

Further, it is easy to see that if q_1, q_2 are two similar prime quanta then $q_2 \in Q_{q_1}$ and as every element in the integral domain R , non co-prime to q_2 is also non co-prime to q_1 , $Q_{q_2} \subseteq Q_{q_1}$ and similarly $Q_{q_1} \subseteq Q_{q_2}$, that is $Q_{q_1} = Q_{q_2}$ and conversely if $Q_{q_1} = Q_{q_2}$ then $q_2 \in Q_{q_1}$ and so $q_2 \in Q_{q_1}$ which implies that q_1, q_2 are non co-prime and hence are similar.

We note that if in an integral domain R , a prime quantum q is contained in a prime ideal P then every non unit factor q_1 of q is in P . The proof follows from the fact that q is a quantum. This observation suggests that if a prime quantum q is in a prime P then $Q_q \subseteq P$.

For further references we record the above observations and their easy consequences as the

Proposition 5. Let q, q_1, q_2 be prime quanta in an integral domain R then

- (1) $Q_{q_1} = Q_{q_2}$ iff q_1 and q_2 are similar.
- (2) If P is a prime ideal in R and $q \in P$ then $Q_q \subseteq P$ and if P is minimal then $Q_q = P$.
- (3) If P is a minimal prime ideal and $q \in P$ then $q_1 \in P$ iff q_1 is similar to q .

Note . By a minimal prime ideal we mean a minimal non zero prime ideal.

We recall that an integral domain R with quotient field

K is called completely integrally closed if for a and u in K with $a \neq 0$, $au^n \in R$ for all n implies that $u \in R$ (cf [23] p.53). From Remark 2, it follows that if x and y are two elements of a GUPD R then $x^n | y$ for all n implies that x has no prime quantum as a factor i.e. x is a unit. Now a GUPD R is an HCF domain and if K is the quotient field of R then for every $u \in K - \{0\}$, $u = x/y = x_1/y_1$ where $(x_1, y_1) = 1$.