Nodal Analysis

Laboratory I

Patrycja Nazim, Adrian Król, Kamil Chaj

1 Aim of exercise

The aim of our exercise was to experimentally verify the nodal analysis in RLC circuits. We have achieved it by measuring the voltages on different nodes of the chosen circuits using a dedicated evaluation board and vector voltmeter. The obtained measurement results are compared with analytical calculations.

Apart from the values of potentials in individual nodes of the circuits being measured, we calculated the currents flowing through pointed elements.

2 Course of measurements

First step of our measurements was connecting evaluation board(fig.1) to laboratory computer via USB and starting Vector Voltage Meter software. Next we connected both INPUT1 and CON1 to OUTPUT using BNC cables and splitter, last we connected oscilloscope probe to INPUT2 connector. Now with all preparation finished we started voltage measurements of each node in Circuit A(fig.4). Then we disconnected BNC cable connected to CON1 and connected it to CON2 and repeated the same measurements for Circuit B(fig.5)

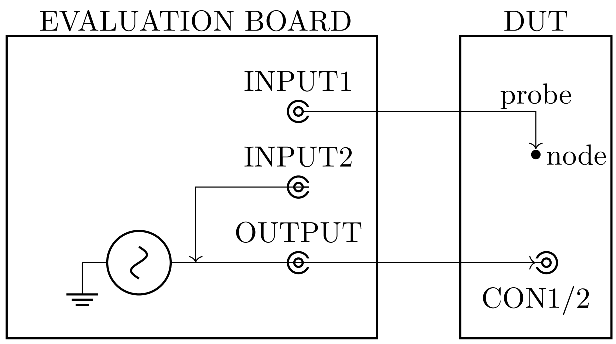

Figure 1: measurements schematic

3 Nodal analysis - method

Method which we are going to use to solve these circuits is known as "Nodal Analysis by Inspection". In this method we need to construct 3 matrices: $\mathbf{i}$ - current vector, $\mathbf{v}$ - voltage vector(unknown), $\mathbf{G}$ - conductance($\mathbf{Y}$ admittance for AC circuits) matrix, with sizes respectively $N\times 1$, $N\times 1$, $N-1\times N-1$.

Size of conductance matrix is always smaller then number of nodes in the circuit by one because in nodal analysis one node is connected to ground as reference node.

$$\mathbf{Gu} = \mathbf{i} \tag{1}$$

$$\begin{bmatrix} G_{11} & -G_{12} & -G_{13} \\ -G_{21} & G_{22} & -G_{23} \\ -G_{31} & -G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{2}$$

Where G_{11}, G_{22}, G_{33} are sums of conductance of each branch connected to the node
$G_{12} = G_{21}$, $G_{13} = G_{31}$, $G_{32} = G_{23}$ are sums of conductance of branches between nodes
I_1, I_2, I_3 are sums of current sources entering or exiting node and V_1, V_2, V_3 are unknown voltages that we are trying to find.

With simple matrix operation we obtain equation

$$\mathbf{v} = \mathbf{G}^{-1}\mathbf{i} \tag{3}$$

which can be easily calculated

4 Theoretical calculations

All calculation are made in Python with NumPy library. Source code for all calculation can be found in the Appendix

Before starting any calculation we modified original circuits' schematics by replacing BNC connectors with AC voltage source and drawing connection between ground(fig. 2a, fig. 3)

for both circuits amplitude of Voltage source is 0.192V and calculations are made for frequencies 1kHz, 5kHz and 9kHz

4.1 Circuit A

In our calculations voltages in nodes 1, 2, 3 and 4 are named V_1, V_2, V_3, V_4 and reference node is the same node that was ground in original circuit schematic. Lone voltage source must be transfigured in order to solve this circuit by nodal analysis, after source transfiguration(fig. 2b) we can see that node 1 is directly connected to reference node therefore in the circuit there are only 2 nodes that have more than 2 elements connected, so $\mathbf{Y}$ will be only 2×2

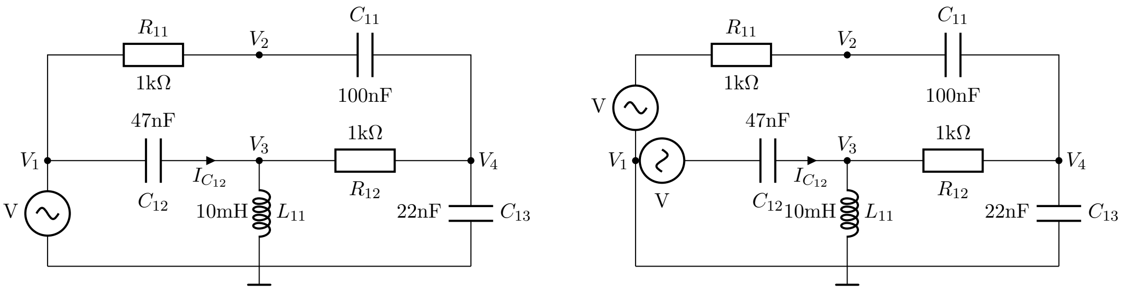

Figure 2: theoretical circuit A

following rules of Nodal Analysis by Inspection(Sec. 3) we obtain $\mathbf{Yu} = \mathbf{i}$ equation

$$\begin{bmatrix} Y_{C_{12}} + Y_{L_{11}} + Y_{R_{12}} & -Y_{R_{12}} \\ -Y_{R_{12}} & Y_{R_{11}+C_{11}} + Y_{R_{12}} + Y_{C_{13}} \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} \frac{V}{Z_{C_{22}}} \\ \frac{V}{Z_{R_{11}}+Z_{C_{11}}} \end{bmatrix} \tag{4}$$

after solving above equation we can easily calculate current of the capacitor C_{12}

$$I_{C_{12}} = \frac{V_1 - V_3}{Z_{C_{12}}} \tag{5}$$

all that is left to do is computing

results of calculations

source code in the Appendix. A

1000.0 Hz	Magnitude	Phase	5000.0 Hz	Magnitude	Phase	9000.0 Hz	Magnitude	Phase
Node 3	0.00742236	146.521	Node 3	0.17592	141.258	Node 3	0.367336	22.0983
Node 4	0.072673	37.3586	Node 4	0.0791113	50.1731	Node 4	0.22825	-17.7003
I of C_12	5.85402e-05	88.8165	I of C_12	0.000512559	71.5092	I of C_12	0.000538848	-47.0308

4.2 Circuit B

Following similar steps as in Circuit A, voltages in notes1, 2, 3 and 4 are named V_1, V_2, V_3, V_4 and reference node is the same node that was ground in original circuit schematic. There is no lone voltage source so source transfiguration is not necessary.

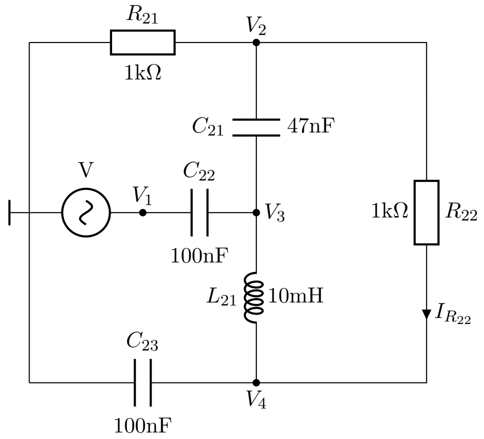

Figure 3: theoretical circuit B

following rules of Nodal Analysis by Inspection(Sec. 3) we obtain $\mathbf{Yu} = \mathbf{i}$ equation

$$\begin{bmatrix} Y_{R_{21}} + Y_{R_{22}} + Y_{C_{21}} & -Y_{C_{21}} & -Y_{R_{22}} \\ -Y_{C_{21}} & Y_{C_{21}} + Y_{C_{22}} + Y_{L_{21}} & Y_{L_{21}} \\ -Y_{R_{22}} & -Y_{L_{21}} & Y_{R_{22}} + Y_{C_{23}} + Y_{L_{21}} \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{V}{Z_{C_{22}}} \\ 0 \end{bmatrix} \tag{6}$$

after solving above equation we can easily calculate current of the capacitor R_{22}

$$I_{R_{22}} = \frac{V_4 - V_2}{Z_{R_{22}}} \tag{7}$$

and these are the results

4.2.1 Results of calculations

Source code in the Appendix B

1000.0 Hz	Magnitude	Phase	5000.0 Hz	Magnitude	Phase	9000.0 Hz	Magnitude	Phase
Node 2	0.0442579	28.1037	Node 2	0.0521098	-53.9736	Node 2	0.16282	63.622
Node 3	0.0837746	22.3054	Node 3	0.0534524	86.6776	Node 3	0.219774	20.3532
Node 4	0.0867875	20.4605	Node 4	0.206963	-12.1633	Node 4	0.0750914	-122.05
I of R_22	4.33246e-05	-167.348	I of R_22	0.000171674	179.512	I of R_22	0.000237659	61.8326

5 Real measurements

Following steps of measurements described in Section 2 we obtain measurements for each circuit

5.1 Circuit A

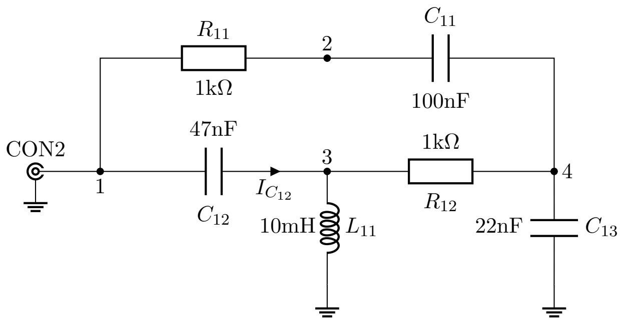

Figure 4: circuit A

Current $I_{C_{12}}$ is measured indirectly using measured voltage and impedance of the component

$$I_{C_{12}} = \frac{V_1 - V_3}{Z_{C_{12}}} \tag{8}$$

where V_3 is voltage in node 3, V_1 is voltage in node 1 and $Z_{C_{12}}$ is impedance of capacitor C_{12}

Circuit A			
Freq	Channel 1[V]	Channel 2 [V]	Angle [°]
node 1			
1kHz	0.192	0.192	0
5kHz	0.192	0.192	-0.1
9kHz	0.192	0.192	-0.1
node 2			
1kHz	0.192	0.145	-19.3
5kHz	0.192	0.59	9.3
9kHz	0.192	0.236	-9.7
node 3			
1kHz	0.192	0.008	140.7
5kHz	0.192	0.154	136.1
9kHz	0.192	0.376	30
node 4			
1kHz	0.192	0.072	38.6
5kHz	0.192	0.082	42.1
9kHz	0.192	0.231	-11.8

(a)

freq	$I_{C_{12}}$[I]	angle[°]
1kHz	0.198	178.5°
5kHz	$7.543 \cdot 10^{-5}$	1.5°
9kHz	$9.954 \cdot 10^{-5}$	-36.9°

(b)

Table 1: evaluation board measurements for Circuit A

5.2 Circuit B

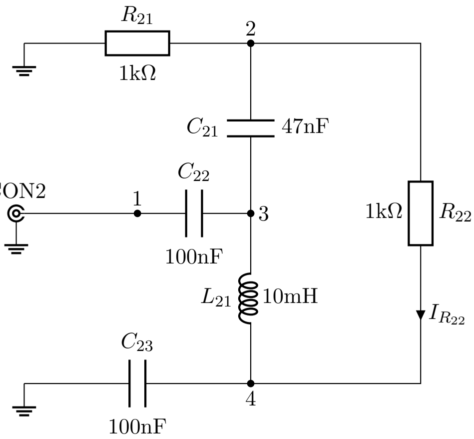

Figure 5: circuit B

Current $I_{R_{22}}$ is measured indirectly using measured voltage and impedance of the component

$$I_{R_{22}} = \frac{V_4 - V_2}{Z_{R_{22}}} \tag{9}$$

where V_4 is voltage in node 4, V_2 is voltage in node 2 and $Z_{R_{22}}$ is impedance of resistor R_{22}

Circuit B			
Freq	Channel 1[V]	Channel 2 [V]	Angle [°]
node 1			
1kHz	0.192	0.192	0
5kHz	0.192	0.192	-0.1
9kHz	0.192	0.192	0
node 2			
1kHz	0.192	0.044	28.7
5kHz	0.192	0.044	-35.7
9kHz	0.192	0.156	69.7
node 3			
1kHz	0.192	0.084	23.3
5kHz	0.192	0.055	62.4
9kHz	0.192	0.22	24.4
node 4			
1kHz	0.192	0.086	21.2
5kHz	0.192	0.18	-11.8
9kHz	0.192	0.09	-113.8

(a)

freq	$I_{R_{22}}$[I]	Angle[°]
1kHz	$4.379{\cdot}10^{-5}$	28.83
5kHz	$1.407{\cdot}10^{-4}$	175.48
9kHz	$2.456{\cdot}10^{-4}$	68.44

(b)

Table 2: evaluation board measurements for Circuit B

6 Comparison

	measurements		calculations		deviation	
	V	phase	V	phase	ΔV	Δphase
Circuit A						
Node 3						
1kHz	0.008	140.7	0.0074	146.52	0.00057	4%
5kHz	0.154	136.1	0.17	141.25	0.021	4%
9kHz	0.376	30	0.36	22.09	0.0086	26%
Node 4						
1kHz	0.072	38.6	0.072	37.35	0.00067	3%
5kHz	0.082	42.1	0.079	50.17	0.0028	19%
9kHz	0.231	-11.8	0.22	-17.7	0.0027	50%
Circuit B						
Node 2						
1kHz	0.044	28.7	0.044	28.10	0.00025	2%
5kHz	0.044	-35.7	0.052	-53.97	0.0081	51%
9kHz	0.156	69.7	0.16	63.62	0.0068	9%
Node 3						
1kHz	0.084	23.3	0.083	22.3	0.00022	4%
5kHz	0.055	62.4	0.053	86.67	0.0015	39%
9kHz	0.22	24.4	0.21	20.35	0.00022	17%
Node 4						
1kHz	0.086	21.2	0.086	20.4605	0.00078	3%
5kHz	0.18	-11.8	0.20	-12.16	0.026	3%
9kHz	0.09	-113.8	0.075	-122.05	0.014	7%

Table 3: Comparison of measurements and calculations

Results of measurements are in general similar to results of nodal analysis. In most cases at least first significant digit of amplitude is the same in both results and phase shift is within few percent of error, but with increasing frequency calculations often become less precise. For some applications first significant digit precision can be not satisfying, but roughly both results are comparable. In order to verify how precise nodal analysis is, more data and more precise measuring equipment are needed

7 Summary

Nodal analysis broadly speaking is good method to evaluate values in a circuit, really simple method for circuits with many branches. But in terms of precision as stated in Section 6 for some applications evaluation can be not good enough.

Appendix

GITHUB repository

A Source code circuit A

```
import numpy as np
from tabulate import tabulate

def get_V(f):
    w=2*np.pi*f
    Zr=1e3 #Zr1 == Zr2
    Zc1=1/(complex(0,100e-9)*w)
    Zc2=1/(complex(0,47e-9)*w)
    Zc3=1/(complex(0,22e-9)*w)
    Zl=complex(0,10e-3)*w
    VS = 0.192
    G3 = 1/Zr + 1/Zc2 + 1/Zl
    G4 = 1/Zc3 + 1/(Zc1+Zr) + 1/Zr
    G34 = -1/Zr
    I3 = VS/Zc2
    I4 = VS/(Zr+Zc1)
    G = np.array([[G3, G34],
                  [G34, G4]])
    I = np.array([[I3],
                  [I4]])
    V = np.matmul(np.linalg.inv(G), I)
    VP = np.angle(V, True)
    VA = np.abs(V)
    # V2 = ((VS-V[1])*Zc1)/(Zr + Zc1)
    IC12 = (VS - V[0])/Zc2
    # VA = np.insert(VA, 0,np.abs(V2), axis=0)
    # VP = np.insert(VP, 0,np.angle(V2,True), axis=0)
    VA = np.append(VA, np.abs(IC12))
    VP = np.append(VP, np.angle(IC12,True))
    headers = [str(f)+' Hz' , 'Magnitude', 'Phase']
    first_col = ['Node 3', 'Node 4', 'I of C_12']
    table1 = np.column_stack((first_col, VA, VP))
    print(tabulate(table1, headers=headers, tablefmt="latex"))
    # print (VA,'\n', VP)

get_V(1e3)
get_V(5e3)
get_V(9e3)
```

B Source code circuit B

```
import numpy as np
from tabulate import tabulate

def get_V(f):
    w=2*np.pi*f
    Zr=1e3
    Zc1=1/(complex(0,47e-9)*w)
    Zc2=1/(complex(0,100e-9)*w)
    Zc3=1/(complex(0,100e-9)*w)
    Zl=complex(0,10e-3)*w
    V = 0.192

    G2 = 2/Zr + 1/Zc1
    G3 = 1/Zc1 + 1/Zc2 + 1/Zl
    G4 = 1/Zc3 + 1/Zl + 1/Zr
    G23 = -1/Zc1
    G24 = -1/Zr
    G34 = -1/Zl

    I1 = 0
    I2 = V / (Zc2)
    I3 = 0
    G = np.array([[G2, G23, G24],
                  [G23, G3, G34],
                  [G24, G34, G4]])
    I = np.array([[I1],
                  [I2],
                  [I3]])

    V = np.matmul(np.linalg.inv(G), I)
    VP = np.angle(V, True)
    VA = np.abs(V)
    IR22 = (V[0]-V[2])/Zr
    VA = np.append(VA, np.abs(IR22))
    VP = np.append(VP, np.angle(IR22, True))
    headers = [str(f)+' Hz', 'Magnitude', 'Phase']
    first_col = ['Node 2', 'Node 3', 'Node 4', 'I of R_22']
    table1 = np.column_stack((first_col, VA, VP))
    print(tabulate(table1, headers=headers,tablefmt='latex'))

get_V(1e3)
get_V(5e3)
get_V(9e3)
```