Laplace Transform
Laboratory III

Patrycja Nazim, Adrian Król, Gabriel Ćwiek, Kamil Chaj

1 Goal of the exercise

2 Laplace Transform

Laplace transform is an integral transform that converts a function of a real variable(time domain) into a function of complex variable(frequency domain). It is powerful tool for solving differential equations, which turns ODEs into algebraic equations and convolution into multiplication. For function $x(t)$, the Laplace transform is the integral

$$\mathcal{L}[x(t)] = X(s) = \int_0^\infty x(t)e^{-st}dt \tag{1}$$

Where $s = \sigma + j\omega \quad \sigma, \omega \in \mathbb{R}$
In order to get solution of differential equation solved in s-domain it is necessary to apply inverse Laplace transform which is given by following complex integral

$$f(t) = \mathcal{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)e^{-st}ds \tag{2}$$

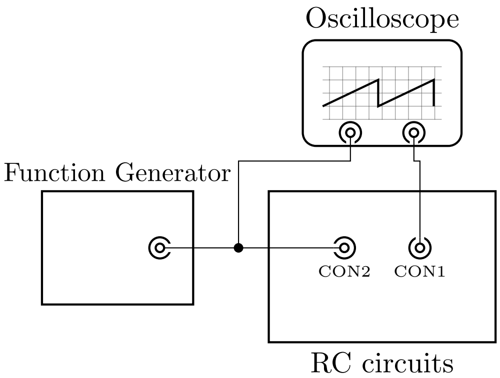
Integrals like these can be quite difficult to solve, that is why lookup table of Laplace transforms and properties can be very handy.

Moving to solving circuits using Laplace transform, there are two approaches, first is constructing differential equation in time domain describing circuit and then solve it using Laplace transform, or second option very similar to solving circuits in steady-state where all components are transformed to s-domain and at the end of calculation transform back to t-domain.

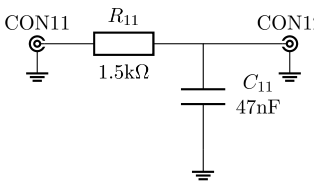
$$R \xrightarrow{\mathcal{L}} R \quad C \xrightarrow{\mathcal{L}} \frac{1}{sC} \quad L \xrightarrow{\mathcal{L}} sL \tag{3}$$

3 Course of measurements

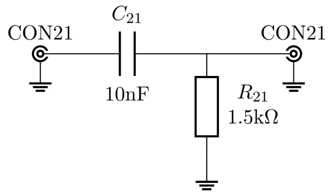
First we tested two circuits: low-pass and high-pass configuration of RC circuit and RLC circuts. After connecting osciloscope to the wave generator and prototype board, we generated square wave with v_{pp} = 1V, v_{offset} = .5V, frequency of 100Hz and duty cycle of 50%. Then we read from the oscilloscope Voltage value at times 1τ, 5τ and 10τ (where 10τ is just as fail-safe) and time when voltage reaches 10% and 90% of the highest value.



(a) Measurement setup



(b) Circuit A

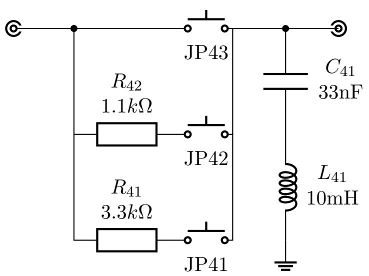


(c) Circuit B

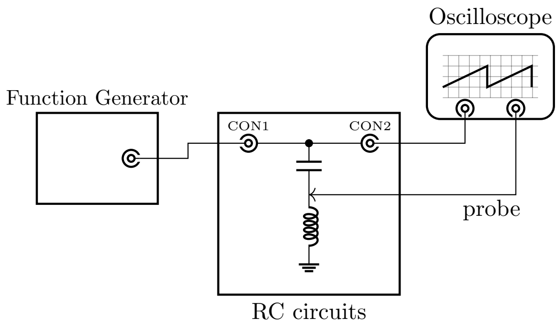
Figure 1: RC Circuits

For RLC circuits we tested two different resistances before RC circuit influence output characteristic. We measured response for 2 cases: Generator out resistance (50 Ω) + resistance selected by jumper wire. In our case we tested jumper on 1.1kΩ resistance path and 3.3kΩ. After that we checked response of the circuit:

- if response was sinusoid with decreasing amplitude - resustance was smaller then RC
- if response was exp. decay if R was higher
- if response was aperiodic critical waveform if resistance was equal RC



(a) Measured Circuit



(b) Measurements setup

Figure 2: RLC circuit

4 Theoretical calculations

For all calculations we used Matlab with Symbolic Math Toolbox. Source code can be found in Appendix. A

In all three circuit input voltage was square wave with 50% duty cycle which can be described in time domain by

$$v_{in}(t) = V_{offset}\mathbf{1}(t) + V_{pp}\mathbf{1}(t - \frac{T}{2}) - V_{pp}\mathbf{1}(t - T) \tag{4}$$

and in frequency domain by

$$V_{in}(s) = \mathcal{L}[v_{in}(t)] = \frac{V_{offset}}{s} + \frac{e^{-\frac{T}{2}s}}{s} - \frac{e^{-Ts}}{s} \tag{5}$$

4.1 Circuit A

First we need to transform circuit to s-domain according to (3)

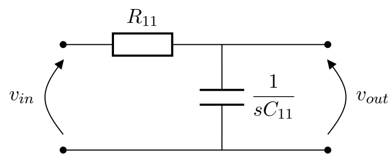
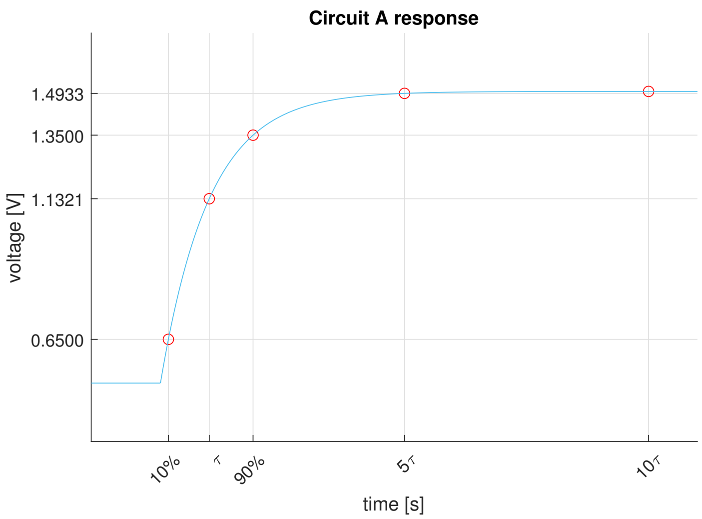


Figure 3: Circuit A schematics

output voltag of circuit A can be described using simple voltage divider

$$V_{out}(s) = V_{in}(s)\frac{\frac{1}{sC_{11}}}{R_{11} + \frac{1}{sC_{11}}} \tag{6}$$

after applying inverse Laplace transform to $v_{out}(s)$ we obtain below plot with marked τ, 5τ, 10τ, 10% and 90% of output voltage.



(a) plot

	time	voltage
τ		y
5τ		y
10τ		y
10%		y
90%		y

(b) table of values

Figure 4: Circuit A output voltage

4.2 Circuit B

steps in circuit B are almost identical to circuit A, first we transformed circuit to s-domain according to (3)

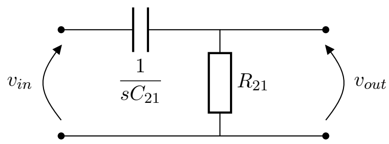
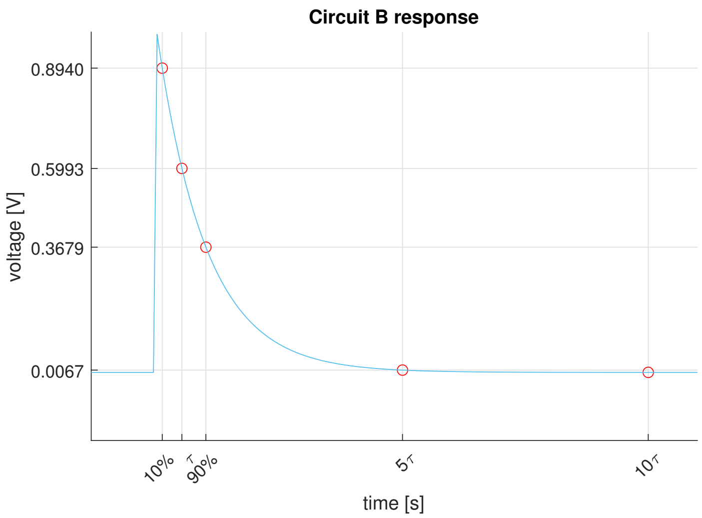


Figure 5: Circuit B schematics

output voltage can be described by voltage divider

$$V_{out}(s) = V_{in}(s)\frac{R_{21}}{R_{21} + \frac{1}{sC_{21}}} \tag{7}$$

after applying inverse Laplace transform to $V_{out}(s)$ we obtain below plot with marked τ, 5τ, 10τ, 10% and 90% of output voltage.



(a) plot

	time	voltage
τ		y
5τ		y
10τ		y
10%		y
90%		y

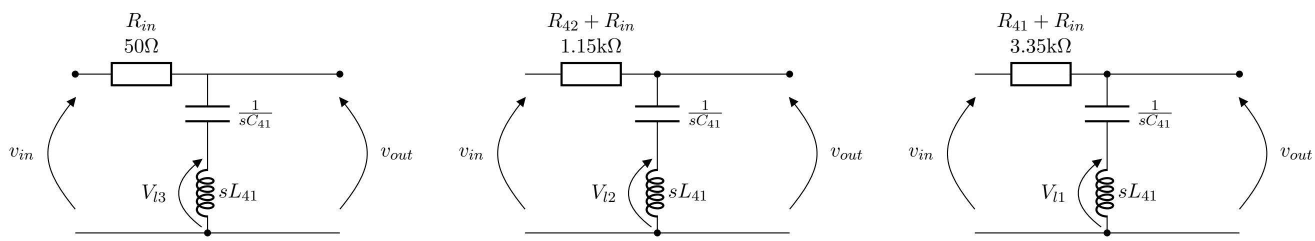
(b) table of values

Figure 6: Circuit B output voltage

4.3 Circuit C

In third circuit we are looking for voltage of coil L_{21}.

Like in previous 2 circuits we transformed circuit to s-domain according to (3)



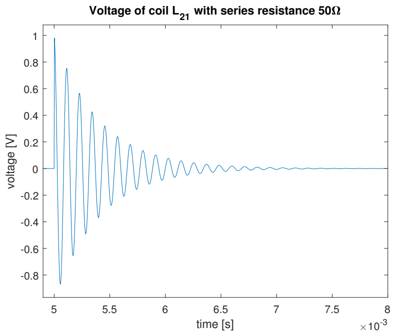
(a) JP43 (b) JP42 (c) JP41

Figure 7: Circuit C schematics

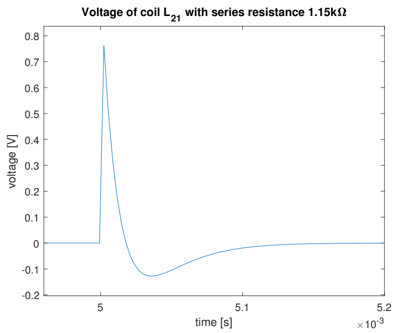
Then solved each circuit for $V_l(s)$

$$V_l(s) = V_{in}(s)\frac{sL_{41}}{R_x + \frac{1}{sC_{21}} + sL_{21}} \tag{8}$$

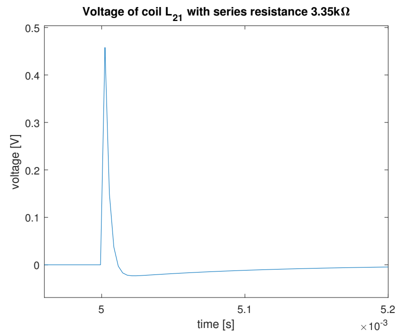
After plugging in value of each component where R_x is series resistance of the circuit, we can transform equation back to time domain and plot results.



(a) Plot JP43



(b) Plot JP42



(c) Plot JP41

	time	voltage
τ		y
5τ		y
10τ		y
10%		y
90%		y

(d) table of values JP43

	time	voltage
τ		y
5τ		y
10τ		y
10%		y
90%		y

(e) table of values JP42

	time	voltage
τ		y
5τ		y
10τ		y
10%		y
90%		y

(f) table of values JP41

5 Comparison

6 Conclusions

A Appendix