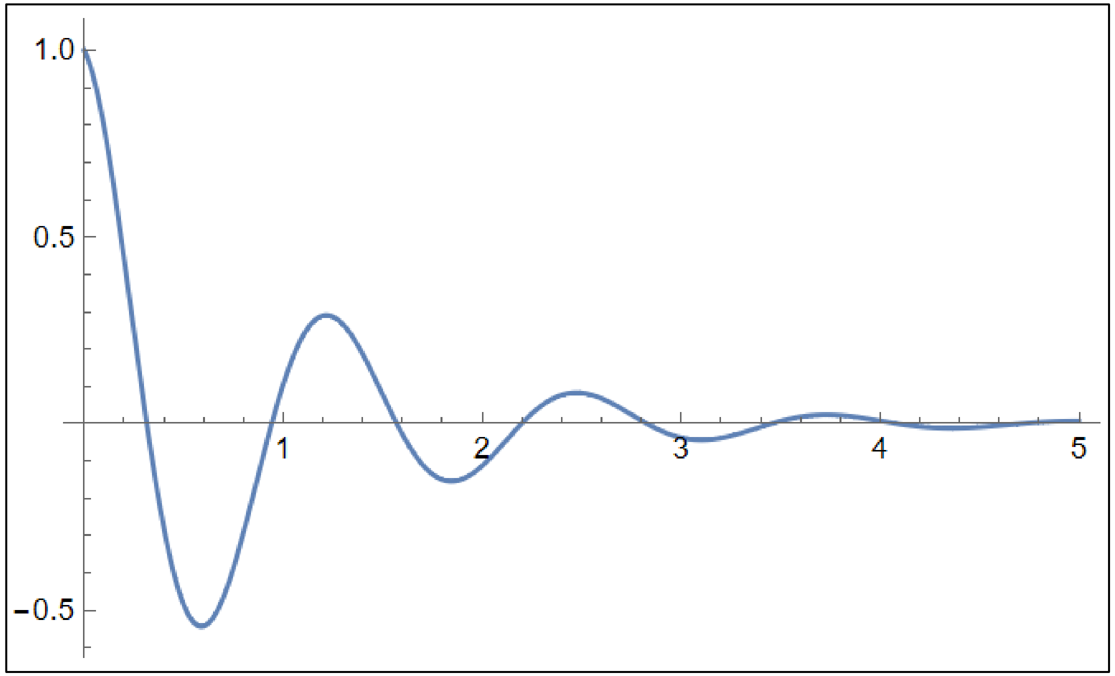
- The torque is not ideally harmonic and doesnt start exactly at a_0

- The chalk bounces from the bloker

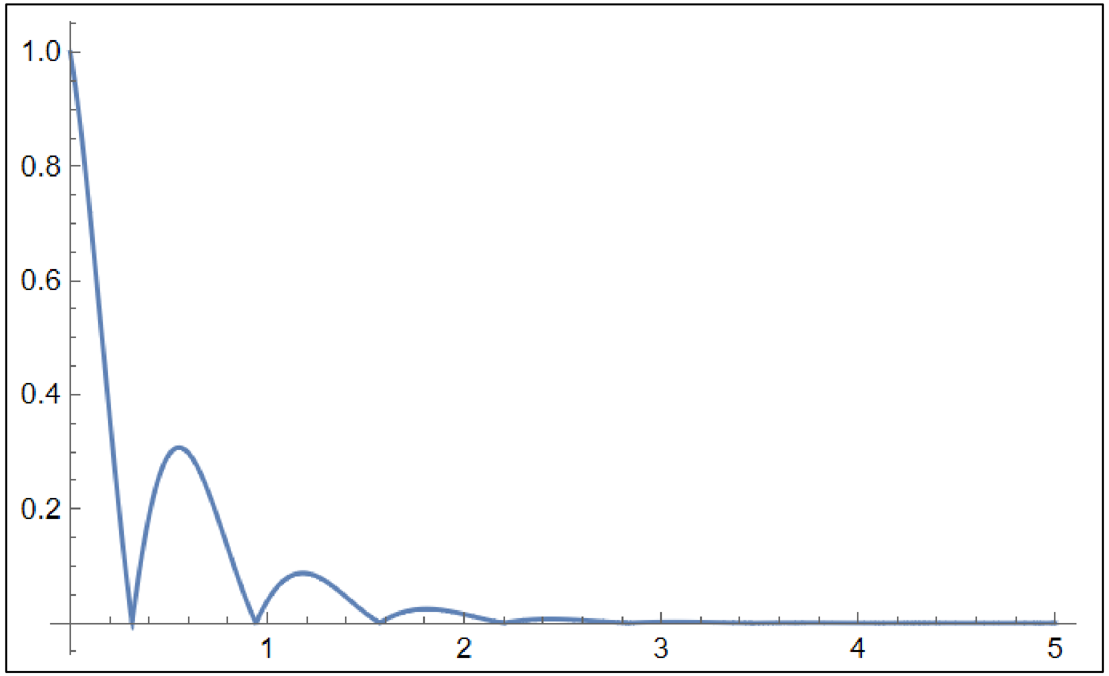
- With higher velocities, the F_d is not constant (which causes a different kind of motion)

Bounce

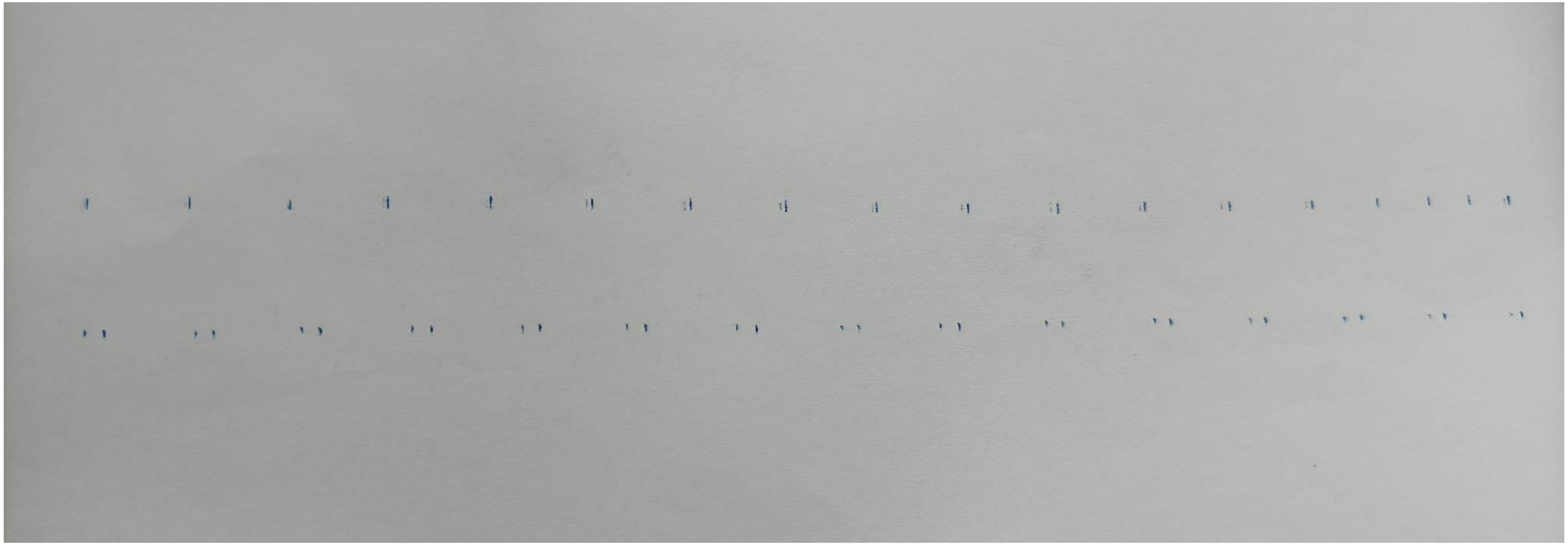
Hand



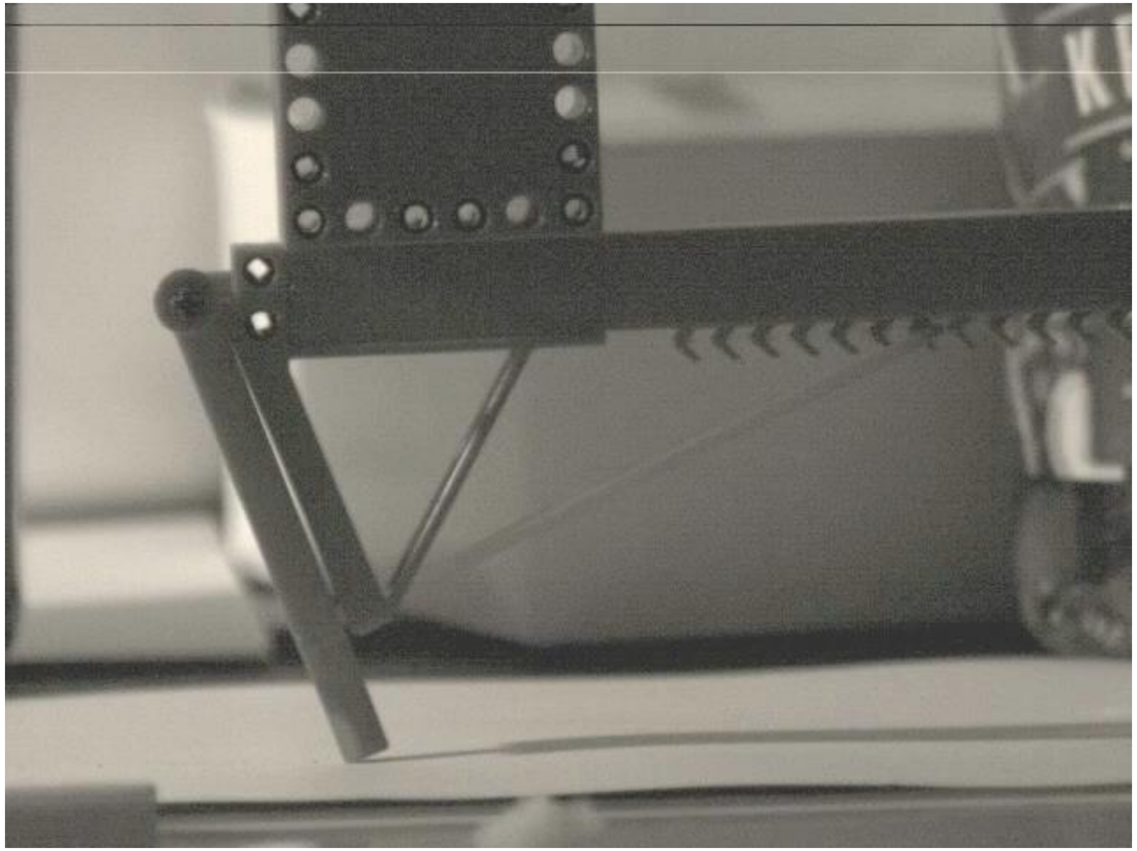
Robot



Bounce



High velocity robot problems



Lower velocity slow motion