Two-port Networks
Laboratory V

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1 Goal of the exercise

The aim of this exercise is to familiarize with experimental methods of determining two-port network parameters by measuring output and input voltages and currents, and then compare theoretical and experimental results

2 Two-port networks

Two-port network can be regarded as "black box" with its properties specified by a characteristic matrix. It allows us to simplify large circuits with those "black boxes" and move to higher level of abstraction, instead of using simple passive element now we can use complex circuits but seeing them as simple 2×2 matrix similar to one in eq. (1).

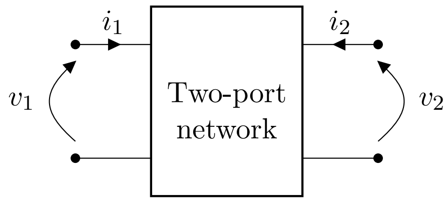


Figure 1: Two-port diagram

$$\mathbf{Z}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{1}$$

Where z_{11}, z_{12}, z_{21}, z_{22} are equal by definition

$$z_{11} = \frac{v_1}{i_1}|_{i_2=0} \quad z_{12} = \frac{v_1}{i_2}|_{i_1=0} \quad z_{21} = \frac{v_2}{i_1}|_{i_2=0} \quad z_{22} = \frac{v_2}{i_2}|_{i_1=0} \tag{2}$$

Characteristic matrix can have form of impedance **Z**, admittance **Y** or chain(ABCD) matrix and many more forms which we are not going to use during this exercise. impedance **Z** matrix can be easily transformed into admittance **Y** and chain(ABCD) matrix using following formulas.

$$\mathbf{Y} = \mathbf{Z}^{-1} = \begin{bmatrix} \frac{z_{11}}{\det \mathbf{Z}} & \frac{-z_{12}}{\det \mathbf{Z}} \\ \frac{-z_{21}}{\det \mathbf{Z}} & \frac{z_{22}}{\det \mathbf{Z}} \end{bmatrix} \tag{3}$$

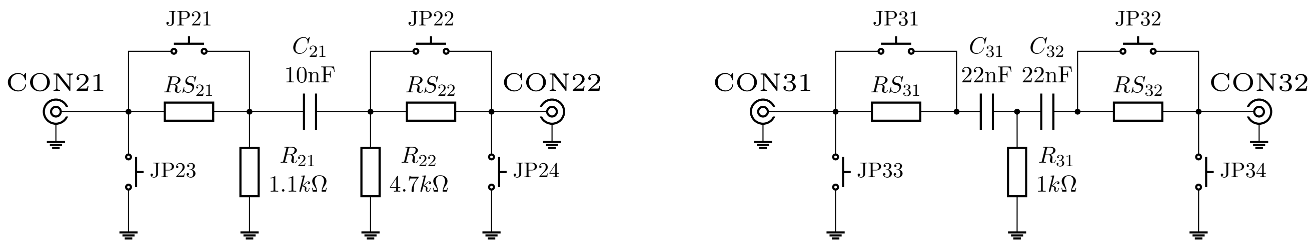
$$\mathbf{Z} = \mathbf{Y}^{-1} = \begin{bmatrix} \frac{y_{11}}{\det \mathbf{Y}} & \frac{-y_{12}}{\det \mathbf{Y}} \\ \frac{-y_{21}}{\det \mathbf{Y}} & \frac{y_{22}}{\det \mathbf{Y}} \end{bmatrix} \tag{4}$$

$$\mathbf{A} = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\det \mathbf{Z}}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} -\frac{y_{11}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\det \mathbf{Y}}{y_{21}} & -\frac{y_{22}}{y_{21}} \end{bmatrix} \tag{5}$$

Two-port networks, similar to simple passive components, can be in parallel and series connection, but also in chain configuration which is unique to two-port networks. Parallel connection can be represented by sum of admittance characteristics, series connection by sum of impedance characteristics and chain connection by multiplication of chain(ABCD) characteristics.

3 Course of measurements

Our goal in the first part of the exercise, was to familiarize with the methods of determining the self-parameters of two-port networks for the given frequencies. We have measured the output voltages and currents. By equating the currents i_1 and i_2 to zero, respectively, we were able to estimate the values of the individual elements of the impedance matrix. After experimentally determining the parameters in the **Z** matrix (impedance), we would calculate parameters of the **Y** matrix (admittance) from the formulas allowing for transformations between the respective matrices. The end of our work would be comparing the obtained results with theoretical calculations.



(a) Circuit no. 2

(b) Circuit no. 3

Figure 2: Measured circuits

In the second part of the exercise, we connected two two-port circuits (2 and 3). Having given frequencies we measured the output voltages and currents. By equating the currents i_1 and i_2 to zero, and by adding adequate resistors to our two port respectively, we were able to estimate the values of the individual elements of the impedance matrix. The final step was to determine the chain (ABCD) parameters of the resulting network.

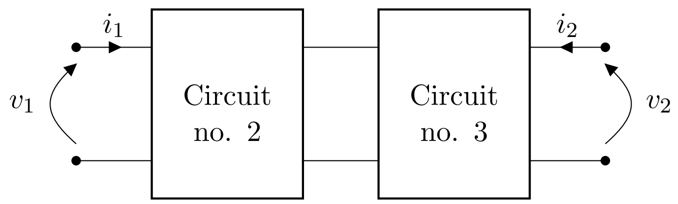


Figure 3: Chain configuration

3.1 Measurements processing

With measured voltage drop over resistor RS we can calculate current flowing in or out of two-port network. Where v_{RS} is voltage over resistor RS and R_in is internal resistance of generator.

$$i_{1/2} = \frac{v_{RS}}{RS + R_{in}} \tag{6}$$

Now following formulas (2) we can obtain impedance characteristics, and then using formulas (3), (4), (5) we can calculate admittance and chain characteristics.

4 Theoretical calculations

First step in our analytical solution is simplifying Circuits by removing connection used to measured voltages and currents.

Some characteristics matrices of two-port network can be solved using mesh or nodal analysis by inspection, and both of our circuit can be solved using one of those methods. Usually mesh or nodal method is quicker and easier way of finding characteristics than using definition, that is why we opted for this method instead of definition.

4.1 Circuit no. 2

In order to solve circuit no. 2 we added current sources i_1 and i_2 between each port, and marked nodes v_1 and v_2 then following rules of nodal analysis by inspection we constructed $\mathbf{Yv} = \mathbf{i}$ matrix equation where $\mathbf{Y}$ is two-port characteristics in admittance form.

$$\begin{bmatrix} \frac{1}{R_{21}} + j\omega C_{21} & -j\omega C_{21} \\ -j\omega C_{21} & \frac{1}{R_{22}} + j\omega C_{21} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{7}$$

Knowing admittance characteristics we can plugin values of each component and transform it into impedance and chain characteristics using formulas (4) and (5). Calculated characteristics in section. 5

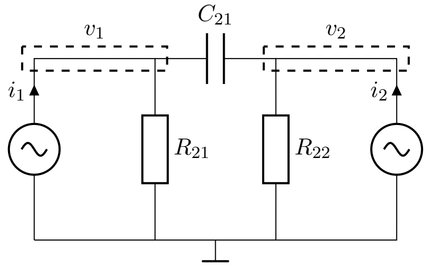


Figure 4: Simplified circuit no. 2

4.2 Circuit no. 3

In circuit no. 3 we added voltage sources v_1 and v_2 between each port, and marked loops i_1 and i_2 in clockwise and counter-clockwise directions respectively, then according to rules of mesh analysis by inspection we constructed $\mathbf{Zi} = \mathbf{v}$ matrix equation, where $\mathbf{Z}$ is two-port characteristics in impedance form.

$$\begin{bmatrix} \frac{1}{j\omega C_{31}} + R_{31} & R_{31} \\ R_{31} & \frac{1}{j\omega C_{32}} + R_{31} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{8}$$

Knowing impedance characteristics we can plugin values and transform it into admittance and chain characteristics using formulas (3) and (5). Calculated characteristics in section. 5

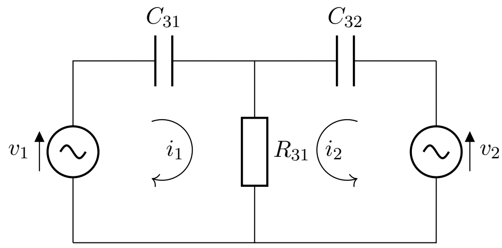


Figure 5: Simplified circuit no. 3

4.3 Chain connection

With ABCD characteristics of each circuit at 13kHz calculated we can get characteristics for chain connection of these two-ports by multiplying $\mathbf{A_2}$ with $\mathbf{A_3}$, which are ABCD characteristics for circuit no. 2 and 3 respectively.

$$\mathbf{A} = \mathbf{A_2}\mathbf{A_3} \tag{9}$$

Calculated characteristics in section. 5

5 Comparison

When comparing Calculated characteristics with measured ones it is clear that they are not exactly equal, this discrepancy between results may come from error in calculation or measurements, but general relation between values in matrices is kept in most cases.

5.1 Circuit no. 2

$$\mathbf{Y} = \begin{bmatrix} 0.0010 & 0.0004 \\ 0.0004 & 0.0005 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 1628.8 & 1466.2 \\ 1466.2 & 3366.6 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 1.1109 & 2273.6 \\ 0.0000 & 2.2961 \end{bmatrix}$$

(a) Calculated characteristics at 7 kHz

$$\mathbf{Y} = \begin{bmatrix} 0.0004 & 0.0068 \\ 0.0024 & 0.0011 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 68.006 & 430.07 \\ 150.49 & 25.817 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 0.1715 & 0.0066 \\ 418.41 & 0.4519 \end{bmatrix}$$

(b) Measured characteristics at 7 kHz

$$\mathbf{Y} = \begin{bmatrix} 0.0012 & 0.0008 \\ 0.0008 & 0.0008 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 2316.4 & 2241.6 \\ 2241.6 & 3354.0 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 1.0000 & 1224.3 \\ 0.0000 & 1.5000 \end{bmatrix}$$

(c) Calculated characteristics at 13 kHz

$$\mathbf{Y} = \begin{bmatrix} 0.0006 & 0.0045 \\ 0.0017 & 0.0011 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 160.48 & 662.34 \\ 242.18 & 90.452 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 0.3735 & 0.0041 \\ 602.40 & 0.6627 \end{bmatrix}$$

(d) Measured characteristics at 13 kHz

Figure 6: Circuit no. 2 characteristics

5.2 Circuit no. 3

$$\mathbf{Y} = \begin{bmatrix} 0001.1 & 0000.7 \\ 0000.7 & 0001.1 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 1645.9 & 1100.0 \\ 1100.0 & 1645.9 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 0001.5 & 1362.6 \\ 0000.0 & 0001.5 \end{bmatrix}$$

(a) Calculated characteristics at 13 kHz

$$\mathbf{Y} = \begin{bmatrix} 0.0007 & 0.0022 \\ 0.0022 & 0.0007 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 170.36 & 515.76 \\ 508.76 & 170.36 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 0.3349 & 0.0020 \\ 458.71 & 0.3349 \end{bmatrix}$$

(b) Measured characteristics at 13 kHz

$$\mathbf{Y} = \begin{bmatrix} 0.0014 & 0.0010 \\ 0.0010 & 0.0014 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 1528.3 & 1100.0 \\ 1100.0 & 1528.3 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 0001.4 & 1023.4 \\ 0000.0 & 0001.4 \end{bmatrix}$$

(c) Calculated characteristics at 15 kHz

$$\mathbf{Y} = \begin{bmatrix} 0.0008 & 0.0020 \\ 0.0019 & 0.0008 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 239.91 & 610.68 \\ 604.45 & 239.91 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 0.3969 & 0.0017 \\ 515.46 & 0.3969 \end{bmatrix}$$

(d) Measured characteristics at 15 kHz

Figure 7: Circuit no. 3 characteristics

5.3 Chain configuration of circuit no. 2 and 3

$$\mathbf{Y} = \begin{bmatrix} 0000.8 & 0000.3 \\ 0000.3 & 0000.8 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 1311.4 & 0493.2 \\ 0493.2 & 1403.8 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 0002.7 & 3239.8 \\ 0000.0 & 0002.8 \end{bmatrix}$$

(a) Calculated characteristics at 13 kHz

$$\mathbf{Y} = \begin{bmatrix} 0.0005 & 0.0075 \\ 0.0061 & 0.0007 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} 14.808 & 165.98 \\ 134.16 & 11.272 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 0.0840 & 0.0075 \\ 164.74 & 0.1104 \end{bmatrix}$$

(b) Measured characteristics at 13 kHz

Figure 8: Equivalent characteristics for chain connection

6 Conclusions

Two-port networks are very useful in circuit design and analysis, ability to describe any linear circuit with one expression is crucial in abstracting complex circuits. Option of find characteristics without looking inside "black box" can be useful.

A Appendix

```
clear all; close all; clc;

%% theoretical calculations
% circuit 2
R21 = 1.1e3;
R22 = 4.7e3;
C21 = 10e-9;
w21 = 2*pi*7e3;
w22 = 2*pi*13e3;
% 7kHz
tY2_7 = abs([[1/R21 + 1j*w21*C21, 1j*w21*C21];
     [-1j*w21*C21, 1/R22 + 1j*w21*C21]]);
tZ2_7 = abs(Y_to_Z(tY2_7));
tA2_7 = abs(Z_to_chain(tZ2_7));
% 13kHz
tY2_13 = [[1/R21 + 1j*w22*C21, 1j*w22*C21];
     [-1j*w22*C21, 1/R22 + 1j*w22*C21]];
tY2_13 = abs(tY2_13);
tZ2_13 = abs(Y_to_Z(tY2_13));
tA2_13 = abs(Z_to_chain(tZ2_13));

% circuit 3
R3 = 1.1e3;
C3 = 10e-9;
w31 = 2*pi*13e3;
w32 = 2*pi*15e3;
% 13kHz
tZ3_13 = abs([[1/(1j*w31*C3) + R3, R3];
     [R3, 1/(1j*w31*C3) + R3]]);
tY3_13 = abs(Z_to_admittance(tZ3_13));
tA3_13 = abs(Z_to_chain(tZ3_13));
% 15kHz
tZ3_15 = abs([[1/(1j*w32*C3) + R3, R3];
     [R3, 1/(1j*w32*C3) + R3]]);
tY3_15 = abs(Z_to_admittance(tZ3_15));
tA3_15 = abs(Z_to_chain(tZ3_15));

%chain connection
tA_chain = abs(tA2_13 * tA3_13);
tZ_chain = abs(chain_to_Z(tA_chain));
tY_chain = abs(Z_to_admittance(tZ_chain));

%% circuit 2
% 7kHz
Z2_7 = abs(import_csv('csv/C2_7.csv'));
Y2_7 = abs(Z_to_admittance(Z2_7));
chain2_7 = abs(Z_to_chain(Z2_7));
% 13kHz
Z2_13 = abs(import_csv('csv/C2_13.csv'));
Y2_13 = abs(Z_to_admittance(Z2_13));
chain2_13 = abs(Z_to_chain(Z2_13));
```

```
%% circuit 3
% 13kHz
Z3_13 = abs(import_csv('csv/C3_13.csv'));
Y3_13 = abs(Z_to_admittance(Z3_13));
chain3_13 = abs(Z_to_chain(Z3_13));
% 15kHz
Z3_15 = abs(import_csv('csv/C3_15.csv'));
Y3_15 = abs(Z_to_admittance(Z3_15));
chain3_15 = abs(Z_to_chain(Z3_15));

%% chain circuit 2-3 13kHz
% measured chain configuration
Z23_13 = abs(import_csv('csv/23_13.csv'));
Y23_13 = abs(Z_to_admittance(Z23_13));
measured_chain23_13 = abs(Z_to_chain(Z23_13));
% calculated from matrix multiplication
calculated_chain23_13 = chain2_13 * chain3_13;

function [Z] = import_csv(file)
CSV = readtable(file);
deg = table2array(CSV(:,4));
deg = reshape(deg, [2, 2])';
Z = table2array(CSV(:, 7));
Z = reshape(Z, [2, 2])';
% Z = Z .* (cos(deg) + 1i * sin(deg));

% UR = table2array(CSV(:,3));
% Uin = table2array(CSV(:,2));
% deg = table2array(CSV(:,4));
% rs21 = table2array(CSV(:,6));
% for k = [1:4]
%      I = UR(k)/rs21(k);
%      Z = Uin(k)/I

end

function [Z] = Y_to_Z(Y)
Z = inv(Y);
end


function [Z] = chain_to_Z(A)
det_A = det(A);
Z = [[A(1, 1)/A(2, 1), det_A/A(2, 1)];
         [1/A(2, 1), A(2, 2)/A(2, 1)]];
end


function [Y] = Z_to_admittance(Z)
det_Z = det(Z);
Y = [[Z(2, 2)/det_Z, -Z(1, 2)/det_Z];
     [-Z(2, 1)/det_Z, Z(1, 1)/det_Z]];
end

function [chain] = Z_to_chain(Z)
det_Z = det(Z);
```

```
chain = [[Z(1, 1)/Z(2, 1), det_Z/Z(2, 1)];
         [1/Z(2, 1), Z(2, 2)/Z(2, 1)]];
end
```