Circuit Theory II Laboratory

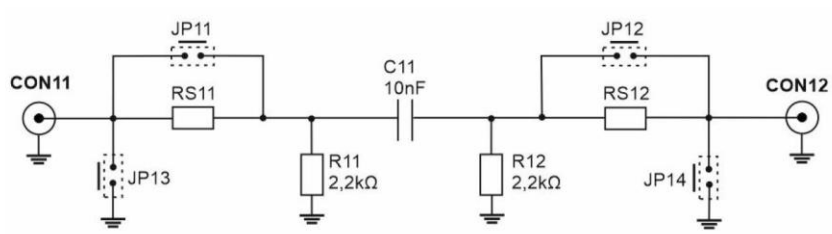
TWO-PORT NETWORKS

Aim of the exercise

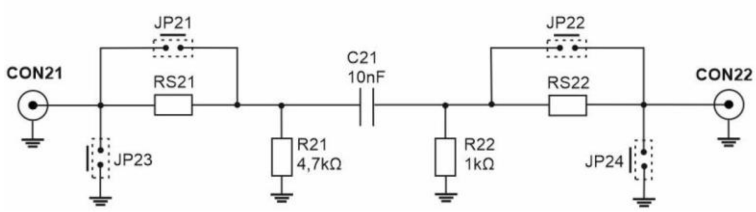
The aim of the exercise is to familiarize with the methods of determining the self-parameters of two-port networks (circuits). The aim of the exercise is carried out by experimentally determining the parameters by measuring the output voltages and currents, and then comparing the obtained results with theoretical calculations.

Schematics of measured networks

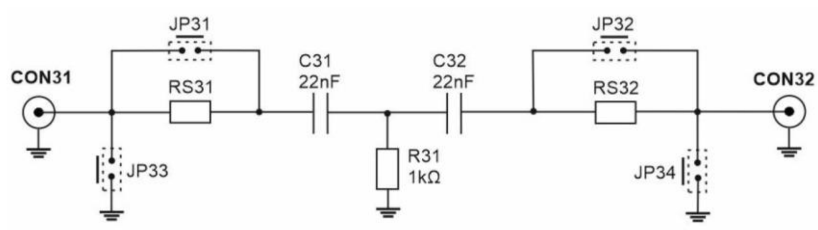
Each of the two-port circuits has 4 jumpers to connect a 100 Ω serial resistor (**RS***XX*) for current measurement to a given input, or to short-circuit a given potential to ground. During the exercise, you will only use jumpers for series connection of resistors.



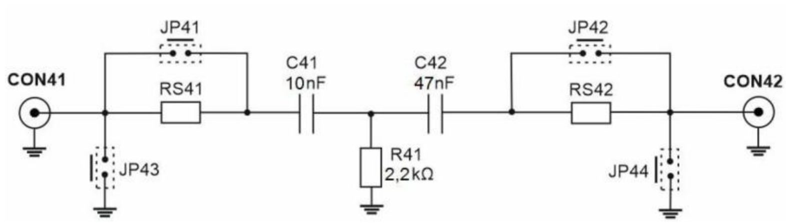
Circuit no. 1



Circuit no. 2



Circuit no. 3



Circuit no. 4

The method of determining the parameters of two-port networks

From the analytical point of view, each electrical circuit can be considered as a so-called "black box". Suppose that there is a two-port circuit of unknown topology, at the input of which there is voltage U_1 and current I_1, whereas the output is defined by U_2 and I_2.

Considering only the above voltages and currents, we are able to model the analyzed electrical circuit using a specific type of parametric matrix (e.g. admittance). Let we consider calculations of the impedance matrix. As is well known, the matrix notation for such a case is as follows:

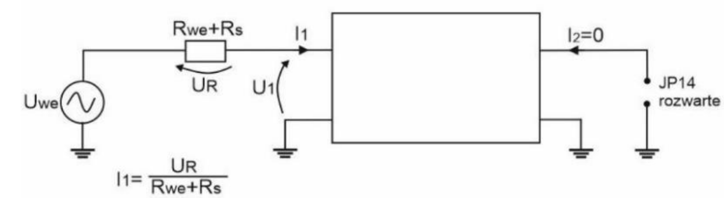
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

The above form of notation can be represented by a set of equations:

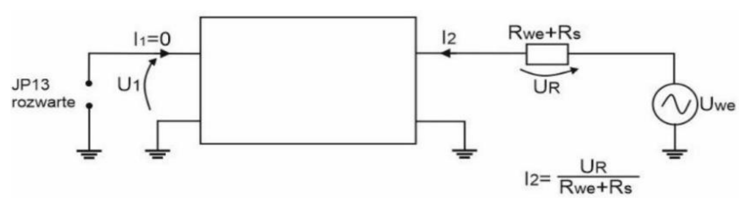
$$U_1 = z_{11}I_1 + z_{12}I_2$$
$$U_2 = Z_{12}I_1 + Z_{22}I_2.$$

By equating the currents I_1 and I_2 to zero, respectively, we are able to estimate the values of the individual elements of the impedance matrix.

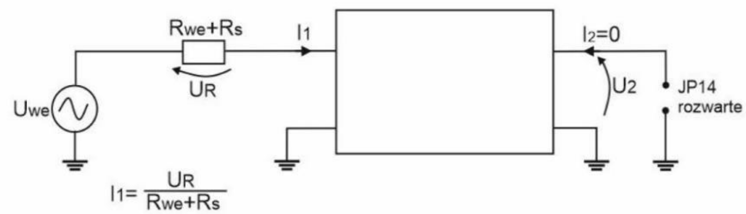
$$z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0}$$



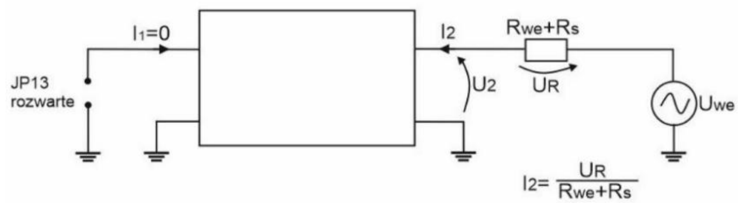
$$z_{12} = \left.\frac{U_1}{I_2}\right|_{I_1=0}$$



$$z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0}$$



$$z_{22} = \left.\frac{U_2}{I_2}\right|_{I_1=0}$$



An analogous procedure can be carried out for the admittance description of the two-port circuit:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = Y \begin{bmatrix} U_1 \\ U_2 \end{bmatrix},$$

And for the abcd parameters also called chain parameters:

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} U_2 \\ -I_2 \end{bmatrix} = A \begin{bmatrix} U_2 \\ -I_2 \end{bmatrix}.$$

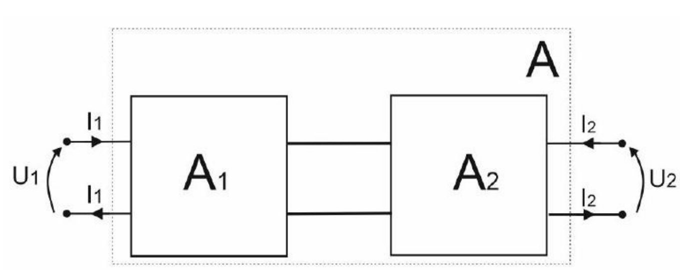
The formulas allowing for transformations between the respective matrices are presented below:

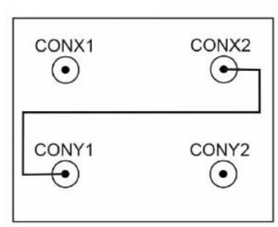
	$\underline{\mathbf{Z}}$	$\underline{\mathbf{Y}}$	$\underline{\mathbf{A}}$
$\underline{\mathbf{Z}}$		$\begin{bmatrix} \frac{\underline{z}_{22}}{\det \underline{\mathbf{Z}}} & \frac{-\underline{z}_{12}}{\det \underline{\mathbf{Z}}} \\ \frac{-\underline{z}_{21}}{\det \underline{\mathbf{Z}}} & \frac{\underline{z}_{11}}{\det \underline{\mathbf{Z}}} \end{bmatrix}$	$\begin{bmatrix} \frac{\underline{z}_{11}}{\underline{z}_{21}} & \frac{\det \underline{\mathbf{Z}}}{\underline{z}_{21}} \\ \frac{1}{\underline{z}_{21}} & \frac{\underline{z}_{22}}{\underline{z}_{21}} \end{bmatrix}$
$\underline{\mathbf{Y}}$	$\begin{bmatrix} \frac{\underline{y}_{22}}{\det \underline{\mathbf{Y}}} & \frac{-\underline{y}_{12}}{\det \underline{\mathbf{Y}}} \\ \frac{-\underline{y}_{21}}{\det \underline{\mathbf{Y}}} & \frac{\underline{y}_{11}}{\det \underline{\mathbf{Y}}} \end{bmatrix}$		$\begin{bmatrix} -\frac{\underline{y}_{22}}{\underline{y}_{21}} & -\frac{1}{\underline{y}_{21}} \\ -\frac{\det \underline{\mathbf{Y}}}{\underline{y}_{21}} & -\frac{\underline{y}_{11}}{\underline{y}_{21}} \end{bmatrix}$
$\underline{\mathbf{A}}$	$\begin{bmatrix} \frac{\underline{a}_{11}}{\underline{a}_{21}} & \frac{\det \underline{\mathbf{A}}}{\underline{a}_{21}} \\ \frac{1}{\underline{a}_{21}} & \frac{\underline{a}_{22}}{\underline{a}_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{\underline{a}_{22}}{\underline{a}_{12}} & -\frac{\det \underline{\mathbf{A}}}{\underline{a}_{12}} \\ -\frac{1}{\underline{a}_{12}} & \frac{\underline{a}_{11}}{\underline{a}_{12}} \end{bmatrix}$	

Measurements

In the first part of the exercise, for the given frequencies, measure the selected parameters of the networks using a vector voltmeter (the list of circuits is at the end of the manual in *List of the networks*). In the report, compare the obtained results with the analytically calculated matrix parameters of the given circuits.

In the second part of the exercise, you should connect two selected two-port circuits (list at the end of the manual), and then determine the chain (abcd) parameters of the resulting network.





In the report, compare the obtained results with the formula for the chain matrix of the chain connection of the circuits:

$$A = A_1 A_2.$$

List of the the networks

Section	**Frequencies**	**Part I** (*Z and Y matrices*)	**Part II** (*abcd matrix*)
I + V	[Z]: 7, 11 kHz [Y]: 11, 14 kHz	[Z]: Schematic no. 1 [Y]: Schematic no. 4	Connection of networks: 1 → 4
II	[Z]: 8, 10 kHz [Y]: 10, 12 kHz	[Z]: Schematic no. 1 [Y]: Schematic no. 2	Connection of networks: 1 → 2
III	[Z]: 7, 13 kHz [Y]: 13, 15 kHz	[Z]: Schematic no. 2 [Y]: Schematic no. 3	Connection of networks: 2 → 3
IV	[Z]: 8, 14 kHz [Y]: 14, 16 kHz	[Z]: Schematic no. 1 [Y]: Schematic no. 2	Connection of networks: 2 → 1

Report requirements

The report should include a comparison of the obtained measurement results with theoretical calculations as well as comments on the obtained measurement results and calculations.