Gas Turbine CO Emission Analysis

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Introduction

The combined cycle power plant, also known as combined cycle gas turbine plant, is an assembly of heat engines that combine to generate electricity (Tüfekci). A combined-cycle power plant (CCPP) is made up of gas turbines, steam turbines, and heat recovery steam generators. The electricity is generated and combined in one cycle by gas and steam turbines and then transferred from one turbine to another.

We are interested in identifying the process variables that impact carbon monoxide emissions. By determining the process variables that impact carbon monoxide emissions we will be able to find opportunities to reduce carbon monoxide emissions.

Gas Turbine CO and NOx Emission Data Set

The data comes from a gas turbine located in Turkey that studies the flue gas emissions of specifically carbon monoxide (CO) and nitrogen oxide (NOx) gases. The data set provides hourly statistics of 11 sensors. Data points were collected from a gas turbine from Jan 01 2011 to Dec 13 2015.

Description

The data file `gt_2015.csv` has 7384 observations and 11 variables from the UCI Gas Turbine CO and NOx Emission Data Set. We are going to explore and analyze the following variables:

- `AT` - Ambient Temperature
- `AP` - Ambient Pressure
- `AH` - Ambient Humidity
- `AFDP` - Air Filter Difference Pressure
- `GTEP` - Gas Turbine Exhaust Pressure
- `TIT` - Turbine Inlet Temperature
- `TAT` - Turbine After Temperature
- `TEY` - Turbine Energy Yield
- `CDP` - Compressor Discharge Pressure

Here's a quick peek at the data set:

AT	AP	AH	AFDP	GTEP	TIT	TAT	TEY	CDP	CO	NOX
1.95320	1020.1	84.985	2.5304	20.116	1048.7	544.92	116.27	10.799	7.4491	113.250
1.21910	1020.1	87.523	2.3937	18.584	1045.5	548.50	109.18	10.347	6.4684	112.020
0.94915	1022.2	78.335	2.7789	22.264	1068.8	549.95	125.88	11.256	3.6335	88.147
1.00750	1021.7	76.942	2.8170	23.358	1075.2	549.63	132.21	11.702	3.1972	87.078
1.28580	1021.6	76.732	2.8377	23.483	1076.2	549.68	133.58	11.737	2.3833	82.515
1.83190	1021.7	76.411	2.8410	23.495	1076.4	549.92	133.58	11.829	2.0812	81.193

AT	AP	AH	AFDP	GTEP	TIT	TAT	TEY	CDP	CO	NOX
1.007500	1021.7	76.942	2.8170	23.358	1075.2	549.63	132.21	11.702	3.1972	87.078
2.074000	1022.0	75.974	2.7981	22.945	1073.7	549.98	131.53	11.687	2.2529	83.171
0.052442	1024.0	64.823	2.7916	23.298	1070.9	550.23	130.43	11.546	3.6518	86.895
-1.084100	1022.3	70.733	2.8280	22.604	1071.9	550.31	130.41	11.526	1.7751	83.696
11.353000	1006.9	72.516	3.0802	26.554	1076.3	550.04	131.77	11.838	3.5554	69.506
7.573300	1009.0	71.839	2.9088	23.677	1076.0	550.17	132.51	11.760	3.5993	73.652

AT	AP	AH	AFDP	GTEP	TIT	TAT	TEY	CDP	CO	NOX
0.68481	1026.7	56.029	4.0703	34.213	1100.0	527.71	168.83	14.358	2.9790	59.354
1.99950	1026.3	54.000	3.9830	34.171	1100.1	530.64	166.13	14.182	2.5793	59.432
3.16630	1025.7	51.350	4.0683	35.162	1099.8	528.21	167.49	14.384	2.2228	58.432
3.64860	1025.3	51.649	4.0375	35.282	1100.0	530.04	165.89	14.257	2.2119	60.172
3.92070	1025.2	49.619	4.0455	34.648	1099.9	529.73	166.00	14.253	2.3487	60.255
3.91930	1025.1	51.181	4.0400	33.944	1100.0	529.56	166.46	14.283	2.3095	59.778

Here's some descriptive statistics of the data set:

```
##        AT               AP               AH              AFDP
##  Min.   :-6.235   Min.   : 989.4   Min.   :24.09   Min.   :2.369
##  1st Qu.:11.073   1st Qu.:1009.7   1st Qu.:59.45   1st Qu.:3.117
##  Median :17.456   Median :1014.0   Median :70.95   Median :3.538
##  Mean   :17.225   Mean   :1014.5   Mean   :68.65   Mean   :3.599
##  3rd Qu.:23.685   3rd Qu.:1018.3   3rd Qu.:79.65   3rd Qu.:4.195
##  Max.   :37.103   Max.   :1036.6   Max.   :96.67   Max.   :5.239
##       GTEP            TIT            TAT             TEY
##  Min.   :17.70   Min.   :1016   Min.   :516.0   Min.   :100.0
##  1st Qu.:23.15   1st Qu.:1070   1st Qu.:544.7   1st Qu.:126.3
##  Median :25.33   Median :1080   Median :549.7   Median :131.6
##  Mean   :26.13   Mean   :1079   Mean   :546.6   Mean   :134.0
##  3rd Qu.:30.02   3rd Qu.:1100   3rd Qu.:550.0   3rd Qu.:147.2
##  Max.   :40.72   Max.   :1100   Max.   :550.6   Max.   :179.5
##       CDP               CO               NOX
##  Min.   : 9.871   Min.   : 0.2128   Min.   : 25.91
##  1st Qu.:11.466   1st Qu.: 1.8082   1st Qu.: 52.40
##  Median :11.933   Median : 2.5334   Median : 56.84
##  Mean   :12.097   Mean   : 3.1300   Mean   : 59.89
##  3rd Qu.:13.148   3rd Qu.: 3.7026   3rd Qu.: 65.09
##  Max.   :15.159   Max.   :41.0970   Max.   :119.68
```

```
##        AT               AP               AH              AFDP
##  Min.   :-5.785   Min.   : 990.8   Min.   :31.62   Min.   :2.644
##  1st Qu.:10.505   1st Qu.:1010.6   1st Qu.:63.24   1st Qu.:3.207
##  Median :15.460   Median :1015.2   Median :73.42   Median :3.344
##  Mean   :15.661   Mean   :1015.1   Mean   :71.45   Mean   :3.441
##  3rd Qu.:20.797   3rd Qu.:1019.2   3rd Qu.:81.10   3rd Qu.:3.751
##  Max.   :35.406   Max.   :1035.3   Max.   :96.67   Max.   :4.287
##       GTEP            TIT            TAT             TEY             CDP
##  Min.   :21.70   Min.   :1052   Min.   :529.2   Min.   :127.0   Min.   :11.29
##  1st Qu.:23.59   1st Qu.:1074   1st Qu.:549.8   1st Qu.:129.7   1st Qu.:11.65
```

```
## Median :24.20   Median :1077   Median :550.0   Median :130.4   Median :11.76
## Mean   :24.52   Mean   :1077   Mean   :549.9   Mean   :130.3   Mean   :11.77
## 3rd Qu.:24.99   3rd Qu.:1079   3rd Qu.:550.1   3rd Qu.:131.1   3rd Qu.:11.88
## Max.   :30.89   Max.   :1087   Max.   :550.5   Max.   :133.0   Max.   :12.28
##       CO                NOX
## Min.   : 0.4843   Min.   : 35.60
## 1st Qu.: 2.0296   1st Qu.: 52.57
## Median : 2.6196   Median : 57.35
## Mean   : 2.7564   Mean   : 59.60
## 3rd Qu.: 3.4285   3rd Qu.: 66.56
## Max.   :36.4540   Max.   :102.33
```

```
##       AT                AP              AH             AFDP
## Min.   :-6.235   Min.   :1006   Min.   :44.25   Min.   :3.608
## 1st Qu.: 3.041   1st Qu.:1021   1st Qu.:67.35   1st Qu.:4.084
## Median : 5.996   Median :1024   Median :75.36   Median :4.201
## Mean   : 5.277   Mean   :1023   Mean   :74.19   Mean   :4.293
## 3rd Qu.: 8.255   3rd Qu.:1025   3rd Qu.:82.20   3rd Qu.:4.516
## Max.   :15.830   Max.   :1037   Max.   :93.52   Max.   :5.239
##      GTEP            TIT             TAT             TEY             CDP
## Min.   :30.99   Min.   :1096   Min.   :516.0   Min.   :160.0   Min.   :13.63
## 1st Qu.:32.81   1st Qu.:1100   1st Qu.:529.7   1st Qu.:162.0   1st Qu.:14.00
## Median :33.43   Median :1100   Median :531.6   Median :163.8   Median :14.13
## Mean   :33.72   Mean   :1100   Mean   :530.9   Mean   :164.7   Mean   :14.17
## 3rd Qu.:34.21   3rd Qu.:1100   3rd Qu.:533.3   3rd Qu.:166.4   3rd Qu.:14.25
## Max.   :40.72   Max.   :1100   Max.   :538.5   Max.   :179.5   Max.   :15.16
##       CO              NOX
## Min.   :0.2128   Min.   :45.92
## 1st Qu.:2.1237   1st Qu.:50.27
## Median :2.4638   Median :57.37
## Mean   :2.3864   Mean   :55.55
## 3rd Qu.:2.7363   3rd Qu.:59.70
## Max.   :4.0948   Max.   :69.88
```

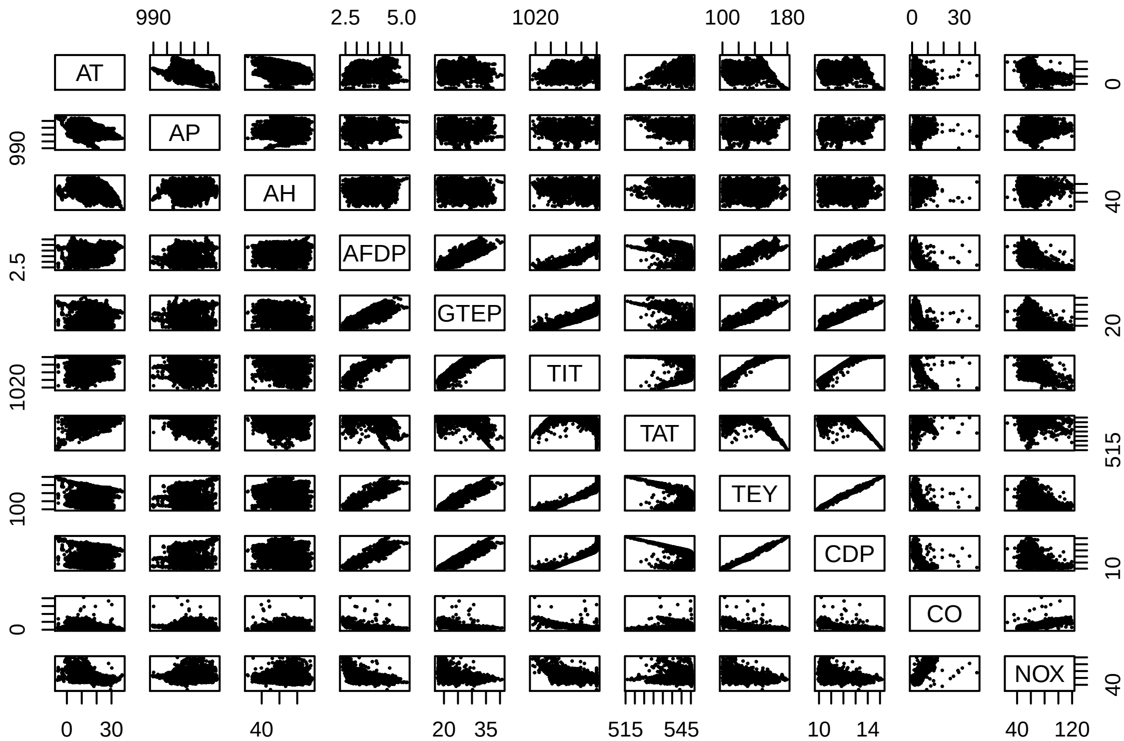
Goals

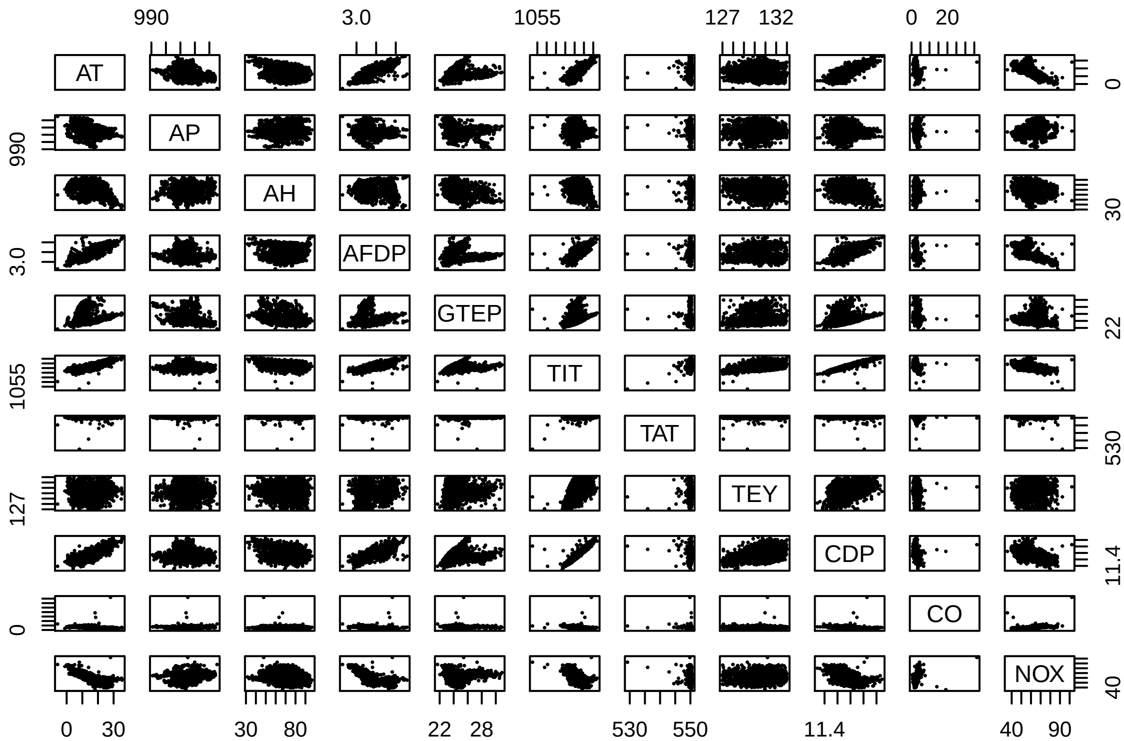
The goal for this project is to utilize this data set for the purpose of studying flue gas emissions, specifically carbon monoxide(CO) and nitrogen oxides (NOx). Our focus will be to find statistically significant relationships between the ambient and turbine variables and the emissions variables. We will limit the size of our model to more clearly demonstrate these relationships. Ultimately we will suggest which variables make the biggest impact on emission levels in order to decrease emissions overall.

Exploratory Data Analysis

Relationships between feature variables

Figure 1: Scatterplot Matrices to decide which feature variables have a linear relationship


AT
AP
AH
AFDP
GTEP
TIT
TAT
TEY
CDP
CO
NOX
990
2.5
5.0
1020
100
180
0
30
40
20
35
515
545
10
14
120


AT
AP
AH
AFDP
GTEP
TIT
TAT
TEY
CDP
CO
NOX
990
3.0
1055
127
132
0
20
30
22
530
11.4
40
90
80
28
550

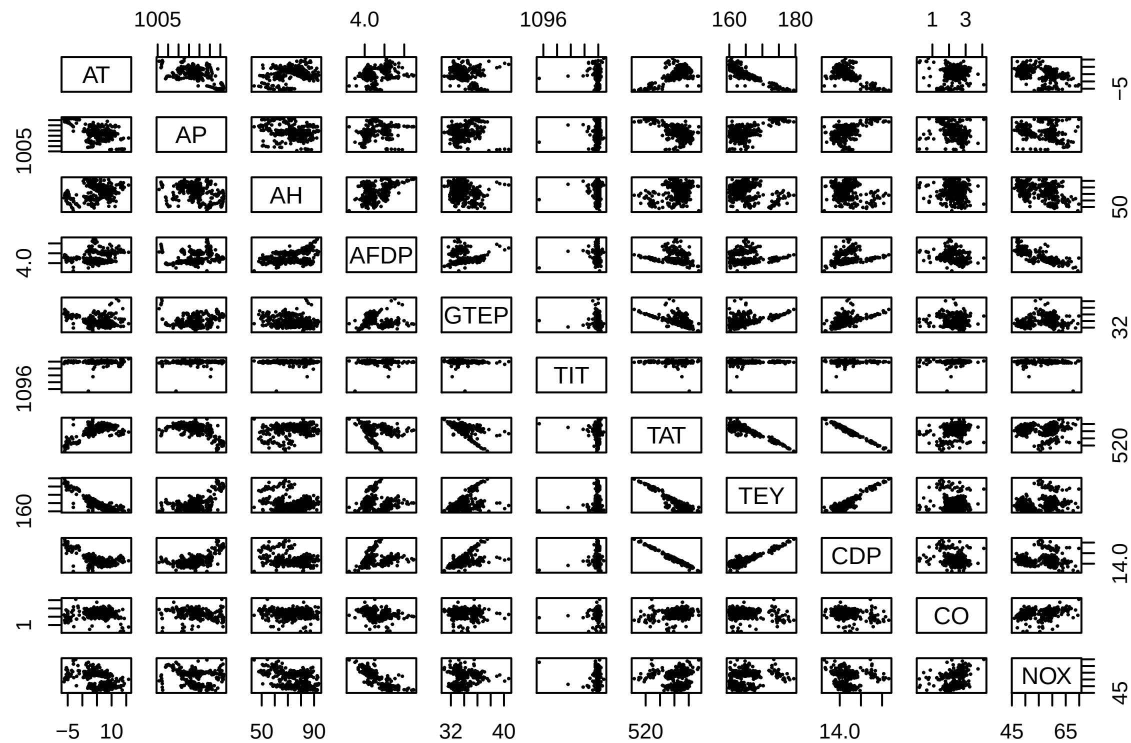


Figure 2:

Table 4: Pairwise Correlation Between Variables (All Data)

	AT	AP	AH	AFDP	GTEP	TIT	TAT	TEY	CDP	CO	NOX
AT	1.00	-0.49	-0.47	0.47	0.19	0.33	0.21	0.11	0.20	-0.39	-0.59
AP	-0.49	1.00	0.08	-0.09	-0.04	-0.08	-0.29	0.05	0.03	0.20	0.21
AH	-0.47	0.08	1.00	-0.25	-0.30	-0.26	0.03	-0.18	-0.22	0.16	0.07
AFDP	0.47	-0.09	-0.25	1.00	0.84	0.92	-0.52	0.88	0.92	-0.64	-0.58
GTEP	0.19	-0.04	-0.30	0.84	1.00	0.89	-0.62	0.93	0.94	-0.56	-0.37
TIT	0.33	-0.08	-0.26	0.92	0.89	1.00	-0.40	0.95	0.95	-0.74	-0.52
TAT	0.21	-0.29	0.03	-0.52	-0.62	-0.40	1.00	-0.63	-0.66	0.03	0.05
TEY	0.11	0.05	-0.18	0.88	0.93	0.95	-0.63	1.00	0.99	-0.62	-0.40
CDP	0.20	0.03	-0.22	0.92	0.94	0.95	-0.66	0.99	1.00	-0.61	-0.44
CO	-0.39	0.20	0.16	-0.64	-0.56	-0.74	0.03	-0.62	-0.61	1.00	0.68
NOX	-0.59	0.21	0.07	-0.58	-0.37	-0.52	0.05	-0.40	-0.44	0.68	1.00

Table 5: Pairwise Correlation Between Variables (Typical Energy Yield)

	AT	AP	AH	AFDP	GTEP	TIT	TAT	TEY	CDP	CO	NOX
AT	1.00	-0.33	-0.34	0.84	0.28	0.84	0.02	0.03	0.83	-0.45	-0.75
AP	-0.33	1.00	0.00	-0.18	-0.31	-0.13	-0.05	-0.03	-0.13	0.14	0.20
AH	-0.34	0.00	1.00	-0.13	-0.42	-0.33	0.01	-0.07	-0.32	-0.02	-0.13

	AT	AP	AH	AFDP	GTEP	TIT	TAT	TEY	CDP	CO	NOX
AFDP	0.84	-0.18	-0.13	1.00	0.12	0.82	0.00	0.18	0.81	-0.41	-0.67
GTEP	0.28	-0.31	-0.42	0.12	1.00	0.23	-0.04	0.15	0.23	0.01	0.04
TIT	0.84	-0.13	-0.33	0.82	0.23	1.00	0.21	0.48	0.91	-0.41	-0.61
TAT	0.02	-0.05	0.01	0.00	-0.04	0.21	1.00	0.06	-0.06	-0.06	-0.11
TEY	0.03	-0.03	-0.07	0.18	0.15	0.48	0.06	1.00	0.41	-0.05	0.06
CDP	0.83	-0.13	-0.32	0.81	0.23	0.91	-0.06	0.41	1.00	-0.40	-0.59
CO	-0.45	0.14	-0.02	-0.41	0.01	-0.41	-0.06	-0.05	-0.40	1.00	0.55
NOX	-0.75	0.20	-0.13	-0.67	0.04	-0.61	-0.11	0.06	-0.59	0.55	1.00

Table 6: Pairwise Correlation Between Variables (High Energy Yield)

	AT	AP	AH	AFDP	GTEP	TIT	TAT	TEY	CDP	CO	NOX
AT	1.00	-0.52	0.20	0.24	-0.29	0.03	0.67	-0.89	-0.70	0.01	-0.42
AP	-0.52	1.00	-0.22	0.23	0.11	0.07	-0.55	0.55	0.59	-0.18	-0.22
AH	0.20	-0.22	1.00	0.30	-0.21	-0.01	0.23	-0.24	-0.24	-0.03	-0.34
AFDP	0.24	0.23	0.30	1.00	-0.08	0.00	-0.17	-0.05	0.15	-0.40	-0.82
GTEP	-0.29	0.11	-0.21	-0.08	1.00	-0.02	-0.59	0.51	0.56	-0.03	0.26
TIT	0.03	0.07	-0.01	0.00	-0.02	1.00	-0.04	0.06	0.09	0.06	-0.04
TAT	0.67	-0.55	0.23	-0.17	-0.59	-0.04	1.00	-0.92	-0.99	0.29	-0.06
TEY	-0.89	0.55	-0.24	-0.05	0.51	0.06	-0.92	1.00	0.94	-0.15	0.27
CDP	-0.70	0.59	-0.24	0.15	0.56	0.09	-0.99	0.94	1.00	-0.26	0.06
CO	0.01	-0.18	-0.03	-0.40	-0.03	0.06	0.29	-0.15	-0.26	1.00	0.39
NOX	-0.42	-0.22	-0.34	-0.82	0.26	-0.04	-0.06	0.27	0.06	0.39	1.00

Remove variables that are highly correlated.

```
##       AT       AP       AH     AFDP     GTEP      TAT
## 3.866424 1.600597 1.718769 7.412520 5.909197 2.301015
```

```
##       AP       AH     AFDP     GTEP      TAT      TEY      CDP
## 1.175646 1.434192 3.568001 1.400706 1.032173 1.383516 4.562918
```

```
##       AT       AP       AH     AFDP     GTEP      TIT      TAT
## 3.084771 1.948954 1.321587 1.996187 1.897431 1.027967 4.191605
```

Exploratory analysis shows possible linear relationships between the response variable CO and the feature variables CDP, TEY, TIT, GTEP and AFDP. Collinearity between some of the feature variables (TIT, CDP, and TEY) could cause some problems in our analysis and will likely lead to the removal of the redundant variables.

Methods

Linear Regression

We will create a multiple linear regression model using all feature variables mentioned in the description of Section 1. The implementation and parameters of this model can be obtained by the following equation where we will find estimates for the parameters β using:

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

Key assumptions are stated as:

- **L**inearity: can be written as a linear combination of the predictors.
- **I**ndependence: the errors are independent of each other (not highly correlated).
- **N**ormality: the distribution of the errors follow a normal distribution.
- **E**qual Variance: the error variance is the same.[1]

We will then use model selection using backward BIC to tune our model and remove any insignificant predictor variables. This selection prefers smaller models which aligns with our goal of limiting the size of our final model.

```
full_model = lm(CO ~ ., data = gt_2015)
linear_model = lm(CO ~ .-NOX - TIT - CDP - TEY, data = gt_2015)
summary(linear_model)
```

```
##
## Call:
## lm(formula = CO ~ . - NOX - TIT - CDP - TEY, data = gt_2015)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -6.839 -0.673 -0.132  0.481 34.242
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 145.099775   4.408695  32.912  < 2e-16 ***
## AT            0.028276   0.004060   6.965 3.57e-12 ***
## AP            0.001918   0.003067   0.625    0.532
## AH           -0.009753   0.001618  -6.026 1.76e-09 ***
## AFDP         -2.531044   0.074576 -33.939  < 2e-16 ***
## GTEP         -0.186308   0.009082 -20.513  < 2e-16 ***
## TAT          -0.237369   0.004619 -51.387  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.436 on 7377 degrees of freedom
## Multiple R-squared:  0.5874, Adjusted R-squared:  0.587
## F-statistic:  1750 on 6 and 7377 DF,  p-value: < 2.2e-16
```

```
#picking a new variable to test
AT_model = lm(CO ~ AT, data = gt_2015)
AP_model = lm(CO ~ AP, data = gt_2015)
AH_model = lm(CO ~ AH, data = gt_2015)
AFDP_model = lm(CO ~ AFDP, data = gt_2015)
GTEP_model = lm(CO ~ GTEP, data = gt_2015)
TAT_model = lm(CO ~ TAT, data = gt_2015)
BIC(AT_model)
```

[1] Dalpiaz David, Applied Statistics in R, https://daviddalpiaz.github.io/appliedstats/model-diagnostics.html

```
## [1] 31634.69
```

```
BIC(AP_model)
```

```
## [1] 32553.05
```

```
BIC(AH_model)
```

```
## [1] 32668.32
```

```
BIC(AFDP_model) #second best
```

```
## [1] 28953.71
```

```
BIC(GTEP_model)
```

```
## [1] 30112.68
```

```
BIC(TAT_model)
```

```
## [1] 32852.49
```

```
BIC(linear_model) #this is the best model
```

```
## [1] 26365.45
```

```
library(MASS)

n = length(resid(linear_model))
BIC_model = step(linear_model, direction = "backward", k = log(n))
```

```
## Start:  AIC=5401.66
## CO ~ (AT + AP + AH + AFDP + GTEP + TIT + TAT + TEY + CDP + NOX) -
##     NOX - TIT - CDP - TEY
##
##        Df Sum of Sq   RSS    AIC
## - AP    1       0.8 15218 5393.1
## <none>              15217 5401.7
## - AH    1      74.9 15292 5429.0
## - AT    1     100.1 15317 5441.1
## - GTEP  1     868.0 16085 5802.4
## - AFDP  1    2376.0 17593 6464.1
## - TAT   1    5447.0 20664 7652.1
##
## Step:  AIC=5393.14
## CO ~ AT + AH + AFDP + GTEP + TAT
##
##        Df Sum of Sq   RSS    AIC
## <none>              15218 5393.1
```

```
## - AH    1      86.8 15304 5426.2
## - AT    1     120.5 15338 5442.5
## - GTEP  1     991.0 16209 5850.1
## - AFDP  1    2564.3 17782 6534.1
## - TAT   1    5608.5 20826 7701.0
```

```r
coef(BIC_model)
```

```
##  (Intercept)           AT           AH         AFDP         GTEP          TAT
## 147.33755305   0.02702808  -0.01004686  -2.51758434  -0.18816259  -0.23782668
```

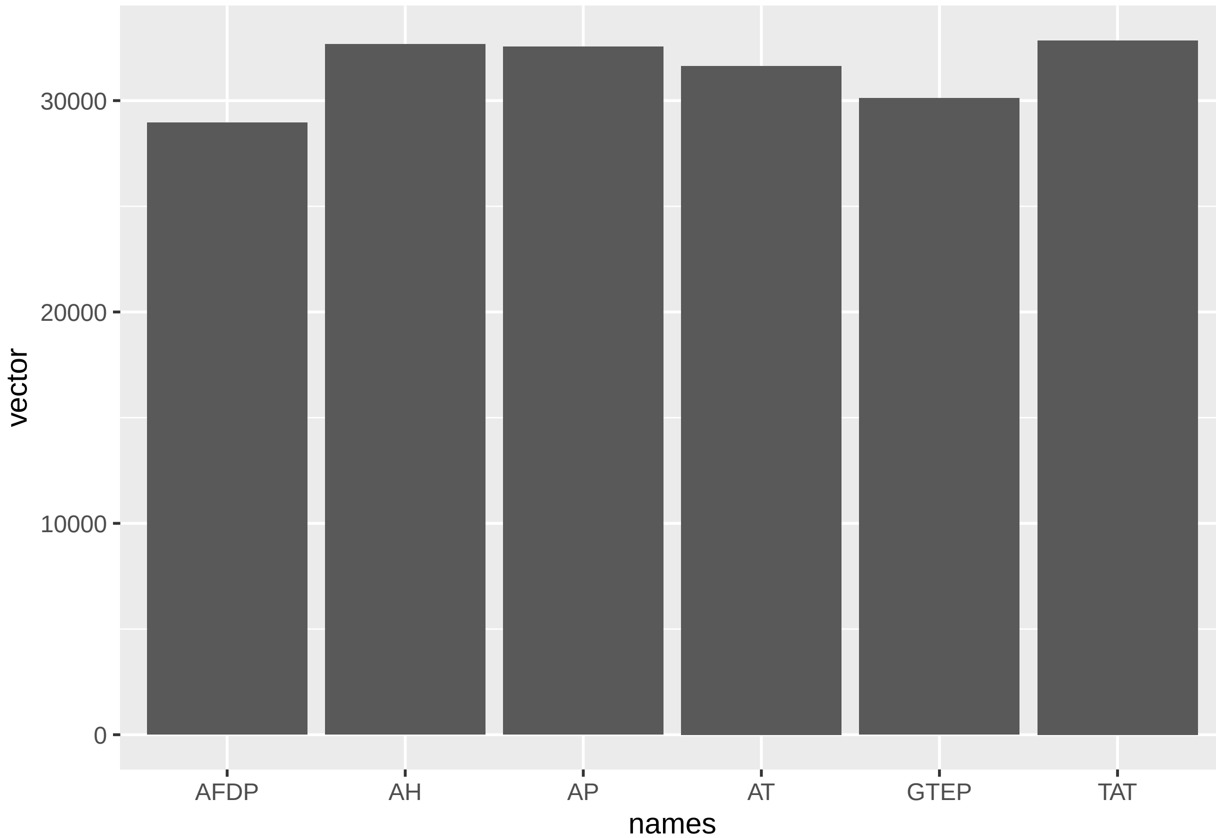
```r
stepAIC(linear_model, direction = "backward")
```

```
## Start:  AIC=5353.31
## CO ~ (AT + AP + AH + AFDP + GTEP + TIT + TAT + TEY + CDP + NOX) -
##     NOX - TIT - CDP - TEY
##
##        Df Sum of Sq   RSS    AIC
## - AP    1       0.8 15218 5351.7
## <none>              15217 5353.3
## - AH    1      74.9 15292 5387.6
## - AT    1     100.1 15317 5399.7
## - GTEP  1     868.0 16085 5760.9
## - AFDP  1    2376.0 17593 6422.6
## - TAT   1    5447.0 20664 7610.7
##
## Step:  AIC=5351.7
## CO ~ AT + AH + AFDP + GTEP + TAT
##
##        Df Sum of Sq   RSS    AIC
## <none>              15218 5351.7
## - AH    1      86.8 15304 5391.7
## - AT    1     120.5 15338 5408.0
## - GTEP  1     991.0 16209 5815.5
## - AFDP  1    2564.3 17782 6499.6
## - TAT   1    5608.5 20826 7666.5
```

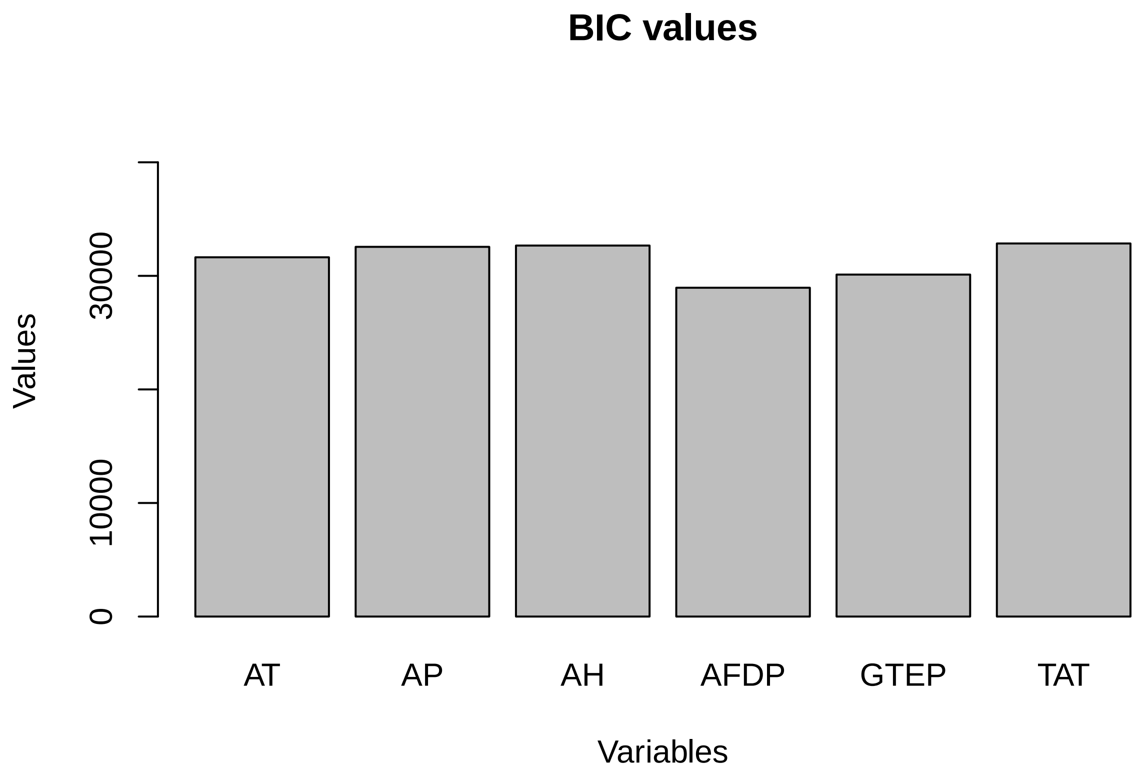
```
##
## Call:
## lm(formula = CO ~ AT + AH + AFDP + GTEP + TAT, data = gt_2015)
##
## Coefficients:
## (Intercept)           AT           AH         AFDP         GTEP          TAT
##   147.33755      0.02703     -0.01005     -2.51758     -0.18816     -0.23783
```

```r
vector <- c(BIC(AT_model), BIC(AP_model), BIC(AH_model), BIC(AFDP_model), BIC(GTEP_model), BIC(TAT_mode

library(ggplot2)
df <- data.frame(vector = c(BIC(AT_model), BIC(AP_model), BIC(AH_model), BIC(AFDP_model), BIC(GTEP_mode
ggplot(data = df, aes(x = names, y = vector), ylim = c(0, 50000)) + geom_bar(stat = "identity")
```




```
barplot(vector, main = "BIC values", xlab = "Variables",ylab = "Values", names.arg = c("AT", "AP", "AH"
```



Linear and Lasso stepwise AIC Models

```
library(caret)
```

```
## Loading required package: lattice

##
## Attaching package: 'lattice'

## The following object is masked from 'package:faraway':
##
##     melanoma
```

```
#5-fold cross validation
cv_5 <- trainControl(method = "cv", number = 5)
```

#All Data

```
set.seed(10)
#AIC stepwise selected linear model
all_linear_mod <- train(
          form = CO ~ . - NOX - TIT - CDP - TEY ,
          data = gt_2015,
          method = "lmStepAIC",
          trControl = cv_5,
```

```
          trace = FALSE
)

#Linear log model
all_log_linear_mod <- train(
          form = log(CO) ~ . - NOX  - TIT - CDP - TEY ,
          data = gt_2015,
          method = "lmStepAIC",
          trControl = cv_5,
          nvmax = 10,
          trace = FALSE
)

#Lasso model
all_lasso_mod <- train(
          form = CO ~ . - NOX - TIT - CDP - TEY ,
          data = gt_2015,
          method = "lasso",
          trControl = cv_5
)
```

#Typical Energy Yield (130-136)

```
set.seed(10)
#AIC stepwise selected linear model
typical_linear_mod <- train(
          form = CO ~ . - NOX - TIT - AT,
          data = gt_2015_typical,
          method = "lmStepAIC",
          trControl = cv_5,
          trace = FALSE
)

#Linear log model
typical_log_linear_mod <- train(
          form = log(CO) ~ . - NOX - TIT - AT,
          data = gt_2015_typical,
          method = "lmStepAIC",
          trControl = cv_5,
          nvmax = 10,
          trace = FALSE
)

#Lasso model
typical_lasso_mod <- train(
          form = CO ~ . - NOX - TIT - AT,
          data = gt_2015_typical,
          method = "lasso",
          trControl = cv_5
)
```

#High Energy Yield (160+)

```r
set.seed(10)

#AIC stepwise selected linear model
high_linear_mod <- train(
          form = CO ~ . - NOX - TEY - CDP,
          data = gt_2015_high,
          method = "lmStepAIC",
          trControl = cv_5,
          trace = FALSE
)

#Linear log model
high_log_linear_mod <- train(
          form = log(CO) ~ . - NOX - TEY - CDP,
          data = gt_2015_high,
          method = "lmStepAIC",
          trControl = cv_5,
          nvmax = 10,
          trace = FALSE
)

#Lasso model
high_lasso_mod <- train(
          form = CO ~ . - NOX - TEY - CDP,
          data = gt_2015_high,
          method = "lasso",
          trControl = cv_5
)
```

#Results

```r
all_linear_mod$results
```

```
##   parameter     RMSE  Rsquared       MAE    RMSESD RsquaredSD      MAESD
## 1      none 1.433269 0.5894359 0.8272737 0.1219639 0.03158297 0.01273652
```

```r
all_log_linear_mod$results
```

```
##   parameter      RMSE  Rsquared       MAE     RMSESD RsquaredSD       MAESD
## 1      none 0.3302711 0.6592231 0.2230641 0.01248279 0.01762377 0.003218697
```

```r
all_lasso_mod$results
```

```
##   fraction     RMSE  Rsquared       MAE    RMSESD RsquaredSD      MAESD
## 1      0.1 1.993589 0.4162352 1.1803507 0.1700911 0.06042455 0.03400057
## 2      0.5 1.548607 0.5508411 0.8209167 0.2118693 0.08098964 0.03102373
## 3      0.9 1.427676 0.5921746 0.8187187 0.2237106 0.08165401 0.02995461
```

```r
typical_linear_mod$results
```

```
##   parameter      RMSE  Rsquared       MAE    RMSESD RsquaredSD      MAESD
## 1      none 0.9697531 0.3377307 0.4738538 0.5080101  0.1978678 0.03731379
```

```r
typical_log_linear_mod$results
```

```
##   parameter      RMSE  Rsquared       MAE     RMSESD RsquaredSD       MAESD
## 1      none 0.2430984 0.4535081 0.1681481 0.02249379 0.06104676 0.007462328
```

```r
typical_lasso_mod$results
```

```
##   fraction      RMSE  Rsquared       MAE    RMSESD RsquaredSD      MAESD
## 1      0.1 1.0797560 0.3035173 0.6724230 0.4957577  0.1996600 0.05480752
## 2      0.5 0.9771476 0.3304005 0.5080683 0.5452057  0.2080158 0.04598878
## 3      0.9 0.9557657 0.3494965 0.4763786 0.5467614  0.2066107 0.04128323
```

```r
high_linear_mod$results
```

```
##   parameter     RMSE  Rsquared       MAE     RMSESD RsquaredSD      MAESD
## 1      none 0.442826 0.2248008 0.3183321 0.04574879 0.05330011 0.02670168
```

```r
high_log_linear_mod$results
```

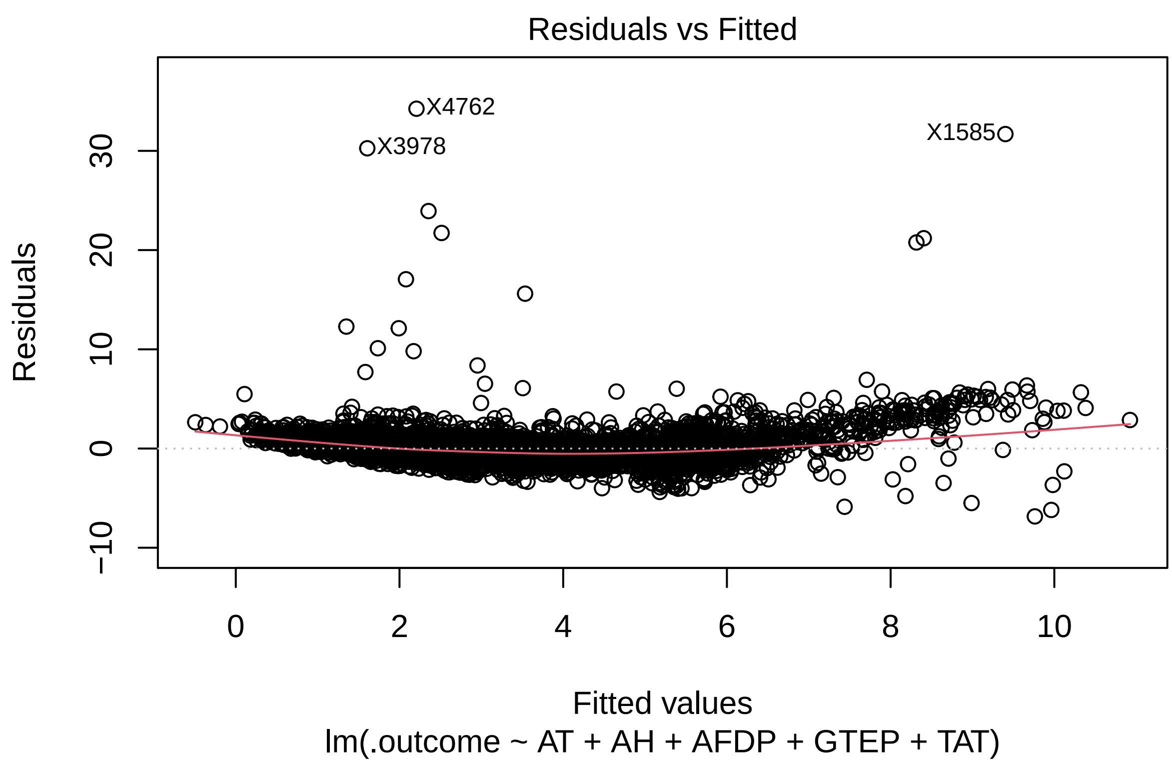
```
##   parameter      RMSE  Rsquared       MAE     RMSESD RsquaredSD      MAESD
## 1      none 0.2796676 0.1914723 0.1751923 0.04989463   0.101253 0.02364425
```

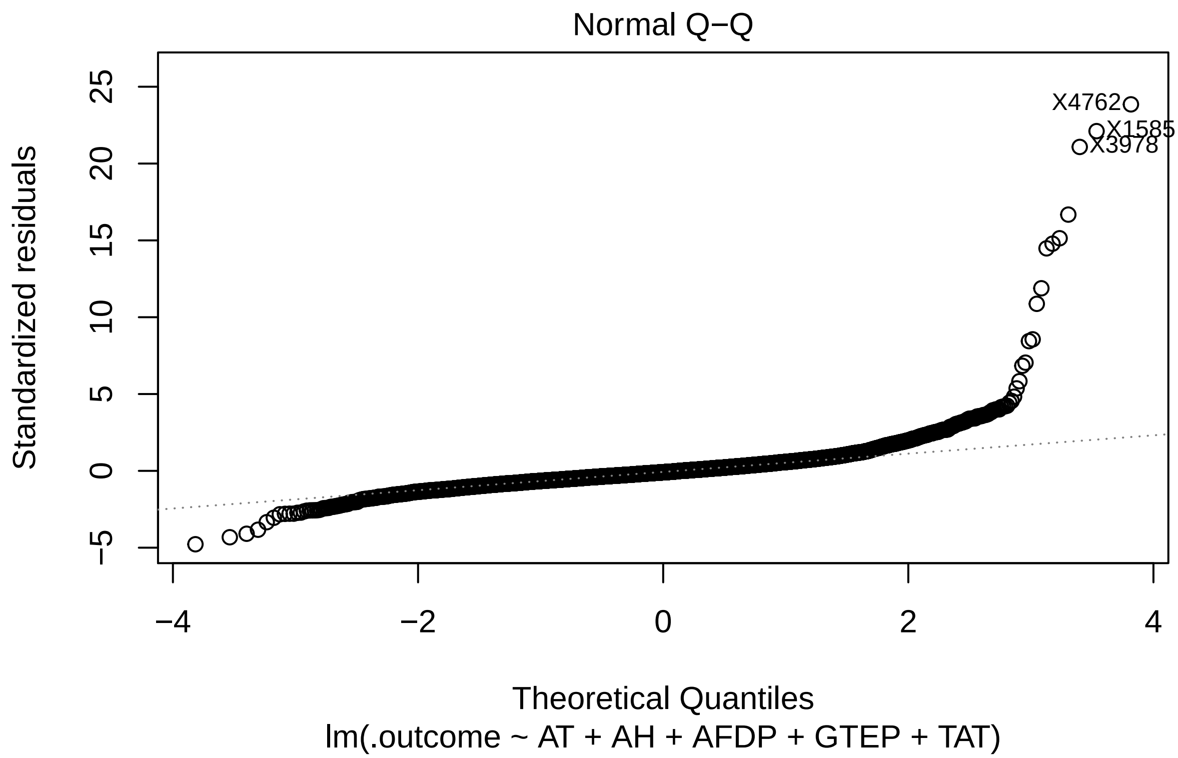
```r
high_lasso_mod$results
```

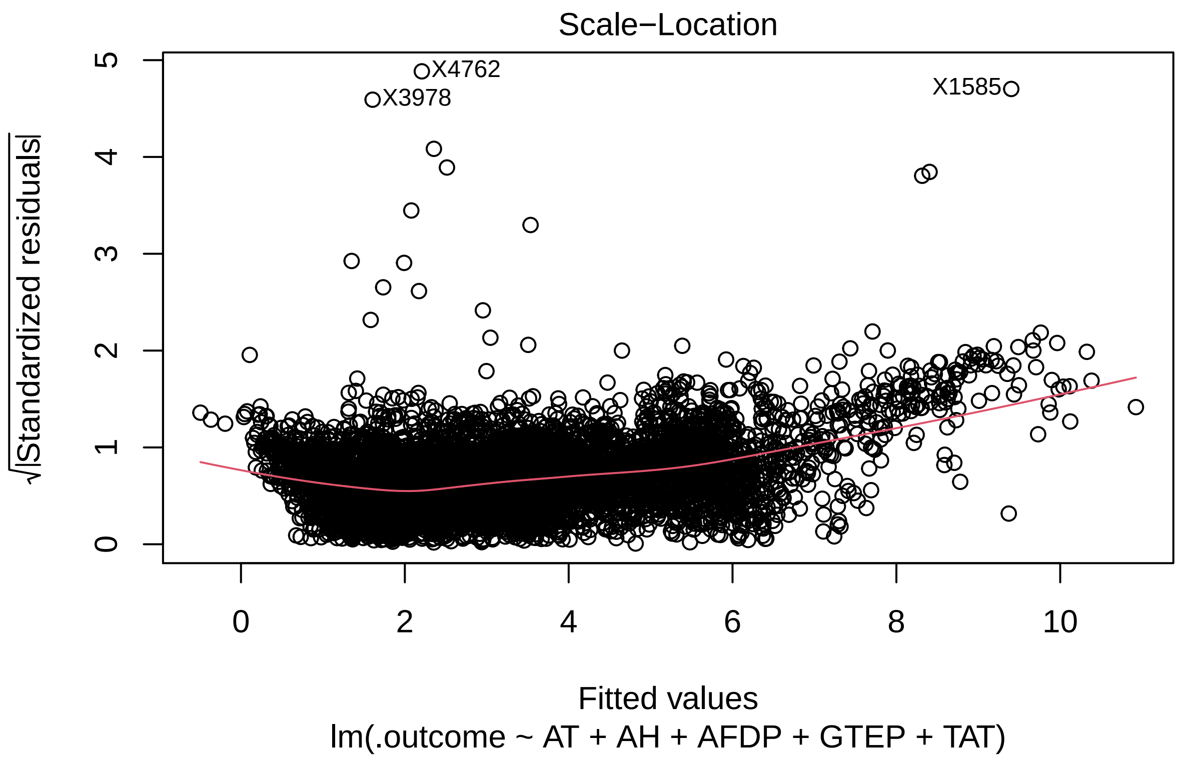
```
##   fraction      RMSE  Rsquared       MAE     RMSESD RsquaredSD      MAESD
## 1      0.1 0.4774531 0.1728564 0.3474956 0.03348756 0.06872672 0.01508288
## 2      0.5 0.4466801 0.2102170 0.3169699 0.04614911 0.08303450 0.02629183
## 3      0.9 0.4452885 0.2143768 0.3204783 0.04854922 0.07978382 0.02675016
```

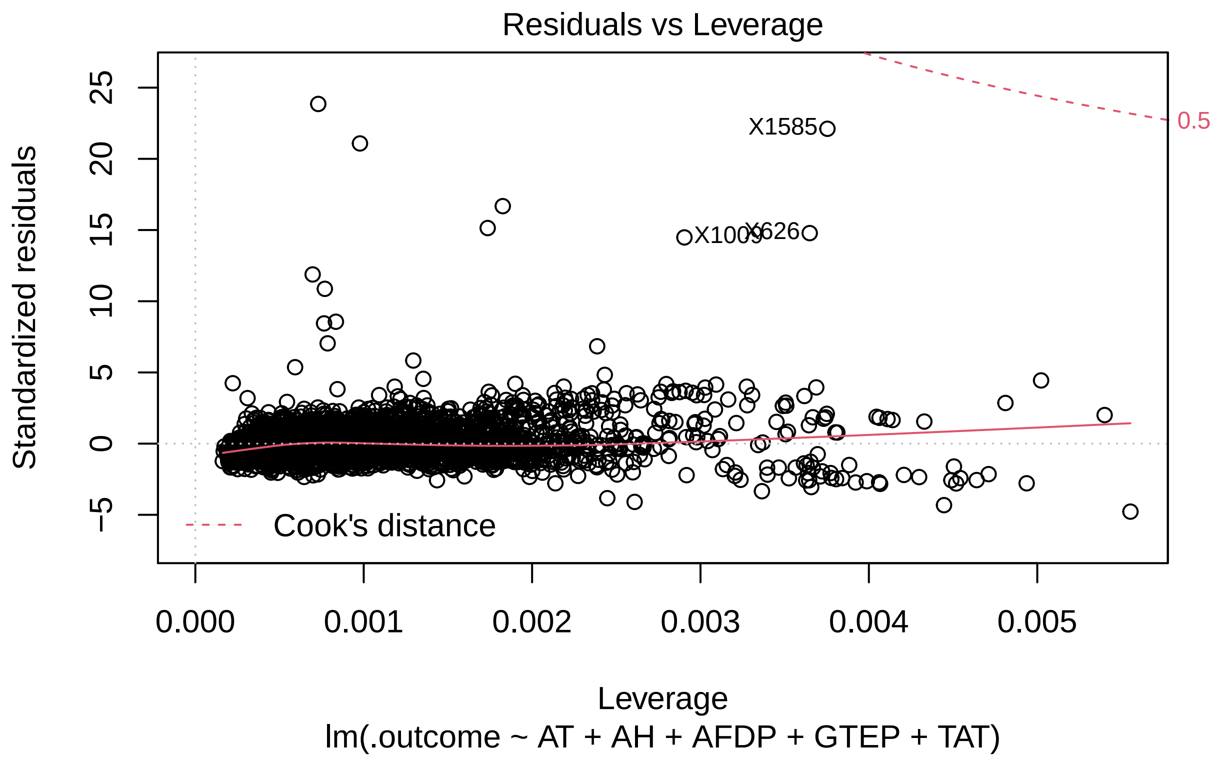
#plots

```
plot(all_linear_mod$finalModel)
```

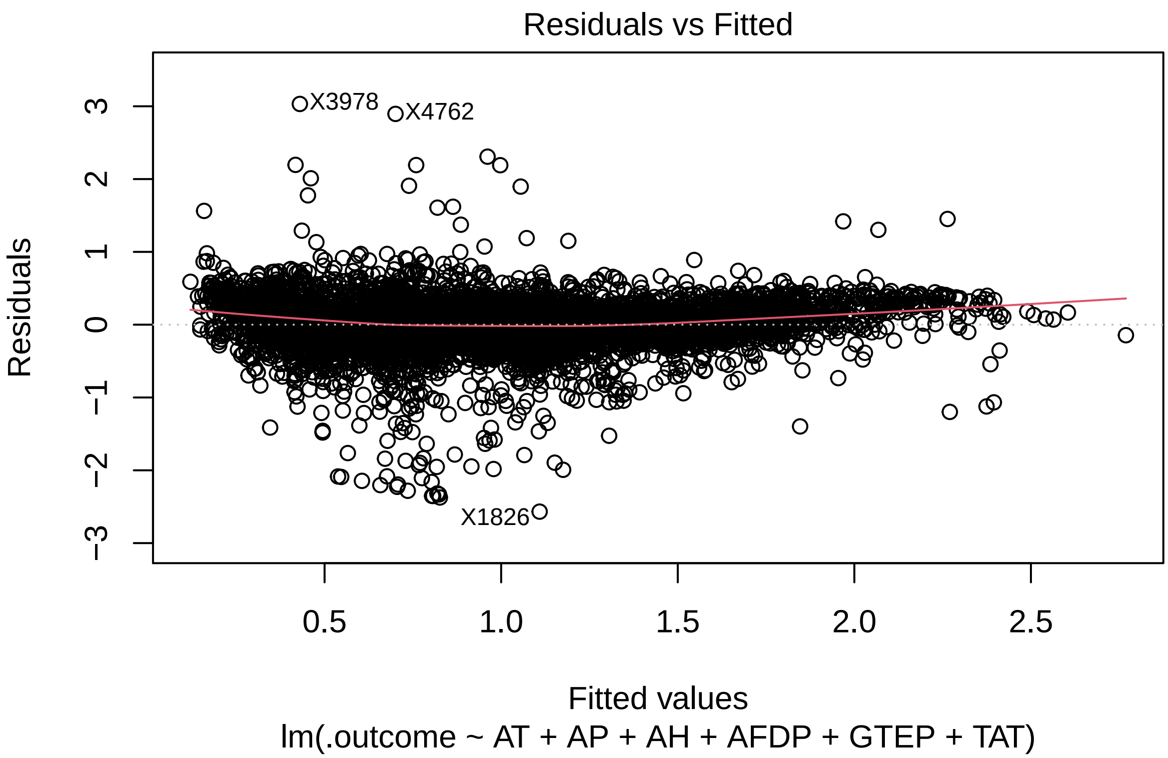


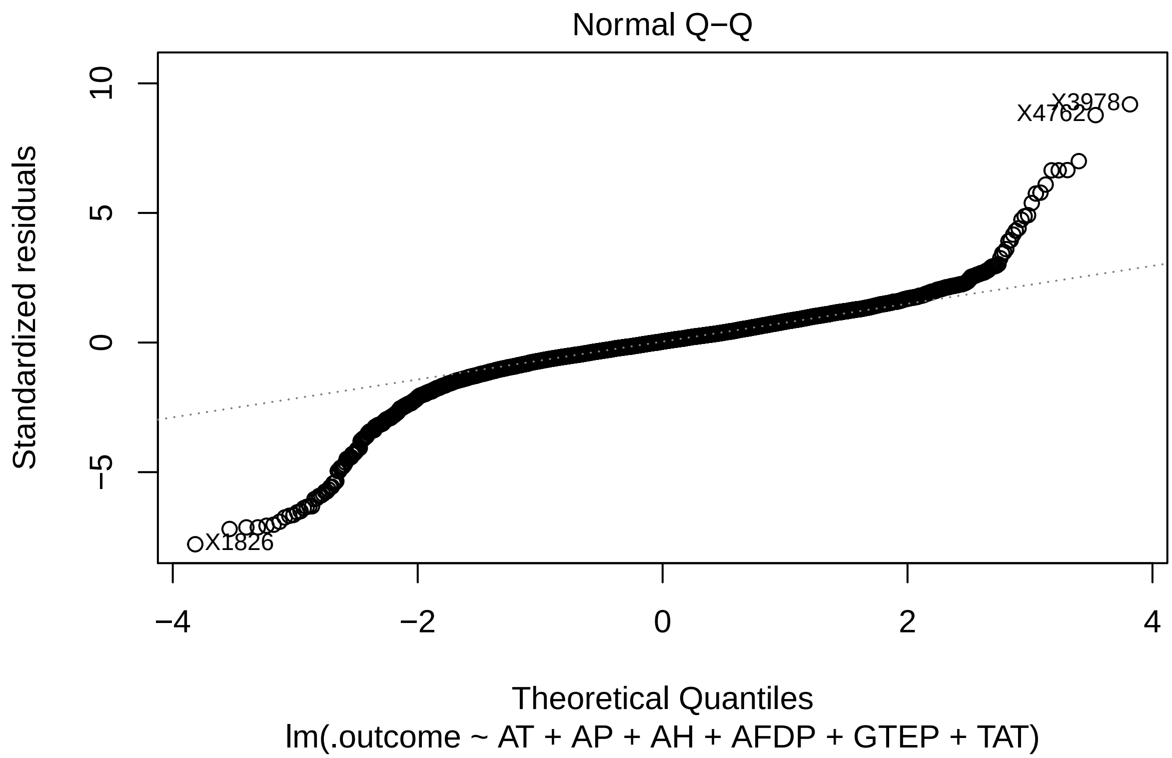

Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(.outcome ~ AT + AH + AFDP + GTEP + TAT)
X4762
X1585
X3978
−5
0
5
10
15
20
25
−4
−2
0
2
4


Scale−Location
X4762
X3978
X1585
√|Standardized residuals|
0
1
2
3
4
5
0
2
4
6
8
10
Fitted values
lm(.outcome ~ AT + AH + AFDP + GTEP + TAT)

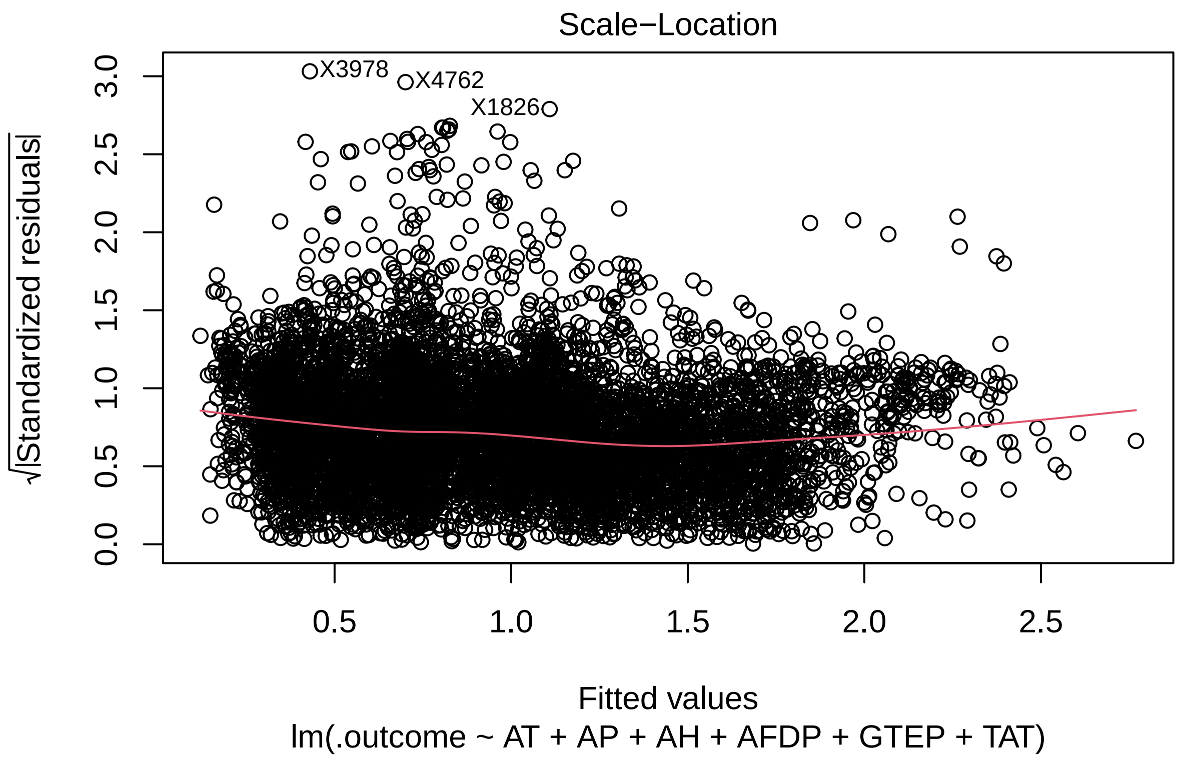


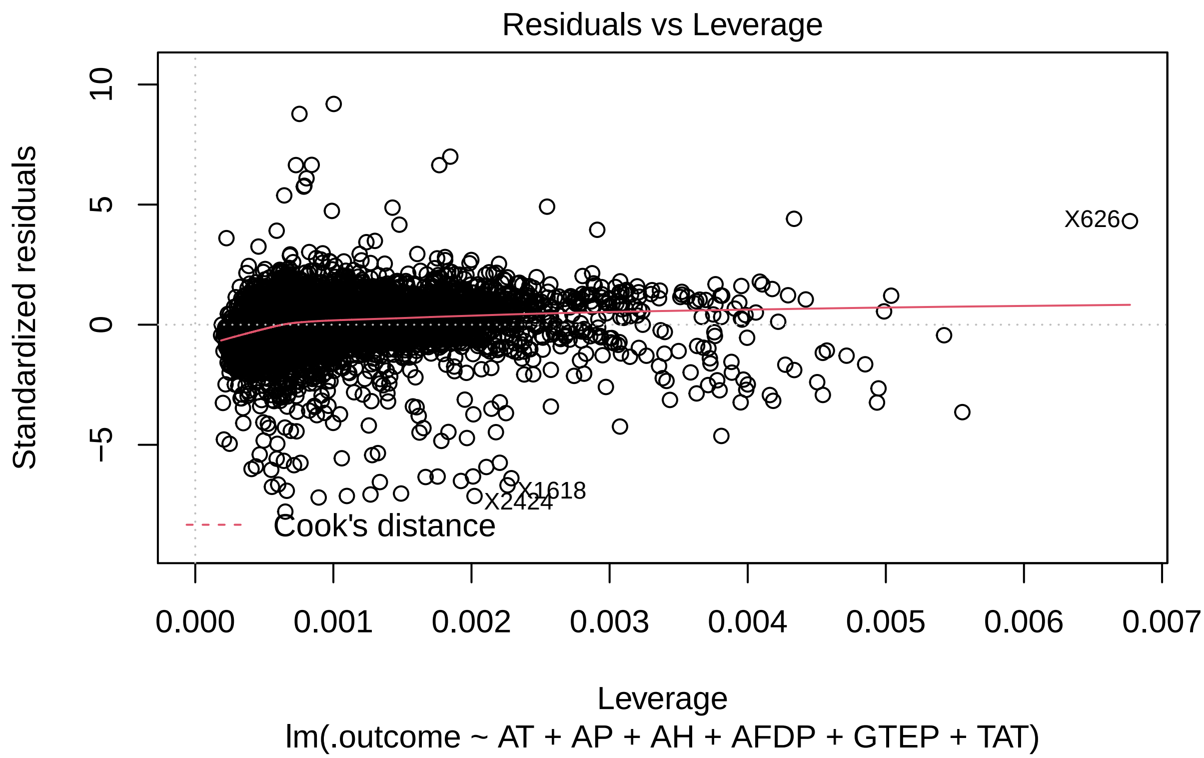

```
plot(all_log_linear_mod$finalModel)
```


Residuals vs Fitted
Residuals
Fitted values
lm(.outcome ~ AT + AP + AH + AFDP + GTEP + TAT)
X3978
X4762
X1826
3
2
1
0
−1
−2
−3
0.5
1.0
1.5
2.0
2.5

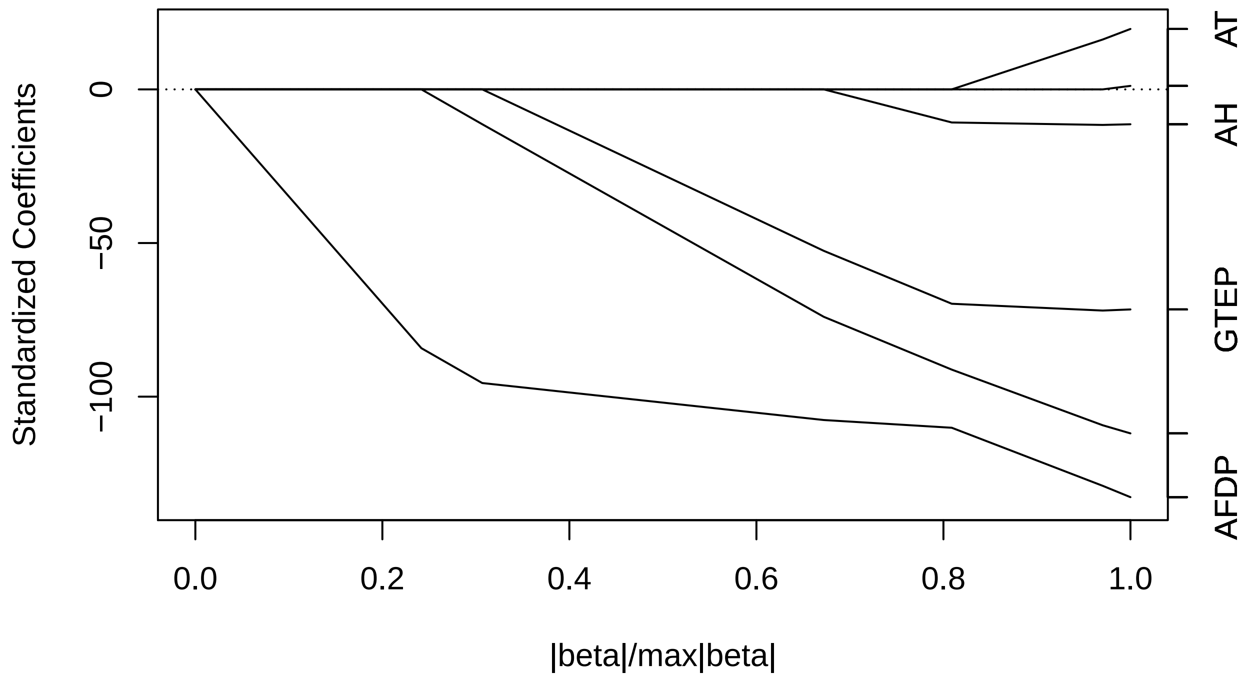


Normal Q−Q
Standardized residuals
10
5
0
−5
X3978
X4762
X1826
−4
−2
0
2
4
Theoretical Quantiles
lm(.outcome ~ AT + AP + AH + AFDP + GTEP + TAT)

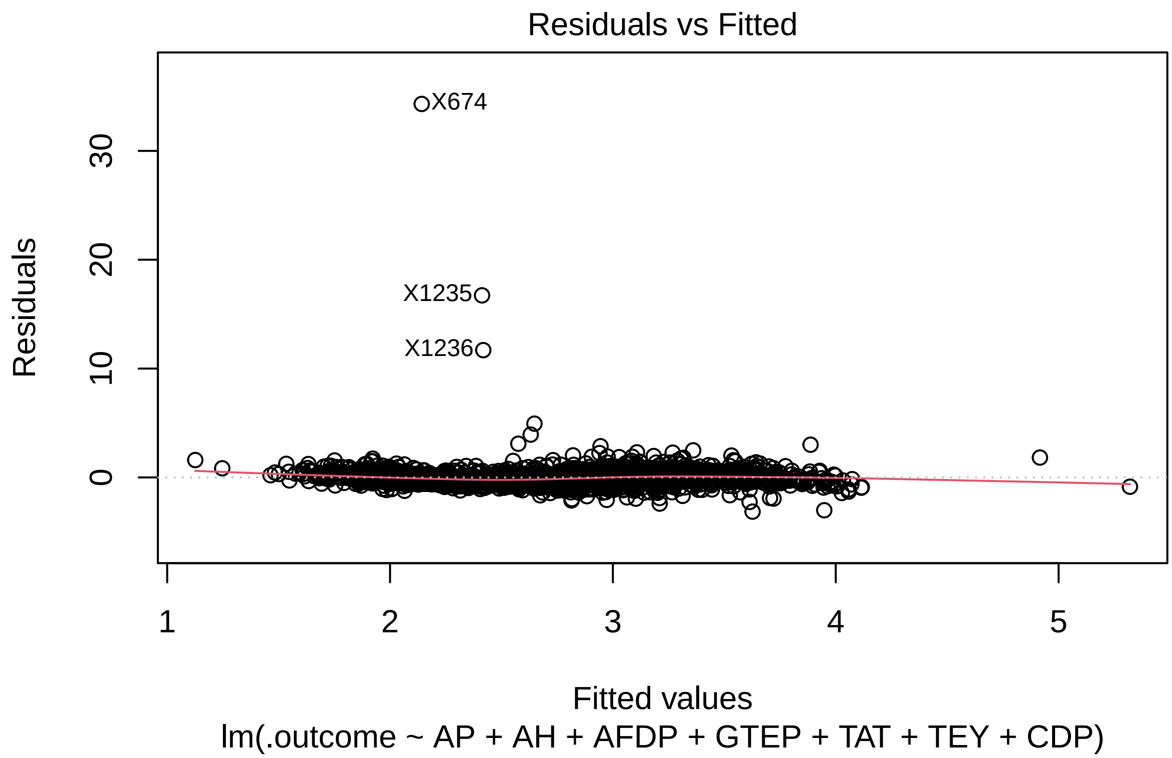

Scale−Location
X3978
X4762
X1826
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.5
1.0
1.5
2.0
2.5
Fitted values
lm(.outcome ~ AT + AP + AH + AFDP + GTEP + TAT)



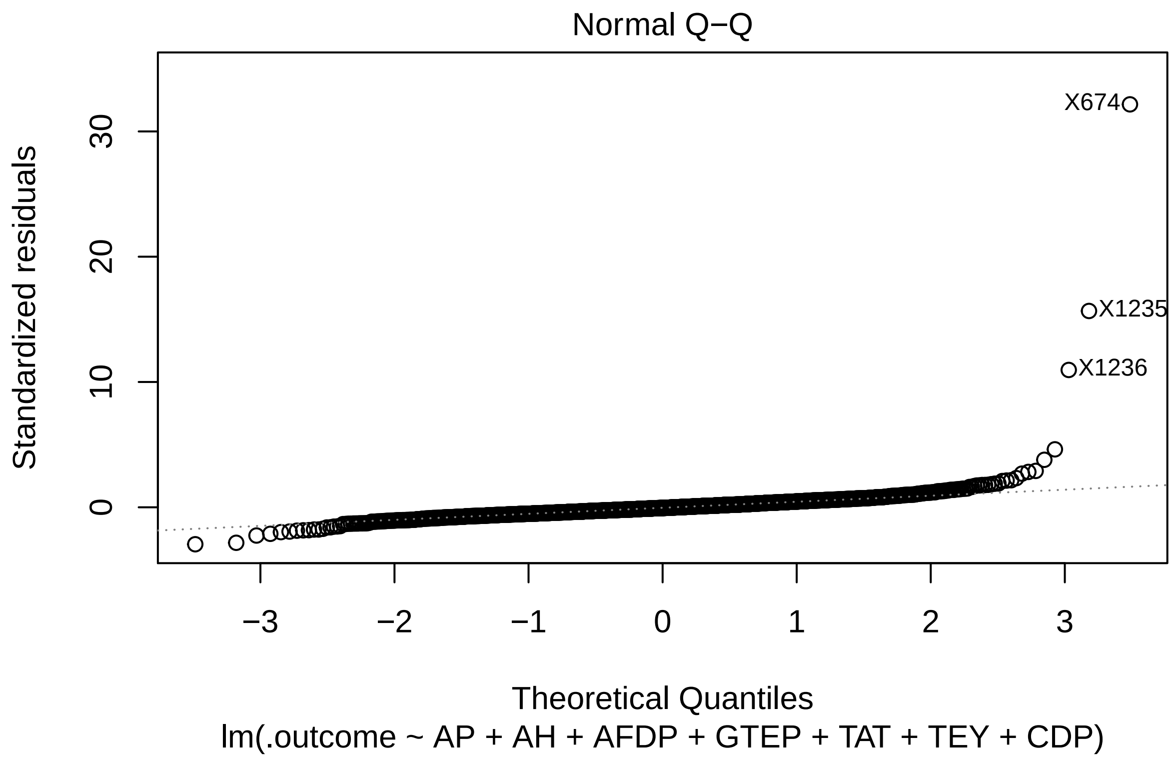

```
plot(all_lasso_mod$finalModel)
```

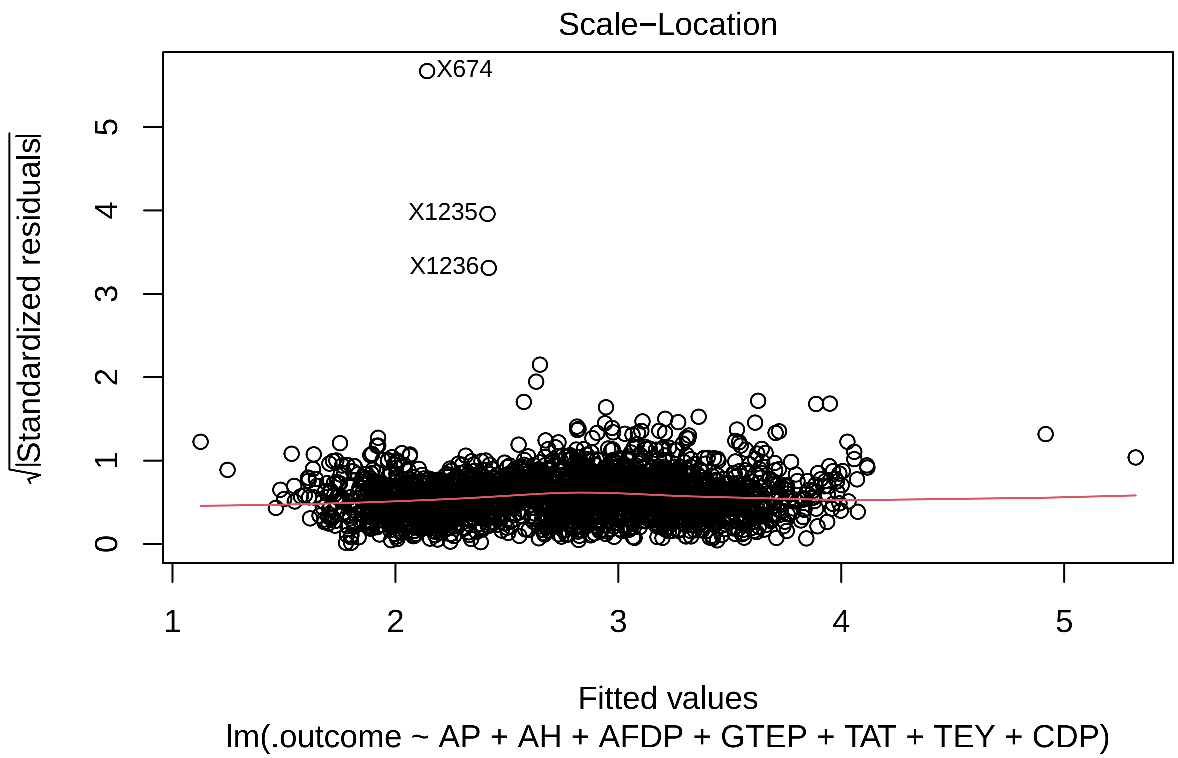


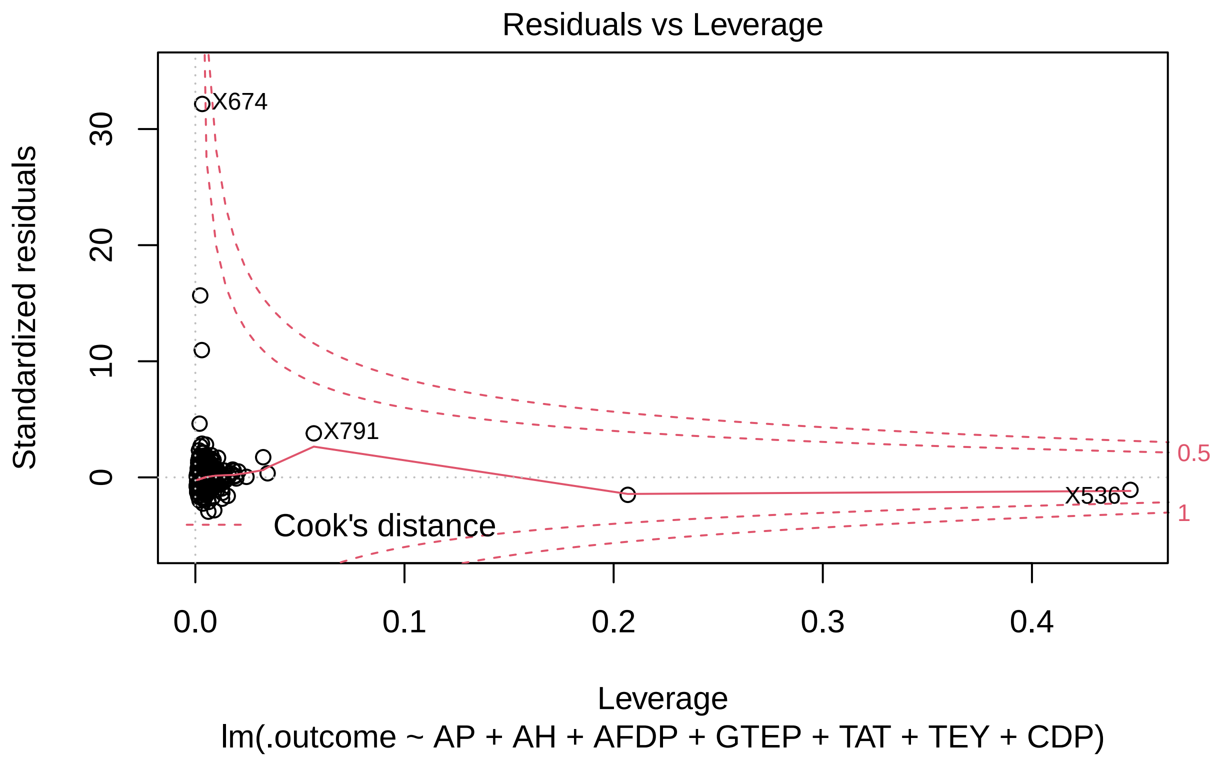

```
plot(typical_linear_mod$finalModel)
```



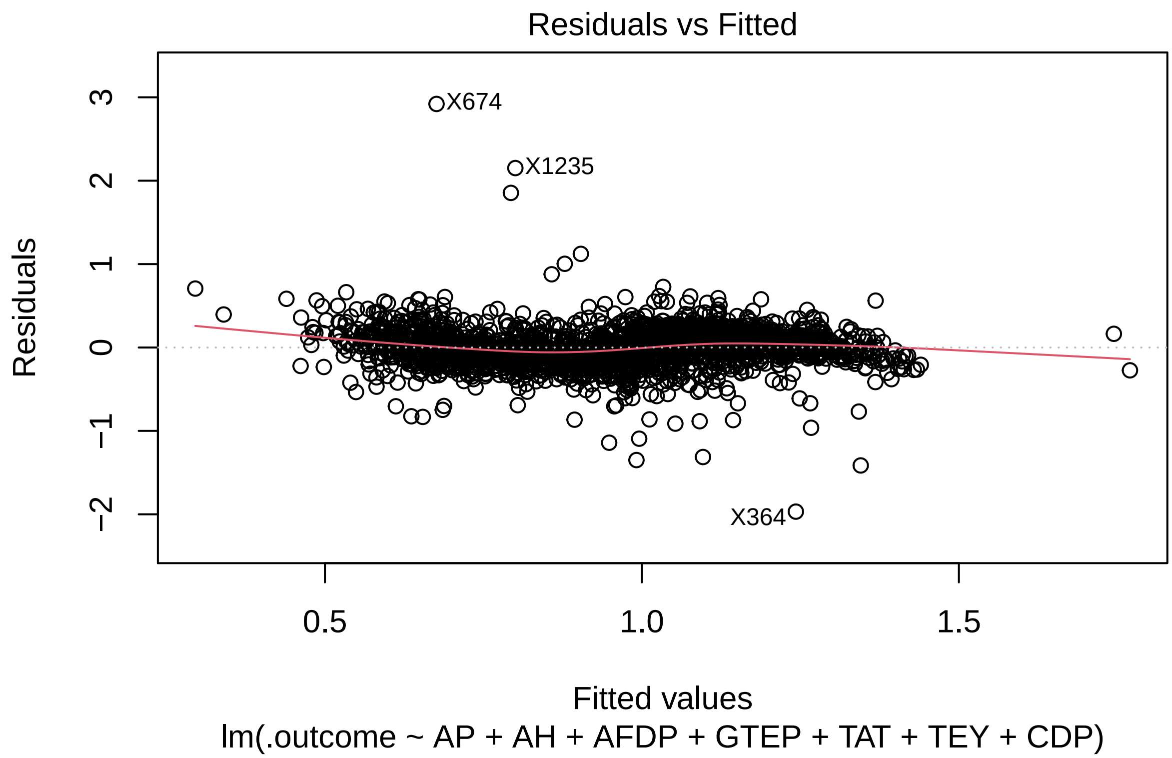
Residuals vs Fitted
Residuals
30
20
10
0
X674
X1235
X1236
1
2
3
4
5
Fitted values
lm(.outcome ~ AP + AH + AFDP + GTEP + TAT + TEY + CDP)

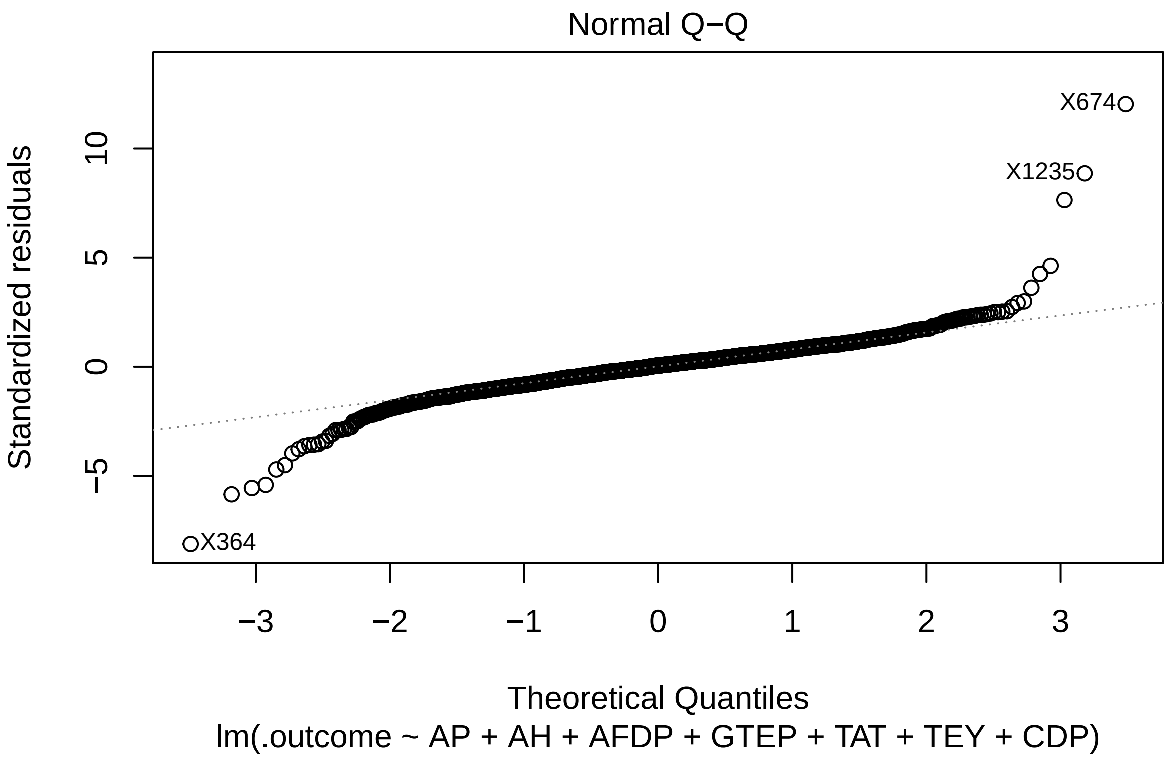

Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(.outcome ~ AP + AH + AFDP + GTEP + TAT + TEY + CDP)
X674
X1235
X1236
0
10
20
30
−3
−2
−1
0
1
2
3

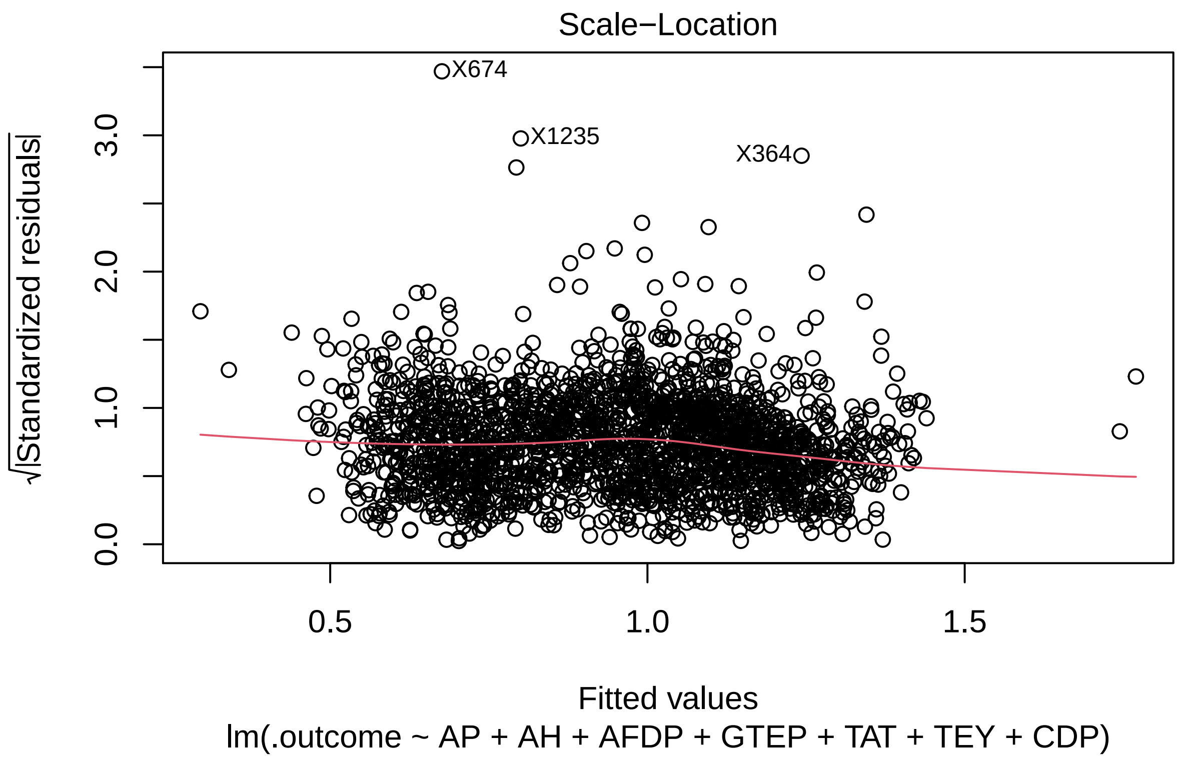

Scale−Location
X674
X1235
X1236
√|Standardized residuals|
0
1
2
3
4
5
1
2
3
4
5
Fitted values
lm(.outcome ~ AP + AH + AFDP + GTEP + TAT + TEY + CDP)

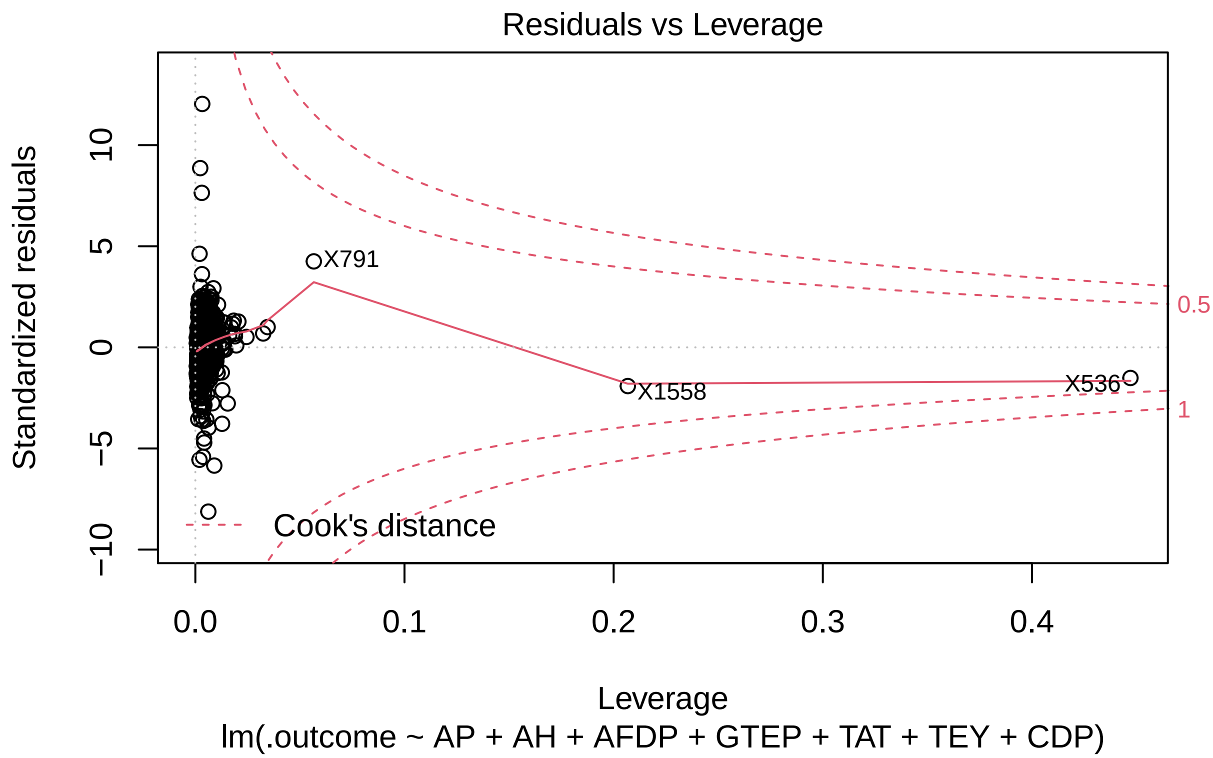



```
plot(typical_log_linear_mod$finalModel)
```

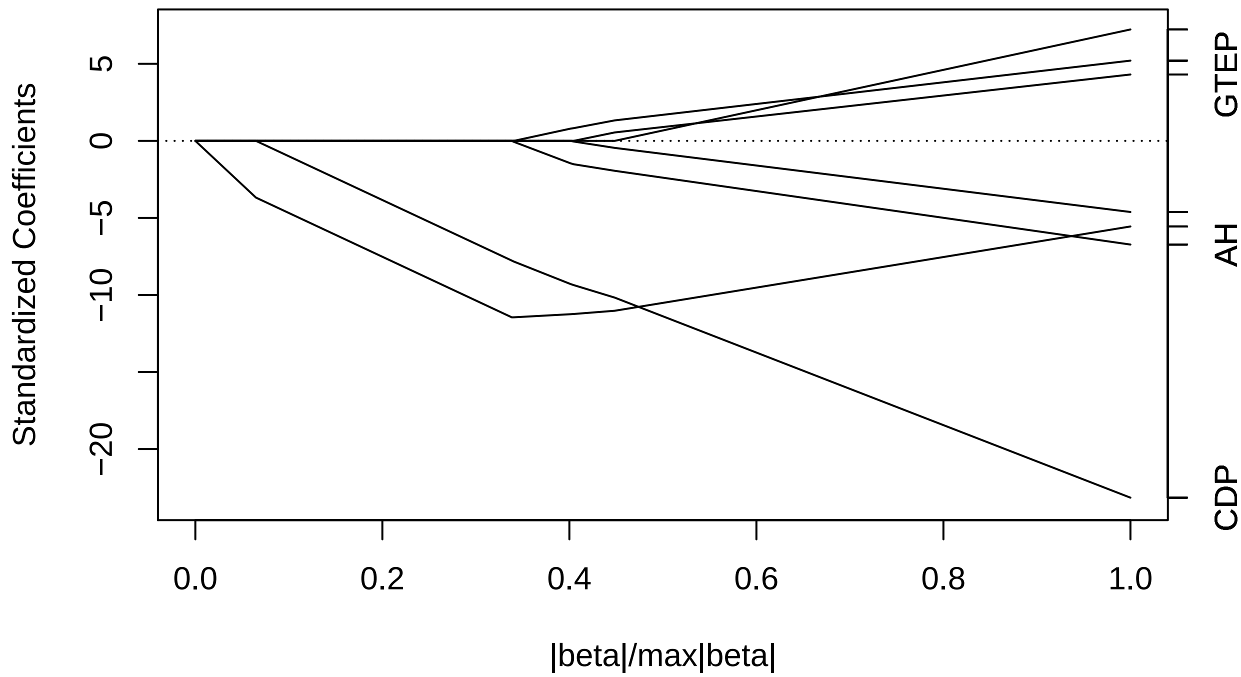

Residuals vs Fitted
Residuals
Fitted values
lm(.outcome ~ AP + AH + AFDP + GTEP + TAT + TEY + CDP)
X674
X1235
X364
3
2
1
0
−1
−2
0.5
1.0
1.5


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(.outcome ~ AP + AH + AFDP + GTEP + TAT + TEY + CDP)
X674
X1235
X364
10
5
0
−5
−3
−2
−1
0
1
2
3

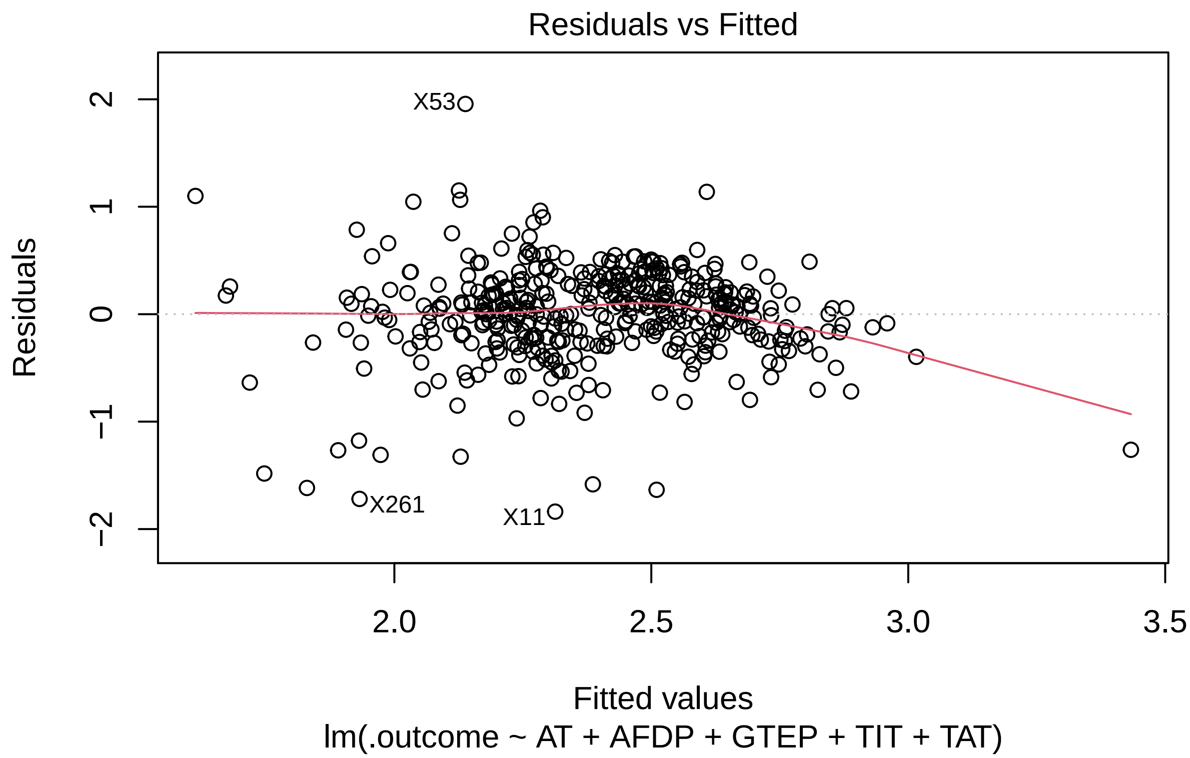

Scale−Location
X674
X1235
X364
√|Standardized residuals|
0.0
1.0
2.0
3.0
0.5
1.0
1.5
Fitted values
lm(.outcome ~ AP + AH + AFDP + GTEP + TAT + TEY + CDP)

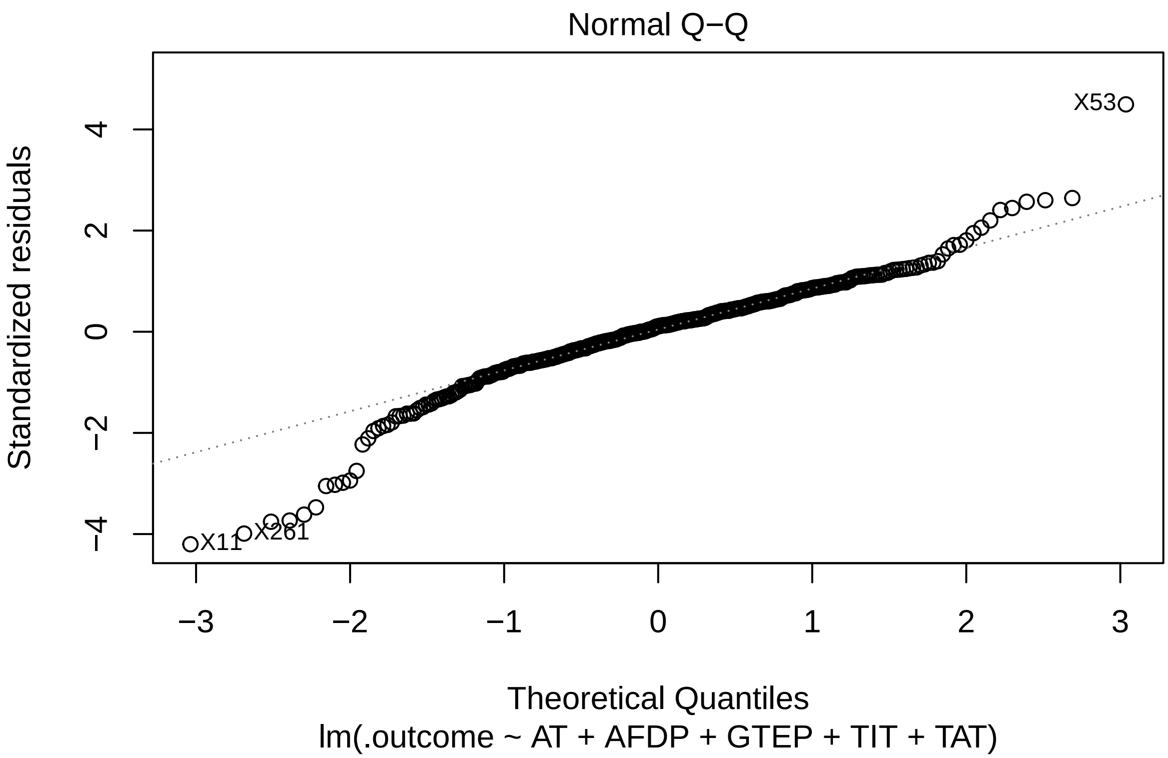


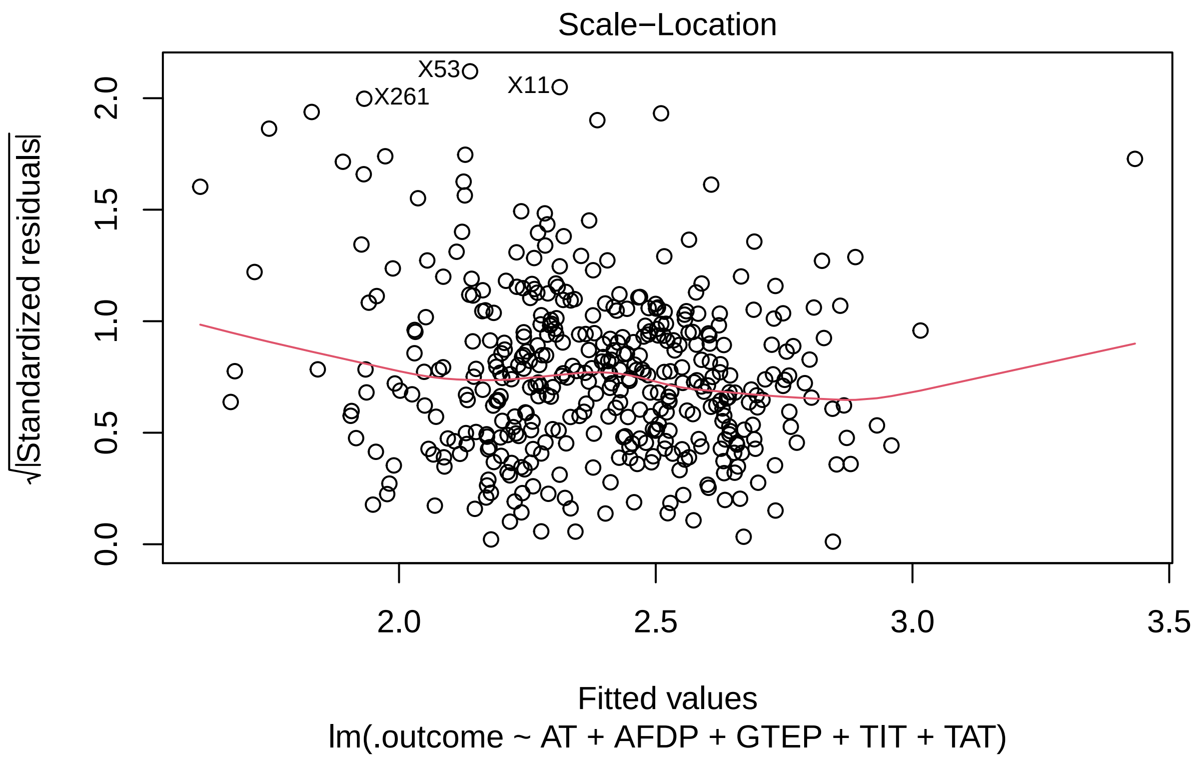

```
plot(typical_lasso_mod$finalModel)
```

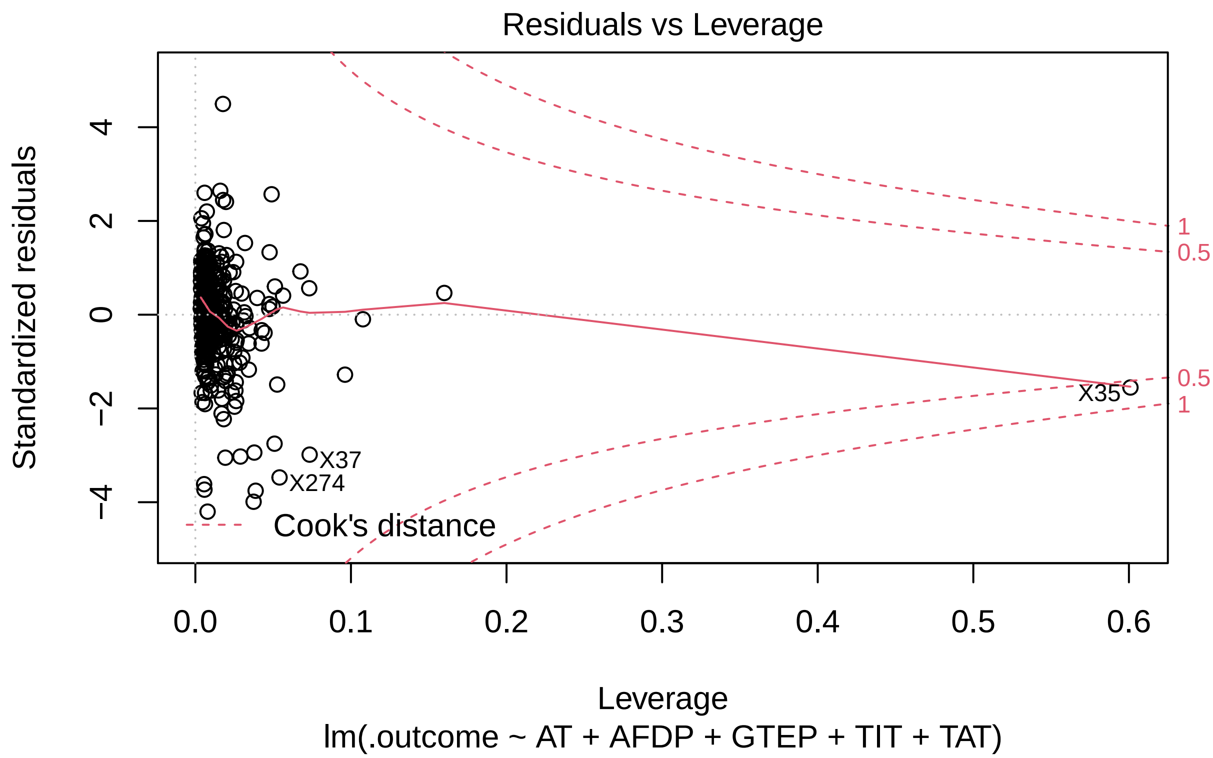



```
plot(high_linear_mod$finalModel)
```

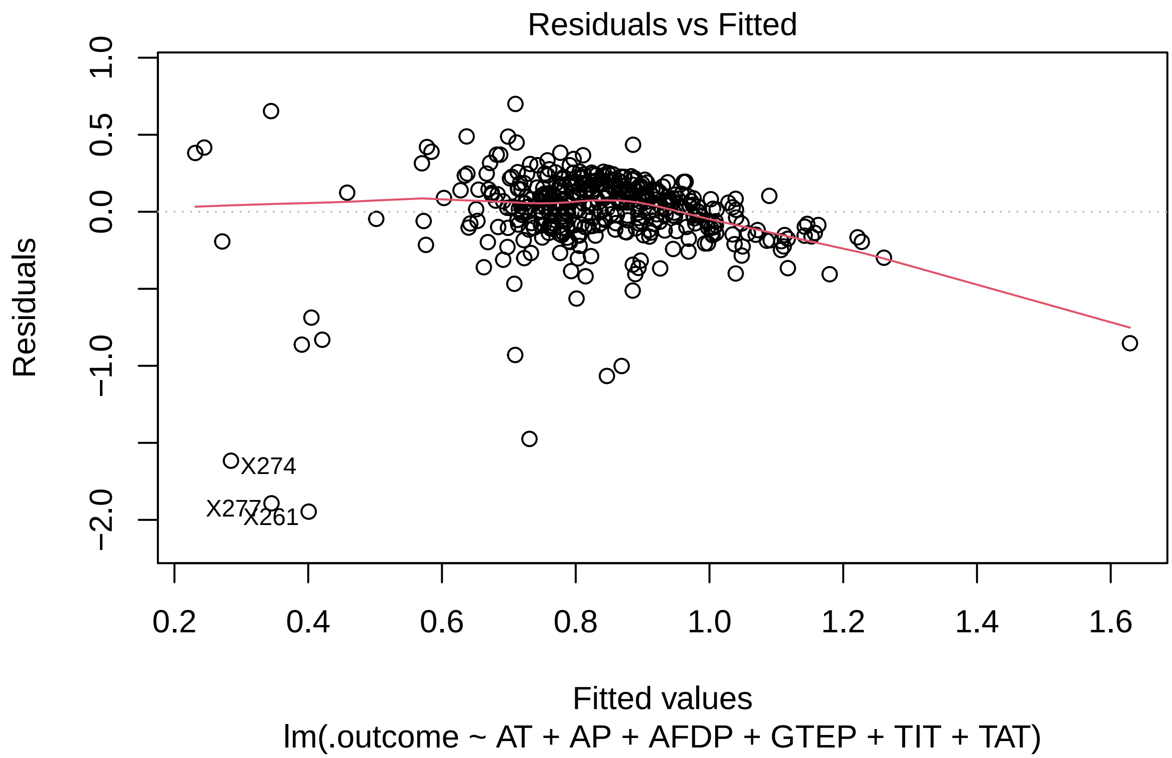

Residuals vs Fitted
Residuals
2
1
0
−1
−2
X53
X261
X11
2.0
2.5
3.0
3.5
Fitted values
lm(.outcome ~ AT + AFDP + GTEP + TIT + TAT)


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(.outcome ~ AT + AFDP + GTEP + TIT + TAT)
X53
X11
X261
−3
−2
−1
0
1
2
3
−4
−2
0
2
4

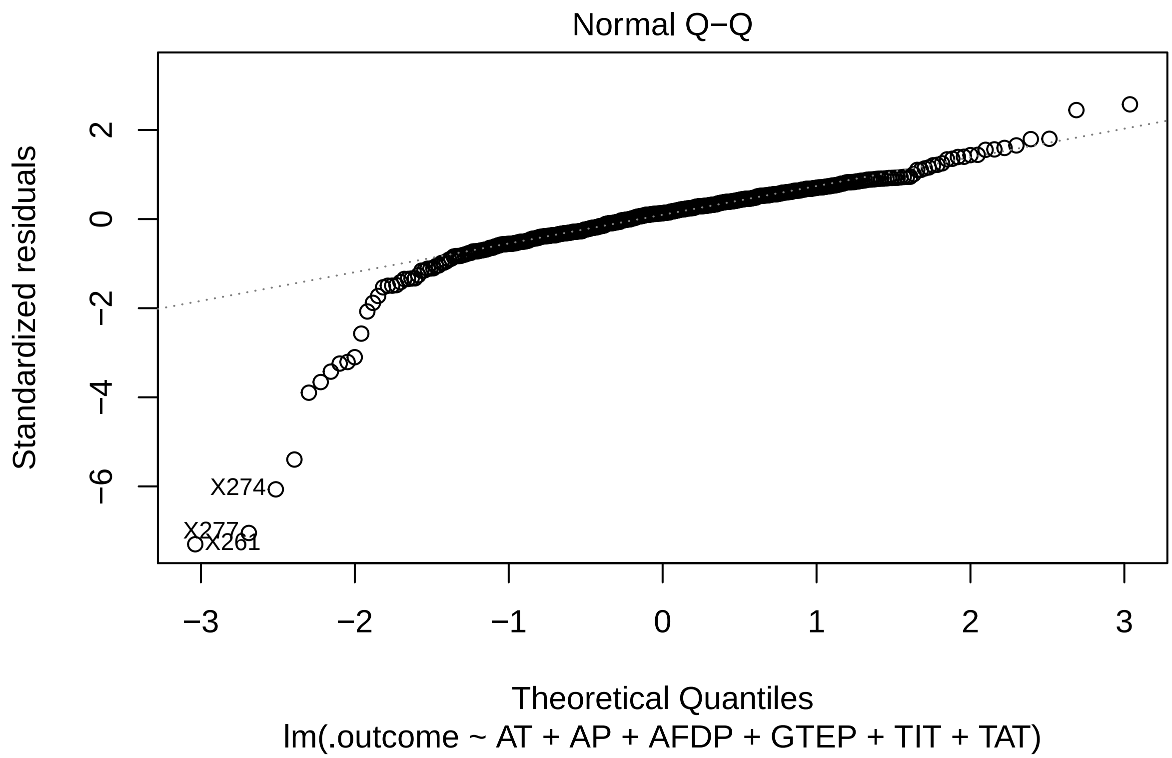

Scale−Location
X53
X261
X11
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
2.0
2.5
3.0
3.5
Fitted values
lm(.outcome ~ AT + AFDP + GTEP + TIT + TAT)

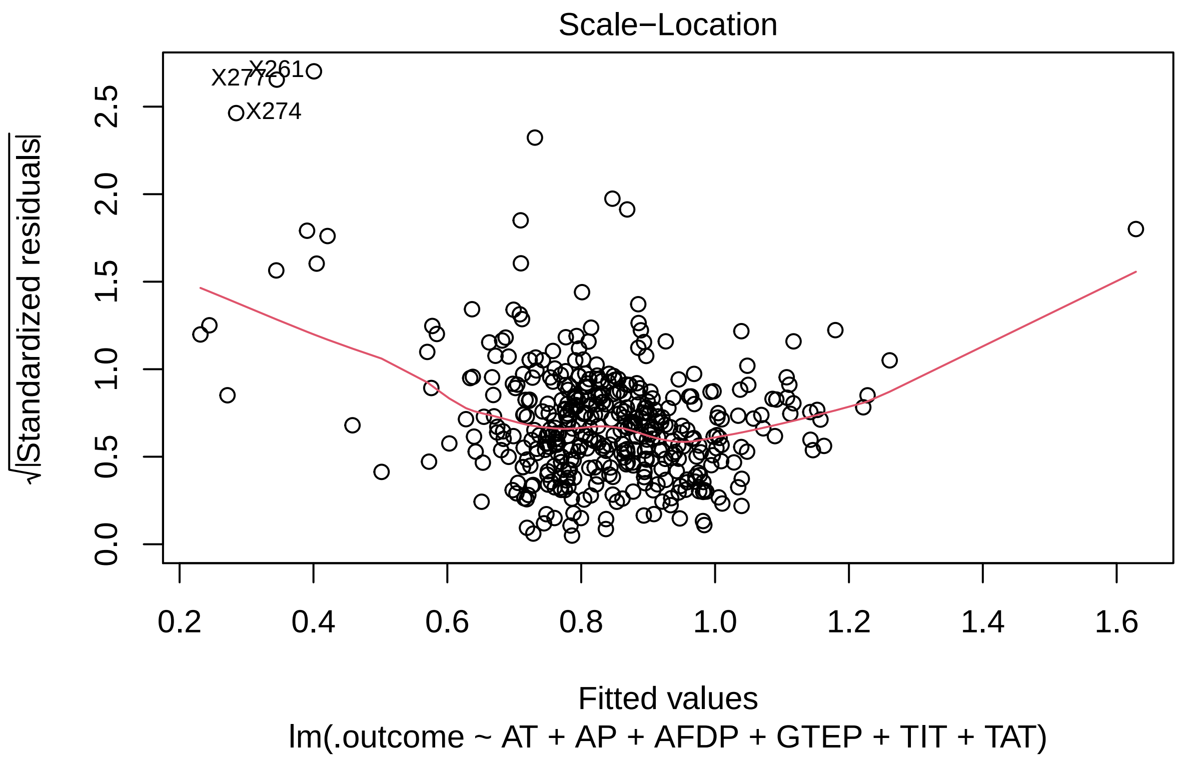



```
plot(high_log_linear_mod$finalModel)
```

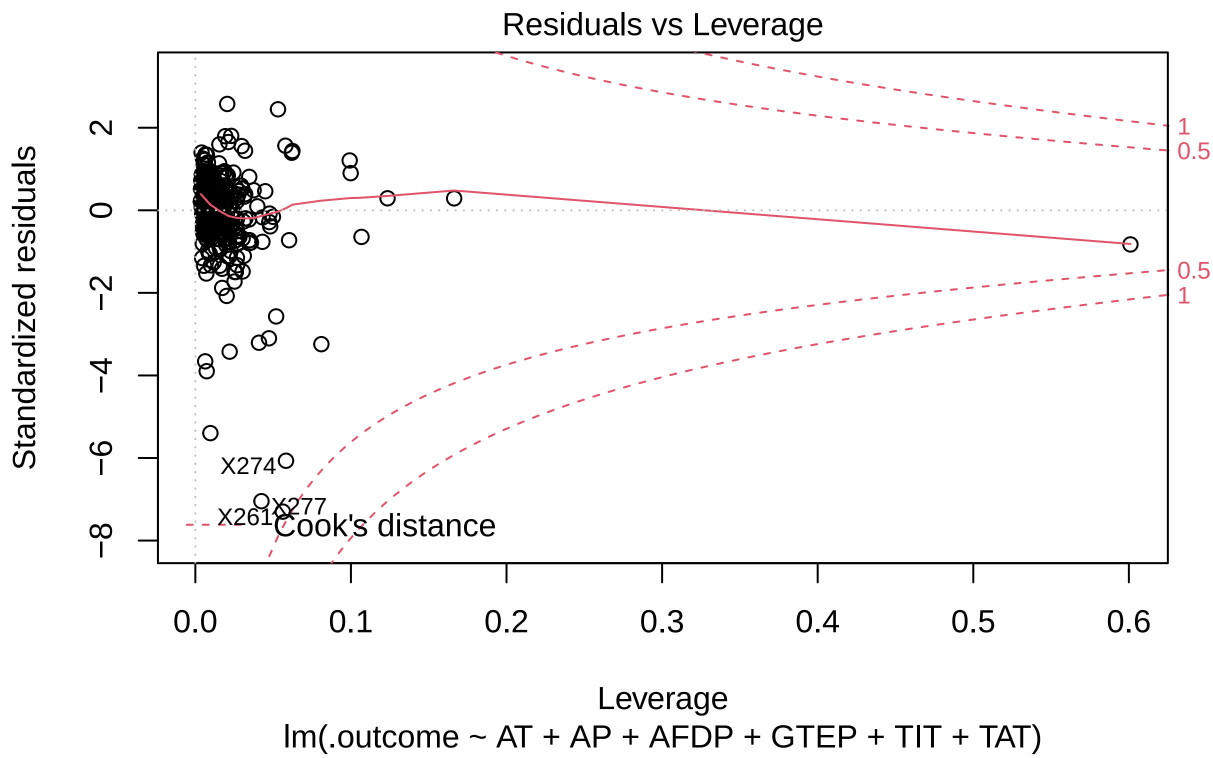


Residuals vs Fitted
Residuals
Fitted values
lm(.outcome ~ AT + AP + AFDP + GTEP + TIT + TAT)
X274
X277
X261
1.0
0.5
0.0
−1.0
−2.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

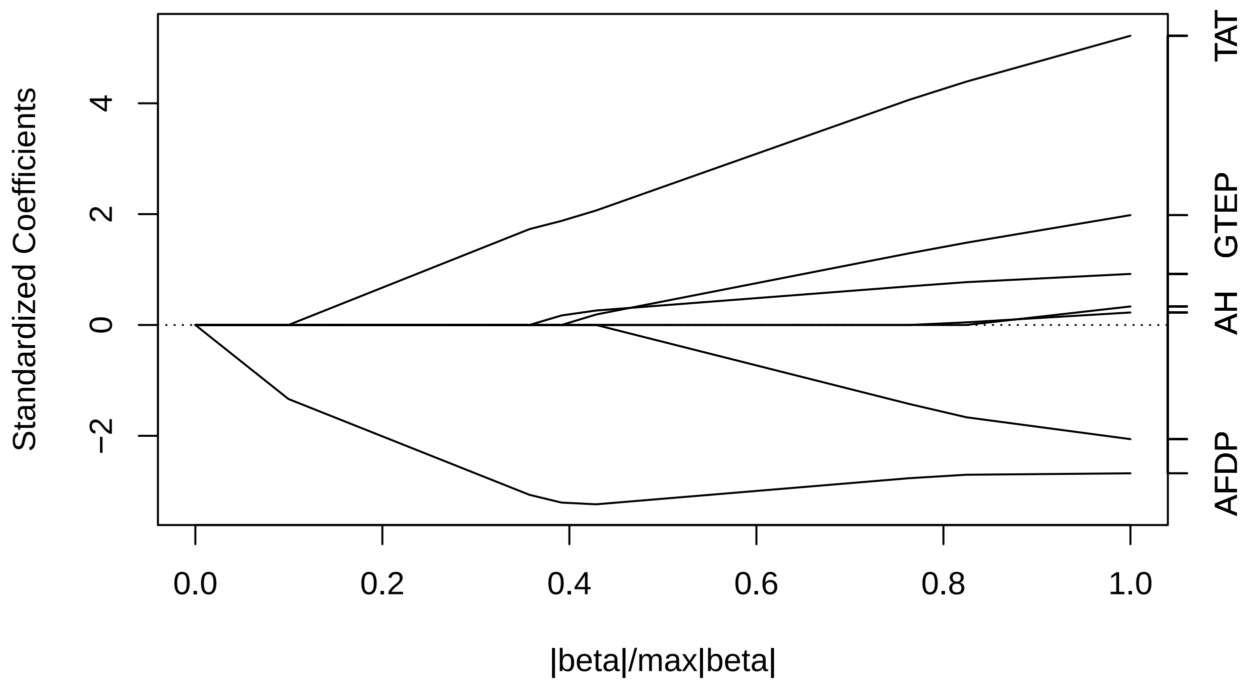

Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(.outcome ~ AT + AP + AFDP + GTEP + TIT + TAT)
X274
X277
X261
−3
−2
−1
0
1
2
3
−6
−4
−2
0
2



Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
2.5
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
X277
X261
X274
Fitted values
lm(.outcome ~ AT + AP + AFDP + GTEP + TIT + TAT)




```
plot(high_lasso_mod$finalModel)
```



##Linear Model Diagnostic Plots

Decision Trees

#All Data

```
# install.packages('tree')
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --
```

```
## v tibble  3.1.0     v dplyr   1.0.5
## v tidyr   1.1.3     v stringr 1.4.0
## v purrr   0.3.4     v forcats 0.5.1
```

```
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## x purrr::lift()   masks caret::lift()
## x dplyr::select() masks MASS::select()
```

```
library(tree)
```

```
## Registered S3 method overwritten by 'tree':
##   method     from
##   print.tree cli
```

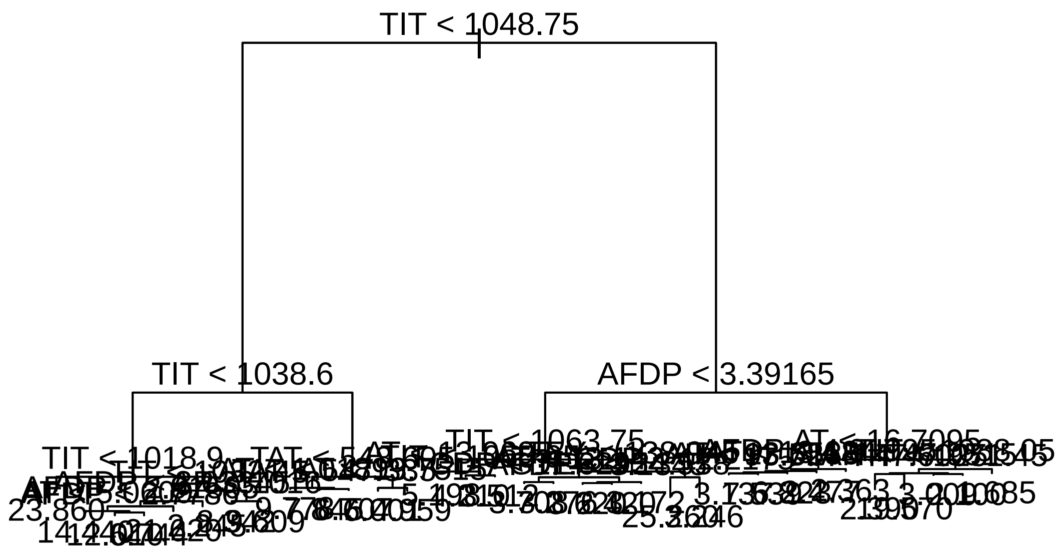
```r
RMSE <- function(y, y_hat) {
  rmse <- sqrt(sum(((y_hat - y)^2)/length(y)))
  print(rmse)
}
```

```r
set.seed(10)
train <- gt_2015 %>% dplyr::select(-NOX) %>% sample_frac(0.8)
test <- gt_2015 %>% dplyr::select(-NOX) %>% setdiff(train)
```

```r
tree_CO <- tree(CO ~ . , train,
                  control = tree.control(nobs = length(train$CO),
                                         minsize = 4, mindev=0.001), method = "recursive.partition")
summary(tree_CO)
```

```
##
## Regression tree:
## tree(formula = CO ~ ., data = train, control = tree.control(nobs = length(train$CO),
##     minsize = 4, mindev = 0.001), method = "recursive.partition")
## Number of terminal nodes:  33
## Residual mean deviance:  1.012 = 5944 / 5874
## Distribution of residuals:
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -16.88000  -0.37240  -0.05792   0.00000   0.28590  30.18000
```

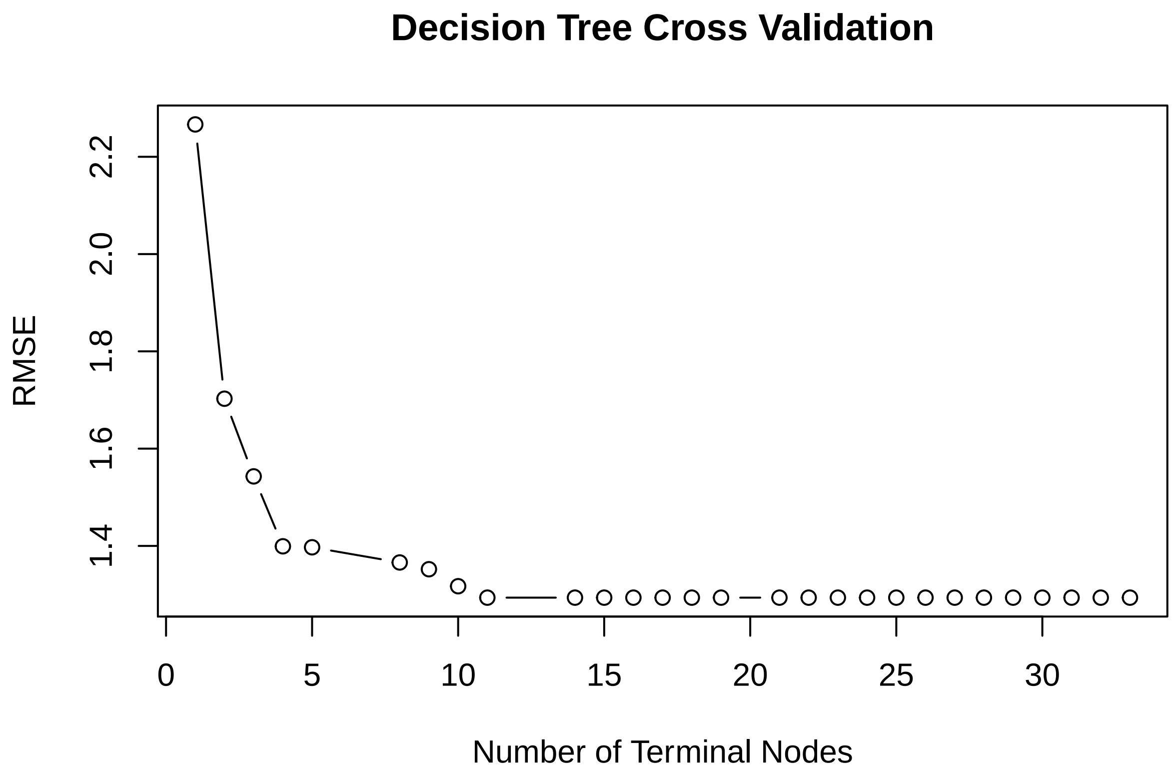
```r
plot(tree_CO)
text(tree_CO, pretty = 0)
```




```
tree_pred <- predict(tree_CO, test)
RMSE(test$CO, tree_pred)
```

```
## [1] 1.316371
```

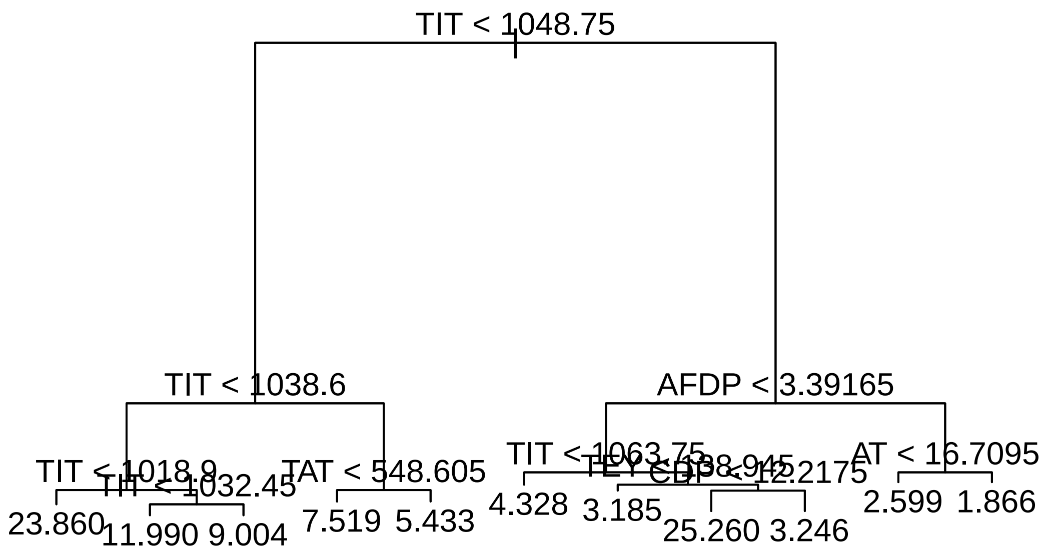
```
cv_info <- cv.tree(tree_CO, FUN = prune.tree)
plot(cv_info$size, sqrt(cv_info$dev / nrow(train)), type = "b", xlab = "Number of Terminal Nodes", ylab
```




```r
pruned_tree <- prune.tree(tree_CO, best = 11)
summary(pruned_tree)
```

```
##
## Regression tree:
## snip.tree(tree = tree_CO, nodes = c(19L, 10L, 14L, 11L, 12L,
## 26L, 18L, 15L))
## Variables actually used in tree construction:
## [1] "TIT"  "TAT"  "AFDP" "TEY"  "CDP"  "AT"
## Number of terminal nodes:  11
## Residual mean deviance:  1.387 = 8180 / 5896
## Distribution of residuals:
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -10.43000  -0.46410  -0.05351   0.00000   0.34680  34.59000
```

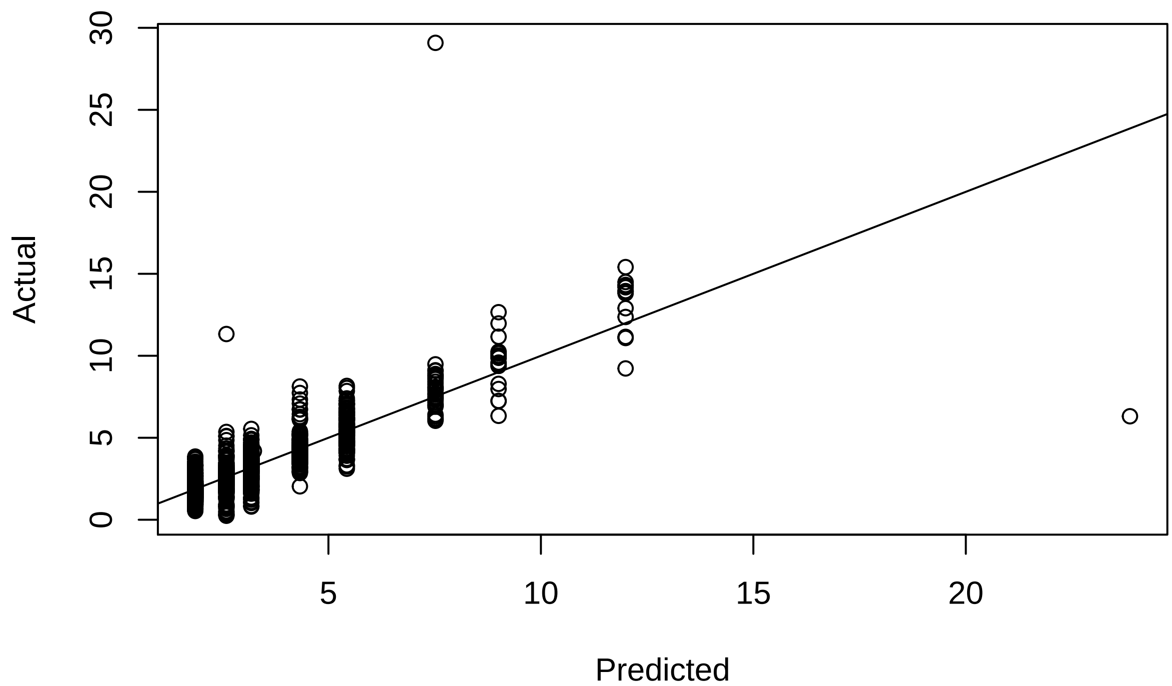
```r
plot(pruned_tree)
text(pruned_tree, pretty = 0)
```




```r
tree_pred <- predict(pruned_tree, test)
RMSE(test$CO, tree_pred)
```

```
## [1] 1.097033
```

```r
plot(tree_pred, test$CO, xlab = "Predicted", ylab = "Actual")
abline(0, 1)
```



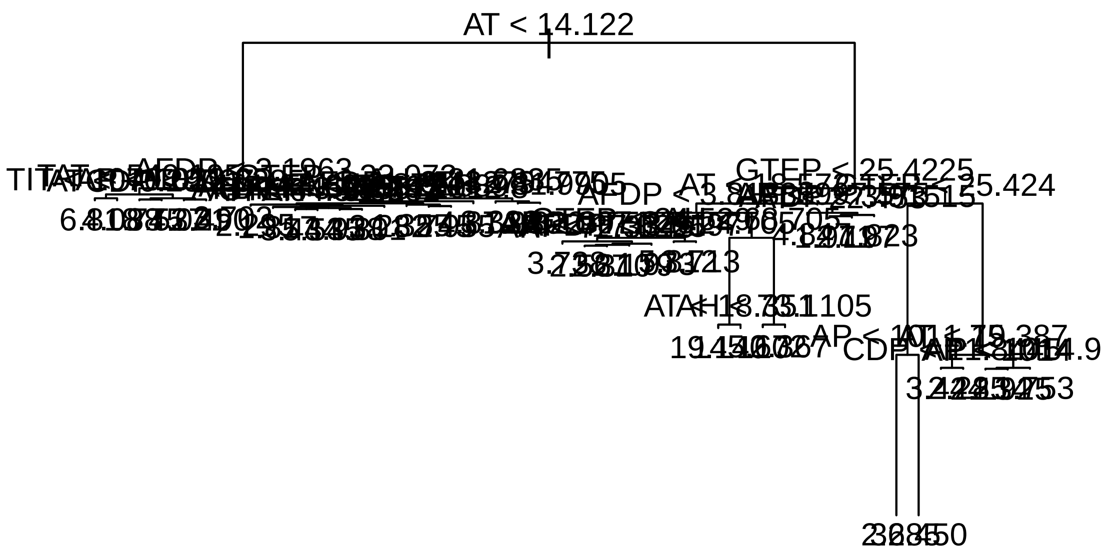
#Typical Energy Yield (130-136)

```
set.seed(10)
train_typical <- gt_2015_typical %>% dplyr::select(-NOX) %>% sample_frac(0.8)
test_typical <- gt_2015_typical %>% dplyr::select(-NOX) %>% setdiff(train_typical)
```

```
tree_CO_typical <- tree(CO ~ . , train_typical,
                        control = tree.control(nobs = length(train_typical$CO),
                                  minsize = 2, mindev=0.001), method = "recursive.partition")
summary(tree_CO_typical)
```

```
##
## Regression tree:
## tree(formula = CO ~ ., data = train_typical, control = tree.control(nobs = length(train_typical$CO),
##     minsize = 2, mindev = 0.001), method = "recursive.partition")
## Variables actually used in tree construction:
## [1] "AT"   "AFDP" "TAT"  "TIT"  "AP"   "CDP"  "GTEP" "AH"
## Number of terminal nodes:  43
## Residual mean deviance:  0.1641 = 260.8 / 1589
## Distribution of residuals:
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -2.29700 -0.23090 -0.01984  0.00000  0.19510  2.31900
```

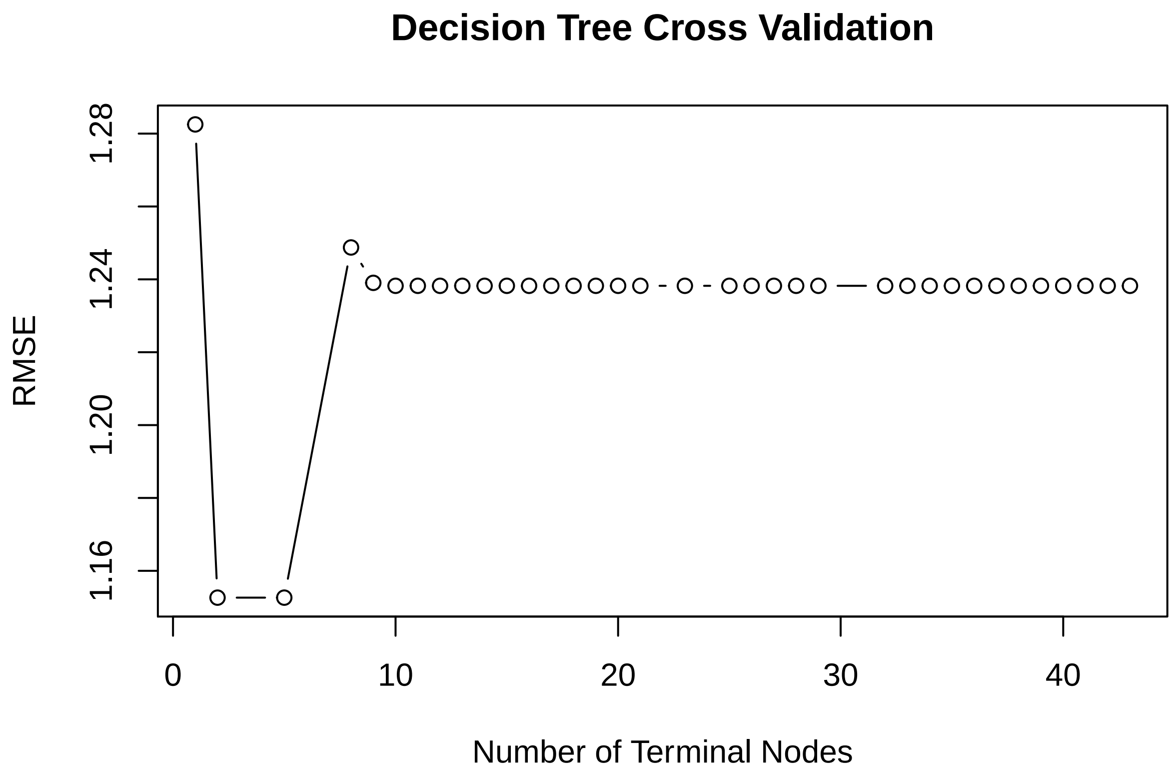
```
plot(tree_CO_typical)
text(tree_CO_typical, pretty = 0)
```




```
tree_pred_typical <- predict(tree_CO_typical, test_typical)
RMSE(test_typical$CO, tree_pred_typical)
```

```
## [1] 0.6818376
```

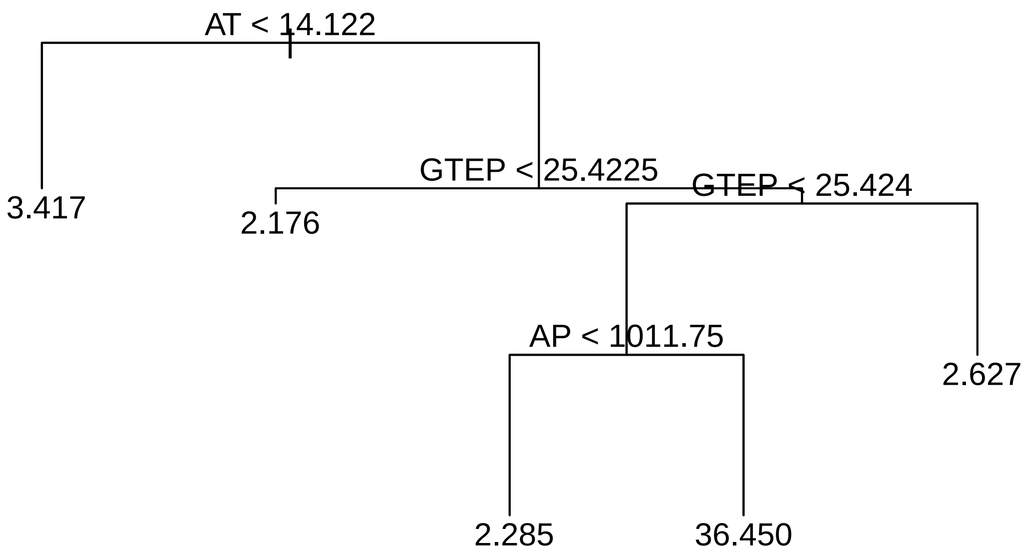
```
cv_info_typical <- cv.tree(tree_CO_typical, FUN = prune.tree)
plot(cv_info_typical$size, sqrt(cv_info_typical$dev / nrow(train_typical)), type = "b", xlab = "Number
```




```
pruned_tree_typical <- prune.tree(tree_CO_typical, best = 5)
summary(pruned_tree_typical)
```

```
##
## Regression tree:
## snip.tree(tree = tree_CO_typical, nodes = c(2L, 15L, 6L))
## Variables actually used in tree construction:
## [1] "AT"   "GTEP" "AP"
## Number of terminal nodes:  5
## Residual mean deviance:  0.5926 = 964.1 / 1627
## Distribution of residuals:
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -2.61200 -0.35520 -0.05681  0.00000  0.27590 16.96000
```

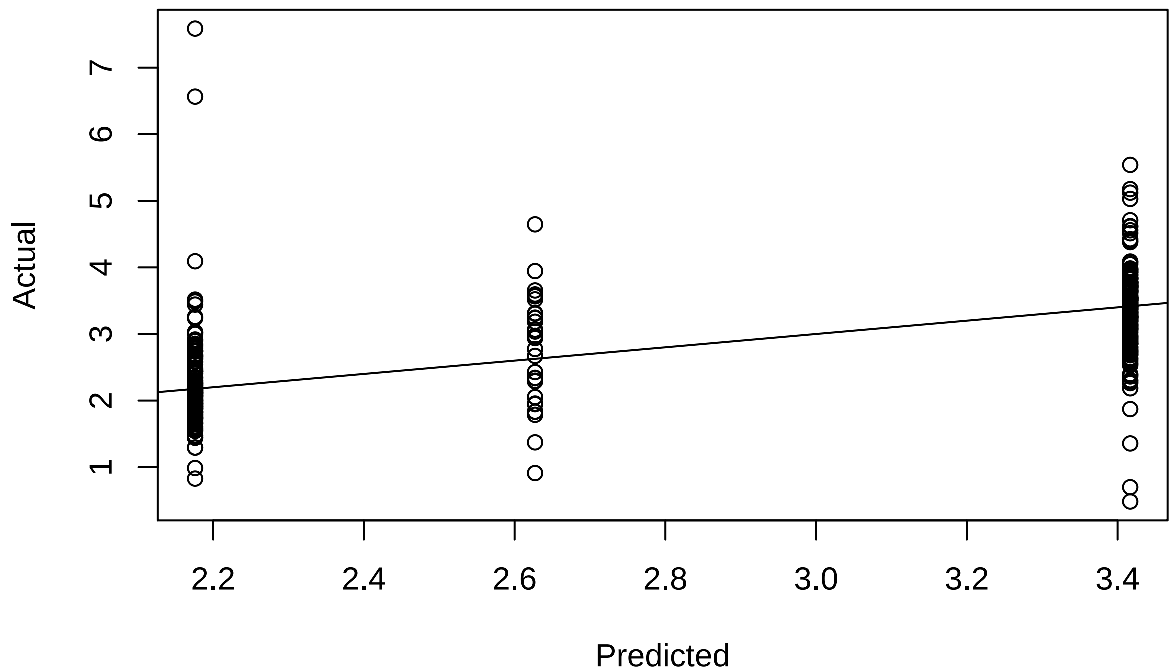
```
plot(pruned_tree_typical)
text(pruned_tree_typical, pretty = 0)
```




```r
tree_pred_typical <- predict(pruned_tree_typical, test_typical)
RMSE(test_typical$CO, tree_pred_typical)
```

```
## [1] 0.6823446
```

```r
plot(tree_pred_typical, test_typical$CO, xlab = "Predicted", ylab = "Actual")
abline(0, 1)
```



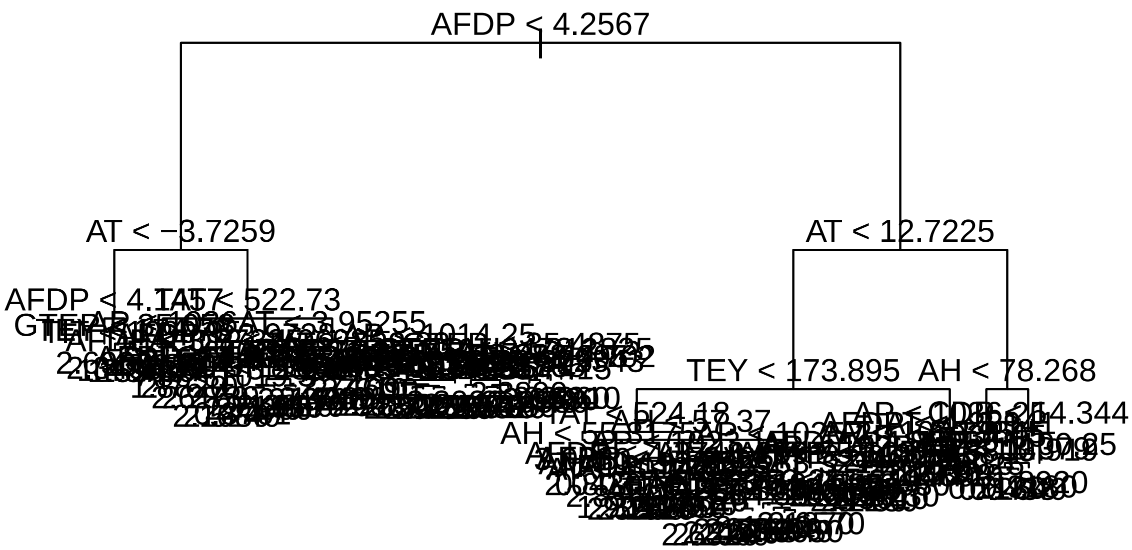
#High Energy Yield (160+)

```
set.seed(10)
train_high <- gt_2015_high %>% dplyr::select(-NOX) %>% sample_frac(0.8)
test_high <- gt_2015_high %>% dplyr::select(-NOX) %>% setdiff(train_high)
```

```
tree_CO_high <- tree(CO ~ . , train_high,
                     control = tree.control(nobs = length(train_high$CO),
                                            minsize = 2, mindev=0.001), method = "recursive.partition")
summary(tree_CO_high)
```

```
##
## Regression tree:
## tree(formula = CO ~ ., data = train_high, control = tree.control(nobs = length(train_high$CO),
##     minsize = 2, mindev = 0.001), method = "recursive.partition")
## Number of terminal nodes:  89
## Residual mean deviance:  0.01502 = 3.679 / 245
## Distribution of residuals:
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -0.28310 -0.06453  0.00000  0.00000  0.06841  0.26280
```

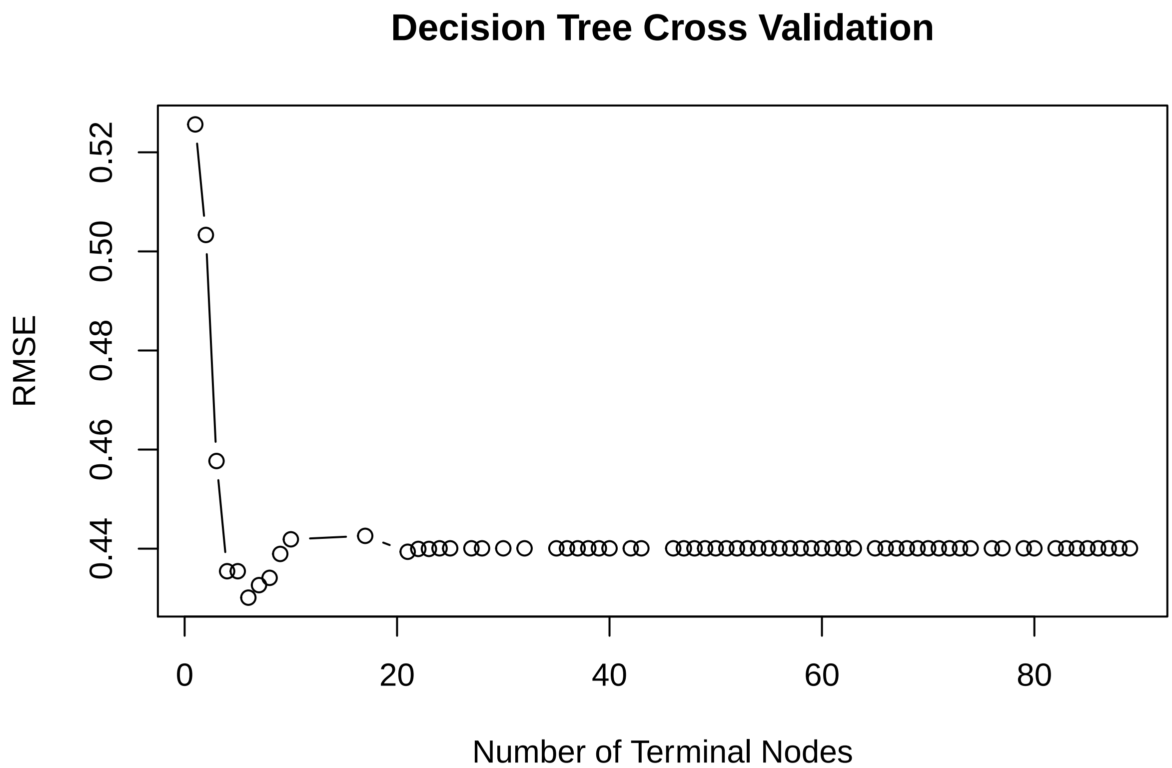
```
plot(tree_CO_high)
text(tree_CO_high, pretty = 0)
```




```r
tree_pred_high <- predict(tree_CO_high, test_high)
RMSE(test_high$CO, tree_pred_high)
```

```
## [1] 0.507176
```

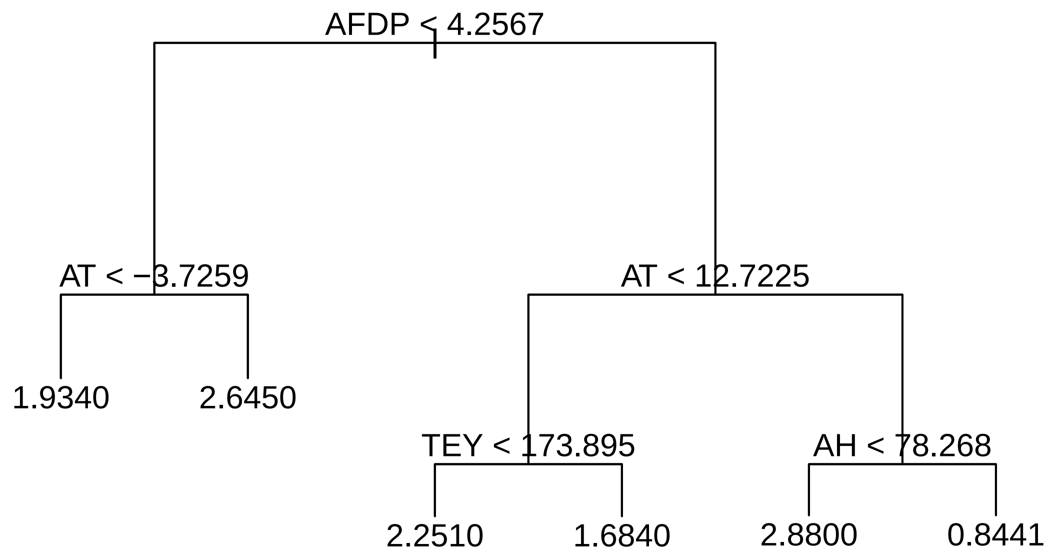
```r
cv_info_high <- cv.tree(tree_CO_high, FUN = prune.tree)
plot(cv_info_high$size, sqrt(cv_info_high$dev / nrow(train_high)), type = "b", xlab = "Number of Termina
```




```r
pruned_tree_high <- prune.tree(tree_CO_high, best = 6)
summary(pruned_tree_high)
```

```
##
## Regression tree:
## snip.tree(tree = tree_CO_high, nodes = c(13L, 12L, 5L, 4L, 15L
## ))
## Variables actually used in tree construction:
## [1] "AFDP" "AT"   "TEY"  "AH"
## Number of terminal nodes:  6
## Residual mean deviance:  0.1334 = 43.76 / 328
## Distribution of residuals:
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -1.76800 -0.19160 -0.01148  0.00000  0.21000  1.45000
```

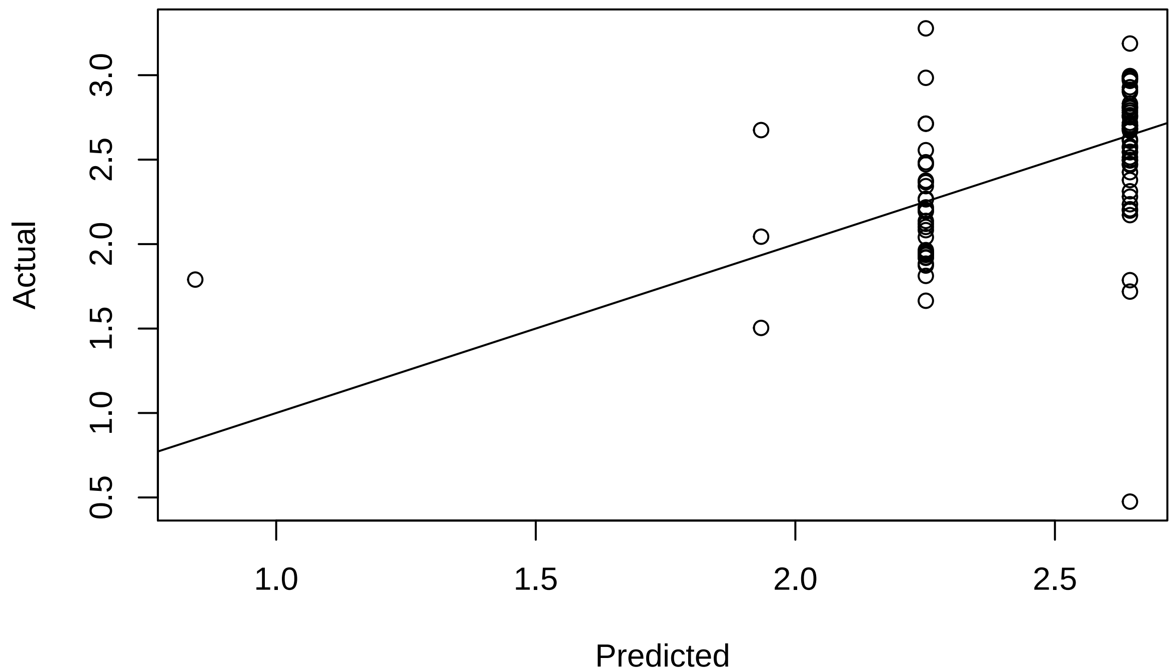
```r
plot(pruned_tree_high)
text(pruned_tree_high, pretty = 0)
```




```r
tree_pred_high <- predict(pruned_tree_high, test_high)
RMSE(test_high$CO, tree_pred_high)
```

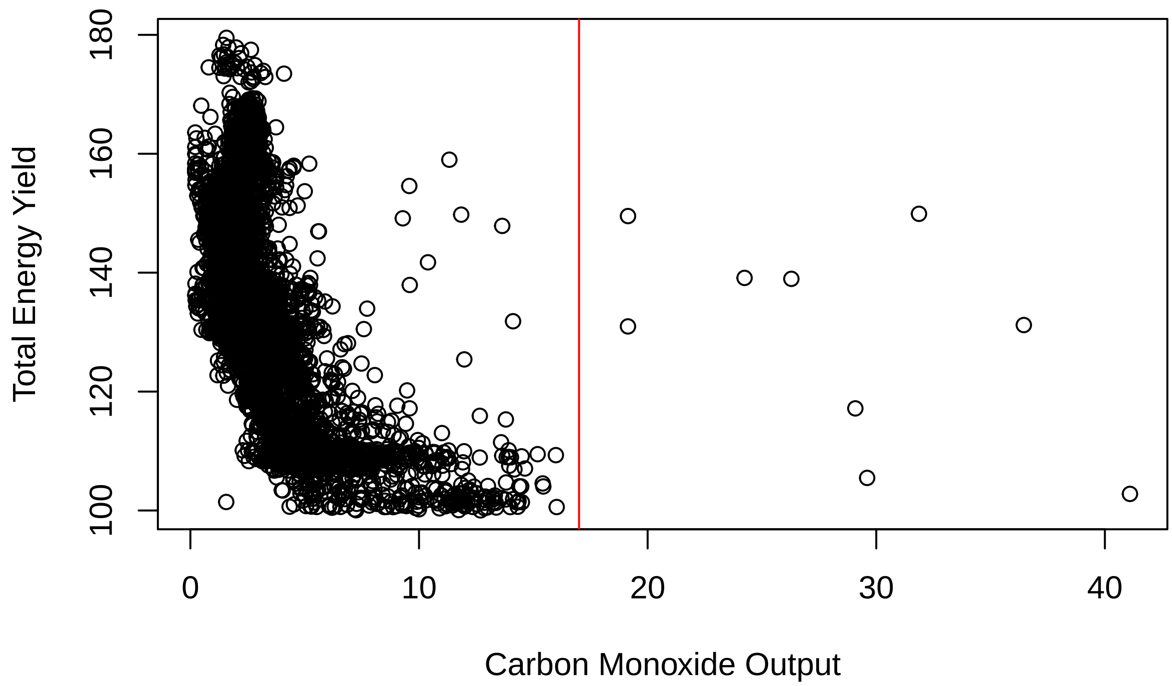
```
## [1] 0.4154548
```

```r
plot(tree_pred_high, test_high$CO, xlab = "Predicted", ylab = "Actual")
abline(0, 1)
```



Tree predicting high vs. low carbon monoxide output

```
plot(gt_2015$CO, gt_2015$TEY, ylab = "Total Energy Yield", xlab = "Carbon Monoxide Output")
abline(v = 17, col = "red")
```



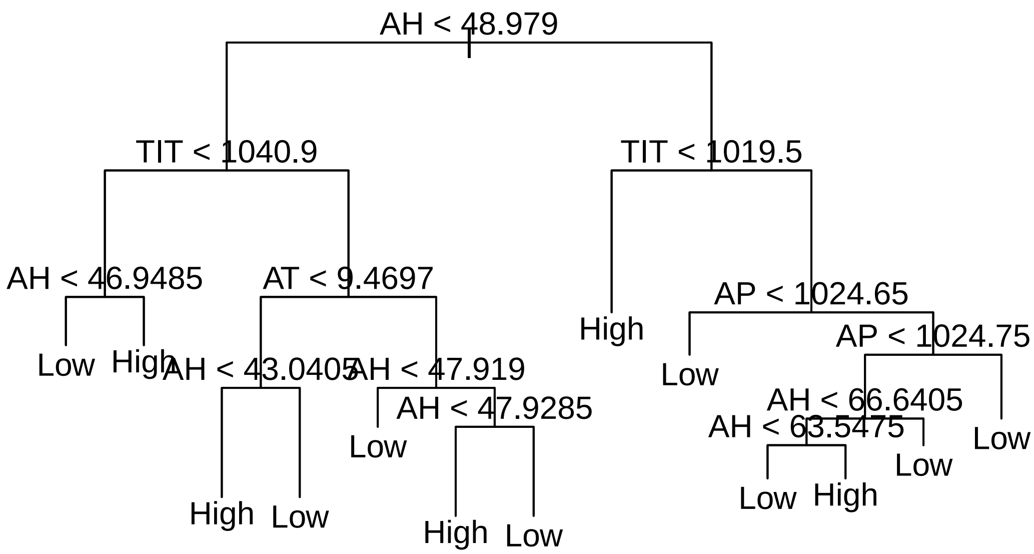

```
data <- gt_2015 %>% mutate(Emissions = as.factor(ifelse(CO > 17, "High", "Low"))) %>% dplyr::select(-NO
high_CO <- data %>% filter(CO > 17) %>% dplyr::select(-CO)
low_CO <- data %>% dplyr::select(-CO) %>% setdiff(high_CO)

set.seed(10)
train <- bind_rows(low_CO %>% sample_frac(7/9), high_CO %>% sample_frac(7/9))
test <- data %>% dplyr::select(-CO) %>% setdiff(train)
```

```
tree <- tree(Emissions ~ . , train,
                  control = tree.control(nobs = length(train$Emissions),
                                         minsize = 1))
summary(tree)
```

```
##
## Classification tree:
## tree(formula = Emissions ~ ., data = train, control = tree.control(nobs = length(train$Emissions),
##     minsize = 1))
## Variables actually used in tree construction:
## [1] "AH"  "TIT" "AT"  "AP"
## Number of terminal nodes:  13
## Residual mean deviance:  0 = 0 / 5730
## Misclassification error rate: 0 = 0 / 5743
```

```
plot(tree)
text(tree, pretty = 0)
```




```
tree_pred <- predict(tree, train, type = "class")
table(predicted = tree_pred, actual = train$Emissions)
```

```
##          actual
## predicted High  Low
##      High    7    0
##      Low     0 5736
```

```
tree_pred <- predict(tree, test, type = "class")
table(predicted = tree_pred, actual = test$Emissions)
```

```
##          actual
## predicted High  Low
##      High    0    3
##      Low     2 1636
```

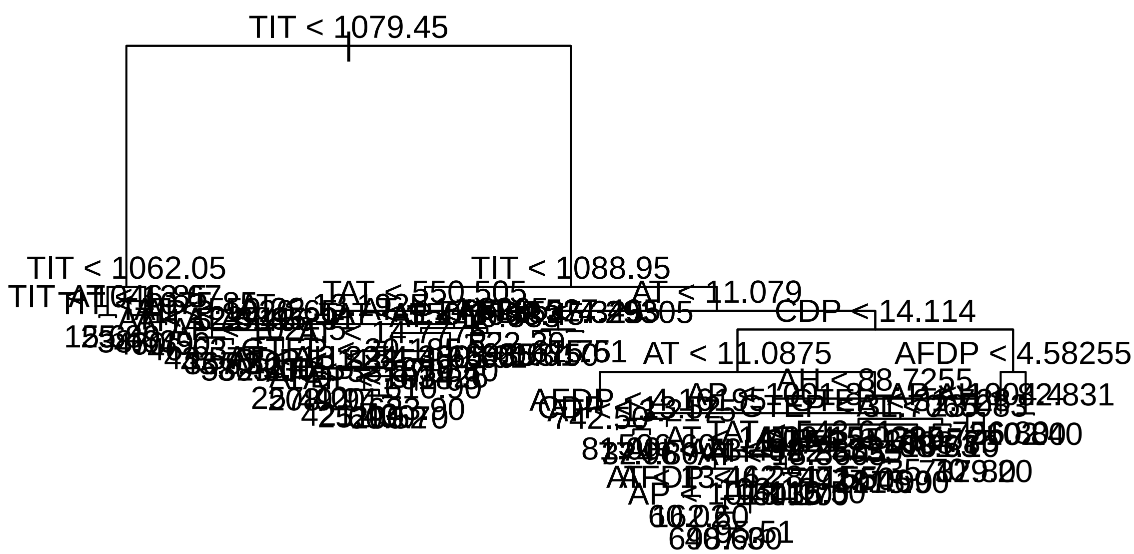
Tree predicting ratio of energy yield over carbon monoxide

```r
set.seed(10)
# train <- gt_2015 %>% mutate(Energy_CO_Ratio = TEY / CO) %>% sample_frac(0.8)
# test <- gt_2015 %>% mutate(Energy_CO_Ratio = TEY / CO) %>% setdiff(train)
train <- gt_2015 %>% mutate(Energy_CO_Ratio = TEY / CO) %>% dplyr::select(-c(NOX, TEY, CO)) %>% sample_
test <- gt_2015 %>% mutate(Energy_CO_Ratio = TEY / CO) %>% dplyr::select(-c(NOX, TEY, CO)) %>% setdiff(
```

```r
tree_Energy_CO_Ratio <- tree(Energy_CO_Ratio ~ . , train,
                  control = tree.control(nobs = length(train$Energy_CO_Ratio),
                                         minsize = 2, mindev=0.001), method = "recursive.partition")
summary(tree_Energy_CO_Ratio)
```

```
##
## Regression tree:
## tree(formula = Energy_CO_Ratio ~ ., data = train, control = tree.control(nobs = length(train$Energy_C
##     minsize = 2, mindev = 0.001), method = "recursive.partition")
## Number of terminal nodes:  57
## Residual mean deviance:  476.9 = 2790000 / 5850
## Distribution of residuals:
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -98.400  -8.495  -1.595   0.000   4.851 385.600
```

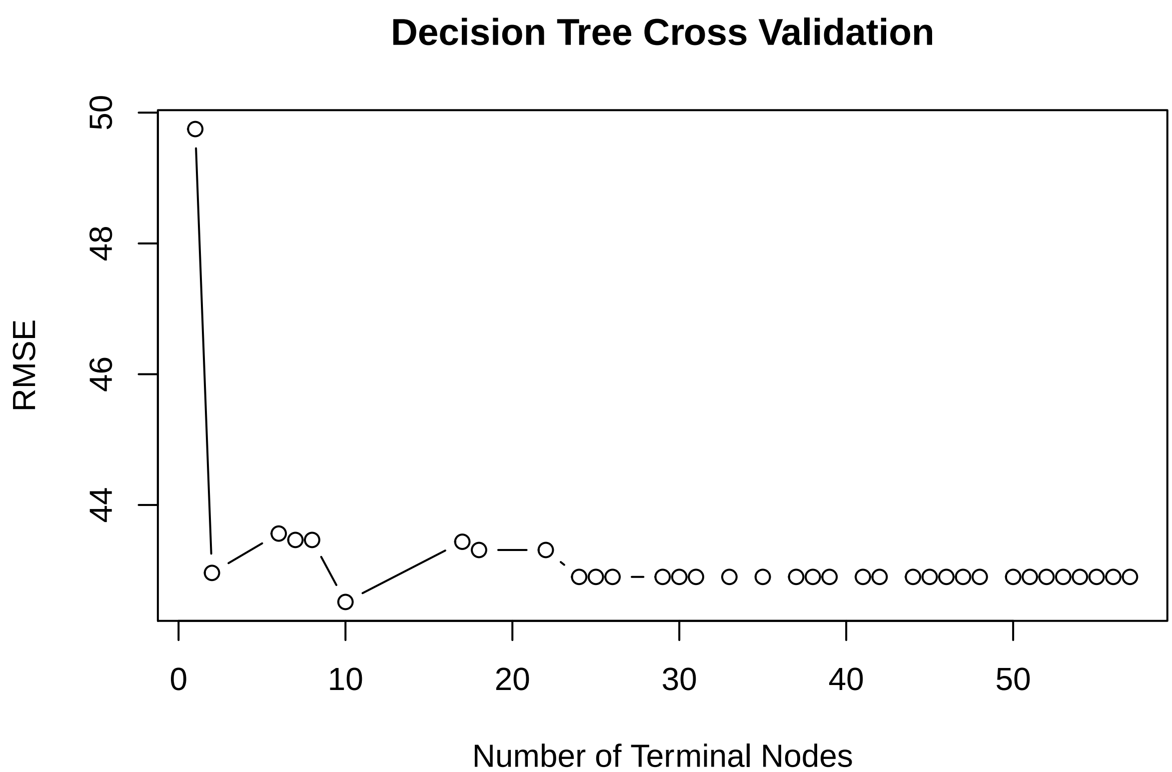
```r
plot(tree_Energy_CO_Ratio)
text(tree_Energy_CO_Ratio, pretty = 0)
```



```r
tree_pred <- predict(tree_Energy_CO_Ratio, test)
RMSE(test$Energy_CO_Ratio, tree_pred)
```

```
## [1] 39.95283
```

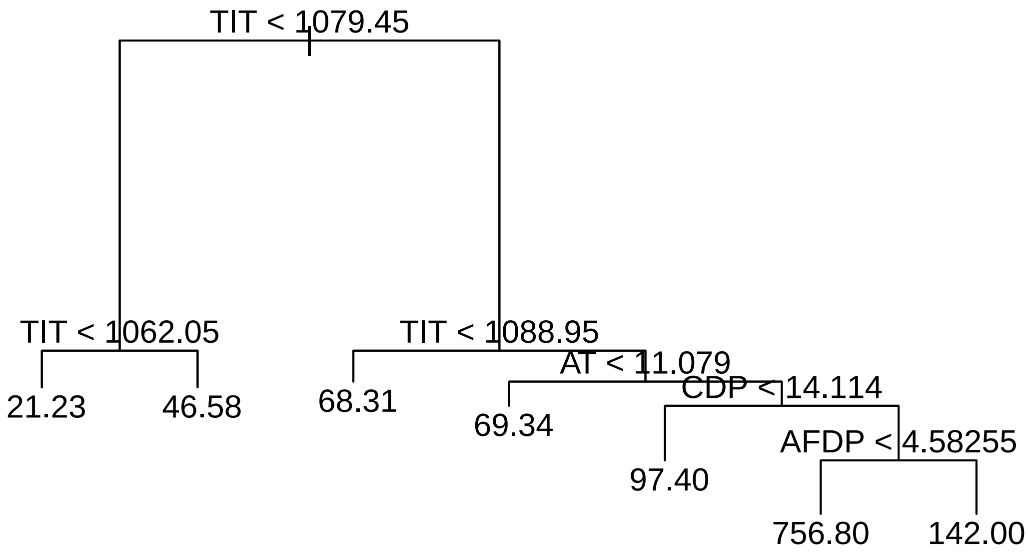
```r
cv_info <- cv.tree(tree_Energy_CO_Ratio, FUN = prune.tree)
plot(cv_info$size, sqrt(cv_info$dev / nrow(train)), type = "b", xlab = "Number of Terminal Nodes", ylab
```




```r
pruned_tree <- prune.tree(tree_Energy_CO_Ratio, best = 7)
summary(pruned_tree)
```

```
##
## Regression tree:
## snip.tree(tree = tree_Energy_CO_Ratio, nodes = c(63L, 4L, 14L,
## 5L, 6L, 30L))
## Variables actually used in tree construction:
## [1] "TIT"  "AT"   "CDP"  "AFDP"
## Number of terminal nodes:  7
## Residual mean deviance:  1430 = 8435000 / 5900
## Distribution of residuals:
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -92.690 -12.730  -4.005   0.000   6.385 645.100
```

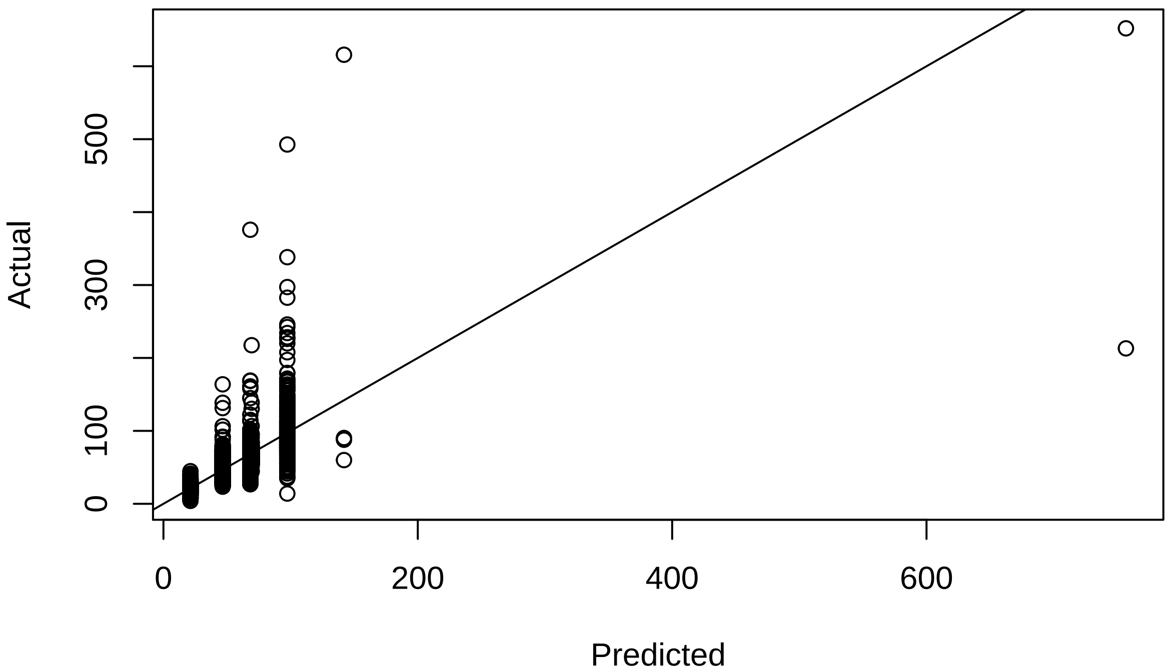
```r
plot(pruned_tree)
text(pruned_tree, pretty = 0)
```




```r
tree_pred <- predict(pruned_tree, test)
RMSE(test$Energy_CO_Ratio, tree_pred)
```

```
## [1] 33.46526
```

```r
plot(tree_pred, test$Energy_CO_Ratio, xlab = "Predicted", ylab = "Actual")
abline(0, 1)
```


Actual
0
100
300
500
0
200
400
600
Predicted