dissimilar if $i \neq j$.

In the case of an element x which is a product of primes we do not need the assumption (2) above, while proving the uniqueness of the factorization because of the fact that a prime is an atom . But as it can be easily verified that every positive integral power of a prime is a prime quantum we can easily achieve the form

$x = up_1^{a_1} p_2^{a_2} \ldots p_r^{a_r}$; where u is a unit and p_i, p_j are non associate primes for $i \neq j$, and hence $p_i^{a_i}, p_j^{a_j}$ are distinct prime quanta. But before accepting the above two restrictive assumptions as a price of generalization we have to be sure that there do exist (1) quanta (2) prime quanta (3) quanta which are not prime quanta (4) Generalized Unique Factorization Domains.

2. Examples.

(1) Quanta: Example 1. Every atom is a quantum.

Obviously every non unit factor of an atom a is an associate of a, and so an atom satisfies the condition of being a quantum.

Example 2. Let R be a quasi-local domain of Krull dimension 1. It is well known that if a,b are two non zero non units of R then there exists a positive integer n such that $b|a^n$ (cf Theorem 108 [23]). And of course the result is symmetric, that is $a|b^m$ for some positive integral m. So if x is a non zero non unit in R and h is a non unit factor of x then there exists n such that $x|h^n$. Thus we conclude that every non zero non unit element of R is a quantum. This example also establishes the existence of quanta which are not atoms e.g. when R is non Noetherian.

(2) Prime quanta: Example 3. A prime is a prime quantum.