$(x,d) = h$ i.e. $x = x_1 h$, $d = d_1 h$ where $(x_1, d_1) = 1$. Since $x \notin q$, $h \notin q$ ($\because h|x$), further since q is a prime and $d_1 h = d \in q$; $d_1 \in q$. Now $(x_1, d_1) = 1$ and we claim that x_1 is a unit, for if not x_1 is a member of at least one of $P_1, P_2, \ldots, P_r$. Suppose that $x_1 \in P_s$, then since $q \subset P_s$; $x_1, d_1 \in P_s$. Further since R is an HCF domain and R_{P_s} is a valuation domain x_1, d_1 are non units in R_{P_s} and so $(x_1, d_1) \neq 1$ in R_P a contradiction implying that x_1 is a unit i.e. $x|d$ and obviously the same procedure holds for each integral power of x.

(3) Let d, d' be as in the hypothesis and let $(d,d') = h$ i.e. $d = d_1 h$, $d' = d_1' h$ such that $(d_1, d_1') = 1$. Obviously $h \in q$ and this leaves us with two possibilities to consider

(a) $d_1, d_1' \notin q$

(b) one of d_1, d_1' is in q.

In case (a) holds $d_1, d_1' | h$ by (2) above and so $d|d'^2$ and $d'|d^2$. And in case (b) holds ; if d_1' is in q then $d|d'$. To show that there exists a positive integer r such that $d'|d^r$ we first prove that there exists an m such that $d^m \nmid d'$. Suppose on the contrary that $d^m | d'$ for each m, then for all m, $d^m | d'$ in R_q. But then R_q being a valuation domain $Q = \bigcap_{n=1}^{\infty} d^n R_q$ is a prime ideal properly contained in dR_q (cf Theorem 17.1 (3) page 187 [11]) that is $d'R_q \subsetneq Q \subsetneq qR_q$ i.e. $Q' = Q \cap R$ contains the minimal prime of d', but since we assumed that q is the minimal prime of d' and this result contradicts our assumption we infer that there exists a positive integer m such that $d^m \nmid d'$. Now if we let $(d^n, d') = d''$ (n greater than m) such that $d^n = ad''$; $d' = bd''$, then $(a,b) = 1$ and $b \notin q$ (for if $b \in q$, $a \notin q$