

Muhammad Zafrullah

From: "Muhammad Zafrullah" <mzafrullah@usa.net>
Date: Monday, November 25, 2002 4:13 AM
To: "Steve McAdam" <mcadam@math.utexas.edu>
Subject: paper#2

Dear Steve,

I have skimmed through your paper. You have done a lot of good, thoughtful and ingenious work and I must congratulate you for it. I have learned quite a few things from your paper such as:

(i) How you got to Distinguished domains (using S-ideals and some inverting) and thanks to your paper now I can say precisely when a distinguished domain is a PVMD. If you are interested I will send you my findings, but it is rough and is mostly in the star operations lingo.

(ii) I had introduced a generalization of UFD's in my thesis and called it "unique representation". The idea was: Impose on a GCD-domain D the condition that each nonzero nonunit of D has finitely many minimal primes and call it a unique representation domain (URD). The result was that a GCD domain D is a URD if and only if every nonzero nonunit x of D is expressible as a product of finitely many packets. A packet was the name I chose for a nonzero element with a unique minimal prime. Of course I also showed that a product of finitely many packets in a GCD domain can be expressed uniquely as a product of mutually coprime packets. I also showed in my thesis that if D is a URD and K the quotient field of D then $D+XK[X]$ is again a URD. Later, I published these results in the form of a paper: [Unique representation domains, J. Natur. Sci. Math. 18 (1978), no. 2, 19--29. MR 82c:13025]. In that paper I had also included the result that if S is a multiplicative set in a URD such that $D+XD[1/S][X]$ is a GCD domain then $D+XD[1/S][X]$ is a URD.

I was in Libya when I published it, then I got busy teaching and running the department, believe me running a department as a foreign worker in Libya, was only slightly easier than getting a job in the US without any friends. In any case I never got round to seeing if the paper was all OK. Now your paper tells me that not only was the theory OK but also that a URD is another example of a UCFD. Thank you. Then there is another something to note: $D+XK[X]$ and $D+XD[1/S][X]$ are examples of pullbacks of a slightly different kind than you have looked into. Of course I had used coprime and not comaximal and I had worked in a GCD domains, but I think this example will hopefully inform you that there were efforts at generalizing UFD's to non atomic set up as far back as 1973. If you would like to read the paper mentioned above I can scan and e-mail you a copy (the paper had a lot of typos and I have kept a corrected copy). In fact there is now quite a bit of literature on extensions of unique factorization. A comprehensive paper to read in this connection is by Dan Anderson: [Extensions of unique factorization: a survey. Advances in commutative ring theory (Fez, 1997), 31--53, Lecture Notes in Pure and Appl. Math., 205, Dekker, New York, 1999. MR 2001h:13026].

(iii) My memory is quite good! Even after an "epileptic seizure" under some very mysterious circumstances in 1990. I was in Rock Hill, South Carolina those days. Some people say it is impossible to do any intellectual work after an epileptic seizure, I have published around thirty papers after that. (I thank God for it.)

Now here are my comments:

Personally, I would like to see the properties of pseudo-prime elements and use them as building blocks, but then the idea occurred to you and not to me. I am just rambling but could it be that x is pseudo-prime element if and only if $x=ab$, a, b nonunits implies that a belongs to every prime ideal containing b or b belongs to every prime ideal containing a ?

I am not too deeply interested in this direction (I might get interested later!) as I am fretting about, among other things, characterizing domains that are locally finite intersections of localizations at primes using tricks similar to the ones developed in the paper I told you about in my last message.

Sincerely,
 Muhammad

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