Task - 1

A complex-valued sequence $x_e(n)$ is called *conjugate-symmetric* if $x_e(n) = x_e^*(-n)$ and a complex-valued sequence $x_o(n)$ is called *conjugate-antisymmetric* if $x_o(n) = -x_o^*(-n)$. Then, any arbitrary complex-valued sequence $x(n)$ can be decomposed into $x(n) = x_e(n) + x_o(n)$ where $x_e(n)$ and $x_o(n)$ are given by

$$x_e(n) = \frac{1}{2}\left[x(n) + x^*(-n)\right] \qquad \text{and} \qquad x_o(n) = \frac{1}{2}\left[x(n) - x^*(-n)\right]$$

respectively.

1. Modify the **evenodd** function discussed in the text so that it accepts an arbitrary sequence and decomposes it into its conjugate-symmetric and conjugate-antisymmetric components by implementing (2.1).
2. Decompose the following sequence:

$$x(n) = 10\exp([-0.1 + \jmath 0.2\pi]n), \quad 0 \leq n \leq 10$$

into its conjugate-symmetric and conjugate-antisymmetric components. Plot their real and imaginary parts to verify the decomposition. (Use the `subplot` function.)

The operation of *signal dilation* (or *decimation* or *down-sampling*) is defined by

$$y(n) = x(nM)$$

in which the sequence $x(n)$ is down-sampled by an integer factor M. For example, if

$$x(n) = \{\ldots, -2, 4, \underset{\uparrow}{3}, -6, 5, -1, 8, \ldots\}$$

then the down-sampled sequences by a factor 2 are given by

$$y(n) = \{\ldots, -2, \underset{\uparrow}{3}, 5, 8, \ldots\}$$

Task - 2

1. Develop a MATLAB function `dnsample` that has the form

```
function [y,m] = dnsample(x,n,M)
% Downsample sequence x(n) by a factor M to obtain y(m)
```

   to implement the above operation. Use the indexing mechanism of MATLAB with careful attention to the origin of the time axis $n = 0$.
2. Generate $x(n) = \sin(0.125\pi n), -50 \leq n \leq 50$. Decimate $x(n)$ by a factor of 4 to generate $y(n)$. Plot both $x(n)$ and $y(n)$ using `subplot`, and comment on the results.
3. Repeat the above using $x(n) = \sin(0.5\pi n), -50 \leq n \leq 50$. Qualitatively discuss the effect of down-sampling on signals.

Task - 3

The following finite-duration sequences are called *windows* and are very useful in DSP:

$$\text{Rectangular: } \mathcal{R}_M(n) = \begin{cases} 1, & 0 \leq n < M \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Hanning: } \mathcal{C}_M(n) = 0.5\left[1 - \cos\frac{2\pi n}{M-1}\right]\mathcal{R}_M(n)$$

$$\text{Triangular: } \mathcal{T}_M(n) = \left[1 - \frac{|M-1-2n|}{M-1}\right]\mathcal{R}_M(n)$$

$$\text{Hamming: } \mathcal{H}_M(n) = \left[0.54 - 0.46\cos\frac{2\pi n}{M-1}\right]\mathcal{R}_M(n)$$

For each of these windows, determine their DTFTs for $M = 10, 25, 50, 101$. Scale transform values so that the maximum value is equal to 1. Plot the magnitude of the normalized DTFT over $-\pi \leq \omega \leq \pi$. Study these plots and comment on their behavior as a function of M.

Task - 4

Using the frequency-shifting property of the DTFT, show that the real part of $X(e^{j\omega})$ of a sinusoidal pulse

$$x(n) = (\cos\omega_o n)\mathcal{R}_M(n) \qquad \text{where } \mathcal{R}_M(n) \text{ is the rectangular pulse}$$

is given by

$$\begin{aligned} X_{\mathrm{R}}(e^{j\omega}) &= \frac{1}{2}\cos\left\{\frac{(\omega-\omega_0)(M-1)}{2}\right\}\frac{\sin\{(\omega-\omega_0)M/2\}}{\sin\{(\omega-\omega_0)/2\}} \\ &\quad + \frac{1}{2}\cos\left\{\frac{(\omega+\omega_0)(M-1)}{2}\right\}\frac{\sin\{[\omega-(2\pi-\omega_0)]M/2\}}{\sin\{[\omega-(2\pi-\omega_0)]/2\}} \end{aligned}$$

Compute and plot $X_{\mathrm{R}}(e^{j\omega})$ for $\omega_o = \pi/2$ and $M = 5, 15, 25, 100$. Use the plotting interval $[-\pi, \pi]$. Comment on your results.

The **deconv** function is useful in dividing two causal sequences. Write a MATLAB function **deconv_m** to divide two noncausal sequences (similar to the **conv** function). The format of this function should be

```
function [p,np,r,nr] = deconv_m(b,nb,a,na)
% Modified deconvolution routine for noncausal sequences
% function [p,np,r,nr] = deconv_m(b,nb,a,na)
%
%  p = polynomial part of support np1 <= n <= np2
% np = [np1, np2]
%  r = remainder part of support nr1 <= n <= nr2
% nr = [nr1, nr2]
%  b = numerator polynomial of support nb1 <= n <= nb2
% nb = [nb1, nb2]
%  a = denominator polynomial of support na1 <= n <= na2
% na = [na1, na2]
%
```

Task - 5

Check your function on the following operation:

$$\frac{z^2 + z + 1 + z^{-1} + z^{-2} + z^{-3}}{z + 2 + z^{-1}} = (z - 1 + 2z^{-1} - 2z^{-2}) + \frac{3z^{-2} + 3z^{-3}}{z + 2 + z^{-1}}$$

Task - 6

A stable system has four zeros and four poles as given below:

$$\text{zeros: } \pm 1, \pm j1 \qquad \text{Poles: } \pm 0.9, \pm j0.9$$

It is also known that the frequency response function $H(e^{j\omega})$ evaluated at $\omega = \pi/4$ is equal to 1, i.e.,

$$H(e^{j\pi/4}) = 1$$

1. Determine the system function $H(z)$ and indicate its region of convergence.
2. Determine the difference equation representation.
3. Determine the steady-state response $y_{ss}(n)$ if the input is $x(n) = \cos(\pi n/4)u(n)$.
4. Determine the transient response $y_{tr}(n)$ if the input is $x(n) = \cos(\pi n/4)u(n)$.