Theorem 1. Let R be an HCF domain and suppose that an element $x \in R$, can be expressed as the product of a finite number of mutually co-prime rigid non units then this factorization is unique up to associates of the rigid non units and up to their order.

Proof. Let R be an HCF domain and let $x \in R$ be such that

$x = r_1 r_2 \dots r_m$; r_i rigid , $(r_i , r_j) = 1$ for $i \neq j$.

Further suppose that

$x = s_1 s_2 \dots s_n$; s_i rigid(non unit) $(s_i , s_j) = 1$, for $i \neq j$.

Since $s_1 | x$, by the HCF property

$$s_1 = s_{11} s_{12} \dots s_{1m} \text{ ; where } s_{1i} | r_i \text{ and since } \{ r_i \}_{i=1}^{i=m}$$

are co-prime, at most one of s_{1i} say s_{1k} is a non unit and so $s_1 | r_k$ for some k ($= 1,2,\dots,n$).

Reversing the process we take $r_k | x$ and so

$$r_k = r_{k1} r_{k2} \dots r_{kn} \text{ where } r_{ki} | s_i \text{ (} i = 1,2,\dots,n \text{)}.$$

By the above argument there exists an s_j such that $r_k | s_j$ and obviously s_j is an associate of s_1, for if not so $(s_1 , s_j) = 1$ while $s_1 | r_k$ and $r_k | s_j$ that is $s_1 | s_j$ a contradiction establishing the fact that s_1 is an associate of r_k.

Repeating the above process for $s_2 , s_3 , \dots , s_n$ we get $m = n$ and each s_i associate of some r_j. In other words the factorization $x = r_1 r_2 \dots r_m$ is unique up to associates of r_i and up to a suitable permutation of the rigid non units.

We can call the non unit of Theorem 1, a <u>Semirigid element</u> and based on this notion we make the following

Definition 2. An HCF domain in which every non zero non unit is semirigid will be called a <u>Semirigid Domain</u>.

We note that in an HCF domain a rigid non unit generalizes a prime quantum (since a prime quantum satisfies the properties of a rigid non unit) and it is easy to see that a