Two-port Networks
Laboratory V

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1 Goal of the exercise

The aim of this exercise is to familiarize with experimental methods of determining two-port network parameters by measuring output and input voltages and currents, and then compare theoretical and experimental results

2 Two-port networks

Two-port network can be regarded as "black box" with its properties specified by a characteristic matrix. It allows us to simplify large circuits with those "black boxes" and move to higher level of abstraction, instead of using simple passive element now we can use complex circuits but seeing them as simple 2×2 matrix similar to one in eq. (1).

$$\mathbf{Z}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{1}$$

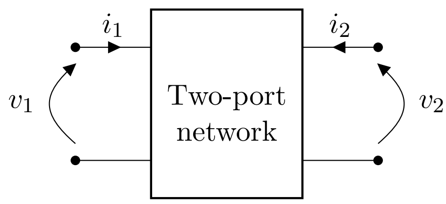


Figure 1: Two-port diagram

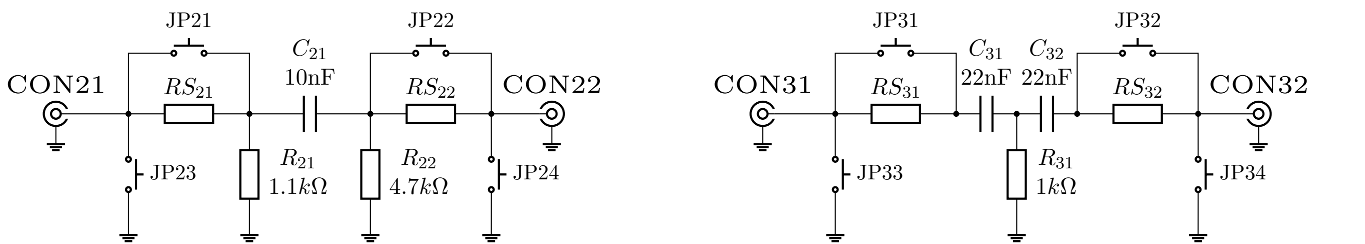
Characteristic matrix can have form of impedance **Z**, admittance **Y** or chain(ABCD) matrix and many more forms which we are not going to use during this exercise. impedance **Z** matrix can be easily transformed into admittance **Y** and chain(ABCD) matrix using following formulas.

$$\mathbf{Y} = \mathbf{Z}^{-1} = \begin{bmatrix} \frac{z_{11}}{\det \mathbf{Z}} & \frac{-z_{12}}{\det \mathbf{Z}} \\ \frac{-z_{21}}{\det \mathbf{Z}} & \frac{z_{22}}{\det \mathbf{Z}} \end{bmatrix} \tag{2}$$

$$\mathbf{Z} = \mathbf{Y}^{-1} = \begin{bmatrix} \frac{y_{11}}{\det \mathbf{Y}} & \frac{-y_{12}}{\det \mathbf{Y}} \\ \frac{-y_{21}}{\det \mathbf{Y}} & \frac{y_{22}}{\det \mathbf{Y}} \end{bmatrix} \tag{3}$$

$$\mathbf{A} = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\det \mathbf{Z}}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} -\frac{y_{11}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\det \mathbf{Y}}{y_{21}} & -\frac{y_{22}}{y_{21}} \end{bmatrix} \tag{4}$$

3 Course of measurements



(a) Circuit no. 2 (b) Circuit no. 3

Figure 2: Measured circuits

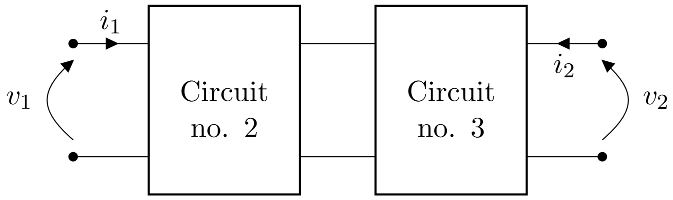


Figure 3: Chain configuration

4 Theoretical calculations

First step in our analytical solution is simplifying Circuits by removing connection used to measured voltages and currents.

Some characteristics matrices of two-port network can be solved using mesh or nodal analysis by inspection, and both of our circuit can be solved using one of those methods. We solved circuit no. 2 using nodal analysis and circuit no. 3 using mesh analysis,

4.1 Circuit no. 2

In order to solve circuit no. 2 we added current sources i_1 and i_2 between each port, and marked nodes v_1 and v_2 then following rules of nodal analysis by inspection we constructed $\mathbf{Yv} = \mathbf{i}$ matrix equation where $\mathbf{Y}$ is two-port characteristics in admittance form.

$$\begin{bmatrix} \frac{1}{R_{21}} + j\omega C_{21} & -j\omega C_{21} \\ -j\omega C_{21} & \frac{1}{R_{22}} + j\omega C_{21} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{5}$$

Knowing admittance characteristics we can transform it into impedance and chain characteristics using formulas (3) and (4)

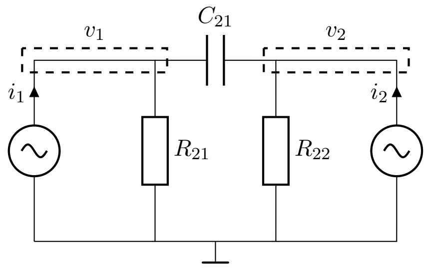


Figure 4: Simplified circuit no. 2

$$Z \tag{6}$$

$$A \tag{7}$$

4.2 Circuit no. 3

In circuit no. 3 we added voltage sources v_1 and v_2 between each port, and marked loops i_1 and i_2 in clockwise and counter-clockwise directions respectively, then according to rules of mesh analysis by inspection we constructed $\mathbf{Zi} = \mathbf{v}$ matrix equation, where $\mathbf{Z}$ is two-port characteristics in impedance form.

$$\begin{bmatrix} j\omega C_{31} + +R_{31} & R_{31} \\ [4pt]R_{31} & j\omega C_{32} + R_{31} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{8}$$

Knowing impedance characteristics we can transform it into impedance and chain characteristics using formulas (2) and (4)

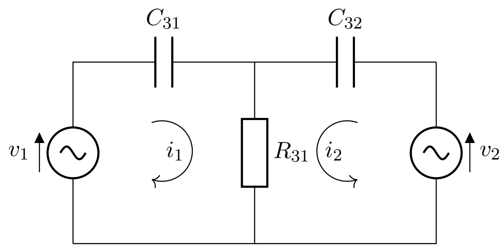


Figure 5: Simplified circuit no. 3

$$Y \tag{9}$$

$$A \tag{10}$$

4.3 Chain connection

5 Comparison

6 Conclusions