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Synchronized Flow as a New Traffic Phase and Related Problems for Traffic Flow Modelling

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Abstract—An empirical study of spatial-temporal features of traffic flow patterns is presented. A critical comparison of model results and results of traffic theories with real features of the traffic patterns is given. Predictable features of congested traffic flow are discussed. Results of an application of methods for the tracing and the prediction of moving traffic jams and of other spatial-temporal patterns in congested traffic are considered. A qualitative theory of congested traffic flow is discussed.

Keywords—Traffic flow patterns, Traffic flow models, Congested traffic, Synchronized flow, Wide moving jams.

1. INTRODUCTION THE FUNDAMENTAL DIAGRAM

Up to now, it is almost self-evident that both homogeneous and time independent solutions of a mathematical traffic flow model should belong to a curve on the flow-density plane which goes through the original and has at least one maximum. This curve is called *the fundamental diagram* for traffic flow (Figure 1a). It represents the result of experimental observations that the higher the vehicle density in traffic flow, the lower the average vehicle speed (e.g., [1–44]).

Recall that real traffic in empirical observations can be in either free or in congested regime. Empirical points measured at some location which corresponds to free flow can really be represented by a curve on the flow density plane. This curve is cut off at some limit (maximum) point $(\rho_{\max}^{(\text{free})}, q_{\max}^{(\text{free})})$ (Figure 1b). Congested traffic can be determined as states of traffic where the average vehicle speed is lower than the minimum possible speed which is related to this limit point [43]. It is well known that within congestion, the flow-density data do not form a neat relationship but show a broad and complex spreading of measurement points, i.e., the flow-density data cover a two-dimensional region in the flow-density plane (Figure 1b) (e.g., [43,44]). This complexity in traffic flow is usually interpreted either as fluctuations, or as an instability, or else as a traffic jam formation. These features of real traffic flow should be explained by traffic models (e.g., [12–15,17–25,30–36,38–41]).

The fundamental diagram will be called as the basic assumption (1) for traffic flow modelling. It is either a result of a traffic flow model or it is used in models explicitly (e.g., [1–41]).

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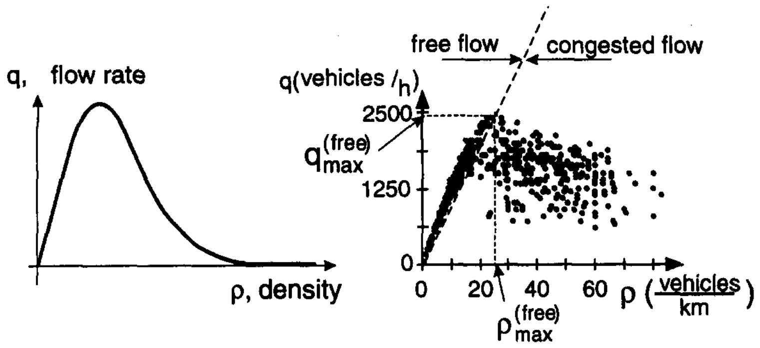


(a) Illustration of the fundamental diagram.

(b) Real patterns which have been observed in different countries (e.g., [43,44]).

Figure 1.

To show the mentioned complexity of traffic, in addition to the basic assumption (1), it is often proposed that a traffic flow model should have the following feature (it will be called the basic assumption (2)). There is a range of the vehicle density where initial hypothetical homogeneous and time independent solutions of the model are *unstable* with respect to infinitesimal perturbations (e.g., [12–15,17–25,30–36,38–41]). In other words, in this range of the vehicle density the related points of the fundamental diagram cannot exist in calculations and simulations of the model. Instead of these points, a complex spatial-temporal dynamics should be the model result (e.g., [12–15,17–25,30–36,38–41]). The mentioned instability should explain and predict results of observations that in congested traffic, a wide spreading of experimental points on the flow-density plane is usually realized, and the breakdown, hysteresis, and the "stop-and-go" phenomena occur (e.g., [12–15,17–25,30–36,38–41]). Either only the basic assumption (1) or both of the basic assumptions (1) and (2) are usually the basic assumptions of traffic flow models which claim to predict and explain real spatial-temporal behavior of traffic flow (see e.g., [12–15,17–25,30–36,38–41] and the references there).

However, the recent empirical discovering of synchronized traffic flow as a new traffic flow phase by Kerner in collaboration with Rehborn [45] has cast doubt on both these basic assumptions of traffic flow models. In particular, empirical investigations allow us to suggest that there is *no* fundamental diagram which can describe hypothetical homogeneous states of synchronized flow. Besides, empirical investigations allow us to suggest that there are no unstable hypothetical homogeneous and stationary states of traffic flow with respect to infinitesimal perturbations [46–49]. This may be the reason why traffic flow models and theories at present (see e.g., [1–19,21–32,34–37,39–41] and the references there) still cannot explain and predict important features of traffic flow patterns in congested regime which have been discovered recently [46–54]. In this paper, empirical features of traffic flow patterns in congested traffic are considered. Some hypotheses, i.e., new basic assumptions for a traffic flow theory by the author [46–49] are discussed.

2. METASTABILITY OF FREE FLOW WITH RESPECT OF MOVING JAM FORMATION

Traffic flow dynamics is a very old field. Up to recent time, theoretical studying has mainly been based on an application and on a further development of such classical traffic models and theories as the Lighthill-Whitham-model [1], the Herman *et al.* car-following models [2,55], the Newell-model [3], the Prigogine kinetic model [4], the Payne macroscopic model [5], the Wiedemann psycho-physical model [6], the Gipps-model [7], different queuing models (e.g., [8,9]), and other theoretical approaches (see papers in [10,11]).

In the past years, a wide community of physicists and mathematicians has been involved in a studying of the traffic flow dynamics [12–14,40,41]. On one hand, new traffic flow models and theoretical approaches, in particular, Nagel-Schreckenberg cellular automata model [15] and a new approach to the optimal velocity (OV) models [3,16] made by Bando *et al.* [15] have been proposed and developed. On the other hand, last results of the nonlinear physics of dissipative distributed systems, in particular, a theory of autosolitons (localized dissipative patterns) [56], have been applied by Kerner and Konhäuser for a development of a nonlinear theory of moving traffic jams. In the frame of this theory, the following has been predicted [18].

(i) Wide moving jams possess some characteristics, i.e., unique, coherent, predictable, and reproducible parameters. Recall that a wide moving jam is an upstream moving localized structure which is restricted by two upstream moving fronts where the vehicle speed and the density change spatially sharply. The width of the wide moving jam in the longitudinal direction is considerably higher than the width of the jam's fronts.

(ii) Free traffic flow is in a metastable state with respect to the jam emergence, if the flow rate in a free flow is higher than the flow rate in the outflow from a wide moving jam, q_{out}. In this case, a wide moving jam can exist and be excited in this free flow. In contrast, if the flow rate in a free flow is lower than q_{out}, then the free flow is stable with respect to the wide moving jam formation. Wide moving jams can neither exist for a long time nor be excited by a short time local perturbation in this free flow. Note that the characteristic flow rate q_{out} is related to the case when free flow is formed in the outflow from a wide moving jam.

These two results have indeed been found in empirical investigations of the jam propagation [57,58]. Therefore, the mentioned Properties (i), (ii) of jams and free flows [18] can be considered as necessary requirements for each traffic flow model [19,59]. Later studying by Bando *et al.* [20], by Krauß *et al.* [21], by Barlovic *et al.* [22], by Helbing and Schreckenberg [23], by Treiber *et al.* [24], and by Mahnke and Kaupužs [25] showed that there can be a lot of traffic flow models and theoretical approaches which arė able to reproduce Properties (i) and (ii) of moving jams and free flows which have first been found out from an investigation of a macroscopic traffic flow model in [18].

Note that the existence of the characteristics, i.e., unique, coherent, predictable, and reproducible parameters of wide moving jams has been used for a development of methods for tracing and prediction of moving traffic jams and time-dependent travel times [60–64]. These methods use essentially the fact that the velocities of the propagation of the moving jams' fronts can be calculated with the available measured traffic data. Besides, the methods work without any validations of model parameters at different highway infrastructure and other road conditions. Some results of the application of the methods [64–66] will be considered below.

3. CONGESTED FLOW

When the vehicle density is high enough, traffic is usually in a congested regime (e.g., [4,8–14,26–30,41]). Often congested flow is defined as a traffic state which occurs only due to a highway bottleneck as the inevitable consequence of an upstream flow that exceeds the downstream capacity of the bottleneck. In this definition, it is obvious to propose that congested flow is not self-organizing, but rigidly controlled by external constraints. However, observations have shown that although bottlenecks are the most frequent reason, they are not the only reason for the occurrence of congested traffic [51]. Besides, in [45–51,57,58], it has been shown that different self-organizing processes are responsible for features of *real* traffic congestion. For these reasons, the mentioned definition of congested traffic will not be used in the paper.

In the paper, phenomena will be defined through their features rather than through the reasons of their occurrence. Congested traffic states will be defined as counter states to states of free flow (e.g., [43]). It is well known that experimental points which are related to free flow can be approximately presented by a curve with a positive slope in the flow-density plane. Congested traffic will be determined, therefore, as a state of traffic where the average vehicle speed is lower than the minimum possible speed which is related to the limit point $(q_{\max}^{(\text{free})}, \rho_{\max}^{(\text{free})})$ in free flow (Figure 1b) (e.g., [43,44]). In this definition, nothing is said about reasons of the occurrence of the congestion. It is linked to the mentioned experimental facts that a congested state can spontaneously occur also away from bottlenecks and that there are a lot of diverse self-organizing effects which are responsible for the congestion [45,49,50,57].

One of the well-known phenomena in congested traffic flow is the 'stop-and-go' phenomenon which has experimentally been studied by a lot of authors, in particular by Treiterer [42] and Koshi *et al.* [43]. Koshi *et al.* [43] found out that the vehicle speed across different highway lanes within the 'stop-and-go' waves can be synchronized. The synchronization of the vehicle speeds on different highway lanes is often observed in congested traffic away from bottlenecks and if only one route for traffic exists. It seems reasonable that in each state of congested flow, vehicles try to change to that lane for which the vehicle speed seems to be higher. This may increase the density on this lane and consequently decreases the vehicle speed, i.e., this process can synchronize the vehicle speed with the speeds on the other lanes leading to synchronized traffic flow. The tendency to equilibrate the speed across lanes in dense enough traffic flow can show almost any multilane traffic flow model.

4. TWO PHASES IN CONGESTED TRAFFIC: WIDE MOVING JAMS AND SYNCHRONIZED FLOW

4A. Objective Criteria for Different Phases in Congested Traffic

It has already been mentioned that within congested traffic, the flow-density data do not form a neat relationship but show a broad and complex spreading of measurement points, i.e., the flow-density data cover a two-dimensional region in the flow-density plane (Figure 1b) (e.g., [43,44]). In [45], it has been shown that if those data are removed from the data of the congested traffic where wide moving traffic jams are realized, nevertheless the remaining congested traffic data show a wide scattering within a two-dimensional area in the flow-density plane. This remaining congested traffic has been called "synchronized flow". Thus, the author suggested that in congested traffic, two different traffic phases should be distinguished, wide moving jams and synchronized flow [45,51]. Before the reasons for this suggestion will be considered, let us formulate an objective criterion for wide moving jams as a separate traffic phase in real congested traffic.

Traffic is a complex nonlinear process where diverse spatial-temporal patterns occur. This process may be considered as a nonequilibrium process. It is well known that in a lot of nonequilibrium physical, chemical, and biological distributed systems, complex spatial-temporal patterns appear. A class of spatial-temporal patterns which possesses qualitative similar spatial-temporal features is usually considered a separate phase of a such nonequilibrium distributed system. Respectively, different classes of spatial-temporal patterns can be considered as different phases of the system. Phase transitions between these different phases of the system and also possible complex dynamics inside each of the phases determine the complexity of the system (e.g., [56,67]). To determine such a separate phase, there should be some objective criteria for this phase. These objective criteria should allow to distinguish such *spatial-temporal features* of the phase which are unique only for this phase. Besides, these unique spatial-temporal features should remain the same for this phase within the whole range of the control parameters of the system where the phase can occur and exist.

This analogy with nonlinear nonequilibrium physical, chemical, and biological distributed systems has been used by the author for a differentiation of the different phases in congested traffic. The related *objective criteria* for the identification of different traffic phases in congested regime are the following [45,46,52,53].

- A local spatial-temporal upstream moving traffic pattern in congested regime, i.e., the pattern which is spatially restricted by two upstream moving (downstream and upstream) fronts is a separate traffic phase "a wide moving jam", if the pattern possesses the following characteristic feature. The pattern as a whole local structure propagates through any other traffic states and through any other spatial-temporal patterns, and also through any bottlenecks (e.g., at on-ramps and off-ramps) *keeping* the mean velocity of the downstream front of the pattern.

- All other states and spatial-temporal patterns in congested traffic which do not possess this characteristic feature belong to the traffic phase "synchronized flow".

Indeed, only wide moving jams possess this characteristic feature [45–47,51,57]. For this reason, a wide moving jam has been distinguished in congested traffic as the separate traffic phase "wide moving jam".

This behavior of wide moving jams can clearly be seen in Figure 2c, where the sequence of two wide jams propagates through at least three bottlenecks (in intersections I1, I2, and I3,

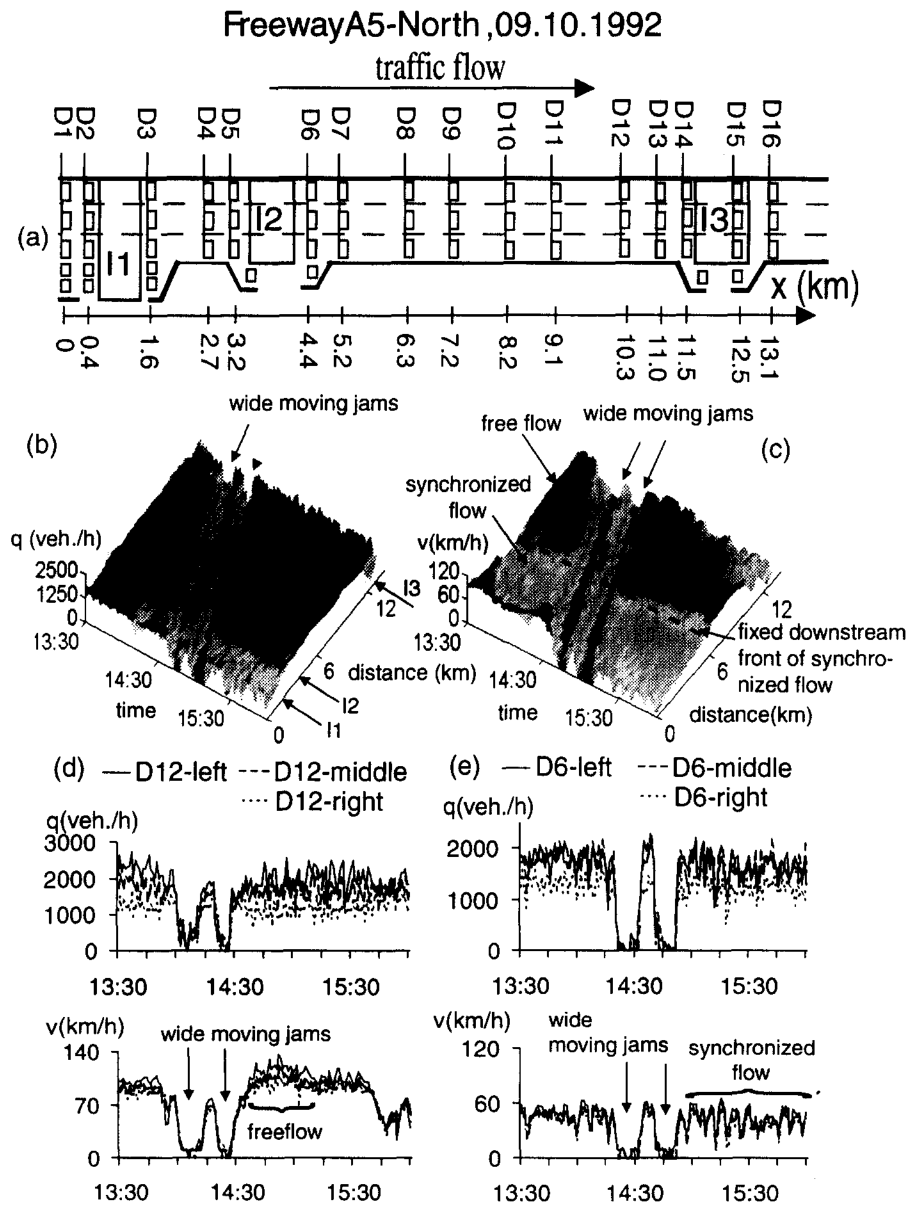


Figure 2. Explanation of traffic phases: free flow, wide moving jams, and synchronized flow (b)–(e) on a section of the highway A5-North (a) in Germany [52,57].

Figure 2a) and through different sometimes very complex states of synchronized flow (Figure 2e, bottom). In contrast to the wide moving jam, after synchronized flow has occurred at an on-ramp, the downstream front of the synchronized flow is *fixed* at the on-ramp. This effect is shown in Figure 2c. In this figure, the downstream front of the synchronized flow is shown by the dotted line. The location of this fixed downstream front determines the effective location of a bottleneck at the on-ramp [50]. Therefore, there are three traffic phases [45,46]:

1. free flow,
2. synchronized flow,
3. a wide moving jam.

4B. Traffic Phases in the Flow-Density Plane

It has already been mentioned that for the identification of the traffic phases "a wide moving jam" and "synchronized flow" in congested traffic, a spatial-temporal analysis of traffic is necessary in general. This analysis must include a simultaneous measurement of traffic variables at different locations on a highway (e.g., Figure 2). After such a spatial-temporal study of traffic has been performed and the traffic phases have been distinguished, some features of the found traffic phases may be represented and in addition investigated in the flow-density plane. Such a representation of the three traffic phases in the flow-density plane is shown in Figure 3. In this figure, the stationary propagation of the downstream front of a wide moving jam is represented by the characteristic line for the downstream front which is called 'line J' [45,57] (Figure 3, right). The slope of line J equals to the velocity of the downstream front of the wide moving jam. It should be noted that in the case shown in Figure 3, right, a synchronized flow is formed in the outflow from the wide moving jam. In this case, the flow rate at the point where the downstream jam front changes into the synchronized flow, $q_{\mathrm{out}}^{(\mathrm{syn})}$, is lower than the flow rate q_{out} which occurs (at other detectors) when a free flow is formed downstream of the jam (Figure 3, right) [46,51]. A more detailed consideration of line J can be found in [46,53].

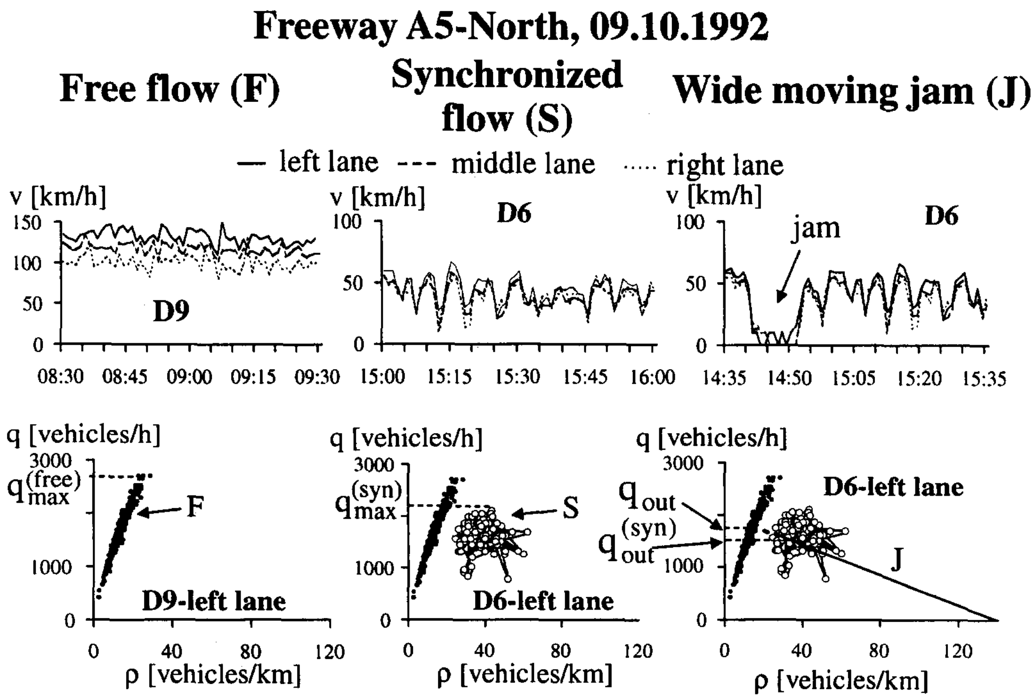


Figure 3. Three phases of traffic in the flow-density plane.

4C. Features of Wide Moving Jams

From the mentioned objective criterion for wide moving jams, it follows that independent from the state of traffic in the outflow from a wide moving jam, the velocity of the downstream jam front is the characteristic, i.e., unique, coherent, predictable, and reproducible parameter. To explain this, note that the moving of the downstream front of a wide moving jam is a deterministic process of driver's escaping from the jam. The velocity v_g of this process is determined by the density $\rho_{\max}$ inside the jam and by the an average delay time τ_{del} between two vehicles following one another escaping from the jam [68]:

$$v_g = -\frac{1}{\rho_{\max}\tau_{\text{del}}}. \tag{1}$$

The vehicle speed and the flow rate inside a wide moving jam are either zero or negligible for the jam propagation. In other words, it can be suggested that each vehicle inside the jam is at least once in a stop during a time-interval very large in comparison with τ_{del}. Therefore, the related average "stop time" τ_{stop} satisfies the condition

$$\tau_{\text{stop}} \gg \tau_{\text{del}}. \tag{2}$$

This suggestion also means that there is no influence of the inflow into the jam on the jam's outflow. A wide moving jam separates the traffic flows upstream and downstream of the jam. No states of synchronized flow possesses the feature of wide moving jams (2).

4D. Spatial-Temporal Patterns of Synchronized Flow

Note that a spatial-temporal pattern consisting only of the traffic phase "synchronized flow" as well the pattern "a wide moving jam" is sometimes also localized spatially. There are downstream and upstream fronts (boundaries) of the spatial-temporal pattern of synchronized flow where the vehicle speed and the density spatially sharply change. The downstream front of the pattern of synchronized flow separates synchronized flow upstream and free flow downstream. The upstream front of the pattern synchronized flow separates the synchronized flow downstream either from free flow or from a wide moving jam upstream. In contrast to a wide moving jam whose downstream front propagates upstream stationary, after a synchronized flow has occurred at an on-ramp (or at another bottleneck), the downstream front of the pattern the synchronized flow is always *fixed* at the bottleneck. Also, in contrast to the wide moving jam whose upstream front propagates upstream continuously, the upstream propagation of the upstream front of the synchronized flow often has a spatial limit.

4E. Examples of Differentiation of Traffic Phases

These differences in the mentioned spatial-temporal features of wide moving jams and patterns of synchronized flow are illustrated in some examples of traffic flow spatial-temporal patterns in Figure 4. In these examples, results of local simultaneous measurements of the vehicle speed (left) and the flow rate (right) at different locations are shown in time and in space. Figures 4b–4d are related to the section of the highway A5-South which schematic configuration is shown in Figure 4a. Each induction double loop detector (see the sets of the detectors $D1, \ldots, D24$ in Figure 4a) records the crossing of a vehicle and measures its speed. A local road computer calculates the flow rate and the average vehicle speed in one minute intervals.

The section has three bottlenecks which effective locations are marked as "B_1", "B_2", and "B_3" in Figures 4b–4d. The effective bottleneck "B_1" is linked to the off-ramp at the dètectors $D23$-off. If too many vehicles would like to leave the highway to the off-ramp either the phase transition from free flow to a wide moving jam (it will be called as the '$F \to J$'-transition) or to synchronized flow (it will be called as the '$F \to S$'-transition) occurs. The effective location of

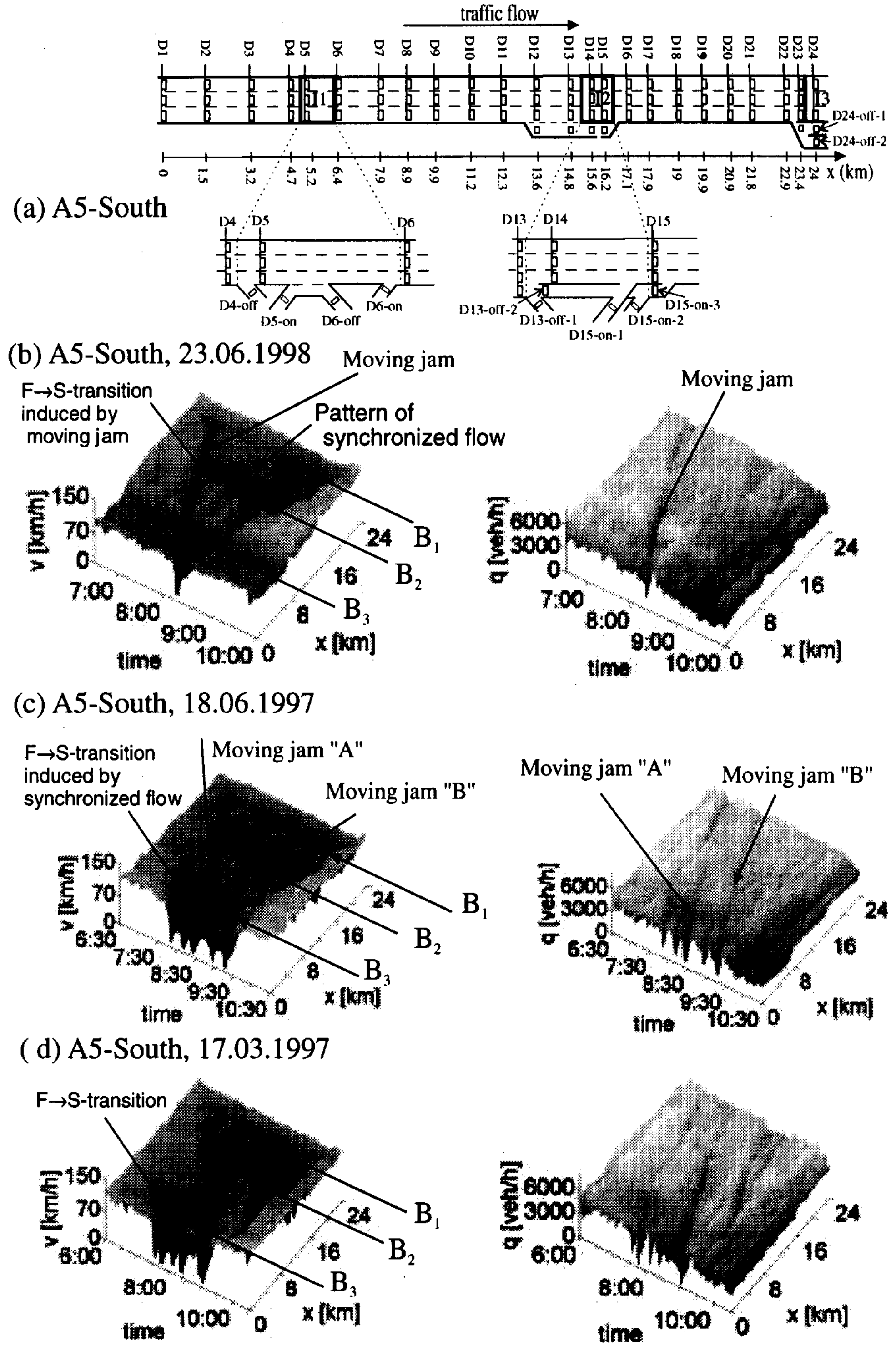


Figure 4. An overview of traffic patterns on three different days observed on the section of the highway A5-South (a): (b)–(d) dependencies of the average (across all lanes) vehicle speed (left) and the total flow rate across the highway (right) on time and space.

the bottleneck "B_1" is at about 23.4 km. The effective bottleneck "B_2" is linked to the on-ramp at the detectors $D16$. The effective location of this bottleneck is at about 17.1 km. The effective bottleneck "B_3" is linked to the on-ramp at the detectors $D6$. The effective location of this bottleneck is at about 6.4 km.

To emphasize the qualitative difference between the traffic phase "a wide moving jam" and the traffic phase "synchronized flow", let us first consider Figure 4b. It can be seen that a moving jam first emerges in the vicinity of the bottleneck "B_1". The physics of this '$F \to J$'-transition has been considered in [54]. The jam propagates further through the whole highway section, where two other bottlenecks "B_2" and "B_3" exist, keeping the velocity of the downstream front of the jam. Therefore, this traffic pattern belongs to the traffic phase "a wide moving jam". Propagating near the effective location of the bottleneck "B_2", the wide moving jam causes the '$F \to S$'-transition at the on-ramp at $D16$ (at $x \approx 17.1$ km). Indeed, the spatial-temporal pattern which appears upstream of the bottleneck "B_2" is further localized upstream of the bottleneck. The downstream front of this pattern is *fixed* at the bottleneck. The upstream front of the pattern is also located at some distance upstream of the bottleneck. Therefore, this pattern does not possess the characteristic feature of wide moving jams. As a result, correspondingly to the criteria mentioned above this pattern is a spatial-temporal pattern of synchronized flow (in Figure 4b the pattern is marked as "pattern of synchronized flow").

The other two examples in Figures 4c and 4d show some more complex patterns. However, the traffic phases "a wide moving jam" and "synchronized flow" can also easily be distinguished from one another. For example, two wide moving jams which are marked as moving jam "A" and moving jam "B" in Figure 4c can be considered. The moving jam "A" propagates through the bottleneck "B_3" and through complex pattern of synchronized flow which is formed at this bottleneck keeping the velocity of the jam downstream front. The same conclusion can be drawn from the propagation of the moving jam "B" through the bottlenecks "B_2" and "B_3" and also through complex patterns of synchronized flow at these bottlenecks. Therefore, the moving jams "A" and "B" belong to the traffic phase "a wide moving jam". In both cases, upstream of each of the bottlenecks "B_1", "B_2", and "B_3" spatial-temporal patterns which do not possess the characteristic feature of wide moving jams occur. The downstream fronts of each of these patterns are fixed at the related bottlenecks. Therefore, they all belong to the traffic phase "synchronized flow".

5. PHASE TRANSITIONS

Due to the existence of three traffic phases, there are three different types of phase transitions in traffic [50]:

(1) 'free flow $\leftrightarrow$ jam',
(2) 'free flow $\leftrightarrow$ synchronized flow',
(3) 'synchronized flow $\leftrightarrow$ jam(s)' which will be called as the '$S \leftrightarrow J$'-transitions.

Observations show that all of these phase transitions are local first-order phase transitions [50]. This means that these transitions are accompanied by sometimes similar looking breakdown, hysteresis, and nucleation effects. However, it must be distinguished between them because the nonlinear features of these phase transitions are qualitatively different. This differentiation determines one of the main difficulties in the understanding of the observations and of the traffic flow theory.

Recently, Kerner found out that the existence of the limit point in free flow $(q_{\max}^{(\text{free})}, \rho_{\max}^{(\text{free})})$ (Figure 1b) is linked to the $F \to S$-transition rather than the $F \to J$-transition [49,51]. As a consequence, it turns out that the cascade of two phase transitions 'free flow $\to$ synchronized flow' $\to$ wide moving jam(s)' (it will be called as the '$F \to S \to J$' transitions) is usually responsible for the jam emergence in free traffic flow [51].

Thus, the $F \to S$-transition is the most frequent phase transition in free flow. Because this transition is the local first-order one, each local perturbation whose amplitude exceeds some critical amplitude should cause this transition (the nucleation effect). Obviously, if these conclusions are correct, then the $F \to S$-transition and the effect of self-maintaining of synchronized flow must occur and show qualitatively the same features in all following cases when for example the $F \rightarrow S$-transition occurs at an on-ramp.

(1) When a random local fluctuation whose amplitude exceeds the critical amplitude occurs in the vicinity of the on-ramp in an initial free flow (the $F \rightarrow S$-transition at the bottleneck "B_3" in Figure 4d). This phase transition is shown in more detailed in Figure 5a.
(2) When an upstream moving jam which has initially occurred far enough downstream of the on-ramp reaches the on-ramp, and besides the $F \rightarrow S$-transition can occur in the initial free flow (the $F \rightarrow S$-transition induced by moving jam at the bottleneck "B_2" in Figure 4b). Indeed, in this case the moving jam plays a role of the local perturbation (a nucleus for the $F \rightarrow S$-transition) of the highest possible amplitude.
(3) When a localized region of synchronized flow propagating upstream which has initially occurred far enough downstream of the on-ramp reaches the on-ramp, and nearby the $F \rightarrow S$-transition can occur in the initial free flow at the on-ramp (the $F \rightarrow S$-transition induced by synchronized flow at the bottleneck "B_3" in Figure 4c). Also in this case the localized region of synchronized flow plays the role of a nucleus for the phase transition.

Case (1) is related to a random (spontaneous) $F \rightarrow S$-transition and Cases (2) and (3) are induced $F \rightarrow S$-transitions. Figure 4b–4d show that all these cases really occur in real traffic flow. After the $F \rightarrow S$-transition has occurred, the behavior of synchronized flow which is self-maintained at the on-ramp is qualitatively the same in all these cases (Figures 4b–4d). This confirms the conclusions made in [50].

6. COMMON FEATURES OF SPATIAL-TEMPORAL TRAFFIC PATTERNS AT BOTTLENECKS

Spatial-temporal traffic patterns can occur away from bottlenecks [47,51]. However, just as defects and impurities are important in physical systems, so are bottlenecks, such as on- and off-ramps, in traffic flow. Observations show that a bottleneck can trigger phase transitions, which is why spatial-temporal traffic patterns are considerably more likely to form near the bottleneck. It can been supposed that the probability of the $F \rightarrow S$-transition has a maximum at a so-called effective location of the bottleneck [52].

6A. General Pattern at Isolated Bottleneck

Let us first consider a bottleneck on a highway which is located far enough from other bottlenecks, so that the effects at other bottlenecks of the highway do not influence the pattern formation and the pattern's features at the bottleneck. Such a bottleneck will be called an *isolated* bottleneck. After the $F \rightarrow S$-transition at the isolated bottleneck has occurred, a spatial-temporal traffic pattern is formed upstream of the isolated bottleneck. This pattern will be called as *the general pattern* at an isolated bottleneck. Empirical investigations of the general pattern which have been made in [51,52] allow us to suggest the following common features of a lot of the general patterns at an isolated bottleneck.

(i) The general pattern consists often of two parts.
    (1) The pattern which consists only of synchronized flow (the region of synchronized flow in Figure 6a).
    (2) The pattern which consists of an alternation of wide moving jams (the region of wide moving jams in Figure 6a).
(ii) The spatial-temporal pattern of synchronized flow (the region of synchronized flow in Figure 6a) consists of two parts.
    (1) A region of synchronized flow where no moving narrow jams are formed (in Figure 6a, the width of this part is R_S).
    (2) The pinch region in synchronized flow, i.e, the region where narrow moving jams emerge (in Figure 6a, the width of this part is R_{PS}).

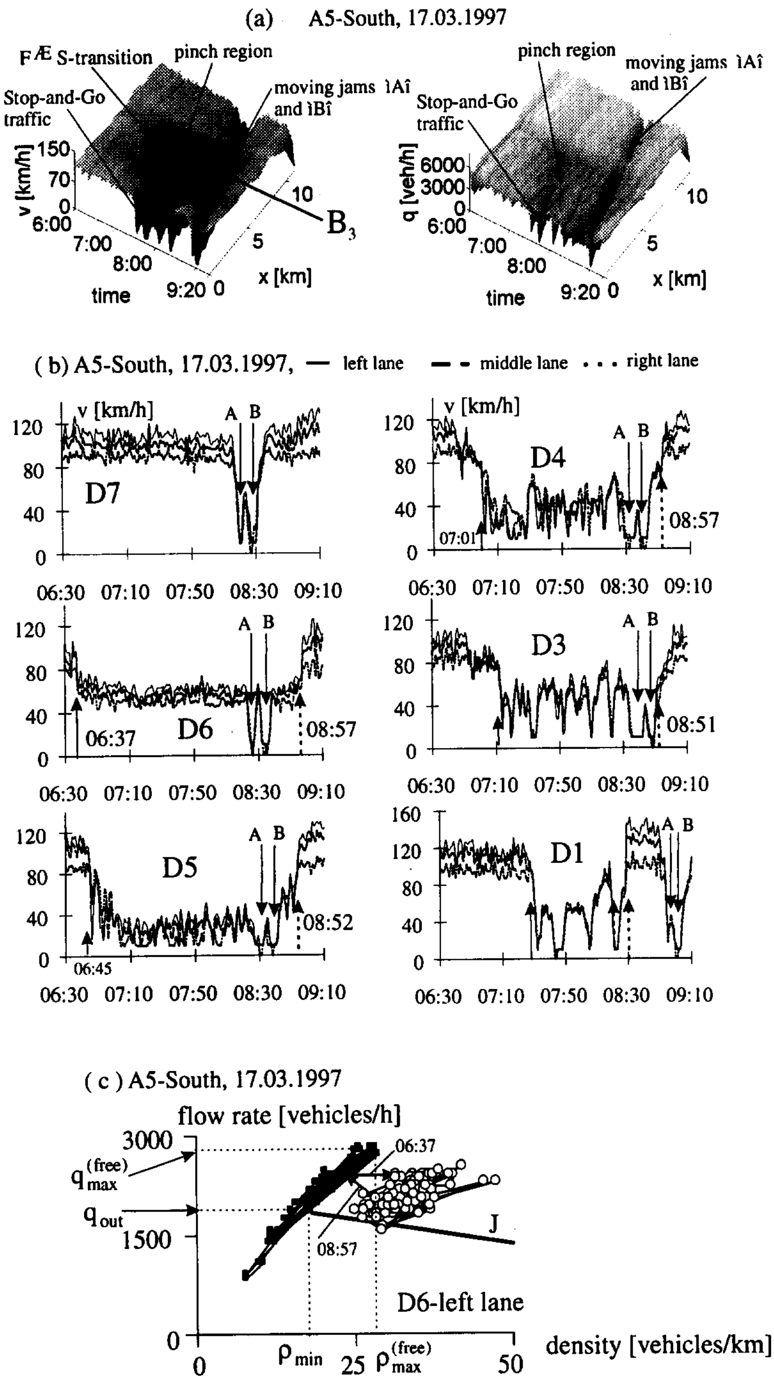


Figure 5. Illustration of common features of spatial-temporal patterns of synchronized flow [45,46,50,51]. (a),(b) Overview of the following spatial-temporal phenomena: (i) the $F \rightarrow S$-transition at the on-ramp at $D6$ (solid up arrow, $D6$, (b)); (ii) the propagation of synchronized flow upstream (solid up arrows at $D5$–$D3$, (b)); (iii) the pinch effect in synchronized flow ((a), (b), $D5$, and $D4$); (iv) the emergence of narrow jams in the pinch region ($D5$–$D3$, (b)); (v) the transformation of the narrow jams into wide moving jams, i.e., the $S \rightarrow J$-transitions ($D1$, (b)); (vi) the return $S \rightarrow F$-transition (dotted up arrows at $D1$–$D6$); (vii) the propagation of wide traffic jams through free flow (the jams "A" and "B", down arrows at $D7$) and through different states of synchronized flow ($D6$–$D3$, (b)). (a) Dependencies of the average (across all lanes) vehicle speed (left) and the total flow rate across the highway (right) on time and space. (b) Dependencies of the vehicle speed for each lane on time at different detectors. (c) Traffic phases in the flow-density plane: free flow (black points), synchronized flow (circles) and a part of line J. Arrows in (c) are related to the $F \rightarrow S$-transition (at $t \approx 06:37$, compare with (b), $D6$) and the return $S \rightarrow F$-transition (at $t \approx 08:57$, (c), $D6$).

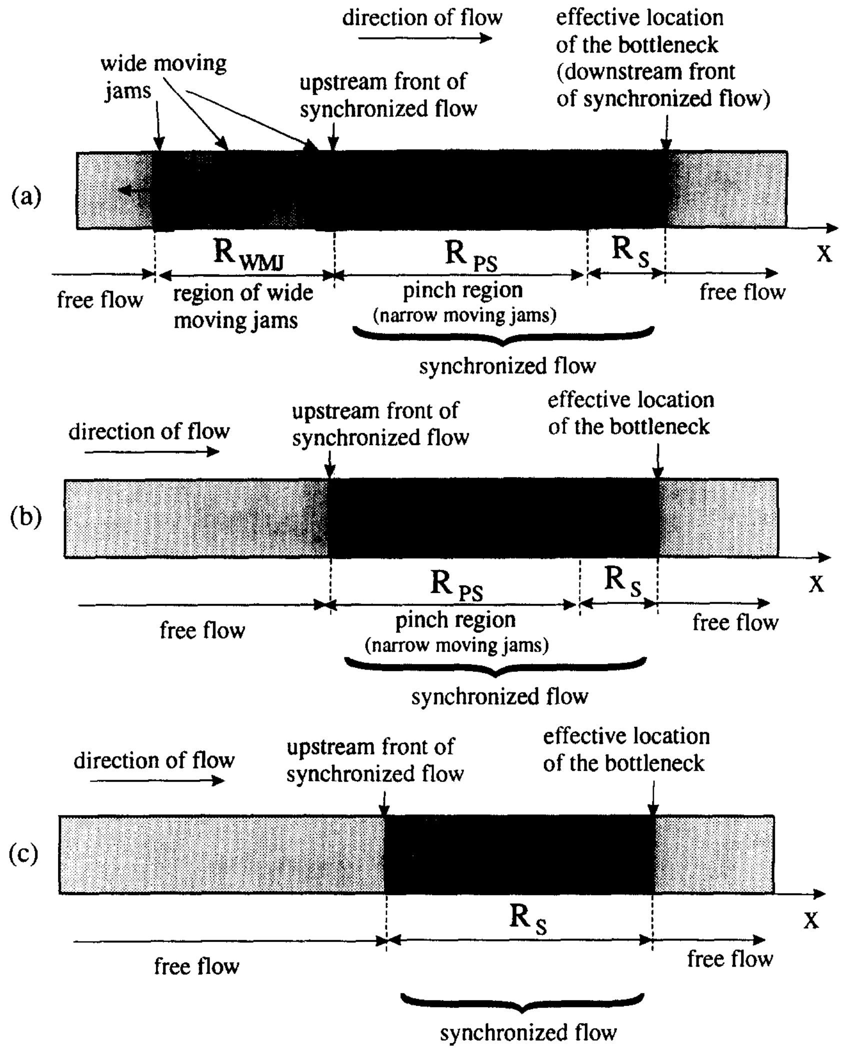


Figure 6. Schematic illustration of the spatial structure of traffic flow patterns at an isolated bottleneck [51,52,62]. (a) The general pattern, (b) the synchronized flow pattern (SP) of Type 1, (c) (SP) of Type 2.

(iii) The downstream front (boundary) of the pattern of synchronized flow (this front is also the downstream front of the general pattern as a whole pattern) is located at the effective location of the bottleneck (Figure 6a).

(iv) The upstream boundary of synchronized flow is determined by the location where a narrow moving jam is just transformed into a wide moving jam. At this place, the $S \rightarrow J$-transition has occurred. Because the transformation of different narrow moving jams into wide moving jams can occur at slightly different locations, the upstream boundary (front) of synchronized flow performs complex spatial oscillations over time. However, these oscillations have usually relatively small amplitude in comparison with the mean length of the pinch region L_{pinch}.

(v) The successive process of the transformation of narrow moving jams into wide moving jams at the upstream boundary (front) of synchronized flow leads to the formation of a sequence of wide moving jams. The sequence of wide moving jams (a "stop-and-go" pattern) may propagate far away from the location where the wide moving jams have first initially been formed (i.e., from the upstream front of synchronized flow).

A narrow moving jam may be considered as a nucleus of the traffic phase "wide moving jam" inside the traffic phase "synchronized flow". At the upstream boundary (front) of synchronized flow where a narrow moving jam has just transformed into a wide moving jam is also the location of the $S \rightarrow J$-transition.

Items (i)–(v) are illustrated in Figures 5a and 5b, where the pattern which has occurred upstream of the on-ramp at $D6$ is shown. This example is qualitatively the same as well as the case of the pattern observed on 13.01.1997 which has been considered in [51].

6B. "Foreign" Wide Moving Jams

The only difference to the case observed on 13.01.1997 [51] is that in the case in Figure 5a and 5b, two other wide moving jams (moving jams "A" and "B" in Figures 5a and 5b) which have initially occurred downstream of the effective location of the bottleneck at the on-ramp $D6$ propagate through the pattern under consideration. Such wide moving jams (moving jams "A" and "B") will be called *"foreign"* wide moving jams in comparison with the wide moving jams which are formed at the upstream front of the synchronized flow inside the pattern under consideration due to the transformation of narrow moving jams into wide moving jams (Item (v)). Thus, until these two "foreign" wide moving jams "A" and "B" do not reach the bottleneck B_3, this bottleneck B_3 can be considered as an isolated one. In the latter case, the features of the pattern at the on-ramp $D6$ (Figures 5a and 5b) are related to the features of the general pattern in Items (i)–(v) above [51]. See also the explanation in the caption in Figure 5.

6C. Synchronized Flow Pattern at Isolated Bottleneck

There are cases when instead of the general pattern at the isolated bottleneck shown in Figure 6a, some synchronized flow patterns, or (SP) for short, are observed (Figures 6b and 6c). There are two types of such (SP):

(i) Type 1, and
(ii) Type 2.

In (SP) of Type 1, the region of wide moving jams does not occur at the upstream boundary of the synchronized flow, i.e., this pattern consists only of the pattern of synchronized flow where the pinch region is formed. In (SP) of Type 2, the pinch region and the region of wide moving jams do not occur at the upstream boundary of the synchronized flow.

A qualitative explanation of (SP) of Type 1 may be the following. Let the flow rate upstream of the pattern be lower than q_{out}. Then, upstream of the upstream front (boundary) of synchronized flow, wide moving jams disappear over time. Such situations can occur if at some distance upstream of the effective location of a bottleneck, a large enough percentage of vehicles leave the highway to an off-ramp. This situation is realized in the case shown in Figure 4b for (SP) which is formed at the bottleneck B_2 (this pattern is marked as "pattern of synchronized flow" in Figure 4b).

7. "EXPANDED" SPATIAL-TEMPORAL TRAFFIC FLOW PATTERNS

A real highway has a lot of bottlenecks. Therefore, it can occur that two or more different separated pinch regions upstream of different highway bottlenecks can appear almost simultaneously. In this case, it can be expected that a complex dynamics of the jams' interaction may occur when some wide moving jams from the downstream sequence of moving jams due to their upstream propagation reach the upstream pinch region where narrow moving jams are just emerging [53]. Recall that such wide moving jams from the downstream sequence of the jams have been called above as "foreign" wide moving jams.

Examples of such cases are shown in Figures 4c and 4d and Figures 5a and 5b. In Figure 4c, the wide moving jam "A" which has initially occurred upstream of the bottleneck B_2 and the wide moving jam "B" which has initially occurred upstream of the bottleneck B_1 are "foreign" wide moving jams for the pattern occurred upstream of the bottleneck B_3. Indeed, both wide moving jams "A" and "B" propagate through the pinch region (upstream of the bottleneck B_3) where narrow moving jams are just emerging. Qualitatively, the same situation is shown in Figure 5b where two "foreign" wide moving jams "A" and "B" propagate through the pattern occurred upstream of the bottleneck B_3 where narrow moving jams are just emerging. As a result of the propagation of "foreign" wide moving jams instead of different sequence of moving jams which have initially occurred at different spatial separated bottlenecks, a united sequence of wide moving jams can be formed [53].

A much more complicated case can occur, if two or more effective locations of different bottlenecks exist on a highway very close one to another. To explain this important case, consider an effective location of a bottleneck on a highway where a spatial-temporal pattern first occurs. Let us assume that downstream of this "first" bottleneck a free flow is realized, i.e., the downstream front of the pattern is located at the bottleneck during the whole time under consideration. Let us further assume that the upstream boundary (front) of synchronized flow in this pattern reaches the effective location of the next (second) upstream bottleneck on the highway. In this case, it can occur that the upstream boundary (front) of synchronized flow propagates further upstream of the second bottleneck. The spatial-temporal pattern which occurs in this case will be called an *"expanded"* pattern.

An empirical example of an "expanded" pattern which occurred on the section of the highway A5-North is shown in Figures 7a–7c. The section has at least three bottlenecks "B_{North1}", "B_{North2}", and "B_{North3}" (Figure 7b). The bottleneck "B_{North1}" may be linked to a curve and a gradient on the highway (a bottleneck without obvious reason) in the vicinity of the detectors $D18$ ($x \approx 15.5$ km). The bottleneck "B_{North2}" is linked to the on-ramp at the detectors $D16$ ($x \approx 13.1$ km). The bottleneck "B_{North3}" is linked to the on-ramp at the detectors $D7$ ($x \approx 5.2$ km). First, the $F \to S$-transition occurs at the bottleneck "B_{North1}" (up-arrow in Figure 7c, $D18$). Then a spatial-temporal pattern begins to form upstream of bottleneck "B_{North1}" (Figures 7b and 7c, $D17$). After the upstream front (boundary) of the synchronized flow in this pattern reaches the bottleneck "B_{North2}" (Figure 7b and 7c, $D16$), this upstream front propagates further upstream. In the "expanded" pattern in Figures 7b and 7c, the region of synchronized flow expands at least over the bottleneck "B_{North2}". In this "expanded" pattern, wide moving jams can already occur downstream of the bottleneck "B_{North2}" because a part of the pinch region has been formed between the bottleneck "B_{North1}" and the bottleneck "B_{North2}" (Figure 7b). These wide moving jams which propagate further upstream can be considered as "foreign" jams for another part of the pinch region upstream of the bottleneck "B_{North2}" where other narrow moving jams emerge.

Therefore, the "expanded" pattern can have qualitatively similar spatial-temporal structure as well the general pattern on an isolated bottleneck shown in Figure 6. However, there are some important peculiarities of "expanded" patterns. First, in an "expanded" pattern, the pinch region can be much longer than in the case of an isolated bottleneck (many of tens or even hundreds kilometer). Indeed, the pinch region in the "expanded" pattern can cover a lot of bottlenecks. Symbolically, this effect is shown by effective locations of bottlenecks $B_2, \ldots, B_n$ in Figure 7d which all are covered by the pinch region. It must be noted that this pinch region has initially begun to form upstream of the bottleneck B_1. Second, a lot of "foreign" wide moving jams which are formed inside different parts of the pinch region between different bottlenecks can propagate through the "expanded" pattern (symbolically this effect is shown in Figure 7d).

8. COMPARISON OF EXPERIMENTAL FEATURES OF TRAFFIC PATTERNS WITH MODEL RESULTS

It has already been noted (see Section 2) that the model results about the existence of the characteristic, i.e., predictable and reproducible parameters of wide moving traffic jams and that states of a free flow are metastable states if the flow rate in the free flow exceeds q_{out} (first

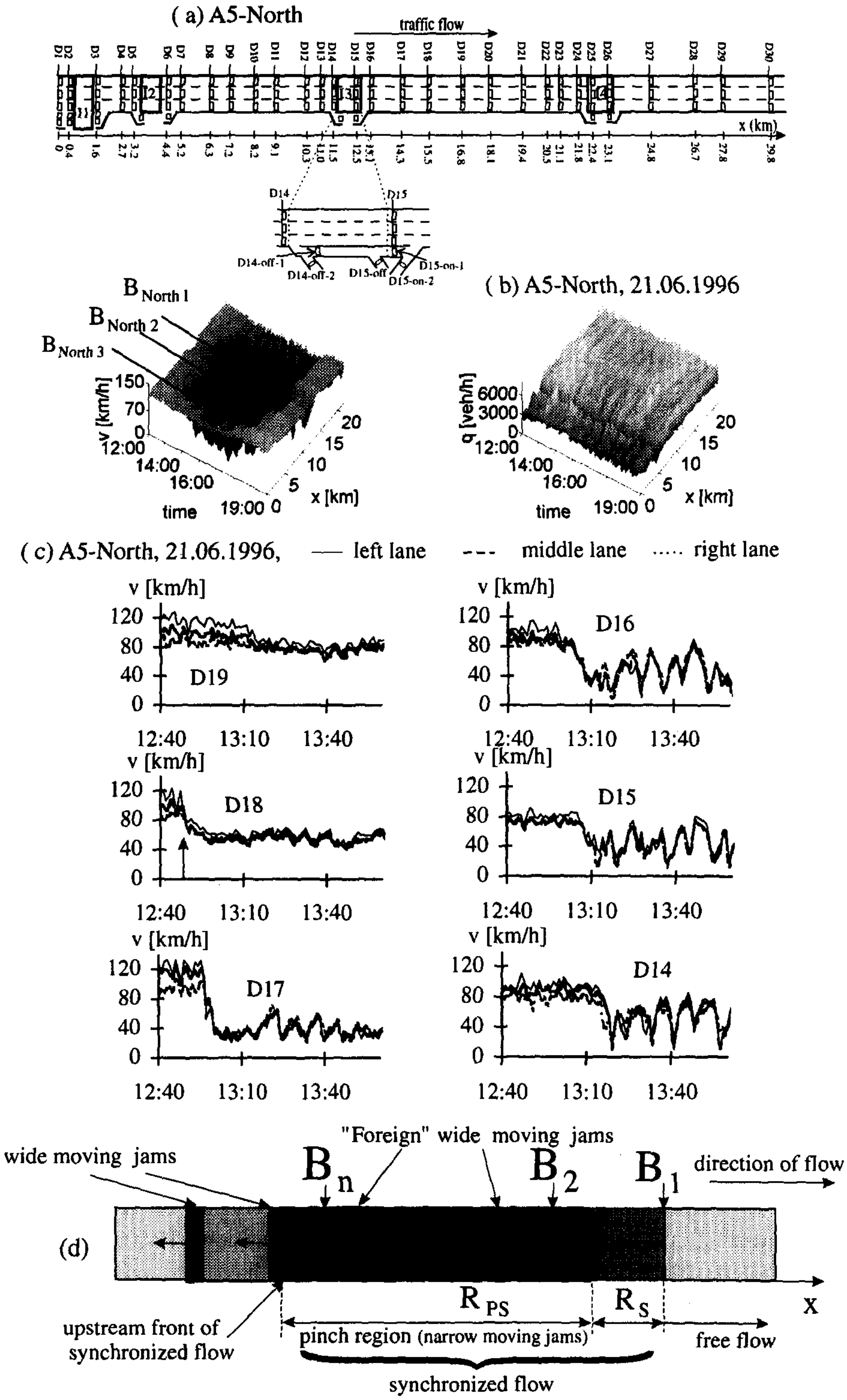


Figure 7. Illustration of features of “expanded” spatial-temporal patterns [62]. (a) Schematic picture of the section of the highway A5-North, (b) overview of an empirical “expanded” spatial-temporal pattern which has initially occurred at the effective location of the bottleneck B_{North1} and later expands over two other upstream bottlenecks B_{North2} and B_{North3}, (c) dependencies of the average vehicle speed at different detectors. (d) Schematic illustration of a spatial structure of “expanded” traffic flow patterns. The up-arrow in (a) symbolically shows the $F \rightarrow S$-transition which occurs at the effective bottleneck B_{North1} in the vicinity of the detectors $D18$ probably due to the curve and the gradient on the highway at this place.

predicted in [18]) have indeed been discovered in empirical investigations [57]. These results can be obtained from a mathematical consideration of traffic as a system which consists of only free flow and moving jams. For an analysis of such traffic, the basic assumptions (1) and (2) which have been considered in the introduction can be used.

However, there are other important empirical results which are linked to the features of spatial-temporal patterns of synchronized flow considered above which apparently cannot be explained in the frame of the basic assumptions (1) and (2). For this reason, the author questioned the basic assumptions (1) and (2) of traffic flow models at present [53,54]. These empirical results [45,49,51–54] are as follows.

(1) Either at on-ramps or away from bottlenecks, the $F \to J$-transition has *never* been observed, wide moving jam(s) emerge *only* due to the $F \to S \to J$-transitions. Therefore, the limit point of free flow $(\rho_{\max}^{(\text{free})}, q_{\max}^{(\text{free})})$ (Figure 1a) is linked to the $F \to S$-transition rather than to the $F \to J$-transition.
(2) Synchronized flow of Types (i) and (ii) [45] can exist on a highway for a long time, i.e., it can be stable at least with respect to small amplitude perturbations.
(3) After the $F \to S$-transition has occurred at an on-ramp, the flow rate $q_{\text{out}}^{\text{bottle}}$ in the outflow from the downstream boundary of synchronized flow (i.e., the boundary where synchronized flow is transferred into free flow downstream) can change in a very wide range. Besides, this discharge flow rate $q_{\text{out}}^{\text{bottle}}$ can considerably exceed the flow rate out from a wide jam, q_{out} (Figure 5c).

9. PREDICTABLE FEATURES OF TRAFFIC FLOW PATTERNS IN CONGESTED TRAFFIC. METHODS FOR TRAFFIC FORECASTING

It must be noted that traffic forecasting based on an application of different dynamic traffic flow models whose parameters should be validated corresponding to real traffic conditions is the well-known approach (e.g., [28]). However, due to very complex dynamics of real traffic flow which strongly depend on the infrastructure of a highway, weather, etc., it is difficult to choose a set of model parameters in a way that the model shows results compatible with real traffic flow.

It has already been stressed that the downstream front of a wide moving jam possesses some characteristics, i.e., predictable and reproducible parameters which do not depend on initial conditions and on time and are also the same for different wide moving jams. Recall that the velocity of the downstream front of a wide moving jam is always the characteristic parameter independent of the traffic parameters downstream from the jam [51,59]. The latter property allows us to make a prediction of the propagation of wide moving jams without any validating of model parameters from available traffic data [60]. Similar conclusion can be made for some important spatial-temporal features of patterns of synchronized flow [45,50,61,62].

Based on these empirical discoveries, Kerner *et al.* [60–62] have proposed an alternative approach to a traffic prediction in contrast to the modelling approach mentioned above (e.g., [28] and references there). In [60–62], the features of traffic patterns are determined experimentally and then used for a tracing and prediction of macroscopic properties of the traffic patterns *without any validation of model parameters*. In this approach [60–62], macroscopic parameters of spatial-temporal traffic patterns based on traffic measurements are determined first. An example of these macroscopic parameters is the velocity of the fronts of moving jams. The second step is to determine, also based on traffic measurements, the characteristic features of spatial-temporal dynamics of these macroscopic parameters of the pattern as functions of traffic flow rate, average speed, and infrastructure peculiarities. In particular, the conditions for the pattern formation, the dependencies of the locations of the fronts and the longitudinal widths of different traffic phases inside the patterns should be found. Finally, the properties of the spatial-temporal dynamics of the macroscopic pattern parameters are used for the predictions of the pattern behaviour.

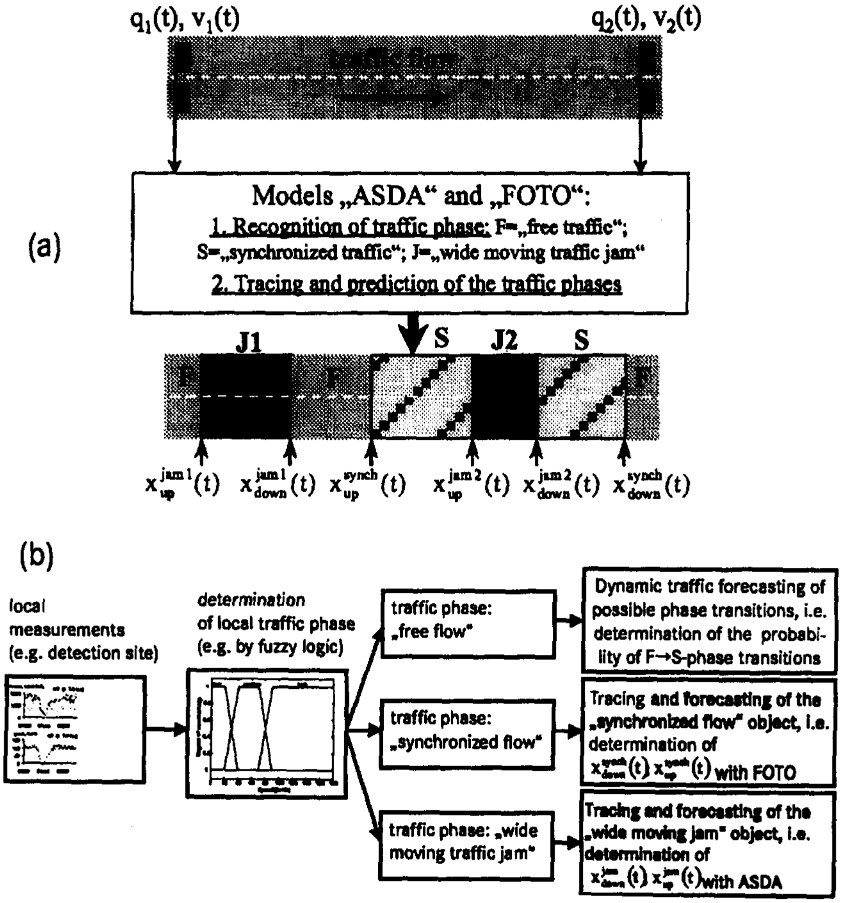
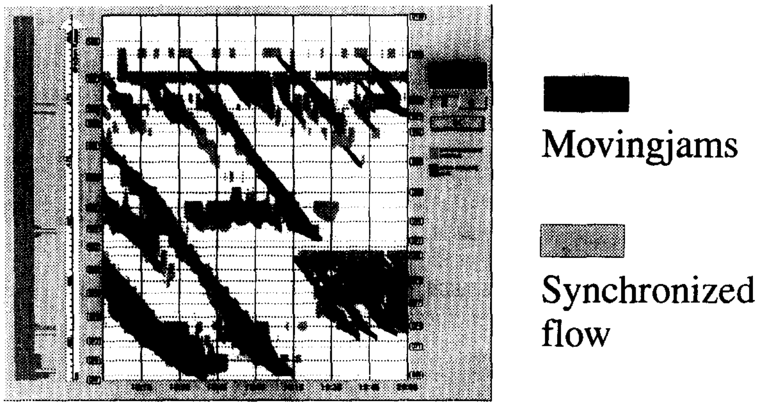


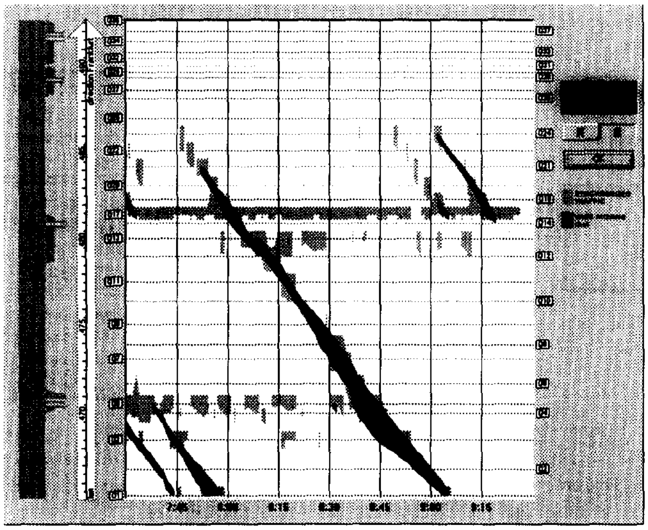
Figure 8. Illustration of the approach to traffic flow pattern tracing and forecasting [60–62]. (a) Illustration of a recognition of different traffic phases and the spatial boundaries (fronts) between these phases. (b) Illustration of the choice of the traffic model for a tracing and prediction of different traffic phases.

Figures 8a and 8b illustrate the approach [60–62], due to local measurements of traffic the recognition of the traffic phases is performed first. Second the initial fronts of wide moving jams $x^{\rm jam}_{\rm up}(t_0), x^{\rm jam}_{\rm down}(t_0)$ and of patterns of synchronized flow $x^{\rm synch}_{\rm up}(t_0), x^{\rm synch}_{\rm down}(t_0)$ are determined. These fronts define the spatial size and location of the related "synchronized flow" object and "wide moving jam" object in congested regime. Finally, with the models ASDA and FOTO, the tracing and forecasting of these objects fronts in time and space is calculated, i.e., the positions of all fronts $x^{\rm jam}_{\rm up}(t), x^{\rm jam}_{\rm down}(t), x^{\rm synch}_{\rm up}(t), x^{\rm synch}_{\rm down}(t)$ as the functions of time are found. Note that for the traffic forecasting historical time series at least for the flow rates are necessary. In other words, after the recognition of the mentioned traffic objects in congested regime, these objects are traced and predicted as *macroscopic single objects*, i.e., the determination of the location and the length of the related objects at any time is the aim of the approach. The knowledge of these parameters of traffic objects as well as the average vehicle speed inside the objects allow us to calculate other traffic characteristics, e.g., travel time or vehicle trajectories.

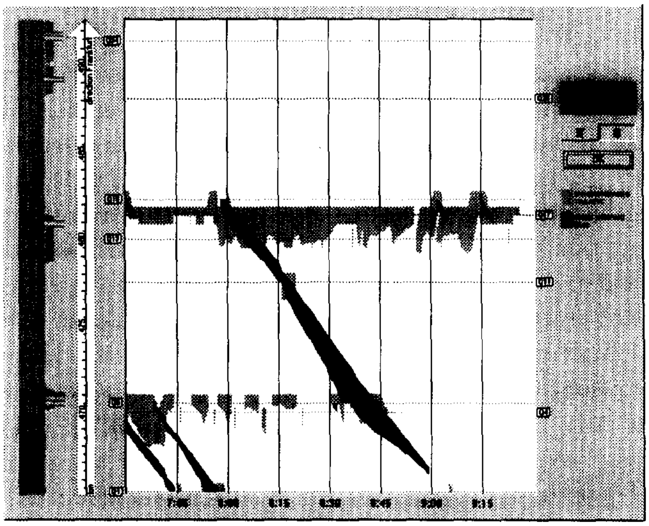
This philosophy has been used in [60–62] for developing a model for tracing and prediction of moving jams which has been called "ASDA" and of a model for tracing and prediction of the $F \to S$-transition and of patterns of synchronized flow. The latter model has been called "FOTO" (forecasting of traffic objects). The test of both models ASDA and FOTO which has been performed in an on-line application on the highway A5 in Germany based on available measured traffic data has shown that these models allow us to predict the $F \to S$-transition, the propagation of moving traffic jams and spatial-temporal features of patterns of synchronized flow without parameter calibration under different infrastructures and environmental conditions [63–66].



(a) Histogram A5-North on 12.04.2000 from 18:00–20:00 (printed from online operation).



(b) Histogram A5-South on 17.07.2000 from 07:30–09:30 using all available traffic data (31 sets of detectors).



(c) the same day and the time interval as well in (b) but using reduced input data (nine of 31 sets of detectors).

Figure 9. Results of the on-line application of the models ASDA and FOTO [63,64].

In the practical application FOTOwin for traffic of traffic objects (wide moving jams and patterns of synchronized flow) [63], which has been realized in a research project for the federal state of Hessen in Germany, both the models for the automatic tracing of jams (ASDA) and for the forecasting of traffic objects (FOTO) have been integrated. Examples of this application are shown in Figure 9. Many useful applications of the models ASDA and FOTO are possible, the output information of the models can be used for driver information systems or for traffic control systems. It is possible to predict the dissolution of moving jams and of patterns of synchronized flow.

The models have been tested on a lot of experimental data and it shows very good results. For the verification of the model results, empirical data of an infrastructure with many detection sites has been chosen. First, the data of all detectors has been used (Figure 9b). Then, the data of some detectors has been omitted and the process of the traffic pattern recognition has been repeated (Figure 9c). The data which has not been used are taken for comparison to the model results. Figure 9c shows the results of ASDA and FOTO with a strongly reduced configuration of the input values, instead of 31 detection sites for the models, only nine detection sites are used, i.e., only 30% of the detection site infrastructure. The histogram A5-South on July 7, 2000 from 07:30–09:30am (Figure 9c) shows a very similar result at the same situation as in Figure 9b with all detectors as input, in spite of the strongly reduced input information in Figure 9c, the analysis of the road section and the generation of traffic objects with the models ASDA and FOTO is very similar.

10. HYPOTHESES TO THEORY OF TRAFFIC FLOW

To explain empirical features of traffic flow patterns which have been considered above, the author has proposed alternative hypotheses (basic assumptions) for traffic flow theory [46–48,51,52]. These basic assumptions will be discussed below.

10A. A Hypothesis About the Multitude of Homogeneous States (Steady States) of Traffic Flow [45,46]

In the flow-density plane, hypothetical homogeneous, and stationary, i.e., time-independent states (steady states) of traffic flow on a multilane-road are related to a curve (curve F) for free flow and to a two-dimensional region for synchronized flow (Figure 10a). The multitude of states of free flow on a multilane-road overlaps hypothetical homogeneous and stationary states (steady states) of synchronized flow in the density. They are separated by a gap in the flow rate at a given density.

The multitude of hypothetical homogeneous and stationary states of synchronized flow on a multilane road (hatched region in Figure 10a) is identical to the related multitude of hypothetical homogeneous and stationary states of traffic flow on a one-lane road (hatched region in Figure 10b).

Note that hypothetical homogeneous and stationary states of synchronized flow cannot exist in a vicinity of the jam density $\rho_{\max}$ (i.e., the vehicle density inside a wide moving jam), exactly if the vehicle speed is lower than some minimal possible vehicle speed $v_{\min}^{(\mathrm{narrow})}$ (Figure 10) which may be about 5–10 km/h. Therefore, the related points in the hatched regions in Figure 10 have been excepted.

10B. A Hypothesis About the Behavior of Infinitesimal Perturbations in Initially Homogeneous States of Traffic Flow [46–48]

Independently of the vehicle density in an initial state of flow, infinitesimal perturbations of traffic flow variables (the vehicle speed and/or the density) do not grow in any hypothetical homogeneous and stationary states of either free or synchronized flow or else of homogeneous-in-speed states of synchronized flow. In the whole possible density range, hypothetical homogeneous

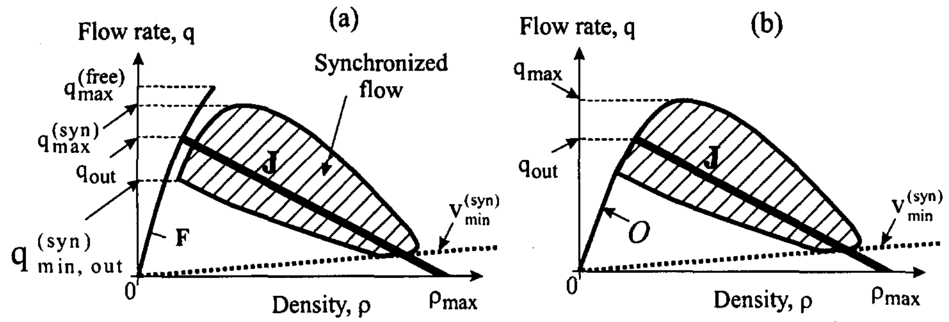


Figure 10. Hypotheses about hypothetical steady states of traffic flow: the concatenation of states of free flow (curve F) with hypothetical homogeneous and stationary (steady) states of synchronized flow (hatched region) and with line J for the downstream front of a wide moving jam (line J) for a multilane (a) and one-lane road (b) [46,48,51,54].

and stationary states of traffic flow can exist. In other words, in the whole possible density range (Figure 10), there are no unstable hypothetical homogeneous and stationary states of traffic flow with respect to infinitesimal perturbations of any traffic flow variables.

10C. A Hypothesis About Continuous Spatial-Temporal Transitions Between Different States of Synchronized Flow [46–48]

Local perturbations in synchronized flow can cause continuous spatial-temporal transitions between different states of both hypothetical homogeneous and homogeneous-in-speed states of synchronized flow (hatched region in Figure 10).

10D. Explanation of the Hypotheses for Hypothetical Homogeneous (Steady) States of Traffic Flow

To explain these hypotheses, note that in synchronized flow, spacing between vehicles is relatively low (i.e., the density is relatively high) in comparison with free flow at the same flow rate (Figure 10). At low spacing, a driver is able to recognize a change in the spacing to the vehicle in front of him, even if the difference in speed is negligible. In other words, the driver is able to maintain a time-independent spacing (without taking fluctuations into account) to the vehicle in front of him in an initially homogeneous state of synchronized flow. The ability of drivers to maintain a time-independent spacing should be valid for a finite range of spacing. Therefore, a given vehicle speed may be related to an infinite multitude of homogeneous states with different densities in a limited range (for example, the range $\rho_{\min}^{(syn)} \leq \rho \leq \rho_{\max}^{(syn)}$ is related to homogeneous-in-speed states of synchronized flow for a fixed vehicle speed, dotted line in Figure 11a). For this reason, the multitude of homogeneous states of synchronized flow covers a two-dimensional region in the flow-density plane (hatched region in Figures 10, 11a, and 11c). States of free flow (curve F in Figure 11c) and states of synchronized flow (hatched region) overlap in densities, i.e., $\rho_{\max}^{(free)} > \rho_S$, where ρ_S is the limit (minimum) density for hypothetical homogeneous and stationary states of synchronized flow (the density ρ_S in Figure 11c is related to the flow rate $q_{\min,out}^{(syn)}$ which is shown in Figure 10a). However, in free flow on a multilane road due to the possibility for vehicles to overtake the average vehicle, speed can be higher than the maximum speed in synchronized flow at the same density. Therefore, there is a gap in the flow rate between states of free flow and synchronized flow at a given density (Figure 10a).

Small enough perturbations in spacing which are linked to an original fluctuation in the braking of a vehicle do not grow. Indeed, small enough changes in spacing are allowed, therefore, drivers should not immediately react on it. For this reason, even after a time delay, which is due to a finite reaction time of drivers τ_{reac}, the drivers upstream should not brake stronger than drivers in front of them to avoid an accident. As a result, a local perturbation of traffic variables (density

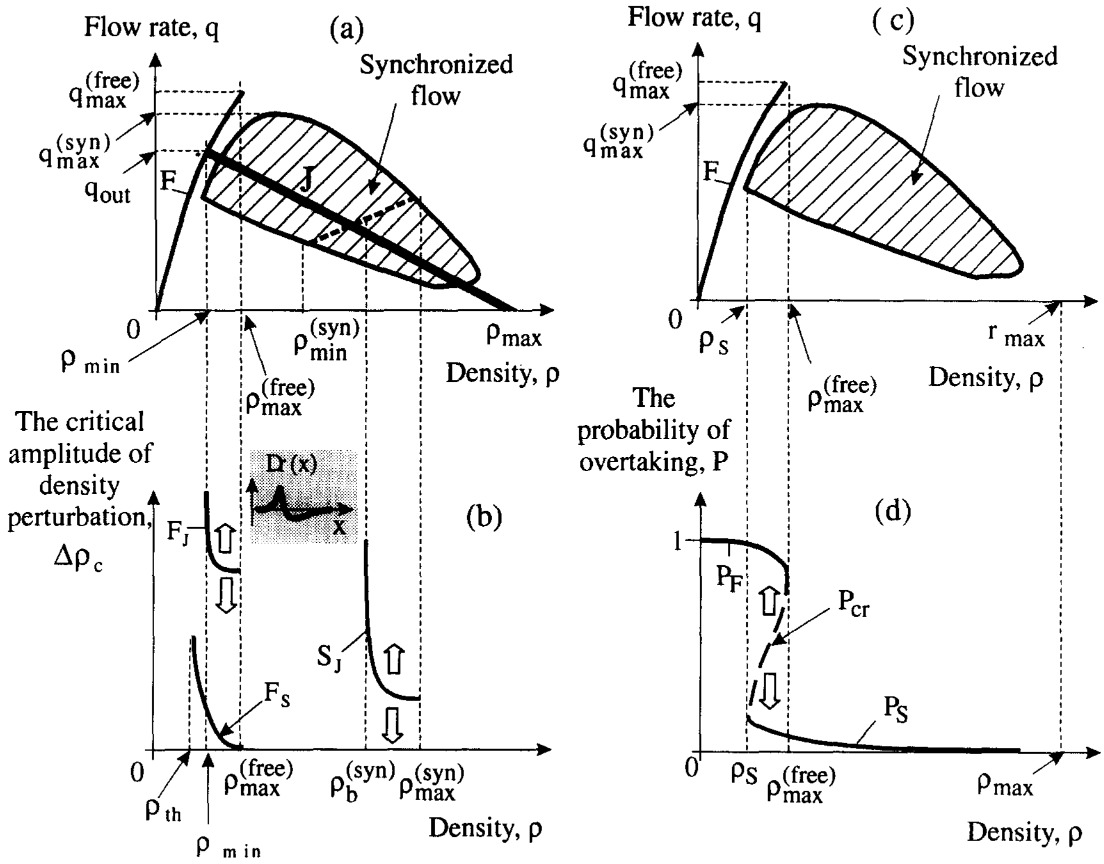


Figure 11. Explanation of hypotheses to the three-phase-traffic-theory [48,52]. States of free (curve F) and synchronized flow (hatched region) in (a),(c) are the same as in Figure 10a.

or vehicle speed) of small enough amplitude does not grow. An occurrence of this perturbation may cause a spatial-temporal transition to another state of synchronized flow. In other words, local perturbations may cause continuous spatial-temporal transitions between different states of synchronized flow.

Independently of the vehicle density on a one-lane road vehicles cannot overtake. Therefore, hypothetical homogeneous and stationary (steady) states of traffic flow on the one-lane road at higher density are identical to hypothetical homogeneous and stationary states of synchronized flow on a multilane road (hatched regions in Figure 10). Lower density states of flow on the one-lane road can be approximately represented by a curve (curve O in Figure 10b) which at low enough density has a slope equal to the same desired speed of vehicles as in free flow on a multilane road (Figure 10a). In real traffic flow a spatial alternation of states of free and synchronized flow on a multilane-road in a middle density range can lead to the occurrence of 'mixture states' [46].

10E. A Hypothesis About Two Different Kinds of Nucleation Effects in Traffic Flow [47–49]

There are two qualitatively different kinds of nucleation effects with two related kinds of metastability effects in traffic flow.

1. The nucleation effect which is responsible for the jam's formation, i.e., for the $F \to J$-transition and the $S \to J$-transition (Figures 11a and 11b).
2. The nucleation effect which is responsible for the $F \to S$-transition (Figures 11c and 11d).

The nucleation effect which is responsible for the $F \to S$-transition is linked to an avalanche self-decrease in the mean probability of overtaking in traffic flow, P. This self-decrease in the mean value of the probability of overtaking will be realized, if such a critical local perturbation of traffic variables (vehicle speed or/and density) occurs which causes an initial local decrease in the mean probability of overtaking below some critical value P_{cr} (dotted curve P_{cr} in Figure 11d). A dependence of the critical amplitude of the density perturbation on the density for the $F \to S$-transition is shown by the curve F_S in Figure 11b. In contrast, the nucleation effect which is responsible for the jam's formation is linked to an avalanche self-growth of the amplitude of a local critical perturbation of the traffic variables (density or/and vehicle speed), i.e., when the initial amplitude of the local perturbation exceeds some critical amplitude. Note that dependencies of the critical amplitude of the density perturbation on the density are shown by the curve F_J for the $F \to J$-transition and by the curve S_J for the phase transition the $S \to J$-transition in Figure 11b. The curve S_J is related to a fixed vehicle speed, dotted line in Figure 11a.

10F. A Hypothesis About the Critical Amplitude of Local Perturbations and the Probability of Phase Transitions in Free Flow [47–49]

At each given density in free flow, the critical amplitude of a local perturbation of traffic variables (density or/and vehicle speed) which is needed for the realization of the $F \to S$-transition (curve F_S in Figure 11b) is considerably lower than the critical amplitude of a local perturbation which is needed for the realization of the $F \to J$-transition (curve F_J in Figure 11a). Therefore, at each given density in free flow, the probability of an occurrence of the $F \to S$-transition is considerably higher than of the $F \to J$-transition. The threshold density ρ_{th} for the $F \to S$-transition can differ both from the threshold density ρ_b (where $\rho_b = \rho_{\text{min}}$) for the $F \to J$-transition and from the limit density ρ_S for homogeneous states of synchronized flow.

Because the critical value of the mean probability of overtaking is an increasing function of the density (Figure 11d, dotted curve P_{cr}), the higher the density in free flow is the lower the amplitude of a local perturbation of the traffic variables (density (curve F_S in Figure 11b) or/and vehicle speed) which causes an avalanche self-decrease in the mean probability of overtaking in the related local region. At the limit density for free flow $\rho = \rho_{\max}^{(\text{free})}$, the critical amplitude of a local perturbation of traffic variables (density or/and vehicle speed) which is needed for the realization of the $F \to S$-transition reaches zero, respectively, the probability of the $F \to S$-transition reaches one.

10G. A Hypothesis About the Nucleation Effect Which is Responsible for the Jam Emergence [51]

Line J (line J in Figure 11a) determines the threshold of the jam's existence and excitation. In other words, all (an infinite number !) homogeneous states of traffic flow which are related to line J in the flow-density plane are *threshold states* with respect to the jam's formation. Line J separates all homogeneous states of both free and synchronized flow into two qualitatively different classes.

1. In states which are related to points in the flow-density plane lying below (see axes in Figure 11a) line J, *no* jams either can continue to exist or can be excited.
2. States which are related to points in the flow-density plane lying on and above line J are *metastable states* with respect to the jam's formation where the related nucleation effect can be realized. Only the local perturbations of traffic variables whose amplitude exceeds some critical amplitude grow and can lead to the jam's formation (up arrows, curves F_J and S_J in Figure 11b), otherwise jams do not occur (down arrows, curves F_J and S_J in Figure 11b). The former critical local perturbations act as nucleation centers (nuclei) for the jam's formation.

The critical amplitude of the local perturbations is maximal at line J and depends both on the density and on the flow rate above line J. The critical amplitude of the local perturbations is considerably lower in synchronized flow than in free flow (compare the curve F_J with the curve S_J in Figure 11b). The lower the vehicle speed in synchronized flow is the lower the critical amplitude of perturbations for states of synchronized flow which are related to the same difference between the densities in these states and the corresponding threshold densities on the line J.

All conclusions about spatial-temporal transitions between different states of synchronized flow (Section 10C), about the metastable and the stable states, and the critical amplitude of the local perturbations in synchronized flow on a multilane road are obviously valid also for the related states of flow on a one-lane road (i.e., for the states in the hatched region in Figure 10b). Thus, the pinch effect where narrow moving jams emerge can also occur on a one-lane road.

10H. Explanation of Hypotheses for Nucleation Effects in Traffic Flow [47–49,51]

To explain the hypotheses, note that in free flow of low enough densities, a driver is not hindered to overtake, i.e., the related mean probability of overtaking is $P = 1$ (Figure 11d). The higher the density in free flow is the lower the mean probability of overtaking. However, in free flow up to the limit point $\rho = \rho_{\max}^{(\text{free})}$, the mean probability of overtaking does not decrease drastically (solid curve P_F, Figure 11d). Indeed, in free flow the difference between vehicie speeds on different lanes of a highway can be high enough. Therefore, the mean probability for a vehicle to be able to overtake is also high. In contrast to that in synchronized flow of high enough density, drivers are not able to overtake at all, i.e., $P = 0$. The lower the density of synchronized flow is the higher the mean probability of overtaking. However, in synchronized flow up to the limit point $\rho = \rho_S$, the mean probability of overtaking cannot increase drastically (solid curve P_S, Figure 11d). Indeed, in contrast to free flow, in synchronized flow the difference between vehicles speeds on different lines of a highway is low. This hinders the overtaking. Therefore, at each given vehicle density in the density range $[\rho_S, \rho_{\max}^{(\text{free})}]$, where states of free flow and synchronized flow overlap in the density, the mean probability of overtaking in free flow is considerably higher than in synchronized flow. As a result, the dependence $P(\rho)$ is **Z**-shaped, i.e., it has a hysteresis loop, and therefore, it consists of three branches.

(i) The branch P_F for free flow.
(ii) The branch P_S for synchronized flow.
(iii) The branch P_{cr} which is related to the critical value of the mean mean probability of overtaking (Figure 11d).

The sense of the branch P_{cr} is the following. If in a local region of free flow due to a local perturbation of traffic variables (speed or/and density) the mean probability of overtaking is decreased below the critical value P_{cr}, then an avalanche self-decrease in the mean probability of overtaking occurs leading to a self-formation of synchronized flow where $P = P_S$ (down arrow in Figure 11d). If on the contrary in a local region of synchronized flow due to a local perturbation of traffic variables the mean probability of overtaking is increased above the critical value P_{cr}, then an avalanche self-increase in the mean probability of overtaking occurs leading to a self-formation of free flow where $P = P_F$ (up arrow in Figure 11d).

Because the critical value P_{cr} of the mean probability of overtaking merges with the value P_F of the mean probability of overtaking for free flow at the limit density $\rho = \rho_{\max}^{(\text{free})}$ for free flow, already a local perturbation of traffic variables of a very small (but finite) amplitude causes an avalanche self-decrease in the mean probability of overtaking at the limit density $\rho = \rho_{\max}^{(\text{free})}$. Therefore, in free flow at the limit density $\rho = \rho_{\max}^{(\text{free})}$, the probability of the $F \to S$-transition reaches one.

Above a threshold of any local first-order phase transition, the higher the distance from the threshold is the higher the related mean probability of the occurrence of the phase transition. In other words, above the related threshold density $\rho = \rho_{\text{th}}$ (Figure 11b) the probability of the $F \to$

S-transition should be an increasing function of the density in free flow reaching one at the limit density $\rho = \rho_{\max}^{(\text{free})}$ for free flow. Such behavior of the probability of the breakdown phenomenon in a freeway bottleneck has recently been found by Persaud *et al.* [69]. This experiment is an additional confirmation of the finding by Kerner and Rehborn in [50] that the latter phenomenon is caused by the first-order local $F \to S$-transition.

Therefore, the existence of the limit point $\rho = \rho_{\max}^{(\text{free})}$ for free flow is indeed linked to the occurrence of the $F \to S$-transition. On the contrary, the critical amplitude of traffic variables (density or/and vehicle speed) which is needed for the $F \to J$-transition (curve F_J in Figure 11b) is a relatively high finite value even at the limit density $\rho = \rho_{\max}^{(\text{free})}$. Indeed, as it has been mentioned, at any density the probability of the $F \to J$-transition is considerably lower than of the $F \to S$-transition. To explain this [49], let us recall that synchronized flow occurs, if due to a local perturbation of traffic variables (density or/and vehicle speed) the mean probability of overtaking decreases below the critical value P_{cr} (Figure 11d). The critical amplitude of a local perturbation which is able to cause the local avalanche self-decrease in the mean probability of overtaking can be considerably lower than the critical amplitude of a local perturbation which is needed for the $F \to J$-transition. Indeed, for the occurrence of synchronized flow an initial perturbation itself does not necessarily have to increase avalanche-like. It must only cause an avalanche-like decrease in the mean value of the probability of overtaking. In contrast, for the jam formation the amplitude of the critical perturbation itself must begin to grow avalanche-like in an initial free flow.

An explanation of the hypothesis about the nucleation effect which is responsible for the jam's formation has been made in [51]. Other hypotheses about features of congested flow can be found in [46,47].

10I. Hypothesis About Three Kinds of Maximal Highway Capacity [51]

There are three kinds of "maximal" capacity on a multilane highway which depend on which phase (free flow, synchronized flow, or wide moving jam) the traffic is in. The related maximal capacity for the phase "free flow" is $q_{\max}^{(\text{free})}$, for the phase "synchronized flow" it is $q_{\max}^{(\text{syn})}$, and downstream of the phase "wide moving jam" it is q_{out} (Figure 10a).

10J. Hypotheses About Spatial-Temporal Features of Traffic Flow Patterns at Isolated Bottlenecks [52,62]

(i) If two Conditions (a) and (b) are fulfilled:
  (a) the traffic demand upstream of the last wide moving jam in the sequence of wide moving jams in the general pattern does not become lower than q_{out}, and
  (b) the synchronized flow is self-maintained at the effective location of the bottleneck, i.e., the downstream front of the synchronized flow remains to be fixed in space over time, then the general pattern (Figure 6a) *eternally* exists after the pattern has been formed. Therefore, there is only *one* type of spatial-temporal traffic flow pattern in this case. The latter also means that the longer the time after the general pattern has been formed, the higher the number of the wide moving jams in the sequence of wide moving jams is. Consequently, the width of the region of wide moving jams, R_{WMJ} (Figure 6a) continuously increases over time.

(ii) The mean flow rate inside the pinch region, $q_{\text{mean}}^{(\text{pinch})}$ (i.e., the flow rate which is averaged during a time interval which is considerable longer than the maximum duration of narrow moving jams) satisfies the condition:

$$q_{\text{mean}}^{(\text{pinch})} < q_{\text{out}}. \tag{3}$$

The flow rate out from a wide moving jam q_{out} is related to the case when a free flow is formed in the outflow from the jam. Correspondingly to (3), a wide moving jam at the upstream boundary of the synchronized flow (Figure 6a) can be considered as a region where 'superfluous' vehicles which cannot immediately pass through the pinch region should be virtually stored. In particular, if the flow rate upstream of the region of the wide moving jams is increasing, then the width of the most upstream wide moving jam is correspondingly increasing.

(iii) Let the general pattern (Figure 6a) occur at an isolated bottleneck which is linked neither to on- and off-ramps (nor to a merger of two or more different roads), nor there are on- and off-ramps in the vicinity of the bottleneck. Then, the mean discharge flow rate $q^{(\text{bottle})}_{\text{out, mean}}$ (the flow rate through the fixed downstream front of synchronized flow which is averaged in the same manner as $q^{(\text{pinch})}_{\text{mean}}$) which in this case $q^{(\text{bottle})}_{\text{out, mean}} = q^{(\text{pinch})}_{\text{mean}}$, corresponding to (3) satisfies the condition $q^{(\text{bottle})}_{\text{out, mean}} < q_{\text{out}}$.

(iv) The average time from the emergence of a narrow wide moving jam in the pinch region till the transformation of the narrow moving jam into a wide moving jam T_{narrow} can be estimated from the formula

$$L_{\text{pinch}} = |v_{\text{narrow, mean}}| T_{\text{narrow}}, \tag{4}$$

where $v_{\text{narrow, mean}}$ is the mean velocity of a narrow jam and L_{pinch} is the mean length of the pinch region. In the representative data in Figure 5b, $L_{\text{pinch}} \approx 3\,\text{km}$, $v_{\text{narrow, mean}} \approx -16\,\text{km/h}$, and $T_{\text{narrow}} \approx 11\,\text{min}$. To explain narrow moving jams which emerge in the pinch region of synchronized flow [51], the following may be supposed. If a vehicle is in a stop inside a narrow moving jam then the related value of τ_{stop} will be either less or of the same order of magnitude as τ_{del}. As a result, time gaps between vehicles remain of the same order of magnitude both inside and outside of the narrow moving jam. Thus, it may be supposed that in contrast to a wide moving jam, the narrow moving jam does not separate the traffic flows upstream and downstream of the narrow moving jam.

The pinch effect can also occur on a one-lane road. Thus, all conclusions made above about the spatial-temporal structure of the general and synchronized flow patterns shown in Figure 6 and also for expanded patterns shown in Figure 7d for a multilane-road are also valid for patterns which can occur upstream of a bottleneck on a one-lane road. The only one difference is that on the one-lane road, no free flow (vehicles cannot overtake on the one-lane road) exist. Therefore, it can be assumed that downstream of an insolated bottleneck on the one-lane road instead of free flow (Figure 6) a traffic flow which is related to the curve O in Figure 10b is realized. The same conclusion may be made for the region on the one-lane road upstream of the general and synchronized flow patterns or upstream of an expanded pattern. Instead of free flow (see Figures 6 and 7d), a traffic flow which is related to the curve O in Figure 10b should be realized on the one-lane road.

10K. Hypotheses About Discharge Flow Rate at On-Ramps [52]

1. Depending on the highway infrastructure in the vicinity of the on-ramp and on the traffic demand the discharge flow rate $q^{(\text{bottle})}_{\text{out}}$, i.e., the flow rate through the fixed downstream front of synchronized flow in a spatial-temporal pattern at the on-ramp can be changed within the range $[q^{(\text{syn})}_{\text{max}}, q^{(\text{syn})}_{\text{min,out}}]$ (Figure 10a)

$$q^{(\text{syn})}_{\text{min,out}} \le q^{(\text{bottle})}_{\text{out}} \le q^{(\text{syn})}_{\text{max}}. \tag{5}$$

2. The discharge flow rate $q^{(\text{bottle})}_{\text{out}}$ can be considerably higher than the flow rate out from a wide moving jam, q_{out}

$$q^{(\text{bottle})}_{\text{out}} > q_{\text{out}}, \tag{6}$$

where q_{out} is related to the case when a free flow is formed downstream of the wide moving jam.

To explain these hypotheses, note that corresponding to (3) and the empirical results [52], the following conditions are valid:

$$q_{\text{max}}^{(\text{syn})} > q_{\text{out}} > q_{\text{mean}}^{(\text{pinch})}, \tag{7}$$

$$q_{\text{out}} > q_{\text{min,out}}^{(\text{syn})}. \tag{8}$$

Besides, the flow rate $q_{\text{mean}}^{(\text{pinch})}$ cannot be lower than some minimum value $q_{\text{mean,min}}^{(\text{pinch})}$ [52]. On one hand, the discharge flow rate $q_{\text{out}}^{(\text{bottle})}$ satisfies conditions (5). On the other hand, the obvious condition for the mean value of the discharge flow rate, $q_{\text{out, mean}}^{(\text{bottle})}$, is valid

$$q_{\text{out, mean}}^{(\text{bottle})} = q_{\text{mean}}^{(\text{pinch})} + q_{\text{on-ramp, mean}}, \tag{9}$$

where $q_{\text{on-ramp, mean}}$ is the mean flow rate through the on-ramp which should be averaged in the same manner as the flow rates $q_{\text{mean}}^{(\text{pinch})}$ and $q_{\text{out, mean}}^{(\text{bottle})}$. Because conditions (7) and (8) are fulfilled, and the discharge flow rate can change at nearly the same vehicle speed in the large range (5) (Figure 10a), if $q_{\text{on-ramp}}$ changes then the discharge flow rate $q_{\text{out}}^{(\text{bottle})}$ can also change in the same manner.

However, it must be noted that empirical observations show [70] that if the pinch effect occurs upstream of the bottleneck on the main road then another pinch effect often occurs on the on-ramp upstream of the effective location of the bottleneck. This effect can lead to a *two-dimensional* development of the traffic flow pattern through the intersection between different roads of the traffic network. Therefore, an independent consideration of the discharge flow rate $q_{\text{out}}^{(\text{bottle})}$ and the flow rate $q_{\text{on-ramp, mean}}$ is usually not possible. A nonlinear dynamics between spatial-temporal effects inside two different pinch regions (on the main road and on the on-ramp) should be taken into account. Often such very complex nonlinear dynamics occurs if the discharge flow rate $q_{\text{out}}^{(\text{bottle})}$ is approaching the maximum capacity of the highway in synchronized flow $q_{\text{max}}^{(\text{syn})}$.

11. CONCLUSIONS

The physics of traffic flow is proving to be much richer than most researchers working in the field had expected. Recent experimental observations of real traffic patterns have discovered new phases of traffic flow, such as synchronized flow, and revealed a large number of features that are qualitatively similar to those observed in other nonlinear systems. The next challenge for mathematicians and physicists in this field is to develop new mathematical and theoretical concepts to explain these features.

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