that $d|d'^n$ and $d'|d^n$.

(4) If $x$ has $q$ as one of its minimal subvalued primes and $d|x$, then there exists a positive integer $n$ such that $d^n \nmid x$. Moreover $x = x_1 x_2$ such that $(x_1, x_2) = 1$ and $x_1$ has $q$ as its only minimal subvalued prime.

(5) If $d = d_1 d_2$; $d_i$ non units ( $i = 1,2$) then there exists a positive integer $n$ such that $d_1 | d_2^n$ or $d_2 | d_1^n$.

Proof. (1) Suppose that $(d_1, d_2) = 1$ and that both $d_i$ are non units. Obviously $(d_1, d_2) = 1$ in any localization of $R$ ( since $R$ is an HCF domain).

Since $q$ is a prime $d_1 d_2 = d \in q$, implies that $d_1 \in q$ or $d_2 \in q$. We note that both of $d_i$ cannot belong to $q$, because if $(d_1, d_2) = 1$ in $R$, $(d_1, d_2) = 1$ in $R_q$ and since $R_q$ is a valuation domain( $q$ is a subvalued prime ) at least one of $d_i$ is a unit in $R_q$, in other words at least one of $d_i$ is not in $q$.

Let $d_2 \notin q$ then since $d_2 | d$ and since we have assumed that $d_2$ is a non unit the set $\{ P_\beta \in \{ P_\alpha \}_{\alpha \in I} \mid d \in P_\beta \}$ is a subset of $\{ P_1, P_2, \ldots, P_r \}$ ( for if not so $\{P_1, \ldots, P_r\}$ is not the set of all the valued primes containing $d$).

Select a member $P_j$ of $\{P_1, \ldots, P_r\}$ such that $d_1, d_2 \in P_j$ but since $(d_1, d_2) = 1$ in $R$ and $(d_1, d_2) = 1$ in $R_{P_j}$ and thus $d_2$ does not belong to $P_j$ i.e. if $(d_1, d_2) = 1$ and $d_2 \notin q$ then there exists no valued prime in the defining family of $R$ which should contain $d_2$, a contradiction to the definition of a ring of Krull type and hence $d_2$ is a unit. Similarly if we had assumed $d_1 \notin q$ we would conclude that $d_1$ is a unit. thus if $(d_1, d_2) = 1$ then either of $d_i$ is a unit( but of course not both).

(2) Let $x$ and $d$ be as in the hypothesis and let