the permutation of distinct prime quanta and up to their associates.

Proof. Let x be a non zero non unit element in an integral domain R and let x be a product of prime quanta q_i i.e.

$$x = q_1 q_2 \dots q_n , \quad q_i , q_j \text{ dissimilar if } i \neq j$$

Suppose that x can also be written as

$$x = p_1 p_2 \dots p_m \; ; \; p_j \text{ prime quanta, } p_i , p_j \text{ dissimilar if}$$

$i \neq j$. Now

$$q_1 q_2 \dots q_n = p_1 p_2 \dots p_m$$

Since q_1 is a factor of the L.H.S.

$$q_1 \mid p_1 p_2 \dots p_m$$

and similarity between prime quanta being an equivalence relation, q_1 can be similar to ~~only one~~ at most one of the p_i ($i = 1 \dots m$) while from the definition of a prime quantum it follows that q_1 is similar to at least one of the p_i . That is there exists a unique p_t such that $q_1 \mid p_t$.

We claim that q_1 and p_t are associates, because reversing the process, that is taking $p_t \mid q_1 q_2 \dots q_n$, we get $p_t \mid q_1$. And combining the two results confirms the claim.

Now we are left with

$$q_2 q_3 \dots q_n = p_1 p_2 \dots p_{t-1} p_{t+1} \dots p_m$$

and repeating the above procedure we conclude that n = m and each q_i is an associate of some p_i for a suitable permutation of $p_1, p_2, \dots p_n$.

Definition 5. An integral domain R will be called a Generalized Unique Factorization Domain (GUFD for short) if every non zero non unit element x in R can be expressed as the product of a finite number of distinct prime quanta.

The proof of Proposition 2, depends heavily on the assumption that we can write $x = q_1 q_2 \dots q_n$, where (1) q_i are prime quanta ($i = 1,2,..n$) and (2) q_i , q_j are