ABSTRACT

This work can be split into two parts. In the first part we generalize the concept of Unique Factorization by viewing Unique Factorization Domains as integral domains, non zero non units of which can be expressed uniquely (up to associates and order) as products of finitely many mutually co-prime associates of prime powers. Our working rule consists of taking a subset Q of the set P of all properties of a general prime power and investigating integral domains, whose non zero non units are expressible uniquely as as products of finitely many non units satisfying the properties in Q. For example we take Q consisting of only one property: of any two factors of a prime power one divides the other and call a non unit x rigid if for each h,k dividing x one divides the other. We find that in a Highest Common Factor domain a product of finitely many rigid elements is expressible uniquely as the product of mutually co-prime rigid elements. And a Highest Common Factor domain with the set of non zeros generated by rigid elements and units is the resulting generalization of a Unique Factorization Domain.

We consider three different Q's which, for suitable integral domains give distinct generalizations of Unique Factorization domains. In each case we provide examples to prove their existence, discuss their points of difference with UFD's and study their behaviour under localization and adjunction of indeterminates. We also study these integral domains in terms of the valuations of their fields of fractions and show that these integral domains are generalizations of Krull