and so by (2) $a|b$ i.e. $d^n|d'$, a contradiction to the fact that $d^m \nmid d'$ for an $m \leq n$.) and so $d'|d^{2n}$ which is the required result.

(4) Let x and d be as in the hypothesis. Using a method similar to the one used in the proof of (3) above, we can prove that there exists an n such that $d^n \nmid x$. Suppose that $d^n \nmid x$ and consider $(d^n, x) = h$, that is $d^n = ah$, $x = bh$ and $(a,b) = 1$ i.e. at least one of a,b is not in q. If $b \in q$ then $a \notin q$ and so $h \in q$ ($\because ah \in q$). Now b has a factor contained in q such that q is the only minimal subvalued prime of this factor (cf Lemma 2) and thus by (2) above $a^m|b$ for each m, and so $d^n = ah|bh = x$ ($\because a|b$ and $h|h$) a contradiction and hence $b \notin q$. If we assume that $a \in q$ then since $(a,b) = 1$ and q is the minimal prime of d and hence of a and h we have $(h,b) = 1$ (since if $(a,b) = 1$ then $(a^n,b^n) = 1$ and by (3) above there exists an n such that $h|a^n$) i.e.

$x = bh$ where $(b,h) = 1$. ------------------------(A)

Similarly if $a \notin q$ we can consider $(d^{n+1}, x) = x'$ and then $d^{n+1} = x'k$, $x = x''x'$, $(k,x'') = 1$ and if $k \notin q$ then $d^n|x'$ and so $d^n|x$ a contradiction establishing that k must be in q. As in (A) above $(k,x'') = 1$ implies that $(x'',x') = 1$ i.e. $x = x''x'$ where x' has q as its only minimal subvalued prime and $(x'',x') = 1$. ---------------------(B)

Now combining (A) and (B) we get the result.

(5) the proof follows as an application of (2) and (3).

The properties (1) and (5) of d in Lemma 3 give rise to the following

Definition 4. A non zero non unit element d in an integral domain R, will be called a <u>packet</u> if every factorization of d into two non units, $d = d_1d_2$ (if it exists) is such that