Laplace Transform

Laboratory III

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1 Goal of the exercise

2 Laplace Transform

Laplace transform is an integral transform that converts a function of a real variable(time domain) into a function of complex variable(frequency domain). It is powerful tool for solving differential equations, which turns ODEs into algebraic equations and convolution into multiplication. For function $x(t)$, the Laplace transform is the integral

$$\mathcal{L}[x(t)] = X(s) = \int_0^\infty x(t)e^{-st}dt \tag{1}$$

Where $s = \sigma + j\omega \quad \sigma, \omega \in \mathbb{R}$

In order to get solution of differential equation solved in s-domain it is necessary to apply inverse Laplace transform which is given by following complex integral

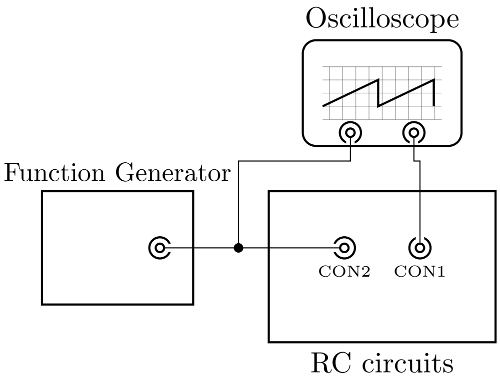
$$f(t) = \mathcal{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)e^{-st}ds \tag{2}$$

Integrals like these can be quite difficult to solve, that is why lookup table of Laplace transforms and properties can be very handy.

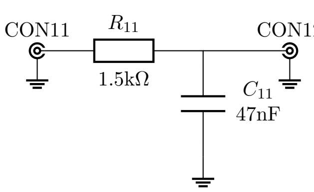
Moving to solving circuits using Laplace transform, there are two approaches, first is constructing differential equation in time domain describing circuit and then solve it using Laplace transform, or second option convert circuit to s-domain with taking into account initial condition.

3 Course of measurements

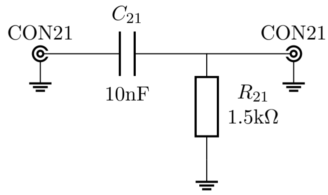
First we tested two circuits: low-pass and high-pass configuration of RC circuit and RLC circuts. After connecting osciloscope to the wave generator and prototype board, we generated square wave with v_{pp} = 1V, v_{offset} = .5V, frequency of 100Hz and duty cycle of 50%. Then we read from the oscilloscope Voltage value at times 1τ, 5τ and 10τ (where 10τ is just as fail-safe) and time when voltage reaches 10% and 90% of the highest value.



(a) Measurement setup



(b) Circuit A

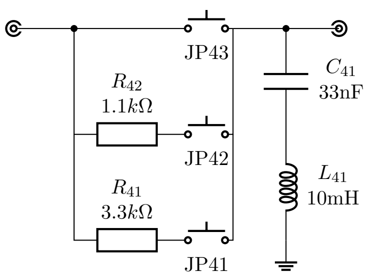


(c) Circuit B

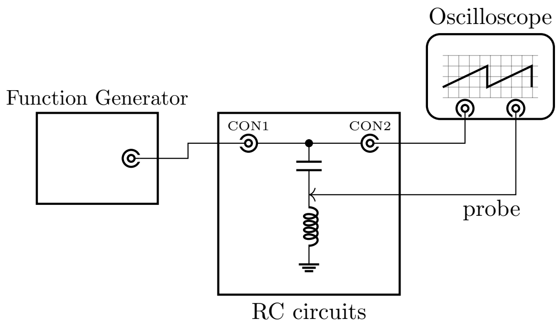
Figure 1: RC Circuits

For RLC circuits we tested two different resistances before RC circuit influence output characteristic. We measured response for 2 cases: Generator out resistance (50 Ω) + resistance selected by jumper wire. In our case we tested jumper on 1.1kΩ resistance path and 3.3kΩ. After that we checked response of the circuit:

- if response was sinusoid with decreasing amplitude - resustance was smaller then RC
- if response was exp. decay if R was higher
- if response was aperiodic critical waveform if resistance was equal RC



(a) Measured Circuit



(b) Measurements setup

Figure 2: RLC circuit

4 Theoretical calculations

For all calculations we used Matlab with Symbolic Math Toolbox. Source code can be found in Appendix. A

In all three circuit input voltage was square wave with 50% duty cycle which can be described in time domain by

$$v_{in}(t) = V_{offset}\mathbf{1}(t) + V_{pp}\mathbf{1}(t - \frac{T}{2}) - V_{pp}\mathbf{1}(t - T) \tag{3}$$

and after Laplace transform into frequency domain

$$V_{in}(s) = \mathcal{L}[v_{in}(t)] = \frac{V_{offset}}{s} + \frac{e^{-\frac{T}{2}s}}{s} - \frac{e^{-Ts}}{s} \tag{4}$$

4.1 Circuit A

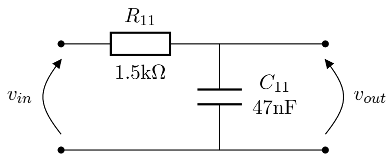


Figure 3: Circuit A schematics

v_{out} of circuit A can be described using simple voltage divider

$$v_{out}(s) = v_{in}(s)\frac{\frac{1}{sC_{11}}}{R_{11} + \frac{1}{sC_{11}}} \tag{5}$$

after applying inverse Laplace transform to $V_out(s)$ we obtain below plot.

Figure 4: Circuit A output voltage

4.2 Circuit B

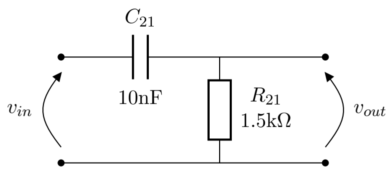
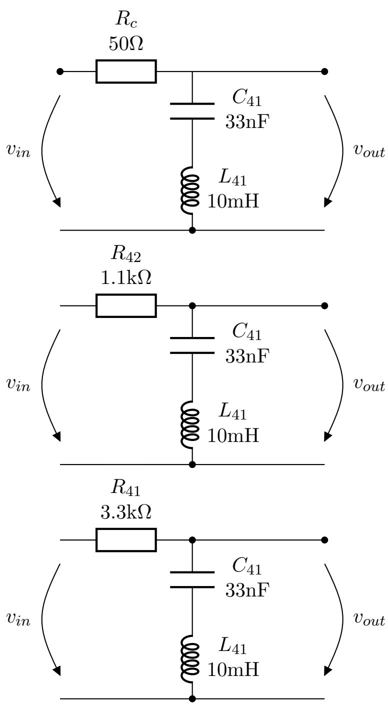


Figure 5: Circuit B schematics

4.3 Circuit C



5 Comparison

6 Conclusions

A Appendix