

Muhammad Zafrullah

From: "Muhammad Zafrullah" <mzafrullah@usa.net>
Date: Monday, November 25, 2002 11:14 PM
To: "Steve McAdam" <mcadam@mail.ma.utexas.edu>
Cc: "Daniel Anderson" <ddanders@math.uiowa.edu>
Attach: Steve3.dvi; Steve3.tex
Subject: Re: paper#2

Dear Steve, (this message also in each of the attached files)

I did realize that. The trouble is that I am still wandering in the realm of divisibility and smoothness. I tried to think about replacing primes by maximal ideals but that too would be somewhat smooth. But in trying to get myself straightened in this set up I stumbled onto something that you might like. I am using the language of star operations, if you do not like the stars just disregard them; the results would still make sense.

Let \ast be a star operation of finite character. The operation d on the set $F(D)$ of nonzero fractional ideals of D , defined by $A^{\ast d} = A^{\ast}$ is also a star operation of finite character. So disregarding the star operation in what follows will take you directly into the realm of ordinary ideals.

Lemma A. Let A and B be any \ast -comaximal integral ideals. If $C^{\ast} \supseteq AB$ then $C^{\ast} = (HK)^{\ast}$ where $H^{\ast} \supseteq A^{\ast}$ and $K^{\ast} \supseteq B^{\ast}$. In particular if A, B, C are principal and $\ast = d$ then $C = HK$ where $H = (C, A)$ and $K = (C, B)$.

Proof. Note that $((C, A)(C, B))^{\ast} = (C^2, CA, CB, AB)^{\ast} = (C^2, (C(A, B))^{\ast}, AB)^{\ast} = (C^2, C^{\ast}, AB)^{\ast} = (C^2, C, AB)^{\ast} = (C, AB)^{\ast} = C^{\ast}$.

This lemma shows that if A and B are comaximal integral ideals and if C is an invertible integral ideal containing the product AB then (A, C) and (B, C) are both invertible and $C = (A, C)(B, C)$. Now if you assume that A, B and C are all principal and suppose that all of a sudden you decide to work in a domain in which every two generated ideal is principal then in such a domain $c \mid ab$, a, b comaximal would directly imply that $c = rs$ where r divides a and s divides b . Now throw in the restriction that c cannot be expressed as a product of two comaximals then r is a unit or

s is a unit. Making c a pseudo prime.

What is amusing is that I can produce the star operation version of this conclusion. This I would do when I can find time and finally a word about Lemma A. It can be stated for any collection of mutually \ast -comaximal set of integral ideals A_1, A_2, \dots, A_n . That is if $C^{\ast} \supseteq A_1 A_2 \dots A_n$ then $C^{\ast} = (\prod (C + A_i))^{\ast}$. I tend to think of it as Multiplicative ideal theory's Chinese remainder theorem.

Now using this for $\ast = d$ and the tacit assumption that, in D , every two generated invertible ideal is principal, it is easy to see that in such a D for $x \mid a_1 a_2 \dots a_n$, where a_i are mutually comaximal we have $x = r_1 r_2 \dots r_n$ such that $r_i \mid a_i$. Now, r_i are mutually comaximal yet, even if D is a CFD, r_i do not have to be pseudo irreducible. Actually, each r_i would have to be a product of mutually comaximal pseudo irreducibles. Boy that is hard (a factor having worse factorization than the factored!) and now I know why I could not find time to go back to my unique representation domains. I see your UCFD's as a generalized d -version of URD's and I know the nooks and crannies of what I created. (By the way, in another paper I showed that a finite intersection of valuations of a field is a URD, but your result that a semilocal domain is a UCFD is far superior.)

You are deciding to branch out, well it is your choice. I can give you my experience, I tried to go into differential equations and then to coding theory. Did not seem to work out, in that every time I seemed to make some progress in branching out I would start having showers of new ideas. Now this semester I decided to read some extra Statistics, while teaching an introductory course on Statistics and I am inundated with ideas in ideal theory! I might keep on trying though and that is what you can do too.

I am sending a copy of this letter to Dan. He can often see some sense in my madness. Hopefully with Dan's help, I would like to produce at least a "t-version" of your paper, and of course we would like to cite your monumental work and of course I would try to keep you informed of what we produce.

Sincerely,

Muhammad

----- Original Message -----

From: "Steve McAdam" <mcadam@mail.ma.utexas.edu>

To: "Muhammad Zafrullah" <mzafrullah@usa.net>

Sent: Monday, November 25, 2002 4:54 PM

Subject: Re: paper#2

> > Dear Muhammad,

>

> I am delighted you found things of interest in the paper.

>

> Distinguished domains? Frankly, I had no idea there was a connection.

> (Distinguished domains are from my far distant past.)

>

> That pair of papers with Swan took a large chunk out of 1 + 1/2 years of my

> life. To be entirely honest, I am exhausted, and a bit sick at the sight

> of them. I plan now to finish up an expository paper I am writing, and

> then set aside all further research (for awhile at least--maybe a long

> while). I am trying to branch out, and learn some new things. (I signed

> up to teach a course in cryptography, and have been sitting in a course

> best described as an introduction to mathematical biology.)

>

> Yet despite the fact that I will not be thinking about research for awhile,

> I would still like to be in the loop, so please do send me copies of

> anything you do pertinent to UCFD's. Sounds like you have lots of ideas.

>

> Concerning your question,

>

> could it be that x is pseudo-prime element if and only if $x=ab$, a, b

> nonunits implies that a belongs to every prime ideal containing b

> or b

> belongs to every prime ideal containing a ?

>

> the answer is no, since every nonzero non-unit in a quasi-local domain is

> pseudo-prime. Thus it fails in $\mathbb{Z}[X]$ localized at any height 2 prime.

(But

> you would have seen that soon enough.)

>

> By The way, Dan sent me a copy of "Factorizations of certain sets of

> polynomials in an integral domain". It looks nice. Boy, you guys sure

> keep busy. Where do you find the time and energy?

>

> Well, back to making up final exams.

>

> Best regards,

> Steve

>

>

>

>

>

>

>

>

> Dear Steve,

> > I have skimmed through your paper. You have done a lot of good,

> > thoughtful and ingenious work and I must congratulate you for it. I have

> > learned quite a few things from your paper such as:

> > (i) How you got to Distinguished domains (using S-ideals and some

> > inverting) and thanks to your paper now I can say precisely when a

> > distinguished domain is a PVMD. If you are interested I will send you my

> > findings, but it is rough and is mostly in the star operations lingo.

> > (ii) I had introduced a generalization of UFD's in my thesis and

> > called it "unique representation". The idea was: Impose on a GCD-domain

D

> > the condition that each nonzero nonunit of D has finitely many minimal

> > primes and call it a unique representation domain (URD). The result was

> > that a GCD domain D is a URD if and only if every nonzero nonunit x of D

> > is expressible as a product of finitely many packets. A packet was the

> > name I chose for a nonzero element with a unique minimal prime. Of

course

> > I also showed that a product of finitely many packets in a GCD domain

can

> > be expressed uniquely as a product of mutually coprime packets. I also

> > showed in my thesis that if D is a URD and K the quotient field of D

then

> > $D+XK[X]$ is again a URD. Later, I published these results in the form of

a

> > paper: [Unique representation domains, J. Natur. Sci. Math. 18 (1978), no.

> > 2, 19--29. MR 82c:13025]. In that paper I had also included the result

> > that if S is a multiplicative set in a URD such that $D+XD[1/S][X]$ is a

> > GCD domain then $D+XD[1/S][X]$ is a URD.

> > I was in Libya when I published it, then I got busy teaching and
 > > running the department, believe me running a department as a foreign
 > > worker in Libya, was only slightly easier than getting a job in the US
 > > without any friends. In any case I never got round to seeing if the
 paper
 > > was all OK. Now your paper tells me that not only was the theory OK but
 > > also that a URD is another example of a UCFD. Thank you. Then there is
 > > another something to note: $D+XK[X]$ and $D+XD[1/S][X]$ are examples of
 > > pullbacks of a slightly different kind than you have looked into. Of
 > > course I had used coprime and not comaximal and I had worked in a GCD
 > > domains, but I think this example will hopefully inform you that there
 > > were efforts at generalizing UFD's to non atomic set up as far back as
 > > 1973. If you would like to read the paper mentioned above I can scan and
 > > e-mail you a copy (the paper had a lot of typos and I have kept a
 > > corrected copy). In fact there is now quite a bit of literature on
 > > extensions of unique factorization. A comprehensive paper to read in
 this
 > > connection is by Dan Anderson: [Extensions of unique factorization: a
 > > survey. Advances in commutative ring theory (Fez, 1997), 31--53, Lecture
 > > Notes in Pure and Appl. Math., 205, Dekker, New York, 1999. MR
 > > 2001h:13026].
 > > (iii) My memory is quite good! Even after an "epileptic seizure"
 under
 > > some very mysterious circumstances in 1990. I was in Rock Hill, South
 > > Carolina those days. Some people say it is impossible to do any
 > > intellectual work after an epileptic seizure, I have published around
 > > thirty papers after that. (I thank God for it.)
 > > Now here are my comments:
 > > Personally, I would like to see the properties of pseudo-prime
 > > elements and use them as building blocks, but then the idea occurred to
 > > you and not to me. I am just rambling but could it be that x is
 > > pseudo-prime element if and only if $x=ab$, a, b nonunits implies that a
 > > belongs to every prime ideal containing b or b belongs to every prime
 > > ideal containing a ?
 > > I am not too deeply interested in this direction (I might get
 > > interested later!) as I am fretting about, among other things,
 > > characterizing domains that are locally finite intersections of
 > > localizations at primes using tricks similar to the ones developed in
 the
 > > paper I told you about in my last message.
 > > Sincerely,
 > > Muhammad

> > ~~~~~
> > Muhammad Zafrullah
> > Department of Mathematics,
> > Idaho State University,
> > Pocatello, ID 83209-8085
> > Phone: (208) 282 2892
> > Fax: (508) 374 6520
> > <http://www.isu.edu/~zafrmuha>
> > ~~~~~
>
>
>