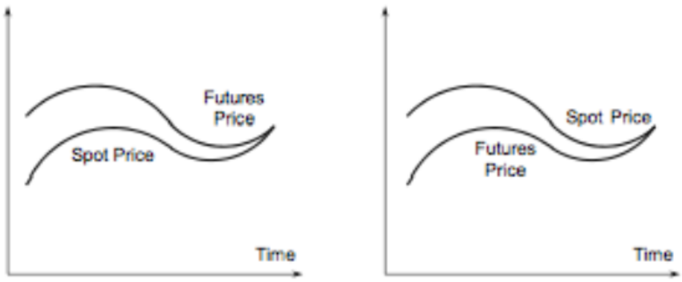
Lecture notes - all lectures on derivatives

Foundations of Finance (Australian National University)

Forwards and Futures Contacts

- Payoff from a <u>long</u> position $S_T - F$
- Payoff from a <u>short</u> position $F - S_T$
- Initial value of the contract is zero
- Forward contracts are similar to futures except they trade in the over-the- counter market
    - Future contracts are exchange traded
    - DRAW GRAPHS

– Convergence of Futures to Spot:



(a) (b)

- If the futures price is above the spot price during the delivery period, traders will have an arbitrage opportunity, will short the futures contract, buy the asset and make delivery- this will force convergence
- If futures price is below the spot price during the delivery period, companies wanting to acquire the asset would enter into a long position in the futures contract and wait for delivery to be made

Key Features of Forward/Futures Contracts:

FORWARDS	FUTURES
Private contract between 2 parties	Exchange traded
Non-standard contract	Standard contract
Usually 1 specified delivery date	Range of delivery dates
Settled at maturity	Settled daily
Delivery or final cash settlement usually occurs	Contract usually closed out prior to maturity

Closing out of a futures position

- Involves entering into an offsetting trade
    - If you hold a long position in wool, to close it out you would take a short position on wool on the same quantity with the same maturity date
- Terms of the contact will stipulate whether there is physical or cash settlement
- <u>Open interest:</u> The total number of contacts outstanding
- <u>Settlement price:</u> the price just before the final bell each day
- <u>Volume of trading:</u> the number of trades in 1 day

Options Contacts

- Payoff to a long call: $S_T - X$
- Payoff to a short call: $X - S_T$
- Payoff to a long put: $X - S_T$
- Payoff to a short put: $S_T - X$

Types of Market Traders

1. Hedgers: want to avoid exposure to adverse movements in the price of an asset
2. Speculators: take a position in the market betting that either the price of an asset will go up or down
3. Arbitrageurs: attempt to lock in a riskless profit by simultaneously entering into transactions in two or more markets

$$F = S(1 + r + q)^T$$

When $F > S(1 + r + q)^T$ BUY LOW, SELL HIGH

- Sell futures
- Borrow and incur r_f
- Buy underlying (S)
- Incur cost of carry (q)

Consumption Vs. Investment Assets

- Investment assets are assets held by significant numbers of people purely for investment purposes
  - Eg. Stock, bonds, gold and silver
- Consumption assets are assets held primarily for consumption and not usually for investment purposes
  - Commodities such as: copper, oil or pork bellies
- We can use arbitrage arguments to determine the forward and futures price of an investment asset from its spot price and other observable market variables- cannot do this for consumption assets

Short Selling

- Selling an asset that is not owned
- The client you borrowed from should be no worse off as a result of lending you their shares
- The investor (the person who has shorted the asset) benefits if the prices fall, as they sell the asset for a higher price than what they buy it back for
- Investor is required to maintain a margin account with the broker
  - The initial margin is required so that possible adverse movements (increases) in the price of the asset that is being shorted is covered
  - Margin account consists of cash or marketable securities with the broker to guarantee that the investor will not walk away from the short position if the share price increases

Interest Rates

- Continuous compounding: EG: $100e^{RT}$
- To discount: $100e^{-RT}$

- Continuous to effective:

$$e^{R}-1$$

$$e^{0.10}-1$$

$$2.71828\ldots^{0.10}-1 = 10.51\%$$

Assumptions regarding market participants

1. They are subject to no transaction costs when they trade
2. They are subject to the same tax rate on all net trading profits
3. They can borrow money at the same risk-free rate of interest as they can lend money
4. They take advantage of arbitrage opportunists as they occur

Arbitrage Relationship Between Spot and Forward Contracts

Position	Initial Cash Flow	Terminal Cash Flow
Borrow and incur cost of carry	S_0	$-S_0e^{rT}$
Buy one unit of commodity	$-S_0$	S_T
Enter 6-month forward sale	0	$F-S_T$
Net portfolio value	***0***	$F-S_0e^{rT}$

- In general if:

$$F_0 > S_0e^{rT}$$

  - Arbitrageurs can make a riskless profit from buying the asset and entering into a short forward contract on the asset. This strategy is financed by borrowing funds at the risk free-rate if interest.

$$F_0 < S_0e^{rT}$$

  - Arbitrageurs can make a riskless profit by shorting the asset and entering into a long forward contract. The excess funds are invested at the risk-free rate of interest until they are needed to buy back the asset.

Forwards and Futures Contract Prices on Assets with Known Incomes

- Consider a forward contract on an investment asset that will provide a perfectly predictable cash income to the holder
  - EG stocks paying known dividend yields and coupon- bearing bonds

$$F = (S - D)e^{rT}$$

  - Where D is the present value of the income

- Must find the present value of D before substituting into the equation

Forward and future contract prices on assets with known yield

- A yield implies that the known income is expressed as a percentage of the asset's price at the time the income is paid
- We define d as the average yield per annum on an asset during the life of a forward contact with continuous compounding

$$F_0 = S_0 e^{(r-d)T}$$

→ Converting nominal to continuous $R_c = m \ln(1 + \frac{R_m}{m})$

Futures and Forwards on Currencies

$$F_0 = S_0 e^{(r-r_f)T}$$

Forwards and Futures on Commodities

- In the absence of storage costs and income the forward price of a commodity is an investment asset given by:

$$F_0 = S_0 e^{rT}$$

- If there are storage costs, Q is the present value of all of the storage costs less all incomes during the life of the forward contract, and the forward price is given by:

$$F_0 = (S_0 + Q)e^{rT}$$

- If storage costs and income are given as a percentage, then q is the percentage storage costs less the percentage income during the life of the forward contract

$$F_0 = S_0 e^{(r+q)T}$$

- Consumption Commodities:
  - Consumption assets rather than investment assets usually provide no income but can be subject to significant storage costs
  - Individuals and companies who keep such a commodity in inventory do so because of its consumption value, not because of its value as an investment
    - Reluctant to sell the commodity and buy forward contacts because forward contracts cannot be consumed
    - There is therefore nothing to keep the previous equations holding
  - As a result:
    - Due to the high storage costs of consumption commodities, Q, is the present value of all of the storage costs

$$F_0 \leq (S_0 + Q)e^{rT}$$

and..

$$F_0 \leq S_0 e^{(r+q)T}$$

  - For this reason we do not have equality in the formula's on the previous slide is because users of a consumption commodity may

feel that ownership of the physical commodity provides benefits that are not obtained by holders of futures contracts
    - The benefits from holding the physical asset is referred to as the convenience yield
  - Can re-write the equations on the previous slide, where y, the convenience yield simply measures the extent to which the LHS is less than the RHS

$$F_0 e^{yT} = (S_0 + Q)e^{rT}$$

$$F_0 = S_0 e^{(r+q-y)T}$$

Valuing Forward contracts

- The value of a long forward contact

$$f = (F - K)e^{-rT}$$

- The value of a short forward contract

$$f = (K - F)e^{-rT}$$

$$F_0 = 25e^{0.1\times 0.5} = \$26.28$$

$$f = (26.28 - 24)e^{-0.1\times 0.5} = \$2.17$$

Forward vs Future Prices

- When interest rates are uncertain forward and future prices and slightly different in theory:
  - A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
    - Due to the person in the long position in a futures contract receiving an immediate gain because of daily settlement
    - Positive correlation indicates that interest rates will also likely to have risen, therefore the gain will be invested at a higher than average rate
  - A strong negative correlation implies the reverse

Delivery:

- In a futures contract, the party in the short position has the right to choose to deliver the asset at any time during a certain period
- The person in the short position has to give at least a few days notice of their intention to deliver

Hedging

- A short hedge involves a short position in a futures contract
  - It is appropriate when the hedger already owns an asset and expects to sell it at some point in the future
  - Allows them to lock in the price they will receive
- A long hedge involves taking a long position in the futures contract
  - Appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in price

- Arguments in favour:
    - Companies should focus on the main business they are in and take steps to minimize risks arising from interest rates, exchange rates and other market variables
    - By hedging, they avoid adverse movements such as sharp rises in the price of a commodity
- Arguments against hedging:
    - Shareholders are usually will diversified and can make their own hedging decisions
    - It may increase risk to hedge when competitors do not
    - Explaining a situation where there is a loss on the hedge and a gain on the underlying can be difficult

Basis Risk

- Hedges are not always perfect and straightforward
    - Asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract
    - The hedger may not be certain of the exact date the asset will be bought or sold
    - The hedge may require the futures contract to be closed out before its delivery month
- What is basis risk?
    - If the asset to be hedged and the asset underlying the futures contract are the same, the basis risk should be zero at the expiration of the futures contract
    - Prior to expiration, the basis may be positive or negative
    - When the spot price increases by more than the futures price, the basis increases
        - Strengthening of the basis
    - When the futures price increases by more than the spot price, the basis declines
        - Weakening of the basis

$$Basis = \ Spot\ price\ of\ asset\ to\ be\ hedged - futures\ price\ of\ contract\ used$$

Basis risk with a long hedge:

– Suppose that:

F_1: Initial Futures Price
F_2: Final Futures Price
S_2: Final Asset Price

- You hedge the future sale of an asset by entering into a short futures contact
- Cost of Asset = $S_2 - (F_2 - F_1) = F_1 + Basis$
- Price of realized= $S_2 + (F_1 - F_2) = F_1 + Basis$
- One key factor affecting basis risk is the choice of the futures contract to be used for hedging
    - The choice has two components:
        - Choose a delivery month that is as close as possible to, but later than, the end of the life of the hedge

- When there is no futures contract on the asset being hedged, choose the contact whose futures price is most highly correlated with the asset price

Cross hedging:

- Occurs when the asset underlying the futures contract is different to the asset whose price is being hedged
    - EG: an airline company may be concerned about the future price of fuel- however, there are no futures contracts on aviation fuel, the company choose to use heating oil futures on contracts to hedge its exposure

Optimal Hedge Ratio

$$h = \rho \frac{\sigma_S}{\sigma_F}$$

Where:

- σ_S: is the standard deviation, the change in the spot price during the hedging period
- σ_F: is the standard deviation, the change in the futures price during the hedge period
- ρ: is the coefficient of correlation between S and F

Hedging Using Index Futures

- Stock index futures can be used to hedge an equity portfolio
- To hedge the risk in a portfolio the number of contacts that should be shorted

$$\beta \frac{P}{A}$$

- Where P is the value of the portfolio and A is the value of the assets underlying one futures contract
- Reasons for using index futures to hedge an equity portfolio include:
    - Desire to be out of the market for a period of time (Hedging may be cheaper than selling the portfolio and buying it back)
    - Desire to hedge systematic risk (Appropriate when you feel that you have picked stocks that will outperform the market)

$$(\beta - \beta^*)\frac{P}{A} = (1.5 - .75)\frac{5m}{87500} = 43$$

- If answer is negative we must go long
- We can use a series of future contracts to increase the life of a hedge
- Each time we switch from 1 futures contact to another we incur a type of basis risk

Interest rate contacts and swaps

- Credit risk: risk that there will be a default by the borrower so that the interest and principal are not paid to the lender as promised
- **Treasury rates:** essentially risk- free
    - The rate that an investor earns on Treasury bills and bonds

- **Libor:** London interbank offered rate
    - Often used as a proxy for the floating interest rate
- **Repo Rates:**
    - A repo or repurchase agreement is a contract where an investment dealer who owns securities agrees to sell them to another company now and buy them back later at a slightly higher price
    - The other company is providing a loan to the investment dealer
    - The difference between the price at which securities are sold and the price at which they are repurchased represents the interest rates
        - Referred to as the repo rate

Bond Pricing

- **Zero rates:**
    - The n- year zero-coupon investment rate is the rate of interest earned on an investment that starts today and lasts for n years
    - No coupons paid
- **Par yield:**
    - The par yield for a certain bond is the coupon rate that causes the bond price to equal its par value (DETERMINING TREASURY RATES)

Forward Rates

- A forward rate is an interest rate which is fixed for a future transaction
- By correct combination of sport transactions we can fix a forward interest rate today
- Rather than using a forward or futures contract, a correct combination of spot rates today can lock in an interest rate in the future
- FORWARD RATES CALCULATIONS SEE SLIDES

– The discrete forward interest rate $r_{x,y}$ on a security that commences in year x and matures in year y can be found by combining spot rates:

$$(r_{x,y})^1 = [(1+r_{0,y})^y / (1+r_{0,x})^x] - 1$$

- Example: If the 3-year spot is 6% and 2-year spot is 5% then $(1+r_{2,3})^1 = (1.06)^3 / (1.05)^2 = 1.0803 - 1 = 0.0803$.

- Simple method:

$$\text{Forward rate} = \frac{R_x T_x - R_y T_y}{T_x - T_y}$$

- Example: 10.8% for 3 years, 11% for 4 years, then forward rate:

$$\text{Forward rate} = \frac{R_x T_x - R_y T_y}{T_x - T_y}$$
$$= \frac{0.11x4 - 0.108x3}{4-3}$$
$$= 0.116$$

Forward Rate Agreements

- A forward rate agreement (FRA) is an agreement that a certain rate will apply to a certain principal during a certain future time period
  - FRAs are over the counter interest rate derivatives
  - Two parties agree to guarantee each other a given interest rate at a future date
  - A notional principal is used but not exchanged
  - When the settlement date arrives on party compensates the other party
- A FRA is equivalent to an agreement where interest at a predetermined rate, R_K is exchanged for interest at the market rate

Interest Rate Swaps

- An agreement where a floating rate is exchanged for a fixed rate or vice versa
- Benefits:
  - They can change your exposure from floating to fixed or fixed to floating
  - They are able to reduce the costs of borrowing
  - SEE SLIDES FOR EXAMPLES

Valuation of an interest rate swap

- When a swap is first entered into it has a value of zero. However, as time proceeds, the swap may obtain a positive or negative value

$$V_{swap} = B_{fl} - B_{fix}$$

- SEE SLIDES

Credit Risk

- Costless to enter
- Credit risk is risk resulting from uncertainty in a counterparty's ability or willingness to meet contractual obligations
- A financial intermediary has credit risk exposure from a swap only when the value of the swap to the financial intermediary is positive

Options Basics

- **At the money:** The strike price equals the asset price
- **In the money:**
  - Call- if the strike price is less than the asset price
  - Put- if the strike price is greater than the asset price
- **Out of the money:**
  - Call- if the strike price is greater than the asset price
  - Put- if the strike price is less than the asset price

Asset Underlying Options

- **Stock options:** most trading on stock options is on exchanges, and they are American in nature
- **Foreign currency options:** mostly occur in over the counter market, although there is some exchange trading
  - Can be either European or American in nature
- **Index options:** traded both OTC and on exchanges

Dividends and Stock Splits

- Suppose you own N options with a strike price of X
  - No adjustments are made to the option terms for cash dividends
  - When there is an n-for-m stock split
    - Strike price is reduced to $\frac{mX}{n}$
    - The number of options is increased to $\frac{nN}{m}$
- Stock dividends are handled in a manner similar to stock splits

Extended Option Topics

- **Warrants**
  - Options that are issues (or written) by a corporation or financial institution
  - The number of warrants outstanding is determined by the size of the original issue and changes only when they are exercised or when they expire
  - Warrants are traded in the same way stocks are
  - The issuer settles up with the holder when a warrant is exercised
  - When call warrants are issues by a corporation on its own stock, exercise will lead to new treasury stock being issued.
- **Executive stock options**:
  - Options issued by a company to its executives as a performance incentive
  - When the option is exercised the company issues more stock
  - Options are usually out of the money when issued to incentivize executives to increase the share price of the company
  - They usually become vested after a period of time and cannot be sold by the executive
  - Often last for as long as 10 or 15 years
- **Convertible bonds**

- Regular bonds than can be exchanged for equity at certain times in the future according to a predetermined exchange ratio
- Very often a convertible is callable. the call provision is a way in which the issuer can force conversion at a time earlier than the holder might otherwise choose

Option Bounds

- No matter what happens, the option can never be worth more than the stock
  - The stock price is an upper bound to the option price
  $$c \leq S_o$$
  - If this relationship did not hold an arbitrageur could easily make a riskless profit by buying the stock and selling the call
- No matter how low the stock price becomes, the option can never be worth more than X
$$p \leq X$$
- For European options, we know that at maturity the option cannot be worth more than the present value of A today, as it cannot be early exercised, hence
$$p \leq Xe^{-rT}$$
- The lower bound for the price of a European call option on a non-dividend paying stock is:
$$c \geq \max(S_0 - Xe^{-rT}, 0)$$
- We can prove this by considering the following two portfolios:
  - Portfolio A
    - One euro call option
    - An amount of cash equal to Xe^{-rT}
  - Portfolio B
    - One share
- The lower bound for the price of a euro put options on a non-dividend paying stock is:
$$p \geq \max(Xe^{-rT} - S_0, 0)$$
- We can prove this by considering the following two portfolios:
  - Portfolio C
    - One euro put option
    - One share
  - Portfolio D
    - An amount of cash equal to Xe^{-rT}

Put-Call Parity

- Both are worth Max (S_T, X) at the maturity of the options
- They must therefore by worth the same today. This means:
$$c + Xe^{-rT} = p + S_0$$

Early exercise of American Options

- Usually there is some chance than an American options will be exercised early
- An exception to this rule as an American call on a non dividend paying stock. These options should never be exercised early:

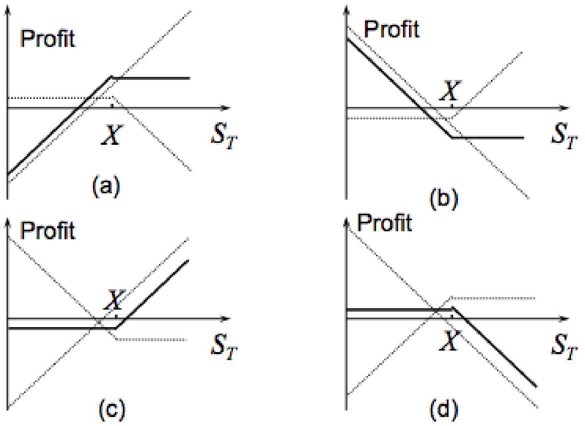
- No income is sacrificed
- We delay paying the strike price
- Holding the call provisions insurance against the stock price falling below the strike price

The impact of dividends on lower bounds to option prices

$$c \geq \max(S_0 - PV\ (D) - Xe^{-rT}, 0)$$
$$p \geq \max(PV(D) + Xe^{-rT} - S_{0,}, 0)$$

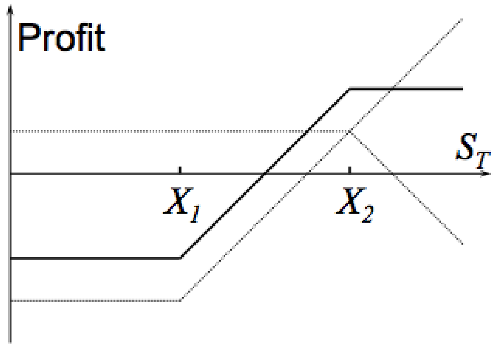
- American options; $D = 0$,
$$S_0 - X < C - P < S_0 - Xe^{-rT}$$
- European options; $D > 0$,
$$c + D + Xe^{-rT} = p + S_0$$
- American options; $D > 0$,
$$S_0 - D - X < C - P < S_0 - Xe^{-rT}$$

Positions in an Option & the Underlying Asset



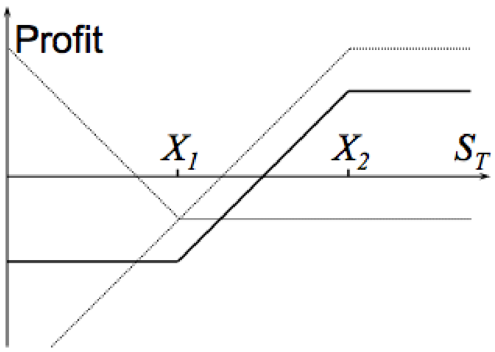
- Figure A- **Covered call option**
  - Selling a call and simultaneously buying the underlying stock
  - The long stock position on the stock covers or protects the investor from the payoff on the short call that becomes necessary is there is a sharp rise in the stock price
    - Used when expecting a price rise
- Figure B- illustrates the reverse of a covered call
  - It consists of buying a call option (a long position) and simultaneously selling the underlying stock
  - It is know as a synthetic long put and is used when expecting the price to drop
- Figure C- illustrates a protective put
  - Involves selling a put option and simultaneously selling the underlying stock
  - It is known as a synthetic short call and is used when expecting the stock price to drop

Bull spread using calls



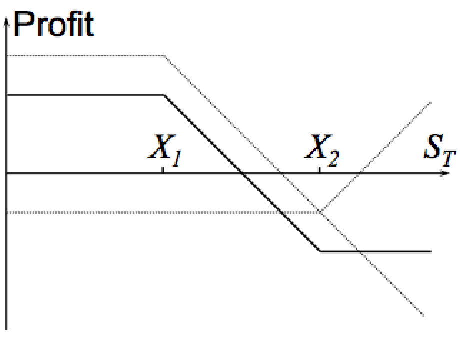
- Created by buying a call option on a stock with a certain strike price and selling a call option on the same stock with a higher strike price
    - Both options have the same expiration date
    - Because a call option price always decreases as the strike price increases, the value of the option sold is always less than the value of the option bought therefore requiring an initial investment
- A bull spread strategy limits both the investors upside as well as downside risk but they hope the stock price will increase to provide them with a positive payoff

Bull spread using puts



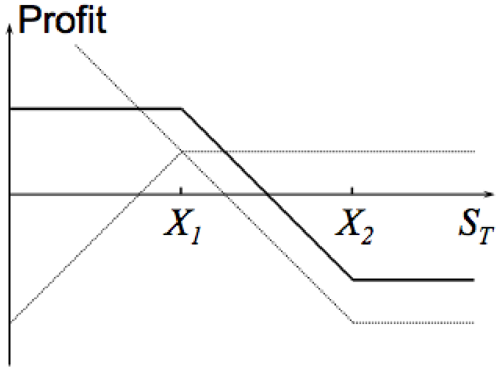
- Created by buying a put with a low strike price and selling a put with a high strike price
- Provides a positive cash flow to the investor up front

Bear spread using calls



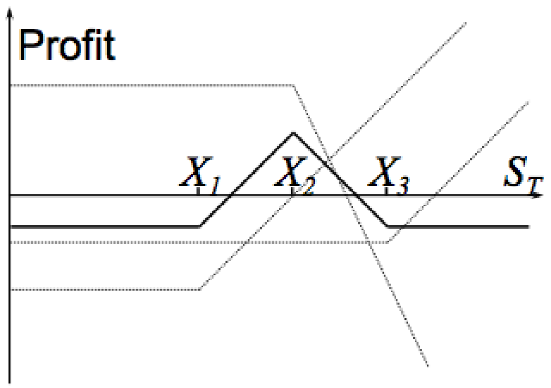
- Buying a call option with a high strike price and selling a put with a high strike price and selling a call with a low strike price, leading to an initial cash inflow
- The investor is hoping that the price of the stock will decrease to give them a positive payoff

Bear Spread Using Puts



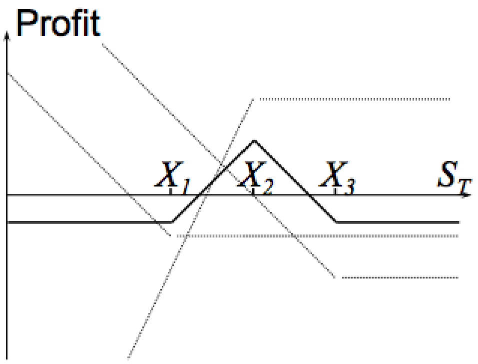
- Buying a put with one strike price and selling a put with a lower strike price
- Using puts involves an initial cash outflow because the price of the put is sold is less than the price of the put purchased

Butterfly spread using calls



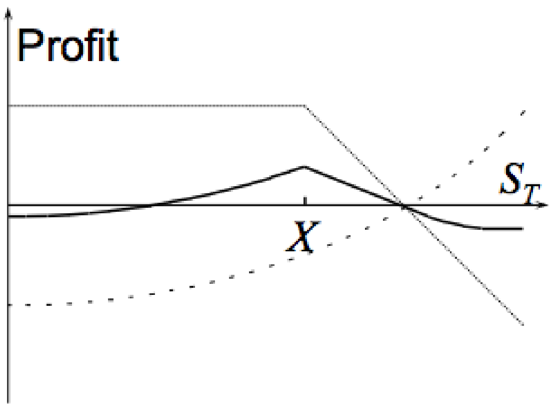
- Involves a position in three different strike prices
  - Long call- low X price
  - Long call- high X price
  - Two short call with X price half way between the other two
    - Generally, the X price of the two call options which are sold, is relatively close to the current stock price
- Butterfly spread leads to a profit if the stock price does not move by much, and gives rise to a small loss if there is a significant movement in the stock price in either direction
- It is an appropriate strategy for an investor who feels that large stock price movements are unlikely
- Strategy requires a small initial outlay

Butterfly spread using puts



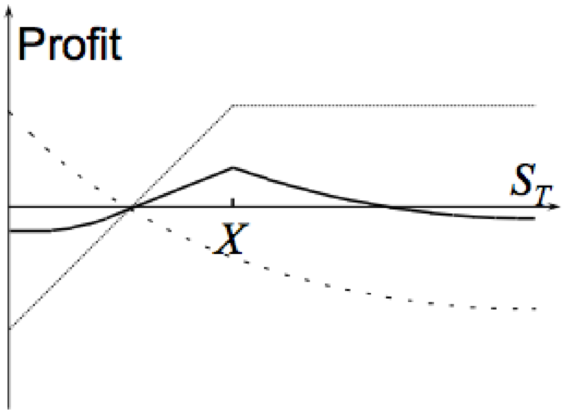
- Can be created by:
  - Buying a put- low X price
  - Buying a put- high X price
  - Selling two puts with an intermediate strike price
- Use of put options would provide the same spread as using call options, the initial outlay would also be identical

Calendar spread using calls



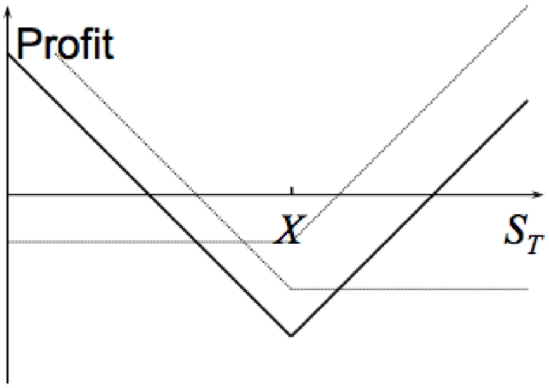
- Use different options that have the same strike price but different expiration dates
  - Selling a call with a certain strike price and buying a longer maturity call option with the same strike price
  - The longer the maturity of an option, usually the more expensive it is
    - Therefore, calendar spread using calls generally requires an initial outlay
- The investor makes a profit if the stock price at expiration of the short maturity option is close to the strike price of the short maturity option
- However, a loss is incurred when the stock price is significantly above or below the strike price

Calendar Spreads Using Puts



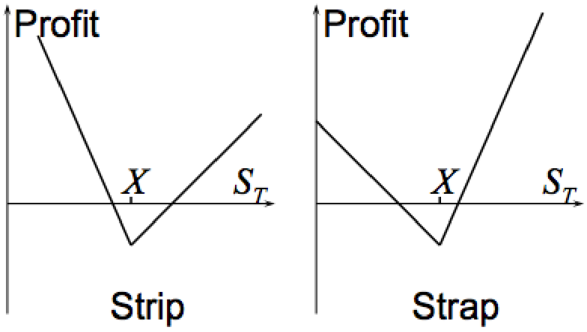
- Created using puts
  - Investor buys a long maturity put option and sells a short maturity put option with the same strike price
- The profit pattern is similar to that obtained from using calls

Straddle Combination



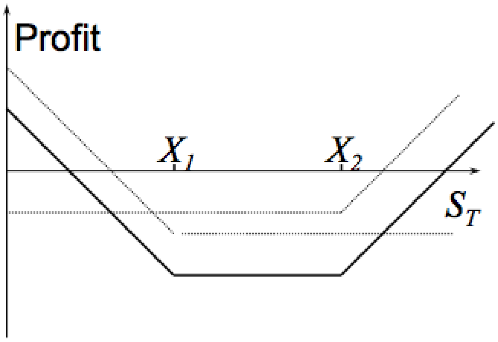
- Involves buying a call option and put option with the same strike price and expiration date
  - If the stock price is close to the strike price at expiration of the option, the straddle leads to a loss
  - If there is sufficiently large move in either direction, a significant profit will result
- Straddle is appropriate when an investor believes there will be a large movement in the stock price, but they are unsure as to the direction
- Very expensive strategy

Strip and Strap



- **Strip:** consists of a long position in a call option and a long position in two put options with the same strike price and expiration date
  - Strip investor is betting that there will be a large stock price move and considers a decrease in the stock price is more likely
- **Strap:** long position a put option and two long positions in a call option with the same strike price and expiration date
  - In a Strap the investor is betting that there will be a large stock price move and considers an increase in the stock price to be more likely
- In both cases you are purchasing three options, this is a very expensive strategy

Strangle



- An investor buys a put and a call options with the same expiration date and different strike prices
  - The call strike price is higher than the put strike price
  - The investor is betting that there will be a large movement in the stock price, but is uncertain as to the direction
- For the investor to make a profit, the stock price has to move much further than in a straddle

One- Step Binomial

- Substituting for Δ we obtain:

$$f = e^{-rT}[pf_u + (1-p)f_d]$$

- Where:

$$p = \frac{e^{rT} - d}{u - d}$$

Risk Neutral Valuation

- The variables p and (1 - p) can be interpreted as the risk- neutral probabilities of up and down movements
- The value of an option is its expected payoff in a risk neutral world discounted at the risk-free rate
- When we are valuing an option in the terms of the underlying stock the expected return on the stock is irrelevant
    - As we are valuing the option in terms of the price of the underlying stock where the chances of future up or down movements are already incorporated into the price of the stock.
    - We do not need to take them into account again when valuing the option in terms of the stock price

How to deal with American Options

- The value of the option at the final nodes is the same as for European options
- At earlier nodes the value of the option is the greater of
    - The value given by the equation we have used previously
    - The payoff from early exercise

Choosing u and d

- Up until now we have assumed values for u and d. In practice, u and d are determined from the stock price volatility, σ. One way of matching the volatility is to set

$$u = e^{\sigma\sqrt{\Delta t}}$$
$$d = \frac{1}{u}$$

Black Scholes Models for Pricing Options

Assumptions underlying evolution of stock prices

- Central assumption of Black- Scholes is that stock prices follow a lognormal distribution
- Black- scholes also implies that in the absence of dividends, stock prices follow a random walk
- Distribution is quite intuitive
    - Stock prices are bounded on the downside by zero, with unlimited upside. Thus they have a right skewed distribution

- Natural logarithm is normally distributed
- The black scholes assumption for stock prices implies that $\ln S_T$ is normal

The mean of $\ln(S_T)$:

$$lnS_0 = (\mu - \frac{\sigma^2}{2})T$$

The volatility of $\ln(S_T)$:

$$\sigma\sqrt{T}$$

Thus:

$$lnS_T: N(lnS_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}$$

- The expected value of S_T:

$$E(S_T) = S_0 e^{\mu T}$$

- The variance of S_T:

$$\text{var}(S_T) = S_0^2 e^{2\mu T}\left(e^{\sigma^2 T} - 1\right)$$

Volatility

- Measure of our uncertainty about stock returns
- The volatility of the stock price is defined as the standard deviation of the return provided by the stock in one year

Assumptions Underlying Black- Scholes

1. Stock price behaviour corresponds to the lognormal model with μ *and* σ constant
2. There are no transaction costs or taxes. All securities are perfectly divisible
3. There are no dividends on the stock during the life of the option
4. There are no riskless arbitrage opportunities
5. Security trading takes place continuously in time
6. Investors can borrow and lend and the same risk free rate
7. The short term risk free rate of interest r is constant

Theory behind black- scholes

- Take a riskless portfolio comprising a position in the option and a position in the underlying stock
- If there are no arbitrate opportunities, the return for the portfolio must be the risk free interest rate
- The reason we can set up this riskless portfolio is that the stock price and option price are both affected by the same underlying uncertainty i.e. stock price movements
- In any <u>short period of time:</u>

- The price of a call is perfectly positive correlated with the price of the underlying stock
- The price of a put is perfectly negatively correlated with the price of the underlying stock
  - In both of these cases, when an appropriate portfolio of the stock and the option is set up, the gain or loss from the stock position will be completely offset by a corresponding loss or gain on the option position so that the overall value of the portfolio at the end of the period is known with certainty
  - As the overall value is known with certainty, and the gains/ losses are completely offset, the return from the riskless portfolio in any short period of time must be the risk free interest rate

Valuing a European option written on a non- dividend paying stock

$$c = S_0\ N(d_1) - X\ e^{-rT} N(d_2)$$

$$p = X\ e^{-rT}\ N(-d_2) - S_0\ N(-d_1)$$

$$\text{Where:}\quad d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / X) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

- $N(d_1)$ and $N(d_2)$ are thus probability terms. Their role is to assess the probability that the stock price will exceed the strike price so that the call option will end up being exercised at maturity

Black Scholes Model

- To test whether black-scholes has the right properties, we consider what happens when the stock price takes extreme values
- When stock price become very large:
  - When stock prices are very large, a call option is almost certain to be exercised. Expect the call price to be $S_0 - Xe^{-rT}$
  - In black scholes, as S_0 becomes very large, both d_1 *and* d_2 become large and positive, so both $N(d_1)$ and $N(d_2)$ become close to 1
  - Therefore…
    $c = S_0N(d_1) - Xe^{-rT}N(d_2)$

    which becomes: $c = S_0 - Xe^{-rT}$
- When the stock price becomes very large, we would expect that a put option would not be exercised and thence, its value should approach zero
  - In Black-Scholes, as S_0 become very large, both $N(-d_1)$ and $N(-d_2)$ are close to zero.
    So, $p = Xe^{-rT}N(-d_2) - S_0N(-d_1)$ becomes $p = 0$ as stated above.
- When the stock price becomes very small:

- Expect the price of the call option to be zero, as the call option is less likely to be exercised
- Both d values become large and negative. $N(d_1)$ and $N(d_2)$ are therefore very close to zero. Thus as expected we get a price of zero for a call option
- Expect the price of the put option to be $Xe^{-rT} - S_0$. As both $N(-d_1)$ and $N(-d_2)$ become close to one as the stock price becomes very small, the price of the put option becomes $Xe^{-rT} - S_0$ as expected

Risk Neutral Vauation

- Any security dependent on another traded security can be valued on the assumption that investors are risk neutral
- Thus, investors risk preferences have no effect on the value of a stock option expressed as a function of the price of the underlying stock
- In a risk neutral world, the following two results hold:
    - The expected return from all securities is the risk free interest rate
    - The risk free interest rate is the appropriate discount rate to apply to any expected future cash flow
- Hence, black scholes uses the risk free interest rate, rather than μ

Increase in:	Effect on Call Value	Effect on Put Value
Stock Price (S)	↑	↓
Strike Price (X)	↓	↑
Time to Expiry (T)	↑	↑
Risk-free Rate (r)	↑	↓
Volatility (σ)	↑	↑

- Deep in the money and out of the money stocks are relatively insensitive to volatility
    - Implied volatilities calculated for these options therefore tend to be unreliable
- Volatility may be caused by the release of new information to the market
- Generally assume that there are 252 trading days per year for stocks

$$Volatility\ per\ annum = volaility\ per\ trading\ day\ x\ \sqrt{number\ of\ trading\ days\ per\ annum}$$

- We can examine European options by assuming that there are two components to a stock's price:
    - A risky component; and,
    - A riskless component that is used to pay the known dividend
        - Thus the riskless component is equal to the PV of all dividends discounted from the ex-dividend dates to the present time at the risk free rate

- Therefore, black- scholes can be used as long as we set S_0equal to the risky component of the stock
- This can be done by subtracting the PV of the dividends from S_0
- Remember, that we only include the dividend in the calculation if its ex-dividend date occurs during the life of the option

$$c = S_0 N(d_1) - X e^{-rT} N(d_2)$$
$$p = X e^{-rT} N(-d_2) - S_0 N(-d_1)$$

Where: $$d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / X) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

How do we treat a stock which pays a known dividend yield?

- Take two identical stocks. Stock 1 pays no dividends, and 2 stock pays a dividend yield at q p.a.
- Both stocks will give an identical overall return. The return from stock 1 will be in the form of capital gains, and the return from stock 2 will be in the form of capital gains from dividends
- The payment of a dividend will cause stock 2's price to drop by an amount equal to the value of the dividend
- So payment of a dividend yield at rate q causes the growth rate to be less than it would otherwise be, by an amount of q.
- Thus if the price of a stock, with a dividend yield of q, grows from S_0 today to S_T at time TL
  - In the absence of dividends it would grow from S_0 to $S_t e^{qT}$ at time T
  - In the absence of dividends it would grow from $S_0 e^{qT}$to S_T
- Therefore, we get the same probability distribution for the stock price at time T in each of the following cases:
  - The stock price starts at price S_0 and pays a dividend yield at rate q
  - The stock price starts at price $S_0 e^{qT}$ and pays no dividend yield

Options and Dividend Yields

- Differences between stock index options and individual stock options:
  - The strike price and premium of an index option are usually expressed in terms of points. A multiplier is then applied to give a dollar value
    - Eg one index contract may be on 25 times the index
  - Index options are cash settled rather than deliverable
  - Index options are usually European in exercise

Options on currencies

- A foreign currency is analogous to a stock paying a known dividend yield
- The owner of the foreign currency option receives a yield equal to the risk free rate of interest in the foreign currency

- The options are traded on both the OTC markets and on an exchange, although they are mainly traded in OTC markets FX options can be either European or American

Lower bounds for options prices

- The payment of a dividend by the underlying asset lowers the price of that asset
- The reduction in price is good for the holder of a put as it increases their payoff, while it is bad news for the folder of a call option as it reduces their payoff
- Thus, the lower bounds for both call and put options are affected by the payment of dividends

Euro call

- <u>Non- dividend paying</u>
$$c \geq S_0 - Xe^{-rT}$$
- <u>Dividend paying</u>
$$c \geq S_0 e^{-qT} - Xe^{-rT}$$

Euro put

- <u>Non- dividend paying</u>
$$p \geq Xe^{-rT} - S_0$$
- <u>Dividend paying</u>
$$p \geq Xe^{-rT} - S_0 e^{-qT}$$

Put- Call Parity

- <u>Non- dividend paying</u>
$$c + Xe^{-rT} = p + S_0$$
- <u>Dividend paying</u>
$$c + Xe^{-rT} = p + S_0 e^{-qT}$$
- Both portfolios must be worth the same today, and the put-call parity result above hold
- Slight variation to the formula is necessary when considering foreign currency options
$$c + Xe^{-rT} = p + S_0 e^{-r_f T}$$

Binomial Trees with dividends

- P must satisfy:

$$pS_0 u + (1-p) S_0 d = S_0 e^{(r-q)T}$$

or

$$p = \frac{e^{(r-q)T} - d}{u - d}$$

Then substitute into general equation:

$$f = e^{-rT} \left[pf_u + (1-p) f_d \right]$$

BSOPM for dividend yielding stocks

$$c = S_0 e^{-qT} N(d_1) - Xe^{-rT} N(d_2)$$

$$p = Xe^{-rT}\ N(-d_2) - S_0 e^{-qT} N(-d_1)$$

As: $$\ln \frac{S_0 e^{-qT}}{X} = \ln \frac{S_0}{X} - qT$$

$$d_1 = \frac{\ln(S_0 / X) + (r - q + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / X) + (r - q - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

- If the dividend yield rate is known but is it not contact during the life of the option, the previous equations are still true, with q equal to the average annualized dividend yield

BSOPM for the FX

$$c = S_0 e^{-r_f T} N(d_1) - Xe^{-rT} N(d_2)$$

$$p = Xe^{-rT}\ N(-d_2) - S_0 e^{-r_f T} N(-d_1)$$

As: $$\ln \frac{S_0 e^{-r_f T}}{X} = \ln \frac{S_0}{X} - r_f T$$

$$d_1 = \frac{\ln(S_0 / X) + (r - r_f + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / X) + (r - r_f - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The forward exchange rate:

$$F_0 = S_0 e^{(r - r_f)T}$$

BSOPM becomes:

$$c = e^{-rT}\left[F_0 N(d_1) - XN(d_2)\right]$$

$$p = e^{-rT}\left[XN(-d_2) - F_0 N(-d_1)\right]$$

Where:

$$d_1 = \frac{\ln(F_0 / X) + \sigma^2 T / 2}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(F_0 / X) - \sigma^2 T / 2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Portfolio Insurance

- Consider a manager of a well diversified portfolio which mimics the ASX 200 Index, and as such has similar returns and dividend yield
    - The percentage changes in the valueof the portfolio can be expected to be approximately the same as the percentage changes in the value of the index
    - We know that an index option on the ASX 200 is on 25 times the index, and as such, a long put option on the ASX 200 would allow a manager to hedge against the risk of the value of their portfolio falling.
- Manager would need to purchase the following number of put options:

$$\frac{Value\ to\ be\ hedged}{Value\ one\ put\ option\ will\ be\ hedge}$$

The total cost of the insurance to protect the $100 million portfolio is calculated as follows:

$$p = 5000e^{-0.09x0.25}\ N(-d_2) - 5000e^{-0.05x0.25}N(-d_1) = 211.36x800x25 = \$4,227,200$$

$$d_1 = \frac{\ln(5000/5000) + (0.09 - 0.05 + 0.25^2/2)0.25}{0.25\sqrt{0.25}} = 0.14$$

$$d_2 = d_1 - 0.25\sqrt{0.25} = 0.02$$

Options on Futures Contracts

- A futures option is the right, but not the obligation, to enter into a futures contract at a certain futures price by a certain date
- The underlying security of a futures option is a futures contract, not a physical commodity. Exercising these options given the holder a position in a futures contract
- Quotations of futures options:
    - We have already stated that most futures options are American
    - They are quoted by the month in which the underlying futures contact matures, not by the expiration month of the option
    - Most futures option contracts mature a few days before the underlying futures contact matures

Call futures options:

- Right to enter a long futures contact at a certain price
- When the holder exercises the option they receive a cash amount equal to the excess of the futures price over the strike price

Put futures option:

- Right to enter into a short futures contact at a certain price
- When the holder exercises the option they receive a cash amount equal to the excess of the strike price over the futures price

Why are Futures Options Popular?

- Futures contracts are generally more liquid than the underlying asset

- Exercising a futures option does not usually lead to the delivery of the underlying asset
    - The underlying futures contact is closed prior to delivery
    - Therefore, futures options are normally settled in cash
- Futures on commodities are easier to trade than the commodities them selves
    - Easier to make or take a delivery of a pork bellies futures contract than it is to make or take delivery of the actual
- Futures and futures options are usually traded in pits side by side in the same exchange
    - This facilitates hedging, arbitrage and speculation
    - The markets tend to be more efficient
- Futures options also tend to entail lower transaction costs than spot options

Put- call parity

- To derive a put- call parity relationship for European futures options we consider the following two portfolios:
    - Portfolio A: a Euro call + cash = Xe^{-rT}
    - Portfolio B: a Euro put futures options + long futures option + cash = F_0e^{-rT}
- The cash in portfolio A is invested at the risk free, and grows to X and time T
- Let F_T be the futures price at maturity the option
    - If $F_T > X$, the call option in portfolio A is exercised and portfolio A is worth F_T
    - If $F_T < X$, the call is not exercised and portfolio A is worth X
- Therefore, the value if portfolio A at time T is:
    - Max (F_T, X)
- The cash in portfolio B is invested at the rf rate and grows to F_0 at time T
- The put option provides a payoff of max $(X - F_T, 0)$
- The futures contact provides a payoff of $F_0 + (F_T - F_0) + \max(X - F_T, 0) = \max(F_T, X)$
- The futures contact is worth zero today due to marking to market, so portfolio B is worth:

$$p + F_0e^{-rT} + 0$$

Therefore, put-call parity for a futures option is:

$$c + Xe^{-rT} = p + F_0e^{-rT}$$

For American futures, the put-call parity relationship is:

$$F_0e^{-rT} - X < C - P < F_0 - Xe^{-rT}$$

Lower bounds for futures options

- As the price of a put option cannot be negative, the lower bound for a euro call must be:

$$c + Xe^{-rT} \geq F_0 e^{-rT}$$

$$or$$

$$c \geq (F_0 - X)e^{-rT}$$

- As the price of a call option cannot be negative, the lower bound for a put option is:

$$Xe^{-rT} \leq F_0 e^{-rT} + p$$

$$or$$

$$p \geq (X - F_0)e^{-rT}$$

- As American options can be exercised at any time, the lower bounds for American options are:

$$C \geq F_0 - X$$

$$and$$

$$P \geq X - F_0$$

Binomial Trees with Futures Options

- If we set q = r, the expected growth rate of the stock price is zero, making it analogous to a futures price
- Therefore, futures prices behave in the same way as a stock paying a dividend yield at the domestic risk free rate

$$puF_0 + (1-p)dF_0 = F_0 e^{(r-r)T}$$

solving for p

$$puF_0 + (1-p)dF_0 = F_0$$

$$p = \frac{1-d}{u-d}$$

- Therefore, we can price any option written on futures with:

$$f_0 = e^{-rT}[pf_u + (1-p)f_d]$$

BSOPM

$$c = e^{-rT}[F_0 N(d_1) - XN(d_2)]$$

$$p = e^{-rT}[XN(-d_2) - F_0 N(-d_1)]$$

Where:

$$d_1 = \frac{\ln(F_0 / X) + \sigma^2 T / 2}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(F_0 / X) - \sigma^2 T / 2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Comparing Futures Options with Spot Option Prices

- The payoff from a euro spot call option with strike price X is:
$$\max(S_T - X, 0)$$
- The payoff from a euro futures call with the same strike price is:
$$\max(F_T - X, 0)$$
- If the euro call futures option matures are the same time as the futures contract $F_T = S_T$ and the two options are equivalent
- If the euro call futures option matures before the futures contact, it is worth more than the corresponding sport option in a normal market, and less in an inverted market

American futures options

- May not be worth the same as the corresponding American options on an underlying asset
  - With some commodities markets, the futures price is consistently higher than the spot prior to maturity
- If an American call futures option is exercised early, then it will provide a greater profit to the holder
- If an American put futures option is exercised early, then it will provide a lower profit to the holder
- Therefore, American call future options are generally worth more than their American spot call option counterparts. American put future options are generally worth less than their spot counterpart

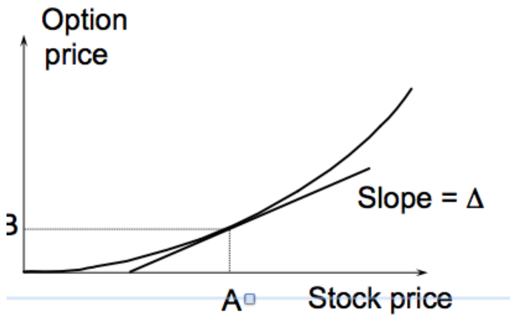
The Greek Letters

- One way in which managers can hedge their positions is through the use of the Greek letters
  - Delta, Gamma, Vega, Theta and Rho
  - They are the partial derivatives of the BSOPM, each eith respect to a different variable
- **Delta:** A measure of an option's sensitivity to changes in the price of the underlying asset

- **Gamma:** A measure of the Delta's sensitivity to changes in the price of the underlying asset
- **Vega:** a measure of an options sensitivity to changes in the volatility of the underlying asset
- **Theta:** a measure of an options sensitivity to time decay
- **Rho:** a measure of an options sensitivity to changes in the risk free interest rate

Delta

- Rate of change of the option price with respect to the underlying asset
    - It is the slop of the curve that relatives the option price to the underlying asset
- If the Delta of a call option on a stock is 0.5
    - This means that when the stock price changes by a small amount, the option price changes by apx 50% of that amount



- When the stock price corresponds to point A, the option price corresponds to point B and Δ is the gradient
- We can see that the relationship between the stock price and the call option is not a straight line. Hence, the value of Delta will change with changes in the stock price.
- In general, the Delta of a call option equals:

$$\frac{\delta c}{\delta S}$$

- Where δS is a small change in the stock price and δc is the resulting change in the call price

Delta Hedging

- A financial institution that trades in options, faces the risk that they may lose money on their options if the price of the underlying asset changes
    - Delta allows us to measure how option prices will change when the underlying asset price changes
- By using out measure of Delta, we can take a position in the underlying stock, so that the combined position in our options and the underlying stock is no longer sensitive to changes in the underlying asset price
    - Delta Neutral portfolio
- Delta hedging involves maintaining a Delta neutral portfolio through time

Delta Hedging with European Stock Options

- The BSOPM can be used to calculate Delta. $N(d_1)$ can be used to calculate the Delta of the European call and put options
- The Delta of a long European call option on a non-dividend paying stock, is $N(d_1)$
- To hedge a short position in a euro call on a stock on a stock that does not pay a dividend, we take a long postion of $N(d_1)$ shares of a stock.
- To hedge a long postion in a euro call on stock that does not pay a dividend we take a short position of $N(d_1)$ shares of stock
- The Delta of a long euro put option on a non- dividend paying stock, is $N(d_1) - 1$
    - To hedge a short position in a euro put on a stock that does not pay a dividend, we take a short position of $N(d_1) - 1$ shares of a stock
    - To hedge a long postion in a euro put on the stock that does not pay a dividend, we take a long position of $N(d_1) - 1$ shares of stock

Where d_1 is defined as:

$$d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

Euro Call

- To hedge a short position in a European call on an asset that pays a dividend at rate q, we take a long position of $e^{-qT}N(d_1)$ of the asset
- To hedge a long position in a European call on an asset that pays a dividend rate at q, we take a short position of $e^{-qT}N(d_1)$ of that asset

Euro put

- The Delta of a euro put option on an asset paying a dividiend yei;d at rate q, is $e^{-qT}[N(d_1 - 1)]$
- To hedge a long position in a European put on an asset that pays a divdened at rate q, we take a long position of $e^{-qT}[N(d_1 - 1)]$ of that asset
- When the asset is a stock index, these formulas are correct with q equal to the dividend yield on the index
- When the asset is a currency, they are correct with q = the foreign rf rate
- When the asset is a futures contract (i.e a European futures option), they are correct with q equal to the domestic risk free rate, r.

Delta hedging with Forward Contacts

- Delta can be applied to other financial instruments other than options
- We know that the payoff for a forward contact is :

$$S_0 - Ke^{-rT}$$

- When the price of the stock changes by δS, with all else remaining the same, the valye of the forward contacrs on the remaining stock also changes by δS
- The delta of a forward contact on one share is thus always 1.

- A short forward contact on one share can be hedged by going long on one share
- A long forward contact on one share can be hedged by shorting one share
- A forward contact on an asset paying a dividend yield at rate q, has a delta of

$$e^{-qT}$$

- For a stock index, q should be set equal to the dividend yield on the index
- For a currency, q should be set equal to the foreign risk-free interest rate

Issues with Delta Hedging

- With large movements in the share price, the value of Delta will change
- To maintain a Delta neutral portfolio, taders must adjust their position to take into account changes in Delta
- This process can be time consuming as the portfolio must be rebalanced ever time Delta changes.
- The process can be quite expensive. If the trader is short in call options:
    - When the spot price rises, they must buy more shares to maintain a Delta neutral portfolio
    - When the spot price falls they must sell shares to maintain a Delta neutral portfolio
    - This strategy is costly as the trader must buy high and sell low

Exotics Options

- Have been developed as they meet a genuine need in the market
    - Tax, accounting, legal or regulatory reasons who corporate treasurers or fund managers find exotic products attract
    - Cometimes the profucs are designed to reflect a corporate treasurer's or fund manager's view on potential futures movements in a particular market variables
    - Occasionally an exotic product is designed by an investment bank to appear more attractive than it is to an unwary corporate treasurer or fund manager

Packages

- A portfolio of standard euro call and puts, forward contracts, cash and the underlying asset itself
    - Eg. Bull spread, bear spread, straddles
    - Packages are often structured by financial engineers to have zero cost

Non- standard American Options

- Traded over the counter
    - A Bermudan option's early exercise may be restricted to certain dates
    - Early exercise allowed during only part of the life of the option (eg there may be an intial "lock out" period)
    - The strike price may change over the life of the option

- Non- standard American options can usually be valued using a binomial tree
    - At each node, if you are allowed to exercise early, and doing so would result in a higher value, you use the early exercise value at that particular node

Forward Start Options

- These are options that will start at some time in the future
    - Employee stock option plans, where a company promises that it will grant at the money options to executives atcetain times in the future
- When the underling asset provides no income, an at the money forward start option is woth the same as a regular at the money start option with the same life
    - Eg. An at the money option that will start in three years and mature in five years is worth the same as a two year at the money option initiated today

Compound Option

- Compound options are options on options
    - Call on call
    - Put on call
    - Call on put
    - Put on put
- With a call on call, on the first exercise date, the holder of the compound option is entitled to pay the first strike price X_1, and receive a call option.
    - The option gives the holder the right to buy the underlying asset for the second strike price X_2, on the second exercise date.
    - The compound option will only be exercised on the first exercise date if the value of the 2^{nd} option on that date is greater than the first strike price
- These compound options are very sensitive to volatility

Chooser Option "As you like it"

- Gives the holder the ability to choose after a specified period of time whether the option is a call or a put
- The value of the chooser option at the time this choice is made is max(c,p)

Barrier Options

- Barrier options are options where the payoff depends on whether the underlying asset's price reaches a certain level during a certain period of time
- These options are attractive as they are less expensive than the corresponding regular option
- Barrier options are classified as either knock- out options or knock in
    - **Knock out:** <u>ceases</u> to exist when the underlying asset price reaches a certain level (barrier) before maturity
    - **Knock in:** <u>Comes into existence</u> only when the underlying asset price reaches a certain level (barrier) before maturity

- **UP=** barrier > X price
- **Down =** barrier < X price

Knock out Options

→ cease to exist until asset price reaches barrier

- **Up and out call:**
  - Barrier > price
- **Down and out call**
  - Barrier < price
- **Up and out put**
  - Barrier > price
- **Down and out put**
  - Barrier < price

Knock in options

→ Comes into existence when asset price reaches a barrier

- **Up and in call**
  - Barrier > price
- **Down and in call**
  - Barrier < price
- **Up and in put**
  - Barrier > price
- **Down and in put**
  - Barrier < price

Lookback options

- The payoff from look back options depend on the maximum or minimum asset price reached during the life of the option
- The payoff from a lookback call is: $S_T - S_{min}$ at time T
  - Allows the buyer to buy back stock at the lowest observed privce in some interval of time
- The payoff from a lookback put is : $S_{max} - S_T$ at time T
  - A lookback put allows a buyer to sell stock at the highest observed price at some interval of time

Shout Options

- European option where the holder can "shout" to the writed at the end of the life of the option is either:
  - Usual option payoff, $\max(S_T - X, 0)$ or:
  - Intrinsic value at time of shout, $S_T - X$
- Payoff: $\max(S_T - S_\tau) + S_\tau - X$
- A shout option has some of the same features as a lookback option, but it is considerably less expensive

Asian Options

- The payoff depends on the average price of the underlying asset during at least some part of the life of the option

- Average Price options pay:
  - Max ($S_{ave} - X, 0$) (call)
  - Max ($X - S_{ave}, 0$) (put)
- Average strike options pay:
  - Max ($S_T - S_{ave}, 0$) (call)
  - Max (($S_{ave} - S_T, 0$) (put)

Options to exchange

- Options to exchange one asset for another in various contexts
  - EG. A stock tender offer is an option to exchange shares in one company for shares in another company
- When an asset with price U can be exchanged for an asset with price V payoff is $\max(V_T - U_T, 0)$

Mortgage backed securities

- Created when a financial institution decides to sell part of its residential mortgage portfolio to investors
  - Mortgages are put into a pool and investors acquire a stake in the pool by buying units
  - A secondary market is usually created for the units so that investors can sell them to other investors as desired
  - An investor who own units representing X% of a certain pool is entitled to X% of the principal and the interest payments received from the mortgages in the pool.
    - This is an example of a MBS where all investors receive the same return and bear the same prepayment risk
- **Collaterized mortgage:** obligation differs from pass through as investors are divided into a number of classes, and rules are developed for determining how principal repayments are channeled to different classes
  - **EG** collaterized mortgage obligations would be one with three classes
    - Principal repayments are channeled to class A until investors in this class have been completely paid off, then channeled to class B until they have been completely paid off and then finally to class C investors
- In a stripped mortgage backed security all principal payments are separated from interest payments
  - All principal payments are channeled into one class of security called: principal only (PO)
  - All interest payments are channeled to a security class known as interest only
  - Both PO and IO are risky investments. As prepayment rates increase, a PO becomes more valuable and IO becomes less valuable
    - As prepayments rates decrease the reverse is true

Non- Standard Swaps

- **Step up swaps:** where the notional principal in an increasing function of time- could be useful for constructing a company that wants to borrow an increasing amount as time progresses at a floating rate and swap it for a fixed rate
- **Amoritizing swap:** where the notional principal is a decreasing function of time
- The principal could also be different on the two sides of the swap, or the payment frequency could be different
- The floating reference rate for a swap is not always LIBOR. It could for example be the commercial paper rate

Compounding swaps and currency swaps

- In a compounded swap, the interest is compounded instead of being paid or received. Usually the payments are made at the end of the life of the swap
    - Enable an interest rate exposure in one currency to be swapped for an interest rate exposure in another currency:
        - Fixed for fixed
        - Floating for floating
        - In a cross- currency interest rate swap, a floating rate in one currency is exchanged for a fixed rate in another currency

More complex swaps

- LIBOR- in- arrears swaps: in a plain vanilla interest rate swap, the floating rate of interest is observed on one payment date and is paid on the next payment date
    - LIBOR in arrears swap, the floating rate paid on a payment date equals the rate observed on the payment date itself
- Constant maturity swap: an interest rate swap where the floating rate equals the swap rate for a swap with a certain life
    - The floating payments on a CMS swap might be made every 6 months at a rate equal to the five year swap rate
- Differential swaps: an interest rate swap where a floating interest rate is observed in one currency and applied to a principal in another currency

Equity Swap:

- One party promises to pay the total return on an equity index applied to a notional principal and the other party promises to pay a fixed or floating return on the notional principal
- The equity index is usually a total return index where dividends are reinvested in the stocks comprising the index

Swaps with Embedded options

- **Accrual swaps**: where the interest on one side accrues only when the floating reference rate is within a certain range. Sometimes the range remains fixed during the entire life of the swap, and sometimes it is reset periodically

- **Cancelable swaps**: a plain vanilla swap where one side has the option to terminate on one or more of the payment dates
- **Indexed principal swap**: where the principal reduces in a way dependent on the level of interest rates. The lower the interest rate, the greater the reduction in the principal
- **Commodity swap**: where one party agrees to pay a certain amount each year in return for a fixed volume of a commodity. This has the effect of locking in the price of the commodity.

Credit, Weather, Energy and Insurance derivatives

Credit derivatives

- A contract where the payoff depends of the credit worthiness of one or more commercial or sovereign entities
- Their purpose is to allow credit risks to be traded and managed in much the same way as market risks

Credit Default Swaps (CDS)

- A contract that provides insurance against the risk of default by a particular company
- Reference company
    - The company which may be at risk of default
- Credit event
    - When default occurs
- The buyer of this type of insurance obtains the right to sell a particular bond issues by the company for its par value when a credit even (default) occurs
- Reference obligation
    - The bond that is issued
- Notional principal
    - The total par value of the bond that can be sold

Mechanics of how a CDS works:

- You buy a bond off a particular company
- In return for buying that bond, you expect to be paid coupons on the bond, and the face value of the bond at maturity
- You fear that the company may default on the payment of those coupons, or the payment of the face value of the bond
    - Hence, you take out a CDS
- In return for buying this CDS, you must made periodic payments to the seller of the CDS until the end of the life of the CDS, or until a credit event occurs (default by the company that issued the bond)
- If a credit even occurs, the party that sold you the CDS must settle the swap either by physical delivery or in cash
- If the terms of the swap require physical delivery, then you (the swap buyer) deliver the bond (originally issued by the company which has

defaulted) to the seller of the swap in exchange for the par value of the swap
- When there is cash settlement, a calculation agent polls various dealers to determine the mid market price (Q), of the reference obligation within a specified number of days after the credit event
- Cash settlement is then (100-Q)% of the notional principal

Total return swap
- Involves the return on one asset or group of assets being swapped for the return on another
- Can be useful to diversify credit risk by swapping one type of exposure for another
    - Plane bank is primarily concerned with lending to the airline industry
    - Oil bank is primarily concerned with lending to the oil industry
    - To reduce its credit risk, plane bank could enter into a total return swap, where the return on one of its loans is exchanged for the return on an oil bank loan
        - This would achieve credit risk diversification for both sides (assume both loans are for the same notional principal
    - Alternatively, each bank could separately exchange the return on one of their loans for a LIBOR based return. This would have the effect of passing credit risk onto someone else

Credit spread options
- This is an option that provides a payoff when the spread between the yields on two assets exceeds some pre-specified level

Weather Derivatives
- As many company's performance may be affected by adverse weather conditions, weather derivatives can be used to hedge this form of risk
- A is the average of the highest and lowest temperature
- A typical product is a forward contract or an option on the cumulative CDD or HDD during a month
- Weather derivative are often used by energy companies to hedge the volume of energy required for heating or cooling during a particular month
- Usually priced using historical data

Energy derivatives
- Energy companies are among the most active and sophisticated users of derivatives
- Energy derivates are traded both over the counter and on exchanges
- Three types of energy derivatives
    - Crude oil
        - Highly volatile
        - OTC
        - Exchange traded
    - Natural gas

- Electricity
    - Demand for electricity is not uniform
    - Greater during the day
    - Typical contract allows one side to receive a specified number of megawatt hours for a specified price at a specific location during a particular month
    - Have daily or monthly exercise

Insurance derivatives

- Used to hedge a particular type of risk, have similar characteristics to insurance contracts
- Both contracts provide some form of protection against an adverse event occurring
- The insurance industry has tired to hedge its exposure to catastrophic (CAT) risks such as earthquakes and typhoons through reinsurance

OTC alternative to traditional reinsurance:

- A bond that is issued by a subsidiary of an insurance company that pays a higher than normal interest rate
- In exchange for the higher interest rate, the holder of the bond agrees to provide an excess- of- cost reinsurance contract
- The interest or principal of the CAT bond could be used to meet any claims with mat arise from the CAT
- CAT bonds generally have a high probability of an above normal interest rate, with a low probability of a high loss
- There are no statistically significant correlations between CAT risks and market returns. CAT bonds are therefore a useful investment as they have no systematic risk so that their total risk can be completely diversified away in a large portfolio

Derivative mishaps

- To mitigate chances of these mishaps:
    - Define risk limits
    - Take these risks limits seriously
    - Do not think you can out guess the market
    - Diversify
    - Stress test
- Lessons for financial institutions
    - Monitor traders
    - Clear separation between departments
    - Do not blindly trust models
    - Be conservative in recognizing inception profits
    - Do not ignore liquidity risk
    - Never ignore risk management
- What are the lessons for non- financial corporations:
    - Make sure you fully understand the trades you are doing
    - Make sure a hedger does not become a speculator