CHAPTER 1

GENERALIZED UNIQUE FACTORIZATION DOMAINS

0. Introduction . The theory of Unique Factorization Domains is well known and the most part of the theory is covered by [30],[31],[32] and by [23].

To start with, we mention that if R is a UFD then every non zero non unit x in R can be expressed as

$$x = up_1^{a_1}p_2^{a_2}\dots p_r^{a_r} \qquad \text{-------} \qquad \text{-------} \ (A)$$

where u is a unit and $p_i^{a_i}$ are powers of primes such that $(p_i^{a_i}, p_j^{a_j}) = 1$ if $i \neq j$ and the expression (A) is unique up to associates of the prime powers and up to a suitable permutation (cf [30] page 16).

We call a non zero non unit a an <u>atom</u> if $a = a_1 a_2$ implies that a_1 or a_2 is a unit and an integral domain is called <u>atomic</u> if every element in it is expressible as a product of a finite number of atoms. A <u>prime</u> is defined to be a non zero non unit p such that $p|ab$, implies that $p|a$ or $p|b$. Obviously if $p = ab$ and $a = a'p$; $p = a'bp$ i.e. $1 = a'b$, that is b is a unit, similarly we could take $b = b'p$ and show that a is a unit. In other words a prime is an atom and a UFD is an atomic integral domain.

Our main aim in this chapter is to replace the prime powers by the more flexible non units; <u>prime quanta</u> which behave like prime powers but are not products of atoms, and to work out a generalized theory of factorization which does not require a generalized unique factorization domain to be atomic.

Section 1, of this chapter mainly deals with the definition of a prime quantum, its properties and with the definition of a Generalized Unique Factorization Domain(GUFD) as an integral domain in which every non zero non unit is