Let $y = p_1 p_2 \ldots p_n$ where p_i are distinct prime quanta. And let x be a non unit factor of y, then by Proposition 3, x is expressible as a product of distinct prime quanta, that is $x = p_1' p_2' \ldots p_s'$ where p_i' are distinct prime quanta each dividing one (and hence only one) of $p_1, \ldots, p_n$. Suppose that for a suitable permutation of p_j, $p_i' | p_i$. And by the definition of a quantum, there exists a positive integer n such that $p_i | p_i'^n$ (properly) that is x^n has at least one prime quantum as a factor which does not divide the prime quantum factor of y which is similar to it and hence $x^n \nmid y$.

Before proceeding further with the analogy, we need an auxiliary arrangement of some new notions and facts. As our first step we introduce the notion of a prime ideal associated to a prime quantum.

Let q be a prime quantum in an integral domain R and put $\quad Q_q = \{ x \in R \mid (x,q) \neq 1 \}$.

Now $x, y \in Q_q$ implies that there are two prime quanta q_1, q_2 such that $x = x_1 q_1$, $y = y_2 q_2$. As similarity between prime quanta is an equivalence relation, q_1 and q_2 are similar and consequently $q_1 | q_2$ or $q_2 | q_1$. If $q_1 | q_2$ say,

$$x + y = x_1 q_1 + y_2 q_2 = q_1 (x_1 + y_2 q_2') \text{ non co-prime to } q,$$

that is $x + y \in Q_q$. And since for every x non co-prime to q rx is non co-prime to q for every r in R, Q_q is an ideal. Moreover $xy \in Q_q$ implies that xy is non co-prime to q and by Def. 3, either x is non co-prime to q or y is i.e. $xy \in Q_q$ implies that $x \in Q_q$ or $y \in Q_q$ and so Q_q is a prime ideal. And this observation provides us the

Definition 6. Let q be a prime quantum in an integral domain R then the prime ideal

$$Q_q = \{ x \in R \; (x,q) \neq 1 \} \text{ will be called the prime}$$

ideal associated to q.