Nodal Analysis

Laboratory I

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1 Aim of exercise

The aim of our exercise was to experimentally verify the nodal analysis in RLC circuits. We have achieved it by measuring the voltages on different nodes of the chosen circuits using a dedicated evaluation board and vector voltmeter. The obtained measurement results are compared with analytical calculations.

Apart from the values of potentials in individual nodes of the circuits being measured, we calculated the currents flowing through pointed elements.

2 Nodal analysis - method

Method which we are going to use to solve this circuit is know as "Nodal Analysis by Inspection". In this method we need to construct 3 matrices: **i** - current vector, **u** - voltage vector(unknown), **G** - conductance matrix with sizes respectively $N \times 1$, $N \times 1$, $N-1 \times N-1$

$$\mathbf{Gu} = \mathbf{i}$$

$$\begin{bmatrix} G_{11} & -G_{12} & -G_{13} \\ -G_{21} & G_{22} & -G_{23} \\ -G_{31} & -G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Where G_{11}, G_{22}, G_{33} are sums of conductance of each branch connected to the node
$G_{12} = G_{21}$, $G_{13} = G_{31}$, $G_{32} = G_{23}$ are sums of conductance of branches between nodes
I_1, I_2, I_3 are sums of current sources entering or exiting node and U_1, U_2, U_3 are unknown voltages that we are trying to find

With simple matrix operation we obtain equation

$$\mathbf{u} = \mathbf{G}^{-1}\mathbf{i}$$

which can be easily calculated

3 theoretical calculations

all calculation are made with Python and NumPy library

3.1 Circuit A

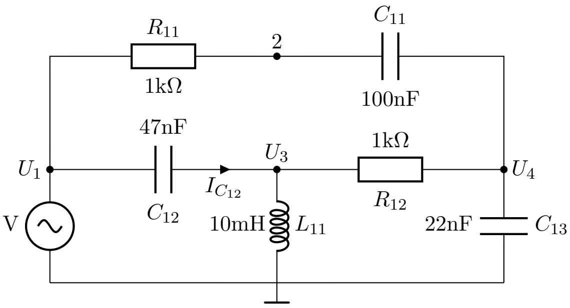

Figure 1: theoretical circuit A

$$\begin{bmatrix} \frac{1}{Z_{C_12}} + \frac{1}{Z_{C_11}+Z_{R_{11}}} & \frac{-1}{Z_{C_{11}}} & \frac{-1}{Z_{R_{11}}+Z_{C_{11}}} \\ \frac{-1}{Z_{C_{11}}} & \frac{1}{Z_{R_{12}}} + \frac{1}{Z_{C_12}} + \frac{1}{Z_{L_{11}}} & \frac{-1}{Z_{R_{12}}} \\ \frac{-1}{Z_{R_{11}}+Z_{C_{11}}} & \frac{-1}{Z_{R_{12}}} & \frac{1}{Z_{C_13}} + \frac{1}{Z_{C_11}+Z_{R_{11}}} + \frac{1}{Z_{R_{12}}} \end{bmatrix} \begin{bmatrix} U_1 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} \frac{-V}{Z_{R_{11}}} - \frac{1}{Z_{C_11}+Z_{R_{11}}} \\ \frac{V}{Z_{C_12}} \\ \frac{V}{Z_{R_{12}}+Z_{C_{11}}} \end{bmatrix}$$

Current of the capacitor C_{12} can be calculated using $I_{C_{12}} = \frac{U_3 - U_2}{Z_{C_{12}}}$

```
import numpy as np
import cmath as cm

f=1e3 # f=5e3 f=9e3
w=2*3.14*f
Zr=1e3
Zc1=1/complex(0,-((100e-9))*w)
Zc2=1/complex(0,-((47e-9))*w)
Zc3=1/complex(0,-((22e-9))*w)
Zl=complex(0,(10e-3)*w)
V = 1.117

G1 = 1/Zc2 + 1/(Zc1+Zr)
G3 = 1/Zr + 1/Zc2 + 1/Zl
G4 = 1/Zc3 + 1/(Zc1+Zr) + 1/Zr
G13 = -1/Zc1
G14 = -1/(Zr+Zc1)
G34 = -1/Zr
I1 = -V/Zr - V/(Zr+Zc1)
I3 = V/(Zc2)
I4 = V/(Zr+Zc1)
G = np.array([[G1, G13, G14],
              [G13, G3, G34],
              [G14, G34, G4]])
I = np.array([[I1],
              [I3],
              [I4]])

V = np.matmul(np.linalg.inv(G), I)

print('V: \n', V)
print('phase: \n', np.angle(V, True))
print('|V|: \n', np.abs(V))
```

3.2 Circuit B

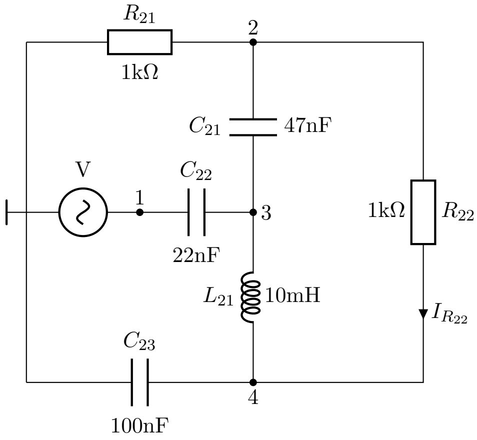

Figure 2: theoretical circuit B

$$\begin{bmatrix} \frac{2}{Z_{R_{21}}} + \frac{1}{Z_{L_{21}}} & \frac{-1}{Z_{C_{21}}} & \frac{-1}{Z_{R_{22}}} \\ \frac{-1}{Z_{C_{21}}} & \frac{1}{Z_{C_{2}1}} + \frac{1}{Z_{C_{22}}} + \frac{1}{Z_{L_{21}}} & \frac{-1}{Z_{L_{21}}} \\ \frac{-1}{Z_{R_{22}}} & \frac{-1}{Z_{L_{21}}} & \frac{1}{Z_{C_{23}}} + \frac{1}{Z_{L_{21}}} + \frac{1}{Z_{R_{22}}} \end{bmatrix} \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{V}{Z_{C_{22}}} \\ 0 \end{bmatrix}$$

Current of the resistor R_{22} can be calculated using $I_{R_{22}} = \frac{U_4 - U_2}{Z_{R_{22}}}$

```python
import numpy as np
import cmath as cm

f=1e3 # f=5e3 f=9e3
w=2*3.14*f
Zr=1e3
Zc1=1/complex(0,-((100e-9))*w)
Zc2=1/complex(0,-((47e-9))*w)
Zc3=1/complex(0,-((22e-9))*w)
Zl=complex(0,(10e-3)*w)
V = 1.117

G1 = 1/Zc2 + 1/(Zc1+Zr)
G3 = 1/Zr + 1/Zc2 + 1/Zl
G4 = 1/Zc3 + 1/(Zc1+Zr) + 1/Zr
G13 = -1/Zc1
G14 = -1/(Zr+Zc1)
G34 = -1/Zr
I1 = -V/Zr - V/(Zr+Zc1)
I3 = V/(Zc2)
I4 = V/(Zr+Zc1)
G = np.array([[G1, G13, G14],
              [G13, G3, G34],
              [G14, G34, G4]])
I = np.array([[I1],
              [I3],
              [I4]])

V = np.matmul(np.linalg.inv(G), I)
```

```
print('V:␣\n', V)
print('phase:␣\n', np.angle(V, True))
print('|V|:␣\n', np.abs(V))
```

4 real measurements

4.1 Circuit A

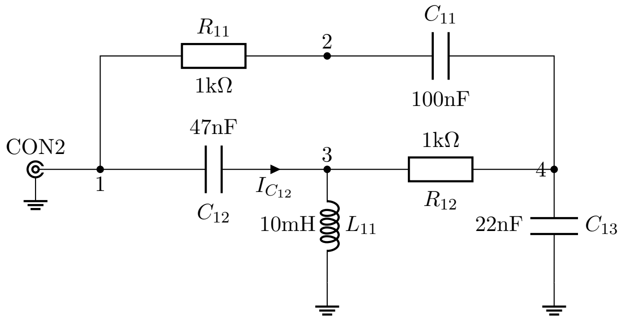

Figure 3: circuit A

Circuit A			
Freq[kHz]	Channel 1[V]	Channel 2 [V]	Angle [°]
node 1			
1kHz	0.192	0.192	0
5kHz	0.192	0.192	-0.1
9kHz	0.192	0.192	-0.1
node 2			
1kHz	0.192	0.145	-19.3
5kHz	0.192	0.59	9.3
9kHz	0.192	0.236	-9.7
node 3			
1kHz	0.192	0.008	140.7
5kHz	0.192	0.154	136.1
9kHz	0.192	0.376	30
node 4			
1kHz	0.192	0.072	38.6
5kHz	0.192	0.082	42.1
9kHz	0.192	0.231	-11.8

Table 1: evaluation board measurements for Circuit A

Current of capacitor C_{12}

freq [kHz]	$I_{R_{22}}$
1kHz	
5kHz	
9kHz	

4.2 Circuit B

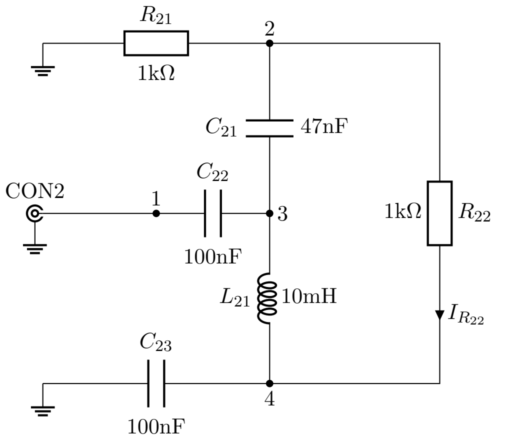

Figure 4: circuit B

Circuit B			
Freq[kHz]	Channel 1[V]	Channel 2 [V]	Angle [°]
node 1			
1kHz	0.192	0.192	0
5kHz	0.192	0.192	-0.1
9kHz	0.192	0.192	0
node 2			
1kHz	0.192	0.044	28.7
5kHz	0.192	0.044	-35.7
9kHz	0.192	0.156	69.7
node 3			
1kHz	0.192	0.084	23.3
5kHz	0.192	0.055	62.4
9kHz	0.192	0.22	24.4
node 4			
1kHz	0.192	0.086	21.2
5kHz	0.192	0.18	-11.8
9kHz	0.192	0.09	-113.8

Table 2: evaluation board measurements for Circuit B

freq [kHz]	$I_{R_{22}}$
1kHz	
5kHz	
9kHz	

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