Nodal Analysis
Laboratory I

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1 Aim of exercise

The aim of our exercise was to experimentally verify the nodal analysis in RLC circuits. We have achieved it by measuring the voltages on different nodes of the chosen circuits using a dedicated evaluation board and vector voltmeter. The obtained measurement results are compared with analytical calculations.

Apart from the values of potentials in individual nodes of the circuits being measured, we calculated the currents flowing through pointed elements.

2 Course of measurements

First step of our measurements was connecting evaluation board(fig.1) to laboratory computer via USB and starting Vector Voltage Meter software. Next we connected both INPUT1 and CON1 to OUTPUT using BNC cables and splitter, last we connected oscilloscope probe to INPUT2 connector. Now with all preparation finished we started voltage measurements of each node in Circuit A(fig.4). Then we disconnected BNC cable connected to CON1 and connected it to CON2 and repeated the same measurements for Circuit B(fig.5)

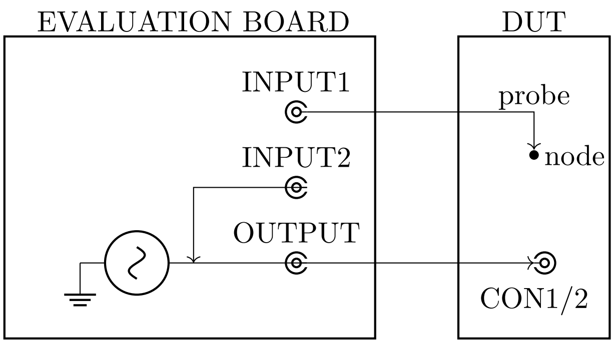


Figure 1: measurements schematic

3 Nodal analysis - method

Method which we are going to use to solve these circuits is known as "Nodal Analysis by Inspection". In this method we need to construct 3 matrices: $\mathbf{i}$ - current vector, $\mathbf{u}$ - voltage vector(unknown), $\mathbf{G}$ - conductance matrix, with sizes respectively $N \times 1$, $N \times 1$, $N-1 \times N-1$.

Size of conductance matrix is always smaller then number of nodes in the circuit by one because in nodal analysis one node is connected to ground as reference node.

$$\mathbf{Gu} = \mathbf{i}$$

$$\begin{bmatrix} G_{11} & -G_{12} & -G_{13} \\ -G_{21} & G_{22} & -G_{23} \\ -G_{31} & -G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Where G_{11}, G_{22}, G_{33} are sums of conductance of each branch connected to the node
$G_{12} = G_{21}$, $G_{13} = G_{31}$, $G_{32} = G_{23}$ are sums of conductance of branches between nodes
I_1, I_2, I_3 are sums of current sources entering or exiting node and U_1, U_2, U_3 are unknown voltages that we are trying to find.

With simple matrix operation we obtain equation

$$\mathbf{u} = \mathbf{G}^{-1}\mathbf{i}$$

which can be easily calculated

4 Theoretical calculations

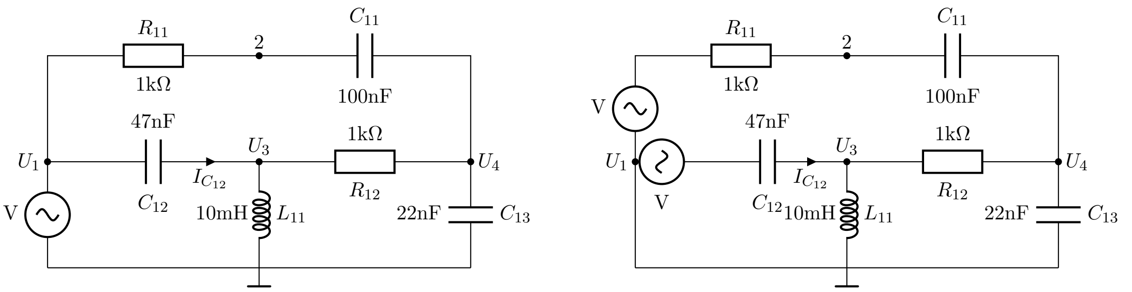
All calculation are made in Python with NumPy library. Source code for all calculation can be found in the Appendix

Before starting any calculation we modified original circuits' schematics by replacing BNC connectors with AC voltage source and drawing connection between ground(fig. 2a, fig. 3)

for both circuits amplitude of Voltage source is 0.192V and calculations are made for frequencies 1kHz, 5kHz and 9kHz

4.1 Circuit A

In our calculations voltages in nodes 1, 3 and 4 are named U_1, U_3, U_4 and reference node is the same node that was ground in original circuit schematic. Lone voltage source must be transfigured in order to solve this circuit by nodal analysis, after source transfiguration circuit looks like this fig. 2b



(a) (b)

Figure 2: theoretical circuit A

following rules of Nodal Analysis by Inspection(Sec. 3) we obtain $\mathbf{Gu} = \mathbf{i}$ equation

$$\begin{bmatrix} G_{R_{11}+C_{11}} + G_{C_{12}} & -G_{C_{12}} & -G_{R_{11}+C_{11}} \\ -G_{C_{12}} & G_{C_{12}} + G_{L_{11}} + G_{R_{12}} & -G_{R_{12}} \\ -G_{R_{11}} - G_{C_{11}} & -G_{C_{12}} & G_{R_{11}+C_{11}} + G_{R_{12}} + G_{C_{12}} \end{bmatrix} \begin{bmatrix} U_1 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} \frac{-V}{Z_{R_{11}+C_{11}}} + \frac{-V}{Z_{C_1 2}} \\ \frac{V}{Z_{C_1 2}} \\ \frac{V}{Z_{R_{11}+C_{11}}} \end{bmatrix}$$

after solving above equation we can easily calculate current of the capacitor C_{12}

$$I_{C_{12}} = \frac{U_3 - U_2}{Z_{C_{12}}}$$

all that is left to do is computing

results of calculations

source code in the Appendix. A

!!!WRONG RESULTS... I THINK!!!

1000.0 Hz	Magnitude	Phase
Node 1	0.192	180
Node 3	1.22663e-18	135
Node 4	0	0
I of C_12	5.66995e-05	-1.5708

5000.0 Hz	Magnitude	Phase
Node 1	0.192	-180
Node 3	2.86098e-17	104.036
Node 4	1.43049e-17	14.0362
I of C_12	0.000283497	-1.5708

9000.0 Hz	Magnitude	Phase
Node 1	0.192	180
Node 3	5.72196e-17	-165.964
Node 4	2.08167e-17	-90
I of C_12	0.000510295	-1.5708

4.2 Circuit B

Following similar steps as in Circuit A, voltages in notes 2, 3 and 4 are named U_2, U_3, U_4 and reference node is the same node that was ground in original circuit schematic. There is no lone voltage source so source transfiguration is not necessary.

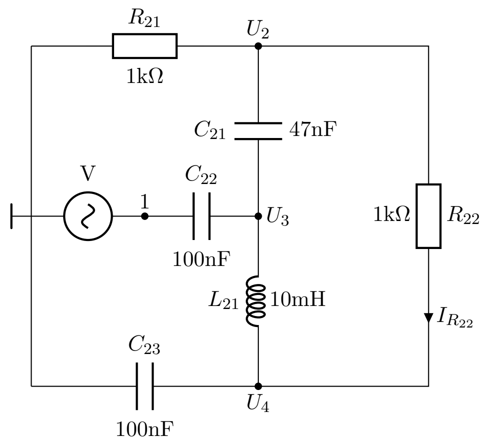


Figure 3: theoretical circuit B

following rules of Nodal Analysis by Inspection(Sec. 3) we obtain $\mathbf{Gu} = \mathbf{i}$ equation

$$\begin{bmatrix} G_{R_{21}} + G_{R_{22}} + G_{C_{21}} & -G_{C_{21}} & -G_{R_{22}} \\ -G_{C_{21}} & G_{C_{21}} + G_{C_{22}} + G_{L_{21}} & G_{L_{21}} \\ -G_{R_{22}} & -G_{L_{21}} & G_{R_{22}} + G_{C_{23}} + G_{L_{21}} \end{bmatrix} \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{V}{Z_{C_{22}}} \\ 0 \end{bmatrix}$$

after solving above equation we can easily calculate current of the capacitor R_{22}

$$I_{R_{22}} = \frac{U_4 - U_2}{Z_{R_{22}}}$$

and these are the results

4.2.1 Results of calculations

Source code in the Appendix B

1000.0 Hz	Magnitude	Phase
Node 2	0.0442579	28.1037
Node 3	0.0837746	22.3054
Node 4	0.0867875	20.4605
I of R_22	4.33246e-05	-2.92078

5000.0 Hz	Magnitude	Phase
Node 2	0.0521098	-53.9736
Node 3	0.0534524	86.6776
Node 4	0.206963	-12.1633
I of R_22	0.000171674	3.13307

9000.0 Hz	Magnitude	Phase
Node 2	0.16282	63.622
Node 3	0.219774	20.3532
Node 4	0.0750914	-122.05
I of R_22	0.000237659	1.07918

5 Real measurements

Following steps of measurements described in Section 2 we obtain measurements for each circuit

5.1 Circuit A

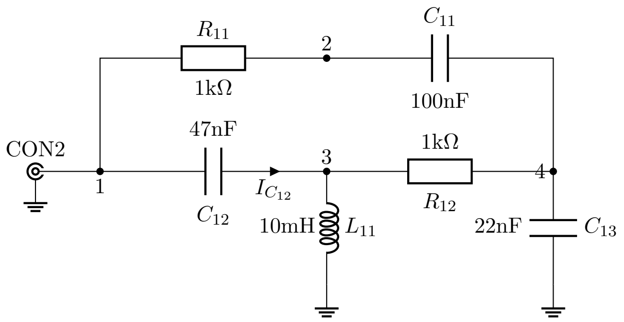


Figure 4: circuit A

Current $I_{C_{12}}$ is measured indirectly using measured voltage and impedance of the component

$$I_{C_{12}} = \frac{U_1 - U_3}{Z_{C_{12}}}$$

where U_3 is voltage in node 3, U_1 is voltage in node 1 and $Z_{C_{12}}$ is impedance of capacitor C_{12}

Circuit A			
Freq	Channel 1[V]	Channel 2 [V]	Angle [°]
node 1			
1kHz	0.192	0.192	0
5kHz	0.192	0.192	-0.1
9kHz	0.192	0.192	-0.1
node 2			
1kHz	0.192	0.145	-19.3
5kHz	0.192	0.59	9.3
9kHz	0.192	0.236	-9.7
node 3			
1kHz	0.192	0.008	140.7
5kHz	0.192	0.154	136.1
9kHz	0.192	0.376	30
node 4			
1kHz	0.192	0.072	38.6
5kHz	0.192	0.082	42.1
9kHz	0.192	0.231	-11.8

(a)

freq	$I_{C_{12}}$[I]	angle[°]
1kHz	0.198	178.5°
5kHz	$7.543 \cdot 10^{-5}$	1.5°
9kHz	$9.954 \cdot 10^{-5}$	-36.9°

(b)

Table 1: evaluation board measurements for Circuit A

5.2 Circuit B

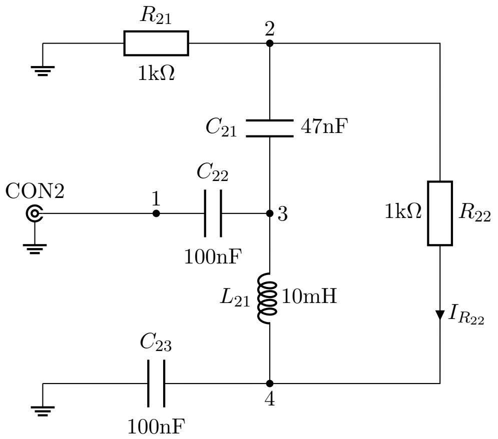


Figure 5: circuit B

Current $I_{R_{22}}$ is measured indirectly using measured voltage and impedance of the component

$$I_{R_{22}} = \frac{U_4 - U_2}{Z_{R_{22}}}$$

where U_4 is voltage in node 4, U_2 is voltage in node 2 and $Z_{R_{22}}$ is impedance of resistor R_{22}

Circuit B			
Freq	Channel 1[V]	Channel 2 [V]	Angle [°]
node 1			
1kHz	0.192	0.192	0
5kHz	0.192	0.192	-0.1
9kHz	0.192	0.192	0
node 2			
1kHz	0.192	0.044	28.7
5kHz	0.192	0.044	-35.7
9kHz	0.192	0.156	69.7
node 3			
1kHz	0.192	0.084	23.3
5kHz	0.192	0.055	62.4
9kHz	0.192	0.22	24.4
node 4			
1kHz	0.192	0.086	21.2
5kHz	0.192	0.18	-11.8
9kHz	0.192	0.09	-113.8

(a)

freq	$I_{R_{22}}$[I]	Angle[°]
1kHz	$4.379 \cdot 10^{-5}$	28.83
5kHz	$1.407 \cdot 10^{-4}$	175.48
9kHz	$2.456 \cdot 10^{-4}$	68.44

(b)

Table 2: evaluation board measurements for Circuit B

6 Comparison

Results of measurements are in general similar to results of nodal analysis. In most cases at least first significant digit of amplitude is the same in both results and phase shift is within few percent of error, but with increasing frequency calculations become less precise. For some applications first significant digit precision can be not satisfying, but roughly both results are comparable. In order to verify how precise nodal analysis is, more data and more precise measuring equipment are needed

7 Summery

Nodal analysis broadly speaking is good method to evaluate values in a circuit, real simple for circuits with many branches and very easy to compute with computer aid because. But in terms of precision as stated in Section 6 for some applications evaluation can be not good enough.

Appendix

GITHUB repository

A Source code circuit A

```
import numpy as np
from tabulate import tabulate

def get_V(f):
    w=2*np.pi*f
    Zr=1e3 #Zr1 == Zr2
    Zc1=1/(complex(0,100e-9)*w)
    Zc2=1/(complex(0,47e-9)*w)
    Zc3=1/(complex(0,22e-9)*w)
    Zl=complex(0,10e-3)*w
    V = 0.192
    G1 = 1/Zc2 + 1/(Zc1+Zr)
    G3 = 1/Zr + 1/Zc2 + 1/Zl
    G4 = 1/Zc3 + 1/(Zc1+Zr) + 1/Zr
    G13 = -1/Zc2
    G14 = -1/(Zr+Zc1)
    G34 = -1/Zr
    I1 = - V/(Zr+Zc1) - V/Zc2
    I3 = V/Zc2
    I4 = V/(Zr+Zc1)
    G = np.array([[G1, G13, G14],
                  [G13, G3, G34],
                  [G14, G34, G4]])
    I = np.array([[I1],
                  [I3],
                  [I4]])
    V = np.matmul(np.linalg.inv(G), I)
    VP = np.angle(V, True)
    VA = np.abs(V)
    IC12 = (V[0]-V[1])/Zc2
    VA = np.append(VA, np.abs(IC12))
    VP = np.append(VP, np.angle(IC12))
    headers = [str(f)+' Hz' , 'Magnitude', 'Phase']
    first_col = ['Node 1', 'Node 3', 'Node 4', 'I of C_12']
    table1 = np.column_stack((first_col, VA, VP))
    print(tabulate(table1, headers=headers, tablefmt="latex"))
    return(V, VA, VP)

(V1, VA1, VP1)=get_V(1e3)
(V5, VA5, VP5)=get_V(5e3)
(V9, VA9, VP9)=get_V(9e3)
```

B Source code circuit B

```
import numpy as np
from tabulate import tabulate

def get_V(f):
    w=2*np.pi*f
    Zr=1e3
    Zc1=1/(complex(0,47e-9)*w)
    Zc2=1/(complex(0,100e-9)*w)
    Zc3=1/(complex(0,100e-9)*w)
    Zl=complex(0,10e-3)*w
    V = 0.192

    G2 = 2/Zr + 1/Zc1
    G3 = 1/Zc1 + 1/Zc2 + 1/Zl
    G4 = 1/Zc3 + 1/Zl + 1/Zr
    G23 = -1/Zc1
    G24 = -1/Zr
    G34 = -1/Zl

    I1 = 0
    I2 = V / (Zc2)
    I3 = 0
    G = np.array([[G2, G23, G24],
                  [G23, G3, G34],
                  [G24, G34, G4]])
    I = np.array([[I1],
                  [I2],
                  [I3]])

    V = np.matmul(np.linalg.inv(G), I)
    VP = np.angle(V, True)
    VA = np.abs(V)
    IR22 = (V[0]-V[2])/Zr
    VA = np.append(VA, np.abs(IR22))
    VP = np.append(VP, np.angle(IR22))
    headers = [str(f)+' Hz', 'Magnitude', 'Phase']
    first_col = ['Node 2', 'Node 3', 'Node 4', 'I of R_22']
    table1 = np.column_stack((first_col, VA, VP))
    print(tabulate(table1, headers=headers,tablefmt='latex'))
    return(V, VA, VP)

(V1, VA1, VP1)=get_V(1e3)
(V5, VA5, VP5)=get_V(5e3)
(V9, VA9, VP9)=get_V(9e3)
```