Fourier Series
Laboratory II

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1 Aim of the exercise

The purpose of exercise was to experimentally familiarize with the Fourier series - an operation that allows to represent any real periodic signal by the sum of sinusoidal signals. We measured the signals generated by the independent function generator with a vector spectrum analyzer.

2 What is Fourier Series

A Fourier series is an expansion of a periodic function $f(x)$ in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonal relationships of the sine and cosine functions. The computation and study of Fourier series is known as harmonic analysis and is extremely useful to break up an arbitrary periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical

3 Course of measurements

During our measurements we skipped first part of exercise and measured all signals in configuration for second part (Fig. 1).

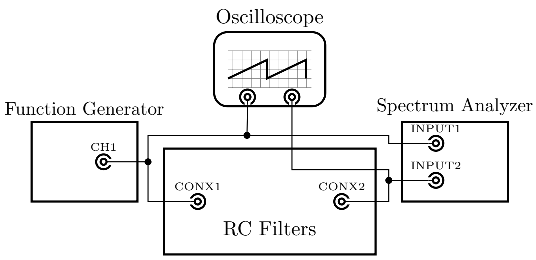


Figure 1: Measurements configuration

After connecting Function Generator, Spectrum Analyzer, Oscilloscope and board with RC filters according to above configuration (Fig. 1) we set Function Generator to Amplitude to $V_{pp} = 5$V and Frequency 1 kHz. With everything set up we proceeded with exercise and recording output of Oscilloscope and Spectrum Analyzer for signals:

- Sin wave
- Square wave 50% duty cycle
- Square wave 25% duty cycle
- Triangle wave 50% symmetry ratio
- Triangle wave 40% symmetry ratio

For Square wave 50% duty cycle and Triangle wave 40% symmetry ratio we also recorded Response for both RC Circuits (Fig. 2)

(a) Circuit A (b) Circuit B

Figure 2: Measured RC Circuits

4 Method for calculating Fourier Series coefficients

All calculations are made using Matlab with Signal Processing Toolbox, source code can be found in Appendix A

First step in our calculation is generating signals that we measured during exercise. Pure signals were obtained with built-in functions of Signal Processing Toolbox and Filtered signals were evaluated using NI Multisim SPICE software.

Next step is finding first 10 coefficients of Fourier series for a signal which we calculated by taking inner product of signal with basis vector. In our calculations we used $\sin nt$, $\cos nt$ and 1 as our basis vectors.

$$a_0 = \langle s(t), 1 \rangle \tag{1}$$

$$a_n = \langle s(t), \cos n\omega t \rangle \tag{2}$$

$$b_n = \langle s(t), \sin n\omega t \rangle \tag{3}$$

During exercise we recorded Fourier series of a signal in Amplitude-Phase form so we need to convert a_n and b_n coefficients into amplitude A_n and phase φ_n

$$A_n = \sqrt{a_n^2 + b_n^2} \tag{4}$$

$$\varphi_n = \arg(a_n - jb_n) \tag{5}$$

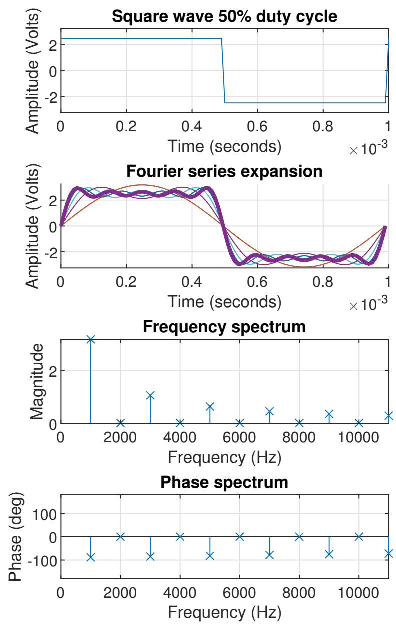


Figure 3: example plot

Knowing first 10 coefficients we can plot approximation of a signal using one of following formulas. Example of approximation of square wave is on the second plot in Figure 3.

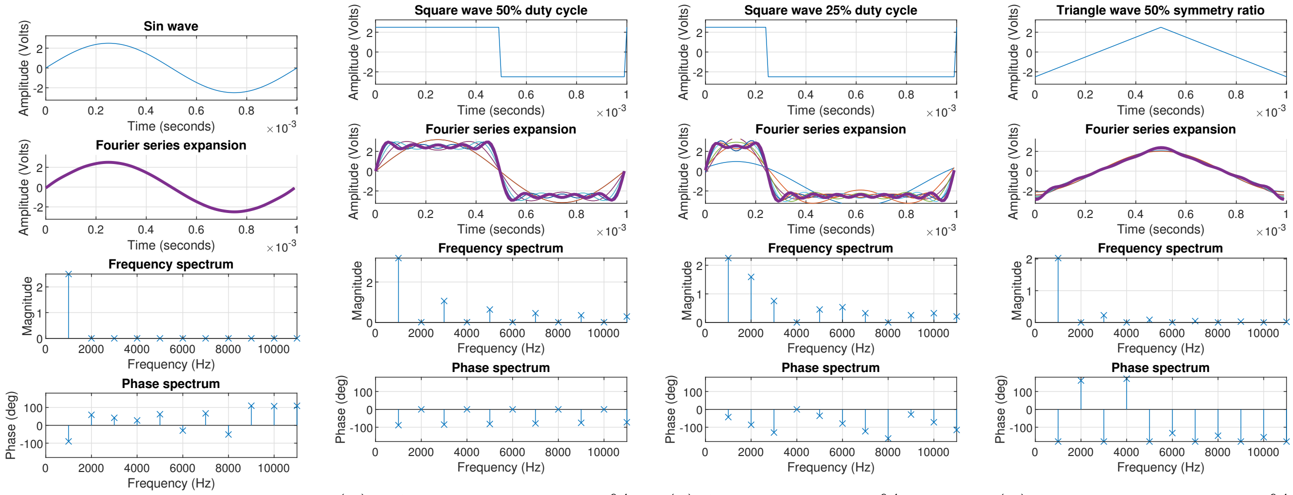
$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{10} (a_n \cos(n\omega t) + b_n \sin(n\omega t)) \tag{6}$$

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{10} (A_n \cos(n\omega t + \varphi_n)) \tag{7}$$

For third and fourth plot in Figure 3 we are calculating Discrete Fourier transform for 10 oscillations of a signal using Fast Fourier transform algorithm from Signal Processing Toolbox and then plotting magnitude and argument of FT both in decibel scale on y axis.

5 Comparison

5.1 Pure signals



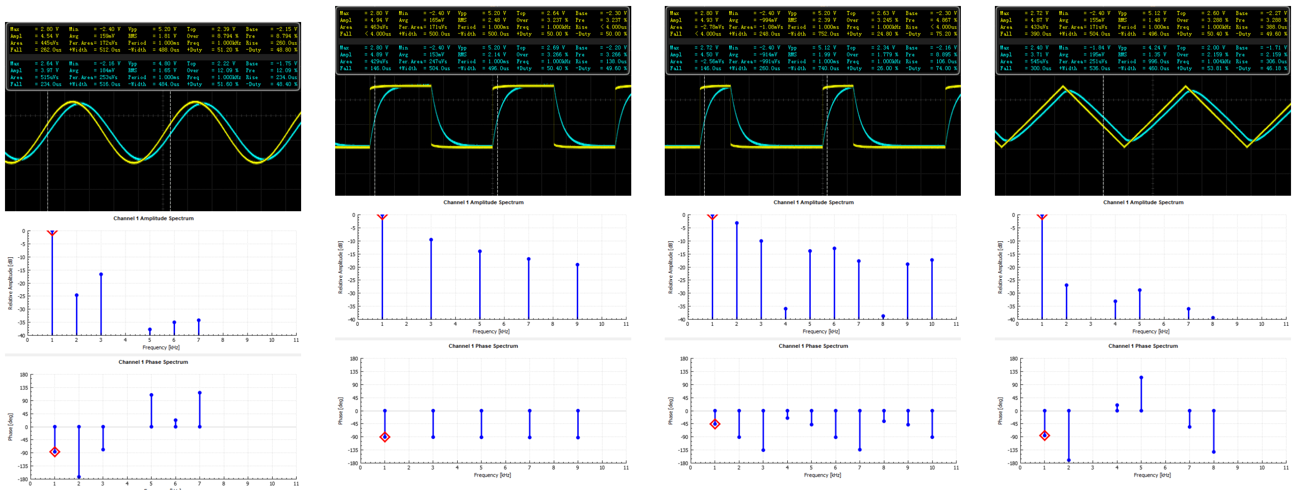
(a) Sin wave

(b) Square wave 50% duty cycle

(c) Square wave 25% duty cycle

(d) Triangle wave 50% symmetry ratio

Figure 4: Calculated signals



(a) Sin wave

(b) Square wave 50% duty cycle

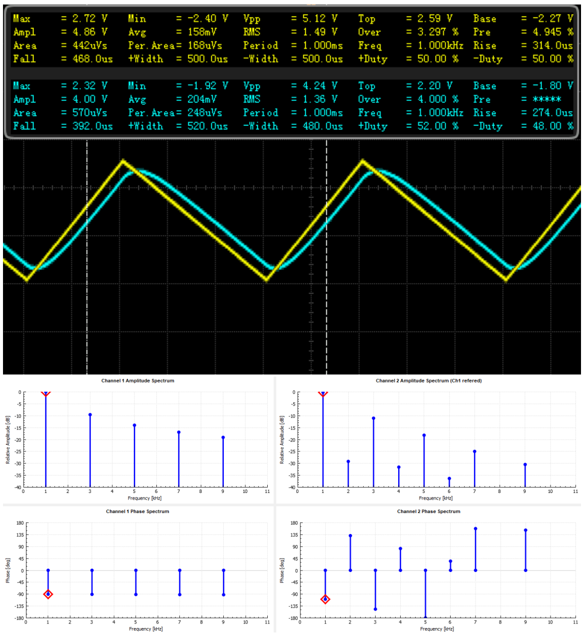
(c) Square wave 25% duty cycle

(d) Triangle wave 50% symmetry ratio

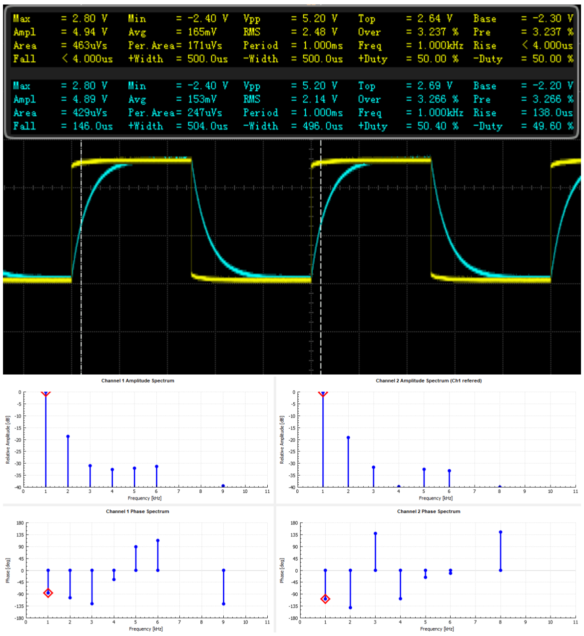
Figure 5: Measured signals

5.2 RC filters

5.2.1 Low-pass filter

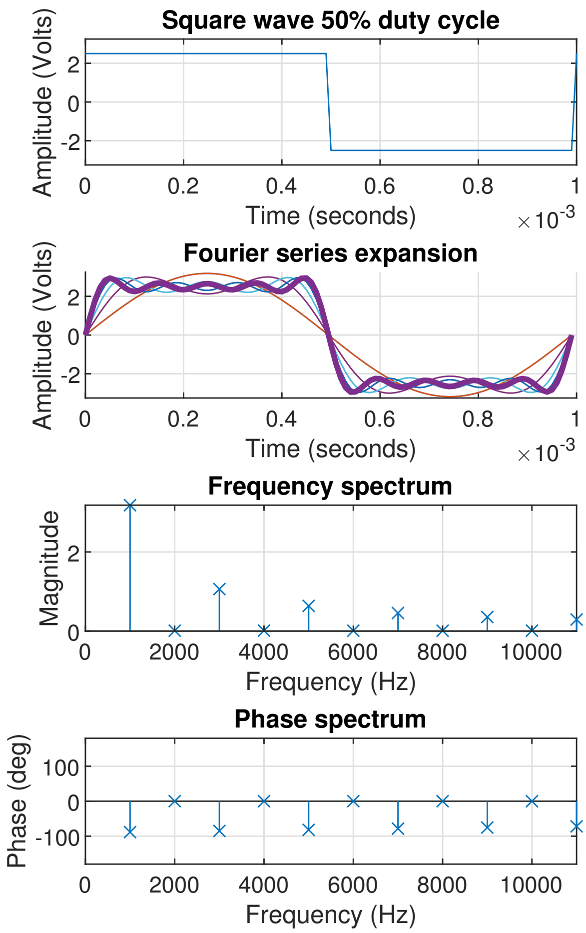


(a) Sin wave

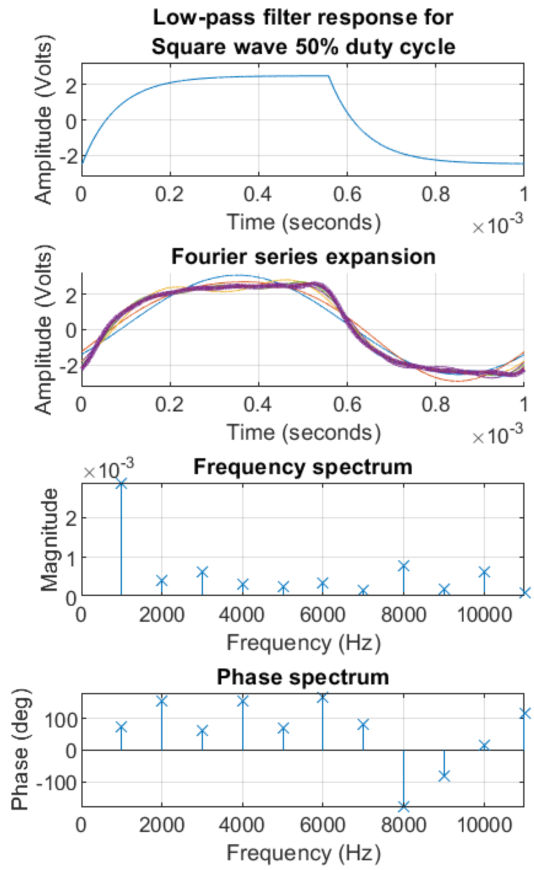


(b) Square wave 50% duty cycle

Figure 6: Measured signals

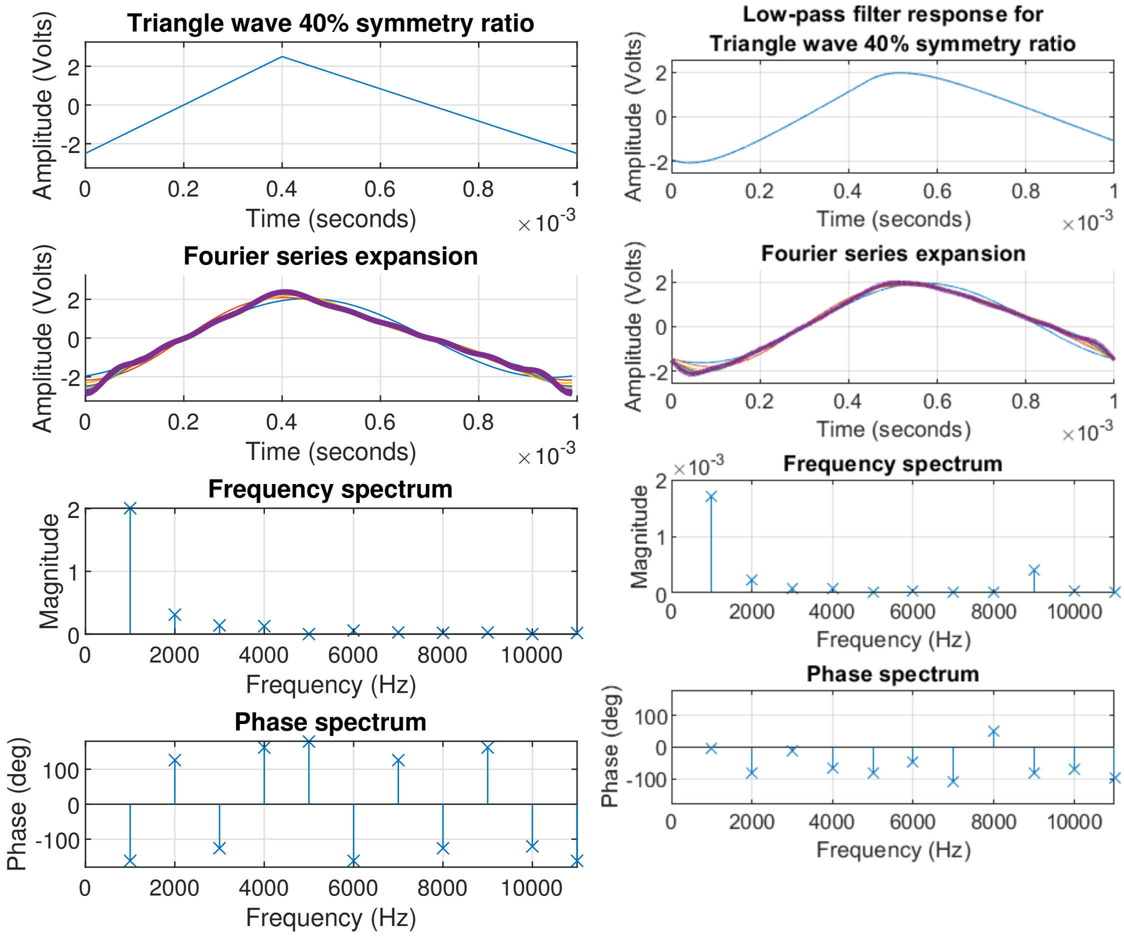


(a) Pure signal



(b) Filtered signal

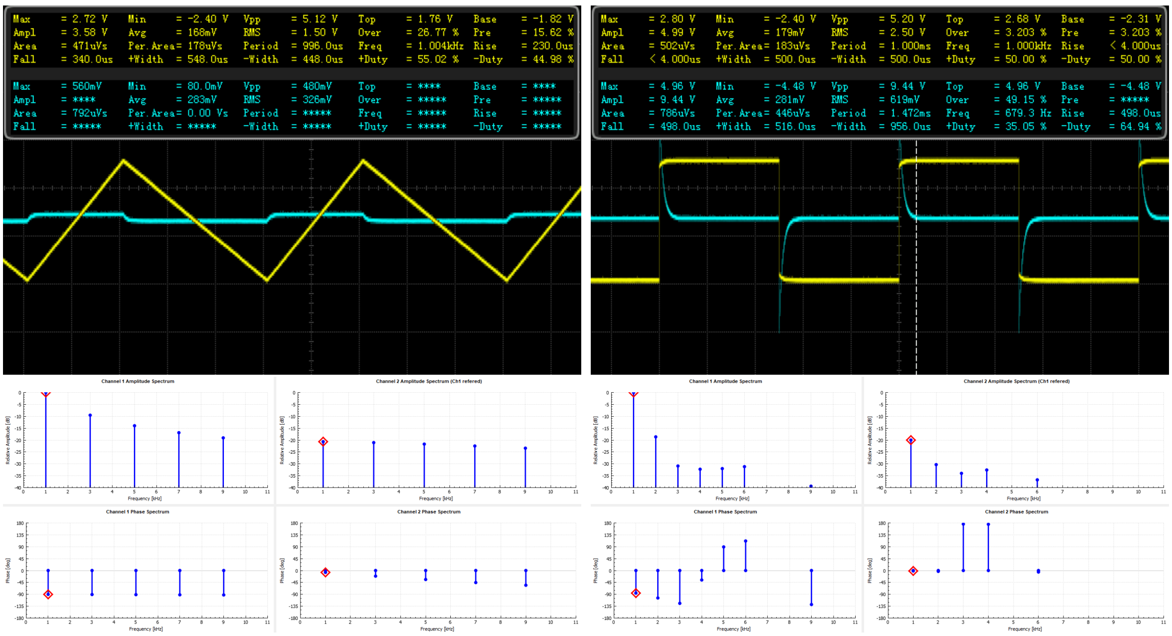
Figure 7: Low-pass filter response for Square wave 50% duty cycle



(a) Pure signal (b) Pure signal

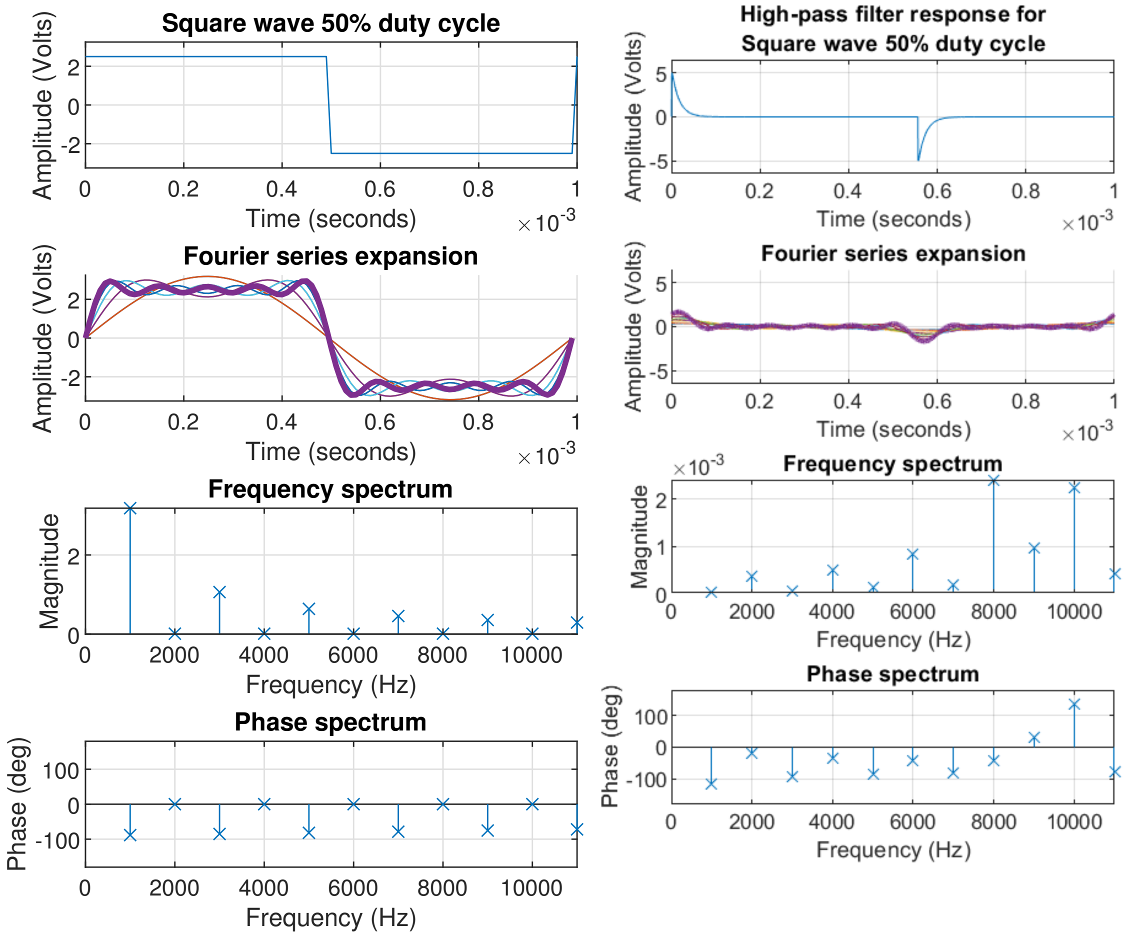
Figure 8: Low-pass filter response for Triangle wave 40% symmetry ratio

5.2.2 High-pass filter



(a) Square wave 25% duty cycle (b) Triangle wave 50% symmetry ratio

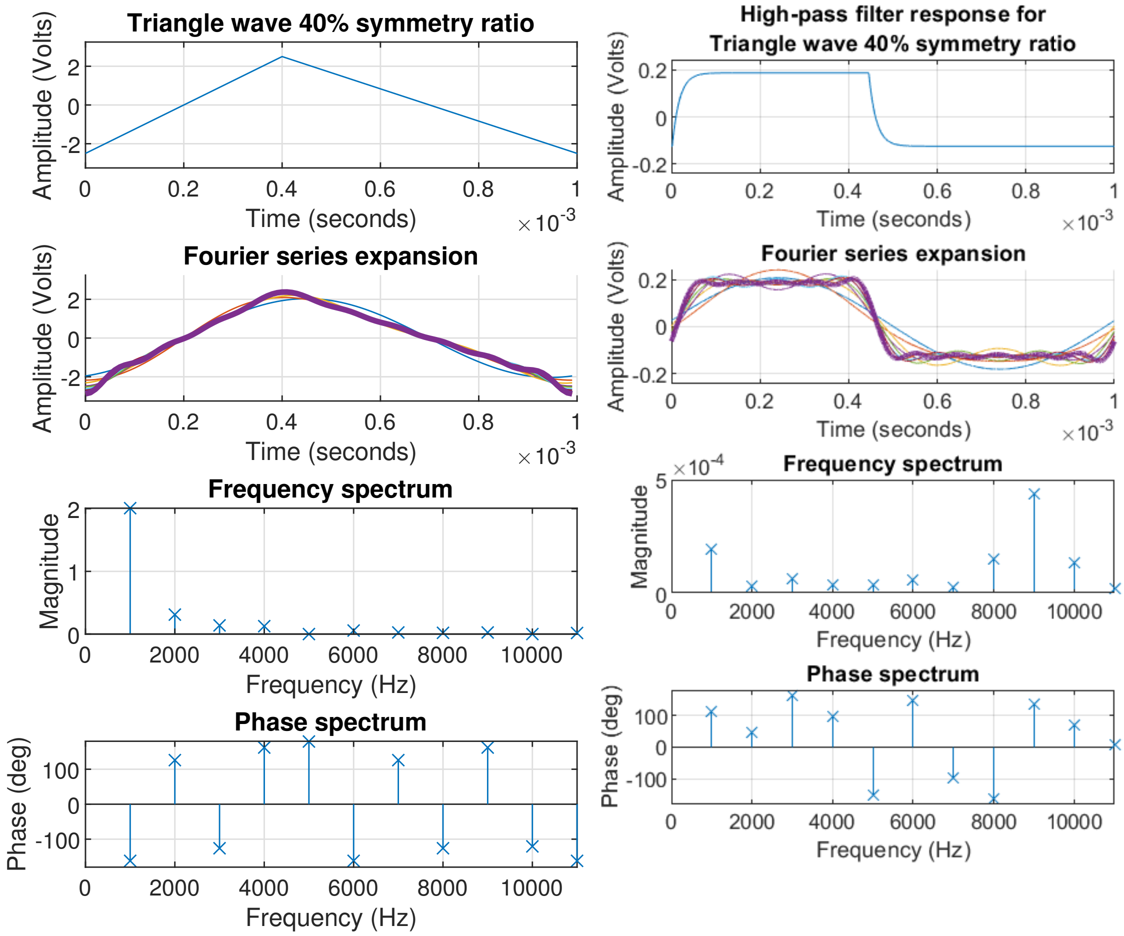
Figure 9: Measured signals



(a) Pure signal

(b) Filtered signal

Figure 10: High-pass filter response for Square wave 50% duty cycle



(a) Pure signal

(b) Filtered signal

Figure 11: High-pass filter response for Triangle wave 40% symmetry ratio

6 Summary

A Source Code

GITHUB repository

lab2.m

```
clear all; close all; clc;
[Square50_integrator, Square50_differentiatior, Triangle40_integrator, Triangle40_differentiat

fs = 100000;
duration = 0.01;
N = fs * duration;
t = 0:1/fs:duration-1/fs;
f = 1000;
a = 2.119;
phi = 0;

a = 2.5;
phi = 0;

s = a * sin(2*pi*t*f + phi);
tr50 = a * sawtooth(2*pi*t*f + phi, 0.5);
tr40 = a * sawtooth(2*pi*t*f + phi, 0.4);
sq50 = a * square(2*pi*t*f + phi, 50);
sq25 = a * square(2*pi*t*f + phi, 25);

Final_Plots(s, 'Sin wave', 'sin');
Final_Plots(tr50, 'Triangle wave 50% symmetry ratio', 'tri50');
Final_Plots(tr40, 'Triangle wave 40% symmetry ratio', 'tri40');
Final_Plots(sq50, 'Square wave 50% duty cycle', 'sqr50');
Final_Plots(sq25, 'Square wave 25% duty cycle', 'sqr25');
Final_Plots(Square50_integrator,['Low-pass filter response for' newline 'Square wave 50% duty
Final_Plots(Square50_differentiatior,['High-pass filter response for' newline 'Square wave 50%
Final_Plots(Triangle40_integrator,['Low-pass filter response for' newline 'Triangle wave 40% s
Final_Plots(Triangle40_differentiatior,['High-pass filter response for' newline 'Triangle wave

%% converted to Final_Plot function
% figure(123);
% %sin
% subplot(4,3,1)
% plot(t,s)
% xlabel('Time (seconds)');
% xlim([0,0.002])
% ylabel('Amplitude (Volts)');
% ylim([-3,3])
% title('Time-domain sinus');
%
% %triangle with 50% duty cycle
% subplot(4,3,2)
% plot(t,tr)
% xlabel('Time (seconds)');
% xlim([0,0.002])
% ylabel('Amplitude (Volts)');
% ylim([-3,3])
```

```
% title('Time-domain triangle');
%
% %square with 50% duty cycle
% subplot(4,3,3)
% plot(t,sq)
% xlabel('Time (seconds)');
% xlim([0,0.002])
% ylabel('Amplitude (Volts)');
% ylim([-3,3])
% title('Time-domain square');
%
% %fft sin
% s_fft = fft(s);
% %fft triangle
% tr_fft = fft(tr);
% %fft square
% sq_fft = fft(sq);
%
% s_oneSide = s_fft(1:N/2);
% frequencies = fs * (0:N/2-1) / N;
% S_magnitude = abs(s_oneSide) / (N/2);
% S_phase = angle(s_oneSide) * 180/pi;
%
% tr_oneSide = tr_fft(1:N/2);
% Tr_magnitude = abs(tr_oneSide) / (N/2);
% Tr_phase = angle(tr_oneSide) * 180/pi;
%
% sq_oneSide = sq_fft(1:N/2);
% Sq_magnitude = abs(sq_oneSide) / (N/2);
% Sq_phase = angle(sq_oneSide) * 180/pi;
%
% freq_of_interest = [1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000, 11000];
% indices = zeros(size(freq_of_interest));
% for i = 1:length(freq_of_interest)
%     [~, indices(i)] = min(abs(frequencies - freq_of_interest(i)));
% end
%
% subplot(4,3,4)
% plot(frequencies, db(S_magnitude));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylim([-40, 10]);
% ylabel('Magnitude');
% title('Frequency spectrum');
%
% subplot(4,3,5)
% plot(frequencies, db(Tr_magnitude));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylabel('Magnitude');
% ylim([-40, 10]);
% title('Frequency spectrum');
%
```

```
% subplot(4,3,6)
% plot(frequencies, db(Sq_magnitude));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylabel('Magnitude');
% ylim([-20, 10]);
% title('Frequency spectrum');
%
%
% subplot(4,3,7)
% stem(frequencies(indices), S_magnitude(indices));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylabel('Magnitude');
% title('Frequency spectrum');
%
% subplot(4,3,8)
% stem(frequencies(indices), Tr_magnitude(indices));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylabel('Magnitude');
% title('Frequency spectrum');
%
% subplot(4,3,9)
% stem(frequencies(indices), Sq_magnitude(indices));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylabel('Magnitude');
% title('Frequency spectrum');
%
%
% subplot(4,3,10)
% stem(frequencies(indices), S_phase(indices));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylabel('Phase (deg)');
% ylim([-180, 180]);
% title('Phase spectrum');
%
% subplot(4,3,11)
% stem(frequencies(indices), Tr_phase(indices));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylabel('Phase (deg)');
% ylim([-200, 200]);
% title('Phase spectrum');
%
% subplot(4,3,12)
% stem(frequencies(indices), Sq_phase(indices));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylabel('Phase (deg)');
% ylim([-200, 200]);
```

```
% title('Phase spectrum');
```

Final Plots.m

```
function [] = Final_Plots(f, name, filename)
frequency = 1000;
duration = 10/frequency;
N = size(f,2);
sampling_frequency = N/duration;
t = 0:1/sampling_frequency:duration-1/sampling_frequency;
fig = figure('Name', name);
fig.Position(3:4) = [300 500];

%% signal
subplot(4,1,1);
plot(t, f);
xlabel('Time (seconds)');
xlim([0,0.001])
ylabel('Amplitude (Volts)');
ylim([-max(f)-(max(f)*0.3),max(f)+(max(f)*0.3)])
title(join(['', name]));
grid on;

%% fourier series
X = t(1:size(t,2)/10);
% f_short = f(1:size(x,2));
L = 1/frequency;
dx = 2*L/(N/10-1);
x = -L:dx:L;
f_short = f(1:size(x,2));
subplot(4,1,2); hold on;
% plot(x, f_short, 'r*');
A0 = (1/L)*sum(f_short.*ones(size(x)))*dx;
fFS = A0/2;
for k=1:10
    A(k) = (1/L)*sum(f_short.*cos(pi*k*x*frequency))*dx;
    B(k) = (1/L)*sum(f_short.*sin(pi*k*x*frequency))*dx;
    fFS = fFS + A(k)*cos(k*pi*x*frequency) + B(k)*sin(k*pi*x*frequency);
    plot(X, fFS);
    pause(.05);
end
plot(X, fFS,'LineWidth',2);
hold off;

xlabel('Time (seconds)');
xlim([0,0.001])
ylabel('Amplitude (Volts)');
ylim([-max(f)-(max(f)*0.3),max(f)+(max(f)*0.3)])
title('Fourier series expansion');
grid on;

%% fft amplitude
% plot
f_fft = fft(f);
```

```
f_oneSide = f_fft(1:N/2);
frequencies = sampling_frequency * (0:N/2-1) / N;
f_magnitude = abs(f_oneSide) / (N/2);
f_phase = angle(f_oneSide) * 180/pi;

freq_of_interest = [1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000, 11000];
indices = zeros(size(freq_of_interest));
for i = 1:length(freq_of_interest)
    [~, indices(i)] = min(abs(frequencies - freq_of_interest(i)));
end

% subplot(4,1,2)
% plot(frequencies, db(f_magnitude));
% xlabel('Frequency (Hz)');
% xlim([0, 11000]);
% ylim([-40, 10]);
% ylabel('Magnitude');
% title('Frequency spectrum');

%stem
subplot(4,1,3)
stem(frequencies(indices), f_magnitude(indices), 'x');
xlabel('Frequency (Hz)');
xlim([0, 11000]);
ylabel('Magnitude');
title('Frequency spectrum');
grid on;

%% fft phase
subplot(4,1,4)
stem(frequencies(indices), f_phase(indices), 'x');
xlabel('Frequency (Hz)');
xlim([0, 11000]);
ylabel('Phase (deg)');
ylim([-180, 180]);
title('Phase spectrum');
grid on;

%% saving img
print(join(['img/',filename]), '-depsc');
end
```

Import csv.m

```
function [Vout] = RC_circuit(frequency, N)
figure('Name', 'RC circuit');
Vpp = 5;
frequency = 1000;
T = 1/frequency;
R = 1.5e3;
C1 = 47e-4;
C2 = 10;
dt = T/(N-1);
```

```
t = 0:dt:T;
Vout = 0*t;
Vout(N/2+1:N) = Vout(N/2+1:N) + Vpp;
Vout(N/2+1:N) = Vout(N/2+1:N) + (-Vpp)*exp(-(1:N/2)/(R*C1));
Vout(1:N) = Vout(1:N) + (Vpp)*exp(-(1:N)/(R*C1));
Vout(1:N) = Vout(1:N) - Vpp/2;
plot(t, Vout);
pause(.1)
end
```