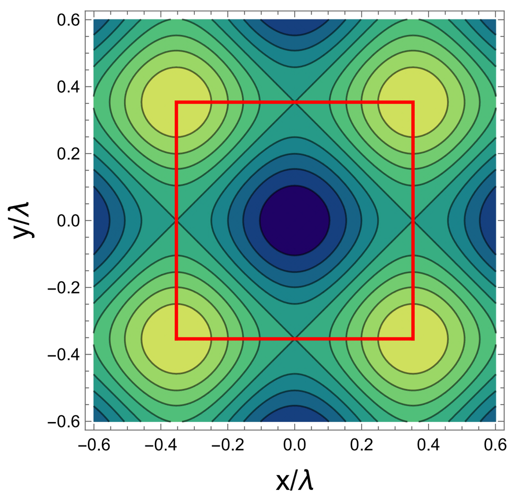
Energy levels in 2D spin dependent optical lattices

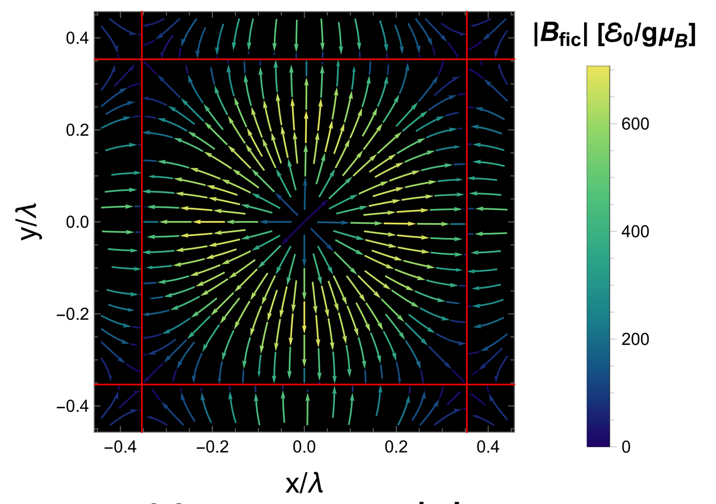
BSc thesis of **Kamil Dutkiewicz** under supervison of **Grzegorz Łach**, with help of **Marek Trippenbach**

FACULTY OF PHYSICS
UNIVERSITY OF WARSAW

Hamiltonian in a 2D Lattice



Scalar potential



Vector potential

Scalar potential and vector potential (in form of a fictitious magnetic field) resulting from the AC Stark effect. In red are marked the edges of the elementary cell, at which the boundary conditions are applied.

Units: $\varepsilon_0 = \frac{4\hbar^2\pi^4}{\lambda^2 M}$, potential later scaled with parameter $a^2 < 0.01$

This study investigates energy levels of a single bosonic atom in a rectangular optical lattice [1]. When the laser beams are polarized at a 45° angle to the XY plane, the resulting AC Stark effect [4,5] potential includes not only a scalar component $\mathrm{V}(x, y)$ but also a spin dependent vector component, which can be represented by a fictitious magnetic field $\mathbf{B}_{\mathrm{fic}}(x, y)$ [2,3]. The resulting Hamiltonian also incorporates a Zeeman interaction with an external magnetic field $B_{ext}\hat{\boldsymbol{z}}$.

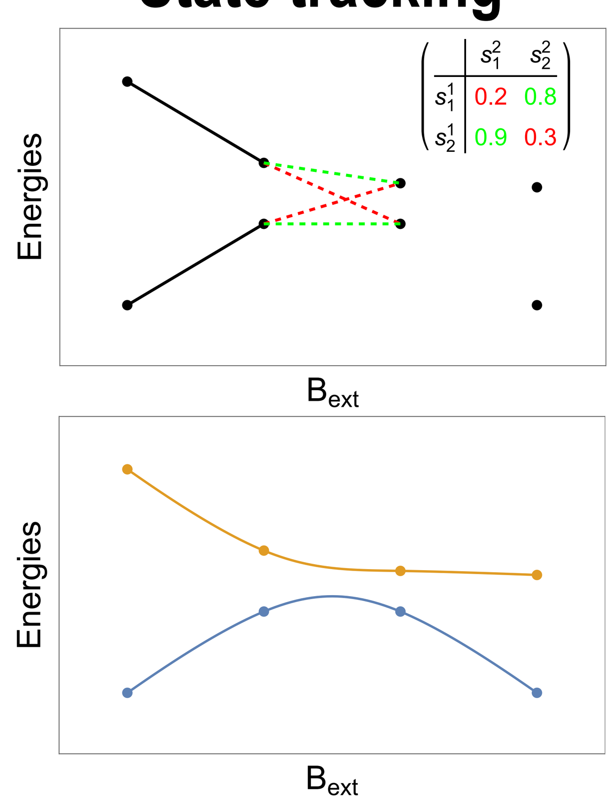
$$\mathrm{V}(x,y) = -\frac{V_0}{2}\cdot(2+\cos(q_0 x)+\cos(q_0\, y))$$

$$\overrightarrow{B_{\mathrm{fic}}}(x,y) = -B_0(\sin(q_0 x)\cos^2(q_0 x/2)\hat{\boldsymbol{x}} + \sin(q_0 x)\cos^2(q_0\, y)\hat{\boldsymbol{y}})$$

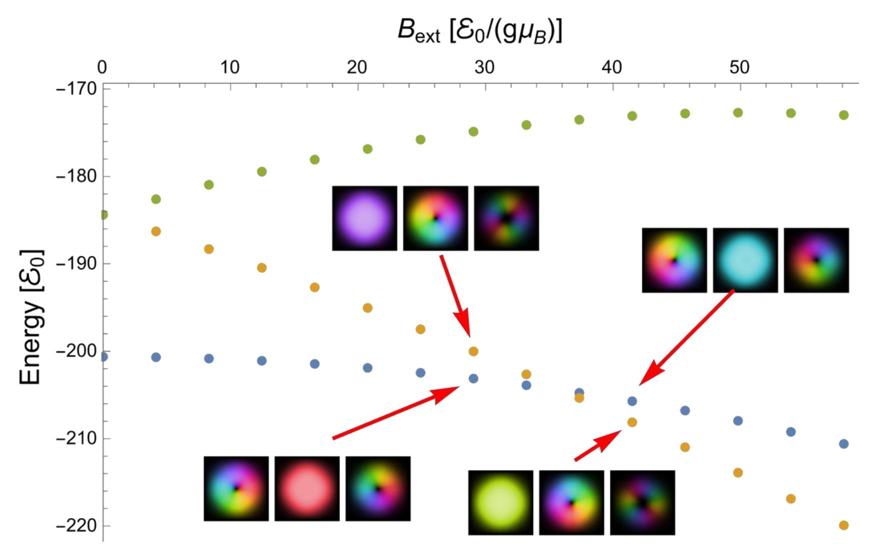
$$\hat{H} = -\frac{\hbar^2}{2M}\Delta + \mathrm{V} - g\mu_B\left(\overrightarrow{B_{fic}} + \overrightarrow{B_{ext}}\right)\cdot\vec{F} \qquad q_0 = 2\sqrt{2}\pi/\lambda$$

Eigenstates and energy levels

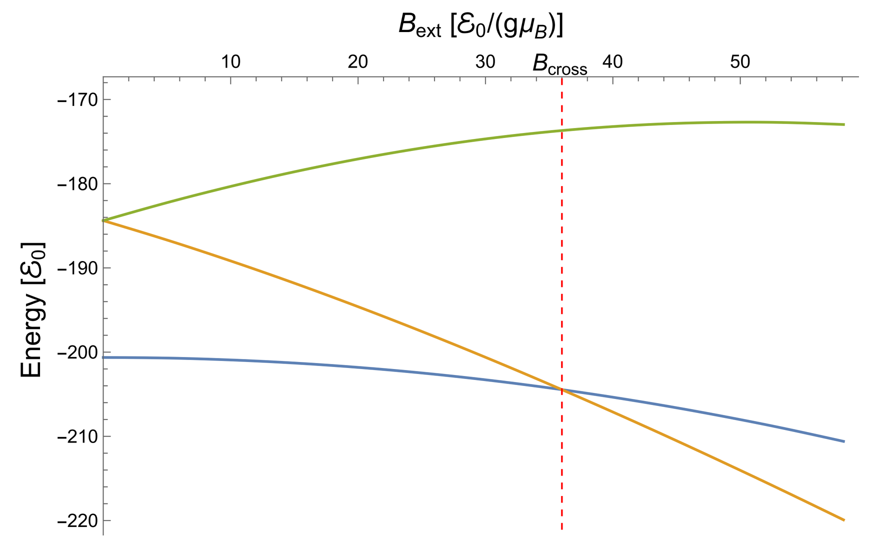
State tracking



To distinguish between regular crossing of energy levels and avoided crossing, a method to track the states is required. A vector $\mathbf{s}^1_i$ and $\mathbf{s}^2_i$ is calculated for each state i by sampling its wave function at two values of the external magnetic field: B_1 and B_2. The *balanced assignment problem* is later solved on Matrix: $\hat{A}^i_j = \mathrm{Correlation}[s^1_i, s^2_j]$ to connect the points on the plot.



Energy levels of the first three states in a 1 by 1 cell with Dirichlet BC, calculated at discrete values of B_{ext}. Displayed are also the three components of the states' wave functions (hue corresponds to phase).



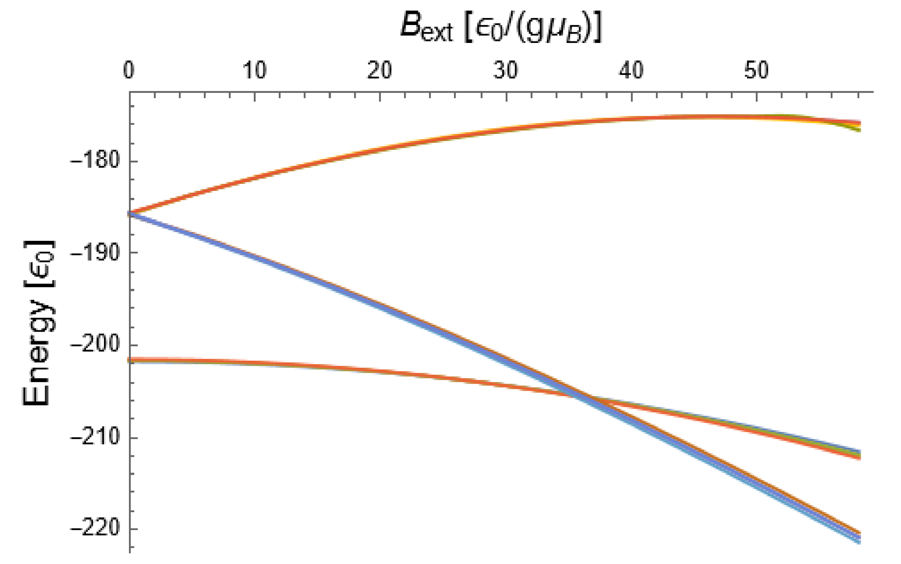
From discrete points shown above, using described methods of state tracking, plots of energy levels from B_{ext} are made.

Eigenstates of the Hamiltonian are computed numerically using **Mathematica**'s *NDEigensolve* [8]. Energy levels as a function of the external magnetic field B_{ext} are investigated.

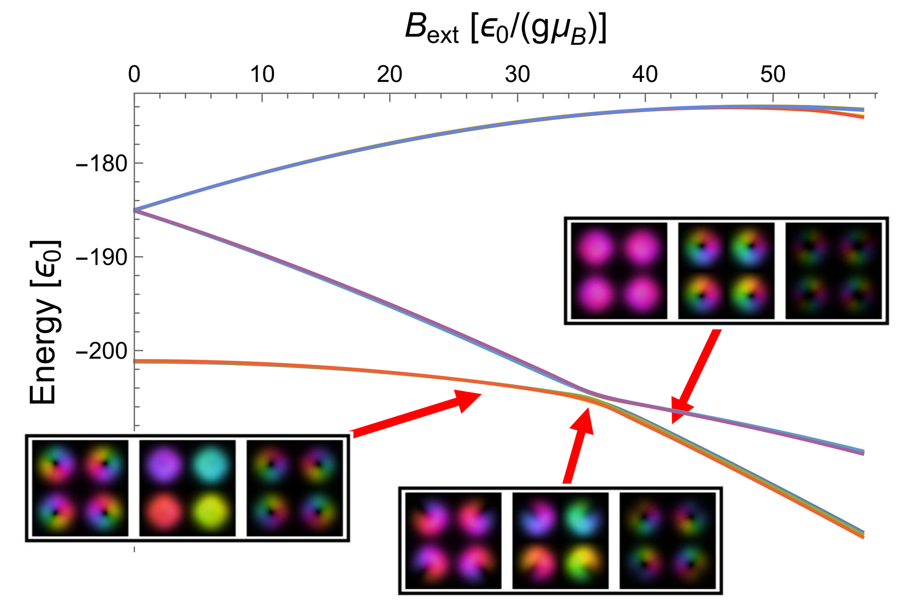
First a 1 by 1 cell is considered, which yields results as displayed on the left (similar for both periodic and Dirichlet BC). Notably, the upper two states begin degenerate at $B_{ext} = 0$, and the lower two cross.

With the same parameters, a 2 by 2 cell with periodic boundary conditions displays similar behavior, but each state is replaced by a multiplet of 4 states.

Remarkably, for 2 by 2 cells with Dirichlet boundary conditions, **avoided crossing** [7,8] is observed between the two lower multiplets, with use of the mentioned state tracking methods.

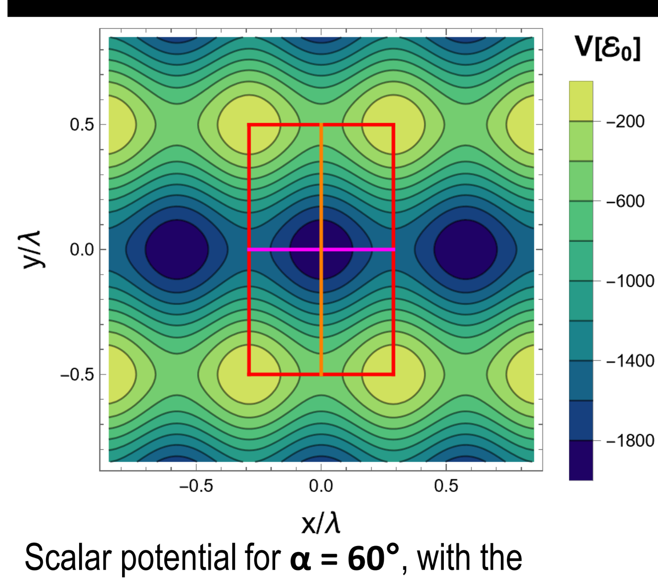


Energy levels from B_{ext} in a 2 by 2 cell with **periodic** BC

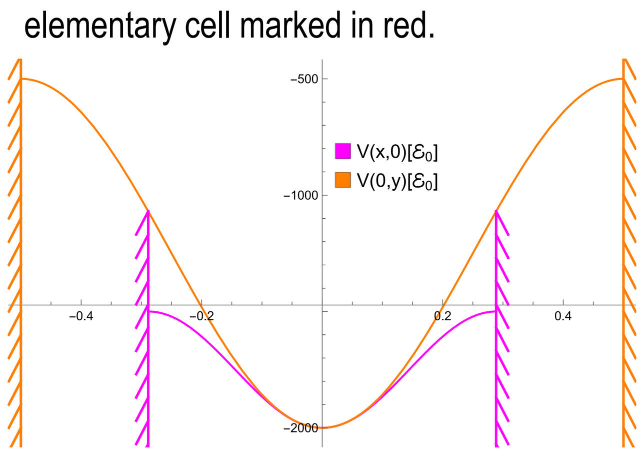


Energy levels from B_{ext} in a 2 by 2 cell with **Dirichlet** BC. **Avoided crossing** is observed. Displayed is the evolution of a single state from the lowest multiplet.

Rectangular lattice



Scalar potential for **α = 60°**, with the elementary cell marked in red.

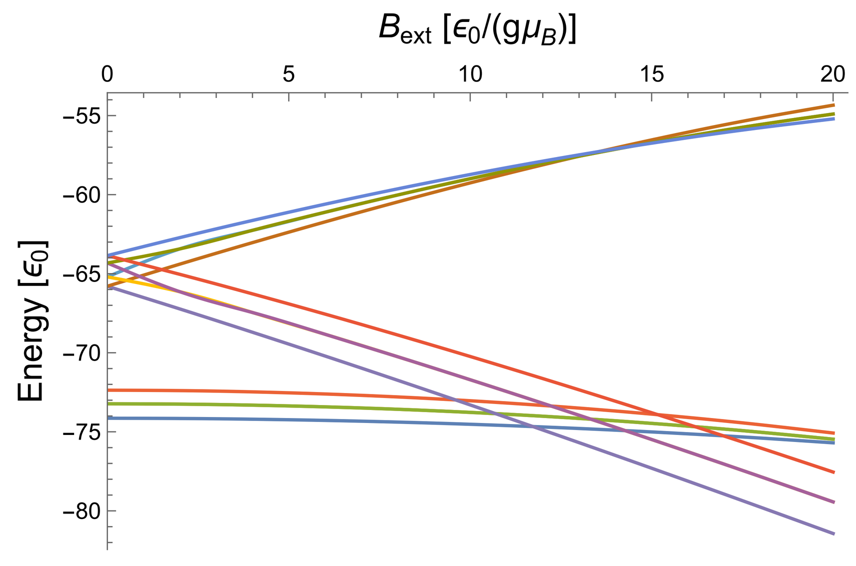


V(x,y) cross-section along y = 0 and x = 0. Marked are the Dirichlet boundary conditions, at different distances from the cells center. The saddle points are lower along the x axis. The shape of the scalar potential near the center is isotropic (not true for B_{fic}).

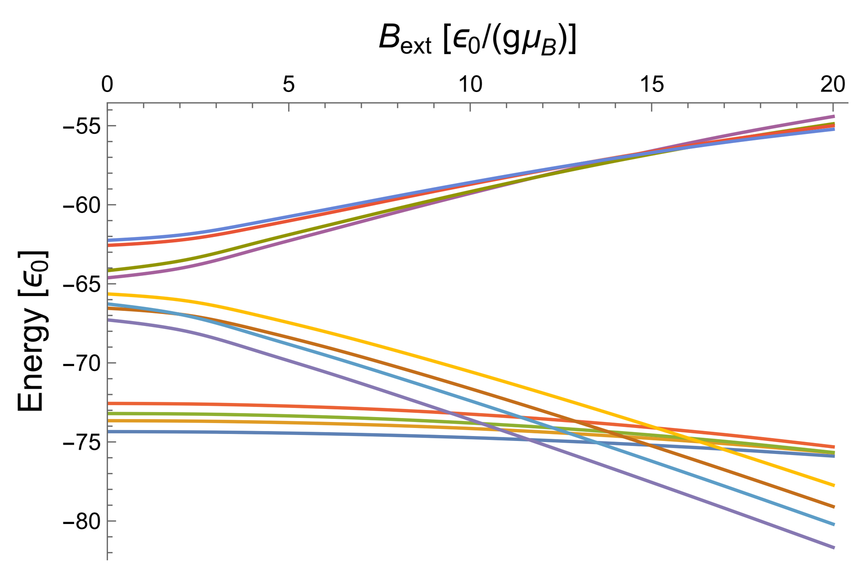
Two pairs of counterpropagating laser beams, crossed at $\alpha = 90°$ result in square cells as pictured before. By varying the angle between the beams, rectangular cells can be obtained. The resulting scalar potential and its cross section (for $\alpha = 60°$) is displayed on the left.

In a 2 by 2 cell, multiplets of 4 states appear, but for $\alpha = 90°$ the middle two are degenerate. At lower angles however, the degeneracy is lifted due to the loss of rotational symmetry. This can be observed in shallower potentials, as demonstrated in Figures on the right.

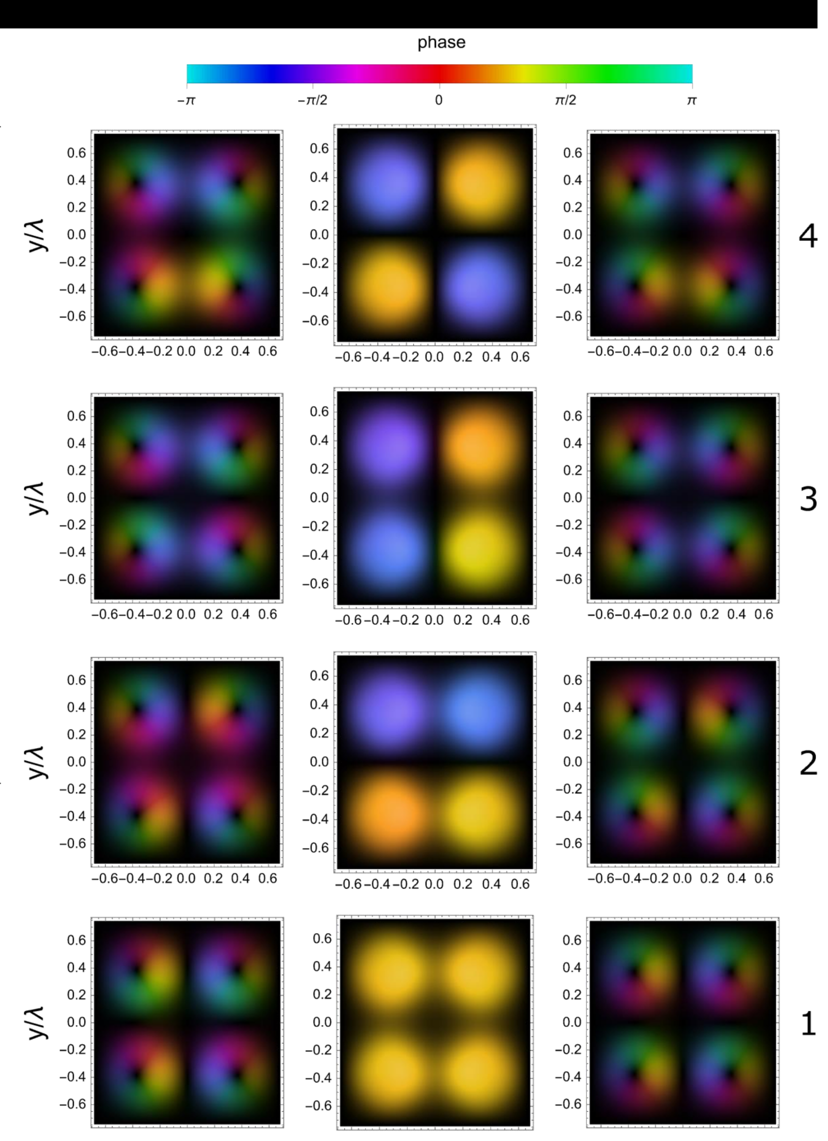
The $B_{ext} = 0$ degeneracy between states 2 and 3 (or multiplets 2 and 3 in the case of a 2 by 2 cell) is lifted upon changing α. Avoided crossing still occurs in a Dirichlet BC 2 by 2 cell, regardless of α.



Energy levels in a 2 by 2 cell, periodic BC, **α = 90°**



Energy levels in a 2 by 2 cell, periodic BC, **α = 85°**



Three components of the lowest 4 states' wave functions. Hue corresponds to phase and brightness to probability density. States 2 and 3 *would* have rotational symmetry, were it not for the elongation of the cell

Results

- lifted degeneracy between the ground and excited state at $B_{ext} = 0$ in a 1 by 1 cell, upon changing the angle α
- lifted degeneracy between the middle two states from a multiplet of four, in a 2 by 2 cell, upon changing the angle α
- avoided crossing between the lowest 2 multiplets appears with Dirichlet boundary conditions, but only for cells larger than 1 by 1

Additionally, probability currents and energy bands were investigated, both of which will be subjects of future work.

References

[1] Oliver Morsch and Markus Oberthaler. *Reviews of Modern Physics*, 78(1):179, 2006.
[2] P. Szulim, M. Trippenbach, Y. B. Band, M. Gajda, and M. Brewczyk. *New Journal of Physics*, 24:033041, 2022.
[3] Igor Kuzmenko, Mirosław Brewczyk, Grzegorz Łach, Marek Trippenbach, and Y. B. Band, 2023. submitted to *Physical Review Letters*, arXiv:2308.07647.
[4] F. Le Kien, P. Schneeweiss, and A. Rauschenbeutel. *European Physical Journal D*, 67:92, 2013.
[5] D. Jaksch. *Contemporary Physics*, 45(5):367381, 2004.
[6] J. von Neumann and E. Wigner. *Phys. Z.*, 30:467, 1929.
[7] L. D. Landau and E. M. Lifshitz. Quantum Mechanics. Pergamon Press, 1981.
[8] NDEigensystem, Mathematica, Wolfram Research Inc. https://reference.wolfram.com/language/ref/NDEigensystem.html