Corrigendum for "Characterizing domains of finite $*$-character"

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There is some confusion in lines 8-15 of the proof of Theorem 1. In the following we offer a fix to clear the confusion and give a rationale for the fix.

The fix: Read the proof from the sentence that starts from line 8 as follows:

Let S be the family of sets of mutually $*$-comaximal homogeneous members of Γ containing I. Then S is nonempty by ($\sharp\sharp$). Obviously S is partially ordered under inclusion. Let $A_{n_1} \subset A_{n_2} \subset ... \subset A_{n_r} \subset ...$ be an ascending chain of sets in S. Consider $T = \cup A_{n_r}$. We claim that the members of T are mutually $*$-comaximal. For take $x, y \in T$, then $x, y \in A_{n_i}$, for some i, and hence are $*$-comaximal. Having established this we note that by ($\sharp$), T must be finite and hence must be equal to one of the A_{n_j}. Thus by Zorn's Lemma, S must have a maximal element $U = \{V_1, V_2, ..., V_n\}$. Disregard the next two sentences and read on from: Next let M_i be the maximal $*$-ideal....

Rationale for the Fix: Using sets of mutually $*$-comaximal elements would entail some unwanted maximal elements as the following example shows: Let $x = 2^2 5^2$ in Z the ring of integers. Then $\mathcal{S} = \{\{(2^2 5^2)\}, \{(25^2)\}, \{(2^2 5)\}\{(2^2)\}, \{(5^2)\}, \{(2^2), (5^2)\}, \{(2)\}, \{(5)\}, \{(2), (5^2)\}, \{(2^2), (5)\}, \{(2), (5)\}\}$. In this case, while $\mathcal{S}$ includes legitimate maximal elements: $\{(2^2), (5^2)\}, \{(2), (5^2)\}, \{(2^2), (5)\}, \{(2), (5)\}$ it also includes $\{(2^2 5^2)\}, \{(25^2)\}, \{(2^2 5)\}$ which fit the definition of maximal elements. The reason why the fix should work is that given any set $T = \{A_1, A_2, ..., A_m\}$ of mutually $*$-comaximal $*$-finite ideals, by ($\sharp\sharp$) there is a set of mutually $*$-comaximal homogeneous $*$-finite ideals $\{H_1, H_2, ..., H_n\}$ in Γ, where $n \geq m$ such that each H_j contains some A_i. Also as a homogeneous ideal cannot be contained in two disjoint ideals we do not face the above indicated problem and Zorn's Lemma gives the required maximal elements.