

to have the  $\mathcal{O}_R$ -property if every over-ring (ring between  $R$

and its quotient field  $K$ ) is a quotient ring. In a similar

way it can be proved that a GUD is a Bezout GKD iff it has

the  $\mathcal{O}_R$ -property, but a more general result is in order and

we state

Proposition 19. A Schreier domain  $R$  is a Bezout domain iff

it has the  $\mathcal{O}_R$ -property.

Proof. If  $R$  has the  $\mathcal{O}_R$ -property, it is a Prüfer domain

[9] p. 319) and  $R$  being Schreier also is Bezout (cf. [5]). Con-

versely it is well known that a Bezout ring has the  $\mathcal{O}_R$  pro-

perty (cf e.g. [5]).

It is obvious that a Bezout GKD (Prüfer GKD) is a  $\mathcal{W}$ -do-

main and so every non zero ideal of a Prüfer GKD has a unique

primary decomposition. The above stated fact makes a Prüfer

Bezout GKD very similar to a Dedekind (Principal ideal) domain.

In fact the only point of difference is that Prüfer (Bezout)

GKD's admit idempotent ideals while Dedekind domains (PID's)

do not. To establish this fact we prove

Proposition 20. A Prüfer GKD  $R$  is a Dedekind domain iff

each non zero prime ideal of  $R$  is non idempotent.

Proof. If  $R$  is a Dedekind domain the result is obvious.

Conversely let  $R$  be a Prüfer GKD such that every non zero

prime ideal of  $R$  is non idempotent. Then if  $P$  is a non zero

prime ideal of  $R$  every  $P$ -primary ideal contains a power of

$P$  (cf [28]) and so every non zero ideal of  $R$  contains a

product of a finite number of maximal ideals, that is  $R$  is a

$\mathcal{W}$ -domain (cf Th. B) but since  $R$  is a GKD and hence complete-

ly integrally closed it is a Dedekind domain by Theorem C.

A Bezout GKD being a GUD, we can state as a corollary

to Theorem 16 the