A,B of R.

(2) T_2-domain if $T(AB) = T(A) + T(B)$ for every pair of finitely generated ideals A,B of R.

(3) T_3-domain if $T(AB) = T(A) + T(B)$ for every pair of principal ideals of R.

Proposition O_1 (cf Prop. 1, [15]) Let $A, A_1, A_2, \ldots, A_n$ and B be ideals of an integral domain R

(a) if k is a positive integer such that $A^k \subseteq B$ then $T(A) \supseteq T(B)$ and $T(AB) = T(A) = T(A) + T(B)$

(b) if e_i and f_i are positive integers for $1 \leqslant i \leqslant n$, then

$$T(A_1^{e_1} \cdot \ldots \cdot A_n^{e_n}) = T(A_1^{f_1} \cdot \ldots \cdot A_n^{f_n})$$

(c) if the hypothesis is as in (b) then

$$T(A_1^{e_1} + A_2^{e_2} + \ldots + A_n^{e_n}) = T(A_1^{f_1} + \ldots + A_n^{f_n}).$$

In particular if $(a_1, \ldots, a_n)$ is an ideal of R then

$$T(a_1^{e_1}, \ldots a_n^{e_n}) = T(a_1^{f_1}, \ldots, a_n^{f_n})$$

(d) $T(AB) \supsetneq T(A) + T(B)$

(e) if A and B are such that there exists an ideal A^* such that $A^* \supseteq B$ and $T(A^*) = T(A)$ then

$$T(AB) = T(B) = T(A) + T(B)$$

(f) if $T(A) = R$ or $T(B) = K$, the field of fractions of R then $T(AB) = T(A) + T(B)$

(g) $T(A \cap B) = T(AB)$

(h) $T(A) \cap T(B) = T(A + B)$.

Note. (a) and (e) of Prop. 1 of [15] are combined to give (a) while (e) of Prop. O_1 is new but easy to verify.

Theorem O_2 (Lemma 1 [15]). (i) Suppose that (A, and B are ideals of R such that $(A + B)\, T(A,B) = T(A,B)$ then for each positive integer k, $(A^k + B^k)\, T(A,B) = T(A,B)$.*1

(ii) If A and B are comaximal ideals of R and if C is any ideal of R then, $T(ABC) = T(AC) + T(BC)$.

*1 $T(A,B) = T(A+B)$