Nodal Analysis

Laboratory I

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1 Aim of exercise

The aim of our exercise was to experimentally verify the nodal analysis in RLC circuits. We have achieved it by measuring the voltages on different nodes of the chosen circuits using a dedicated evaluation board and vector voltmeter. The obtained measurement results are compared with analytical calculations.

Apart from the values of potentials in individual nodes of the circuits being measured, we calculated the currents flowing through pointed elements.

2 Course of measurements

First step of our measurements was connecting evaluation board(fig.1) to laboratory computer via USB and starting Vector Voltage Meter software. Next we connected both INPUT1 and CON1 to OUTPUT using BNC cables and splitter, last we connected oscilloscope probe to INPUT2 connector. Now with all preparation finished we started voltage measurements of each node in Circuit A(fig.4). Then we disconnected BNC cable connected to CON1 and connected it to CON2 and repeated previous measurements for Circuit B(fig.5)

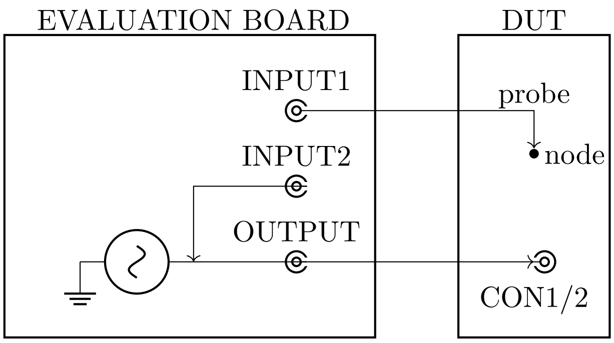


Figure 1: measurements schematic

3 Nodal analysis - method

Method which we are going to use to solve these circuits is known as "Nodal Analysis by Inspection". In this method we need to construct 3 matrices: **i** - current vector, **u** - voltage vector(unknown), **G** - conductance matrix, with sizes respectively $N \times 1$, $N \times 1$, $N-1 \times N-1$.

Size of conductance matrix is always smaller then number of nodes in the circuit by one because in nodal analysis one node is connected to ground.

$$\mathbf{Gu} = \mathbf{i}$$

$$\begin{bmatrix} G_{11} & -G_{12} & -G_{13} \\ -G_{21} & G_{22} & -G_{23} \\ -G_{31} & -G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Where G_{11}, G_{22}, G_{33} are sums of conductance of each branch connected to the node $G_{12} = G_{21}$, $G_{13} = G_{31}$, $G_{32} = G_{23}$ are sums of conductance of branches between nodes I_1, I_2, I_3 are sums of current sources entering or exiting node and U_1, U_2, U_3 are unknown voltages that we are trying to find.

With simple matrix operation we obtain equation

$$\mathbf{u} = \mathbf{G}^{-1}\mathbf{i}$$

which can be easily calculated

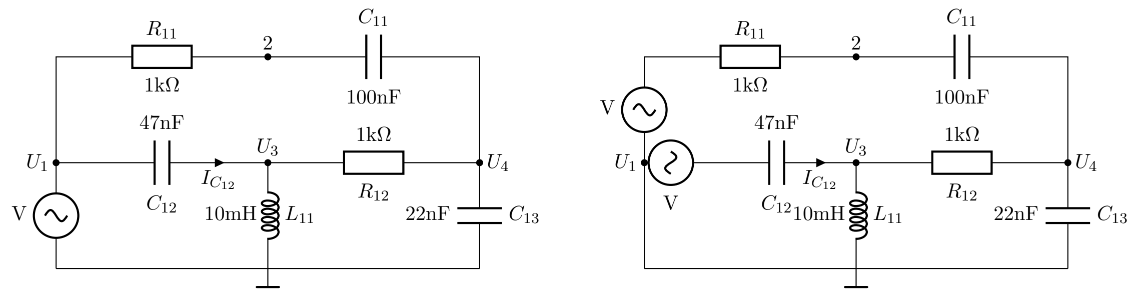
4 Theoretical calculations

all calculation are made in Python with NumPy library. Source code for all calculation can be found in the Appendix

Before starting any calculation we modified original circuits' schematics by replacing BNC connectors with AC voltage source and drawing connection between ground

4.1 Circuit A

In our calculations voltages in nodes 1, 3 and 4 are named U_1, U_3, U_4 and grounded node is the same node that was ground in original circuit schematic. Lone voltage source must be transfigured to solve this circuit with nodal analysis



(a) (b)

Figure 2: theoretical circuit A

$$\begin{bmatrix} G_{R_{11}+C_{11}} + G_{C_{12}} & G_{C_{12}} & G_{R_{11}+C_{11}} \\ G_{C_{12}} & G_{C_{12}} + G_{L_{11}} + G_{R_{12}} & G_{R_{12}} \\ G_{R_{11}} + G_{C_{11}} & G_{C_{12}} & G_{R_{11}+C_{11}} + G_{R_{12}} + G_{C_{12}} \end{bmatrix} \begin{bmatrix} U_1 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} I1 \\ I3 \\ I4 \end{bmatrix}$$

Current of the capacitor C_{12} can be calculated using $I_{C_{12}} = \frac{U_3 - U_2}{Z_{C_{12}}}$

results of calculations

source code in Appendix. A

!!!WRONG RESULTS... I THINK!!!

1000.0 Hz	Magnitude	Phase	5000.0 Hz	Magnitude	Phase	9000.0 Hz	Magnitude	Phase
Node 2	0.192	180	Node 2	0.192	-180	Node 2	0.192	180
Node 3	1.22663e-18	135	Node 3	2.86098e-17	104.036	Node 3	5.72196e-17	-165.964
Node 4	0	0	Node 4	1.43049e-17	14.0362	Node 4	2.08167e-17	-90
I of C_12	5.66995e-05	-1.5708	I of C_12	0.000283497	-1.5708	I of C_12	0.000510295	-1.5708

4.2 Circuit B

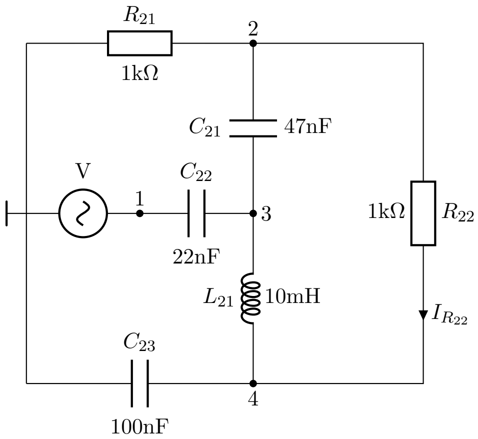


Figure 3: theoretical circuit B

Current of the resistor R_{22} can be calculated using $I_{R_{22}} = \frac{U_4 - U_2}{Z_{R_{22}}}$

5 Real measurements

5.1 Circuit A

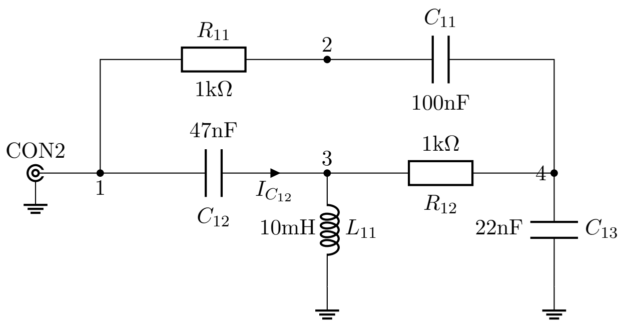


Figure 4: circuit A

Circuit A			
Freq[kHz]	Channel 1[V]	Channel 2 [V]	Angle [°]
node 1			
1kHz	0.192	0.192	0
5kHz	0.192	0.192	-0.1
9kHz	0.192	0.192	-0.1
node 2			
1kHz	0.192	0.145	-19.3
5kHz	0.192	0.59	9.3
9kHz	0.192	0.236	-9.7
node 3			
1kHz	0.192	0.008	140.7
5kHz	0.192	0.154	136.1
9kHz	0.192	0.376	30
node 4			
1kHz	0.192	0.072	38.6
5kHz	0.192	0.082	42.1
9kHz	0.192	0.231	-11.8

Table 1: evaluation board measurements for Circuit A

Current of capacitor C_{12}

freq [kHz]	$I_{R_{22}}$
1kHz	
5kHz	
9kHz	

5.2 Circuit B

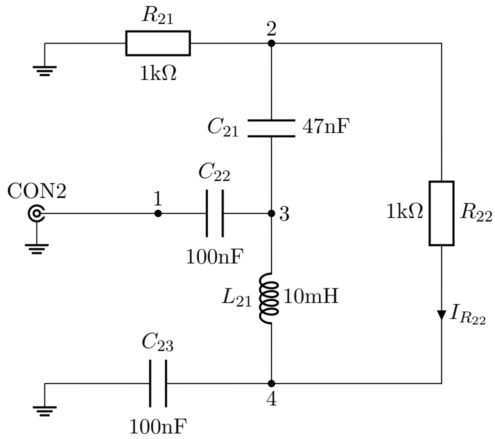


Figure 5: circuit B

Circuit B			
Freq[kHz]	Channel 1[V]	Channel 2 [V]	Angle [°]
node 1			
1kHz	0.192	0.192	0
5kHz	0.192	0.192	-0.1
9kHz	0.192	0.192	0
node 2			
1kHz	0.192	0.044	28.7
5kHz	0.192	0.044	-35.7
9kHz	0.192	0.156	69.7
node 3			
1kHz	0.192	0.084	23.3
5kHz	0.192	0.055	62.4
9kHz	0.192	0.22	24.4
node 4			
1kHz	0.192	0.086	21.2
5kHz	0.192	0.18	-11.8
9kHz	0.192	0.09	-113.8

Table 2: evaluation board measurements for Circuit B

freq [kHz]	$I_{R_{22}}$
1kHz	
5kHz	
9kHz	

6 Comparison

7 Summery

Appendix

A Source code circuit A

```
import numpy as np
from tabulate import tabulate

def get_V(f):
    w=2*np.pi*f
    Zr=1e3 #Zr1 == Zr2
    Zc1=1/(complex(0,100e-9)*w)
    Zc2=1/(complex(0,47e-9)*w)
    Zc3=1/(complex(0,22e-9)*w)
    Zl=complex(0,10e-3)*w
    V = 0.192
    G1 = 1/Zc2 + 1/(Zc1+Zr)
    G3 = 1/Zr + 1/Zc2 + 1/Zl
    G4 = 1/Zc3 + 1/(Zc1+Zr) + 1/Zr
    G13 = -1/Zc2
    G14 = -1/(Zr+Zc1)
    G34 = -1/Zr
    I1 = - V/(Zr+Zc1) - V/Zc2
    I3 = V/Zc2
    I4 = V/(Zr+Zc1)
    G = np.array([[G1, G13, G14],
                  [G13, G3, G34],
                  [G14, G34, G4]])
    I = np.array([[I1],
                  [I3],
                  [I4]])
    V = np.matmul(np.linalg.inv(G), I)
    VP = np.angle(V, True)
    VA = np.abs(V)
    IC12 = (V[0]-V[1])/Zc2
    VA = np.append(VA, np.abs(IC12))
    VP = np.append(VP, np.angle(IC12))
    headers = [str(f)+' Hz' , 'Magnitude', 'Phase']
    first_col = ['Node 1', 'Node 3', 'Node 4', 'I of C_12']
    table1 = np.column_stack((first_col, VA, VP))
    print(tabulate(table1, headers=headers, tablefmt="latex"))
    return(V, VA, VP)

(V1, VA1, VP1)=get_V(1e3)
(V5, VA5, VP5)=get_V(5e3)
(V9, VA9, VP9)=get_V(9e3)
```

B Source code circuit B

```
import numpy as np
from tabulate import tabulate

def get_V(f):
    w=2*np.pi*f
    Zr=1e3
    Zc1=1/(complex(0,47e-9)*w)
    Zc2=1/(complex(0,100e-9)*w)
    Zc3=1/(complex(0,100e-9)*w)
    Zl=complex(0,10e-3)*w
    V = 0.192

    G2 = 2/Zr + 1/Zc1
    G3 = 1/Zc1 + 1/Zc2 + 1/Zl
    G4 = 1/Zc3 + 1/Zl + 1/Zr
    G23 = -1/Zc1
    G24 = -1/Zr
    G34 = -1/Zl

    I1 = 0
    I2 = V / (Zc2)
    I3 = 0
    G = np.array([[G2, G23, G24],
                  [G23, G3, G34],
                  [G24, G34, G4]])
    I = np.array([[I1],
                  [I2],
                  [I3]])

    V = np.matmul(np.linalg.inv(G), I)
    VP = np.angle(V, True)
    VA = np.abs(V)
    IR22 = (V[0]-V[2])/Zr
    VA = np.append(VA, np.abs(IR22))
    VP = np.append(VP, np.angle(IR22))
    headers = [str(f)+' Hz', 'Magnitude', 'Phase']
    first_col = ['Node 2', 'Node 3', 'Node 4', 'I of R_22']
    table1 = np.column_stack((first_col, VA, VP))
    print(tabulate(table1, headers=headers,tablefmt='latex'))
    return(V, VA, VP)

(V1, VA1, VP1)=get_V(1e3)
(V5, VA5, VP5)=get_V(5e3)
(V9, VA9, VP9)=get_V(9e3)
```