Similarly $0 \neq a \in K$ can be written as $a = x_2/y_2$ where $(x_2, y_2) = 1$.

Now $au^n \in R$ for all n implies that $(x_2/y_2)(x_1/y_1)^n \in R$ for all n. By the HCF property $y_1^n | x_2$ for all n, which by the above observation is possible only if y_1 is a unit in R, that is $u \in R$. Thus we have proved the

Proposition 6. A GUFD is a completely integrally closed integral domain.

We go further in our pursuit of analogous results and state the

Proposition 7. An integral domain R is a GUFD iff every non zero prime ideal in R contains a prime quantum.

Proof. Suppose that every prime ideal of R contains a prime quantum and let S be the set generated multiplicatively by prime quanta and units of R. If $S \neq R - \{0\}$ then by Zorn's Lemma, the complement $R - S$ contains a prime ideal and hence a prime quantum, a contradiction and hence $S = R - \{0\}$ i.e. R is a GUFD. Conversely if R is a GUFD and P a prime ideal in R, let x be a non zero element in P. Then $x = q_1 q_2 \ldots q_n$ where q_i are distinct prime quanta. Obviously $q_1 q_2 \ldots q_n \in P$ implies that $q_1 \in P$ or $q_2 q_3 \ldots q_n \in P$, and proceeding in this manner we conclude that at least one of q_i ($i = 1,2,\ldots n$) is in P.

Corollary 3. If q is a prime quantum in a GUFD R then Q_q the prime ideal associated to q is a minimal prime ideal($\neq 0$)

Proof. Obviously Q_q is non zero. Now suppose that Q_q is not minimal and let P be a non zero prime ideal contained in Q_q. By Proposition 7, P contains a prime quantum q' say and by (2) of Prop.5, $Q_{q'} \subset P$. But as $q' \in P \subset Q_q$; q' is similar to q and thus by (1) of Prop. 5, $Q_q = Q_{q'}$, so that $Q_q \subseteq P$ i.e. $Q_q = P$.

Corollary 4. In a GUFD R every non zero prime ideal