Two-port Networks
Laboratory V

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1 Goal of the exercise

The aim of this exercise is to familiarize with experimental methods of determining two-port network parameters by measuring output and input voltages and currents, and then compare theoretical and experimental results

2 Two-port networks

Two-port network can be regarded as "black box" with its properties specified by a characteristic matrix. It allows us to simplify large circuits with those "black boxes" and move to higher level of abstraction, instead of using simple passive element now we can use complex circuits but seeing them as simple 2×2 matrix similar to one in eq. (1).

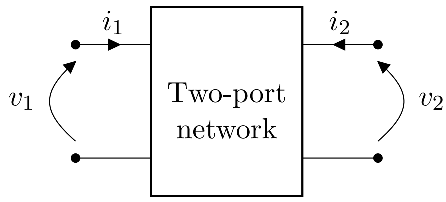


Figure 1: Two-port diagram

$$\mathbf{Z}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{1}$$

Where z_{11}, z_{12}, z_{21}, z_{22} are equal by definition

$$z_{11} = \frac{v_1}{i_1}|_{i_2=0} \quad z_{12} = \frac{v_1}{i_2}|_{i_1=0} \quad z_{21} = \frac{v_2}{i_1}|_{i_2=0} \quad z_{22} = \frac{v_2}{i_2}|_{i_1=0}$$

Characteristic matrix can have form of impedance $\mathbf{Z}$, admittance $\mathbf{Y}$ or chain(ABCD) matrix and many more forms which we are not going to use during this exercise. impedance $\mathbf{Z}$ matrix can be easily transformed into admittance $\mathbf{Y}$ and chain(ABCD) matrix using following formulas.

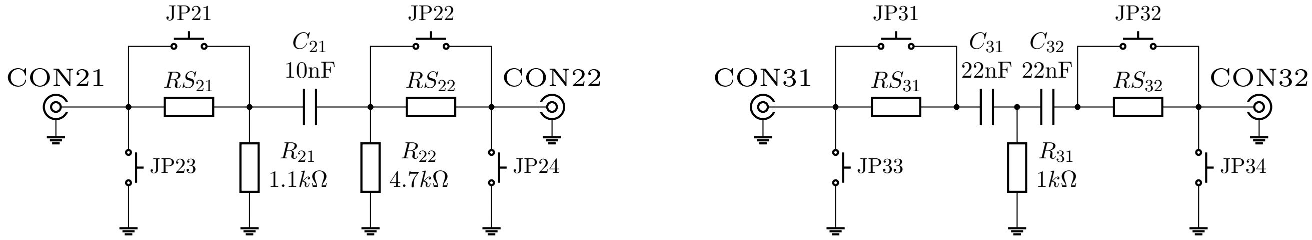
$$\mathbf{Y} = \mathbf{Z}^{-1} = \begin{bmatrix} \frac{z_{11}}{\det \mathbf{Z}} & \frac{-z_{12}}{\det \mathbf{Z}} \\ \frac{-z_{21}}{\det \mathbf{Z}} & \frac{z_{22}}{\det \mathbf{Z}} \end{bmatrix} \tag{2}$$

$$\mathbf{Z} = \mathbf{Y}^{-1} = \begin{bmatrix} \frac{y_{11}}{\det \mathbf{Y}} & \frac{-y_{12}}{\det \mathbf{Y}} \\ \frac{-y_{21}}{\det \mathbf{Y}} & \frac{y_{22}}{\det \mathbf{Y}} \end{bmatrix} \tag{3}$$

$$\mathbf{A} = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\det \mathbf{Z}}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} -\frac{y_{11}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\det \mathbf{Y}}{y_{21}} & -\frac{y_{22}}{y_{21}} \end{bmatrix} \tag{4}$$

Two-port networks, similar to simple passive components, can be in parallel and series connection, but also in chain configuration which is unique to two-port networks. Parallel connection can be represented by sum of admittance characteristics, series connection by sum of impedance characteristics and chain connection by multiplication of chain(ABCD) characteristics.

3 Course of measurements



(a) Circuit no. 2

(b) Circuit no. 3

Figure 2: Measured circuits

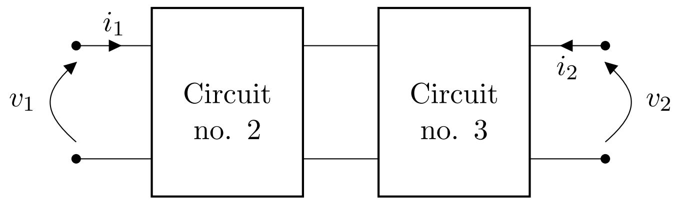


Figure 3: Chain configuration

4 Theoretical calculations

First step in our analytical solution is simplifying Circuits by removing connection used to measured voltages and currents.

Some characteristics matrices of two-port network can be solved using mesh or nodal analysis by inspection, and both of our circuit can be solved using one of those methods. Usually mesh or nodal method is quicker and easier way of finding characteristics than using definition, that is why we opted for this method instead of definition.

4.1 Circuit no. 2

In order to solve circuit no. 2 we added current sources i_1 and i_2 between each port, and marked nodes v_1 and v_2 then following rules of nodal analysis by inspection we constructed $\mathbf{Yv} = \mathbf{i}$ matrix equation where $\mathbf{Y}$ is two-port characteristics in admittance form.

$$\begin{bmatrix} \frac{1}{R_{21}} + j\omega C_{21} & -j\omega C_{21} \\ -j\omega C_{21} & \frac{1}{R_{22}} + j\omega C_{21} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{5}$$

Knowing admittance characteristics we can plugin values of each component and transform it into impedance and chain characteristics using formulas (3) and (4)

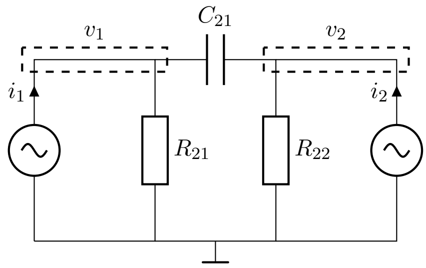


Figure 4: Simplified circuit no. 2

$$\mathbf{Y} = \begin{bmatrix} 0.9091 + 0.0700i & 0.0000 + 0.0700i \\ 0.0000 - 0.0700i & 0.2128 + 0.0700i \end{bmatrix} \cdot 10^{-3}$$
$$\mathbf{Z} = \begin{bmatrix} 1.1174 - 0.0967i & -0.1378 - 0.3223i \\ 0.1378 + 0.3223i & 4.3232 - 1.4677i \end{bmatrix} \cdot 10^{-3}$$
$$\mathbf{A} = \begin{bmatrix} 0.0001 - 0.0003i & 0.0000 - 1.4286i \\ 0.0000 - 0.0000i & 0.0001 - 0.0013i \end{bmatrix} \cdot 10^{-4}$$

(a) frequency = 7 kHz

$$\mathbf{Y} = \begin{bmatrix} 0.9091 + 0.0700i & 0.0000 + 0.0700i \\ 0.0000 - 0.0700i & 0.2128 + 0.0700i \end{bmatrix} \cdot 10^{-3}$$
$$\mathbf{Z} = \begin{bmatrix} 1.1174 - 0.0967i & -0.1378 - 0.3223i \\ 0.1378 + 0.3223i & 4.3232 - 1.4677i \end{bmatrix} \cdot 10^{-3}$$
$$\mathbf{A} = \begin{bmatrix} 0.0001 - 0.0003i & 0.0000 - 1.4286i \\ 0.0000 - 0.0000i & 0.0001 - 0.0013i \end{bmatrix} \cdot 10^{-4}$$

(b) frequency = 13 kHz

Figure 5: Circuit no. 2 characteristics

4.2 Circuit no. 3

In circuit no. 3 we added voltage sources v_1 and v_2 between each port, and marked loops i_1 and i_2 in clockwise and counter-clockwise directions respectively, then according to rules of mesh analysis by inspection we constructed $\mathbf{Zi} = \mathbf{v}$ matrix equation, where $\mathbf{Z}$ is two-port characteristics in impedance form.

$$\begin{bmatrix} \frac{1}{j\omega C_{31}} + R_{31} & R_{31} \\ R_{31} & \frac{1}{j\omega C_{32}} + R_{31} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{6}$$

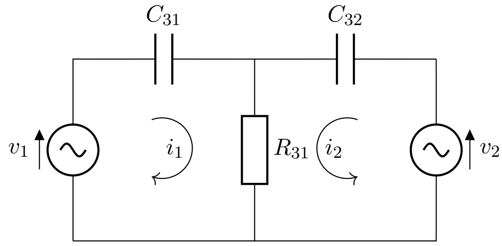


Figure 6: Simplified circuit no. 3

Knowing impedance characteristics we can plugin values and transform it into admittance and chain characteristics using formulas (2) and (4)

$$\mathbf{Y} = \begin{bmatrix} 0.9091 + 0.0700i & 0.0000 + 0.0700i \\ 0.0000 - 0.0700i & 0.2128 + 0.0700i \end{bmatrix} \cdot 10^{-3}$$
$$\mathbf{Z} = \begin{bmatrix} 1.1174 - 0.0967i & -0.1378 - 0.3223i \\ 0.1378 + 0.3223i & 4.3232 - 1.4677i \end{bmatrix} \cdot 10^{-3}$$
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(a) frequency = 13 kHz

$$\mathbf{Y} = \begin{bmatrix} 0.9091 + 0.0700i & 0.0000 + 0.0700i \\ 0.0000 - 0.0700i & 0.2128 + 0.0700i \end{bmatrix} \cdot 10^{-3}$$
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$$\mathbf{A} = \begin{bmatrix} 0.0001 - 0.0003i & 0.0000 - 1.4286i \\ 0.0000 - 0.0000i & 0.0001 - 0.0013i \end{bmatrix} \cdot 10^{-4}$$

(b) frequency = 15 kHz

Figure 7: Circuit no. 3 characteristics

4.3 Chain connection

With ABCD characteristics of each circuit at 13kHz calculated we can get characteristics for chain connection of these two-ports by multiplying $\mathbf{A_2}$ with $\mathbf{A_3}$, which are ABCD characteristics for circuit no. 2 and 3 respectively.

$$\mathbf{A} = \mathbf{A_2}\mathbf{A_3} \tag{7}$$

$$\mathbf{Y} = \begin{bmatrix} 0.9091 + 0.0700i & 0.0000 + 0.0700i \\ 0.0000 - 0.0700i & 0.2128 + 0.0700i \end{bmatrix} \cdot 10^{-3}$$

$$\mathbf{Z} = \begin{bmatrix} 1.1174 - 0.0967i & -0.1378 - 0.3223i \\ 0.1378 + 0.3223i & 4.3232 - 1.4677i \end{bmatrix} \cdot 10^{-3}$$

$$\mathbf{A} = \begin{bmatrix} 0.0001 - 0.0003i & 0.0000 - 1.4286i \\ 0.0000 - 0.0000i & 0.0001 - 0.0013i \end{bmatrix} \cdot 10^{-4}$$

Figure 8: Equivalent characteristics

5 Comparison

6 Conclusions