Circuit Theory II Laboratory

FOURIER SERIES (HARMONIC ANALYSIS)

Aim of the exercise

The purpose of this exercise is to experimentally familiarize with the Fourier series - an operation that allows to represent any real periodic signal by the sum of sinusoidal signals. The aim of the exercise is achieved by measuring the signals generated by the independent function generator with and vector spectrum analyzer.

Introduction

If the periodic function *s*(*t*) of period *T* satisfies the so-called Dirichlet conditions (in practice, every real electrical signal meets such assumption), then it can be represented as a sum of sinusoidal waveforms, the so-called Fourier series:

$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos(n\omega t) + b_n \sin(n\omega t)) \tag{1}$$

where:

$$\omega = \frac{2\pi}{T} \tag{2}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\cos(n\omega t)dt, n = 0,1,2, \dots \tag{3}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\sin(n\omega t)dt, n = 1,2,3, \dots \tag{4}$$

The sum of the cosine and the sine (1) can be more conveniently represented by one trigonometric function, as a result of which we get:

$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n cos(n\omega t + \varphi_n) \tag{5}$$

where:

$$A_n = \sqrt{a_n^2 + b_n^2} \tag{6}$$

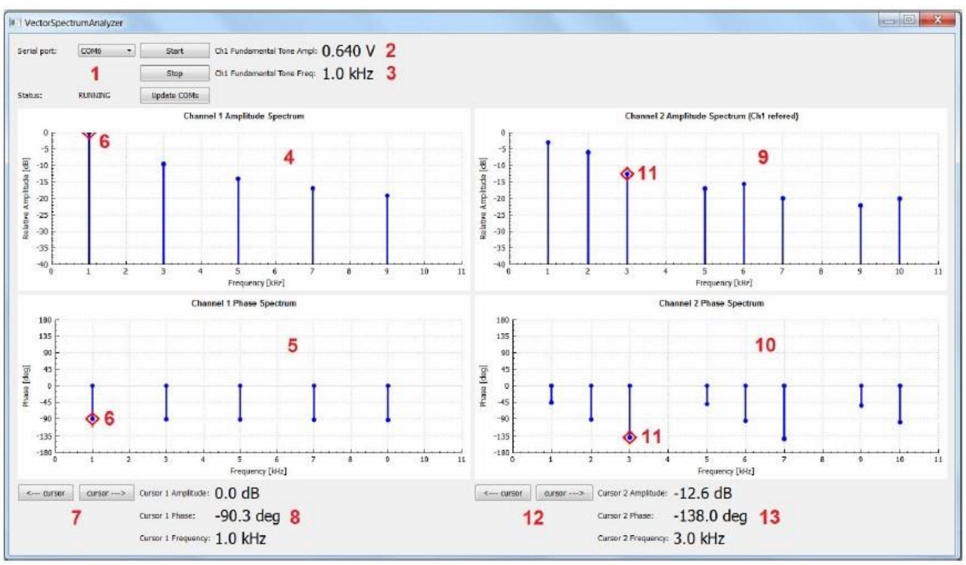
$$\varphi_n = \arg(a_n - jb_n)$$

Vector Spectrum Analyzer

The spectrum analyzer is a device used to present the frequency spectrum (components of sinusoidal waveforms with different frequencies) of the considered signal varying in time. Using a spectrum analyzer, both periodic and non-periodic waveforms can be examined - in the case of non-periodic signals, the interpretation of spectrum analyzer results should be based not on Fourier series, but on *Fourier transform*.

In these exercise, you will be dealing with the harmonic analysis of periodic waveforms only. For this purpose, a multifunctional measurement module operating (the evaluation board) in the vector spectrum analyzer mode will be used. This mode is operated by the ***VectorSpectrumAnalyzer*** application. The measurement module operating in the vector spectrum analyzer mode finds the frequency of the periodic signal under test and calculates its first 10 harmonic components (fourier series coefficients). The device gives both the amplitude A_n of the nth harmonic component and its phase ϕ_n. The module has the option of two-channel operation, but the signal fed to the *INPUT 2* must have the same frequency as the signal fed to the *INPUT 1*. The parameters of the signal from the second measuring channel are related to the signal parameters of the first channel, i.e. the signal on the first measuring channel serves reference waveform.

The interface of the *VectorSpectrumAnalyzer* application with the marked components is presented below:



1- The control panel allows you to start and end the measurement.
2- The amplitude of the fundamental tone (A_1 from equation (5)) of the signal applied to the first channel (*INPUT* 1).
3- Frequency (1/T) of the signal applied to the first channel (*INPUT* 1).
4- Amplitude spectrum (A_n, n=1.2,…10) of the signal fed to the first channel (*INPUT* 1), given in decibels (logarithmic scale)
5- Phase spectrum (ϕ_n, n=1.2,…10) of the signal fed to the first channel (*INPUT* 1).
6- Cursor of the spectrum of the signal given to the first channel (*INPUT* 1).
7- Adjusts the cursor position of the first channel.
8- Readout of the cursor parameters of the first channel.

9- Amplitude spectrum (A_n, n=1.2,…10) of the signal fed to the second channel (*INPUT* 2), given in decibels (logarithmic scale), normalized to the amplitude of the fundamental tone of the signal from the first channel
10- Phase spectrum (ϕ_n, n=1.2,…10) of the signal fed to the second channel (*INPUT* 2).
11- Cursor of the spectrum of the signal fed to the second channel (*INPUT* 2).
12- Adjust the second channel cursor position
13- Readout of the cursor parameters of the second channel.

Since the amplitudes of further harmonics for most of the waveforms encountered quickly decrease, to values of up to one-hundredth of the amplitude of the fundamental tone, the amplitude spectrum is expressed in decibels. Such interpretation is useful to observe values differing from each other by orders of magnitude on one graph, which is not is possible for graphs with a simple linear scale. The decibel is a logarithmic unit of measure used to compare two quantities. When comparing power values, the decibel is defined as:

$$P[dB] = 10\log_{10}\left({}^{P}\!/_{P_0}\right)$$

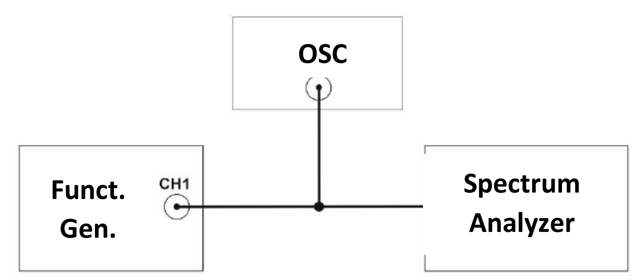
where P_0 is the reference value. However, if we compare the amplitude values (voltages), the decibel is defined as:

$$A[dB] = 20\log_{10}\left({}^{A}\!/_{A_0}\right)$$

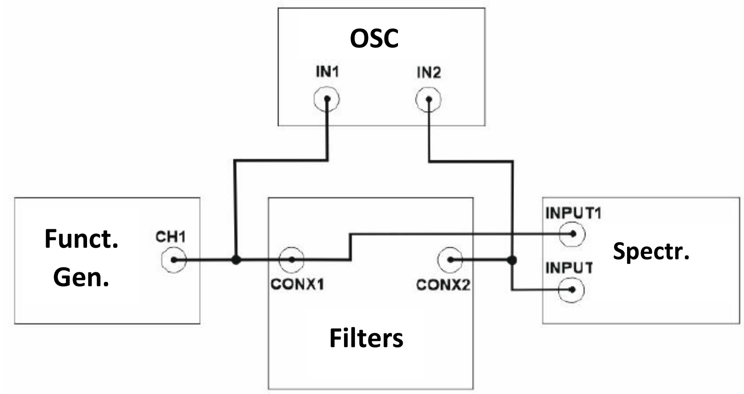
where A_0 is the reference value. To determine the phase of the waveform, it is necessary to specify a reference point in the time domain. The discussed vector spectrum analyzer assumes the moment t = 0 in the middle of the rising edge of the waveform given to the first measurement channel (*INPUT* 1) - just like the oscilloscope trigger circuits ("trigger") in the default state. The factor $a_0/2$ in Fourier expansion (1), (5) is the average value of the signal (DC offset). Since the input circuits of the evaluation board remove the DC component of the signals under test, the $a_0/2$ factor is not displayed in the *VectorSpectrumAnalyzer* application.

Measurements Methodology

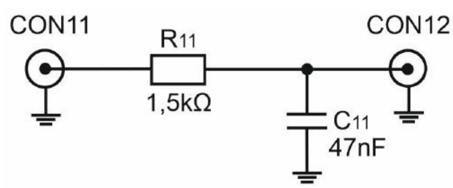
The first part of the exercise consists in the simultaneous observation on the oscilloscope and vector spectrum analyzer of periodic signals of different shapes and the same frequency of 1 kHz and the same amplitude of 1 V_{pp} (circuit assembled according to the diagram below). Record all signal parameters reported by the vector spectrum analyzer.



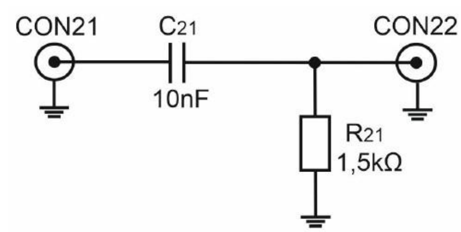
The second part of the exercise consists in filtering periodic non-sinusoidal waveforms using RC circuits (circuit assembled according to the concept schematic shown below). The signals from the input and output of the filter should be measured by connecting the input signal to the first channel of the spectrum analyzer, and the output signal to the second channel.



Measurements should be made for two waveforms (list at the end of the manual) and for two RC circuits - a total of 4 measurements.



Circuit no. 1



Circuit no. 2

Report requirements

The report should include a comparison of the obtained measurement results with theoretical calculations and comments on the obtained measurement results and calculations. It is recommended to prepare a script that will allow you to determine the amplitude and phase spectrum of the signal being the response of the tested filter for the given input signal.

List of configurations to measure

Section	Meaured Signals	Measured RC Circuits Response
I and V	• Square having Duty Cycle 75% • Sinus • Triangle having symmetry ratio 50% • Triangle having symmetry ratio 0%	• Square having Duty Cycle 50%, circuit 1 and circuit 2. • Triangle having symmetry ratio 25%, circuit 1 i circuit 2
II	• Sinus • Square having Duty Cycle 40% • Square having Duty Cycle 33,(3)% • Triangle having symmetry ratio 100%	• Square having Duty Cycle 50%, circuit 1 and circuit 2 • Triangle having symmetry ratio 75%, circuit 1 i circuit 2
III	• Sinus • Square having Duty Cycle 50% • Square having Duty Cycle 25% • Triangle having symmetry ratio 50%	• Square having Duty Cycle 50%, circuit 1 and circuit 2 • Triangle having symmetry ratio 40%, circuit 1 and circuit 2
IV	• Sinus • Square having Duty Cycle 50% • Square having Duty Cycle 40% • Triangle having symmetry ratio 100%	• Square having Duty Cycle 50%, circuit 1 and circuit 2 • Triangle having symmetry ratio 60%, circuit 1 and circuit 2