Muhammad Zafrullah

From: "Muhammad Zafrullah" <mzafrullah@usa.net>
Date: Monday, November 25, 2002 11:14 PM
To: "Steve McAdam" <mcadam@mail.ma.utexas.edu>
Cc: "Daniel Anderson" <ddanders@math.uiowa.edu>
Attach: Steve3.dvi; Steve3.tex
Subject: Re: paper#2

Dear Steve, (this message also in each of the attached files)

I did realize that. The trouble is that I am still wandering in the realm of divisibility and smoothness. I tried to think about replacing primes by maximal ideals but that too would be somewhat smooth. But in trying to get myself straightened in this set up I stumbled onto something that you might like. I am using the language of star operations, if you do not like the stars just disregard them; the results would still make sense.

Let $\ast $ be a star operation of finite character. The operation d on the set $F(D)$ of nonzero fractional ideals of $D,$ defined by $A^{d}=A$ is also a star operation of finite character. So disregarding the star operation in what follows will take you directly into the realm of ordinary ideals.

Lemma A. Let A and B be any $\ast $-comaximal integral ideals. If $%
C^{\ast }\supseteq AB$ then $C^{\ast }=(HK)^{\ast }$ where $H^{\ast
}\supseteq A^{\ast }$ and $K^{\ast }\supseteq B^{\ast }.$ In particular if
$%
A,B,$ C are principal and $\ast =d$ then $C=HK$ where $H=(C,A)$ and $%
K=(C,B).$

Proof. Note that $((C,A)(C,B))^{\ast }=(C^{2},CA,CB,AB)^{\ast
}=(C^{2},(C(A,B))^{\ast },AB)^{\ast }=(C^{2},C^{\ast },AB)^{\ast
}=(C^{2},C,AB)^{\ast }=(C,AB)^{\ast }=C^{\ast }.$

This lemma shows that if A and B are comaximal integral ideals and if
C
is an invertible integral ideal containing the product AB then (A,C) and (B,C) are both invertible and $C=(A,C)(B,C).$ Now if you assume that A,B and C are all principal and suppose that all of a sudden you decide to work in a domain in which every two generated ideal is principal then in such a domain $c\mid ab,$ a,b comaximal would directly imply that $c=rs$ where r divides a and s divides $b.$ Now throw in the restriction that c cannot be expressed as a product of two comaximals then r is a unit or

s is a unit. Making c a pseudo prime.

What is amusing is that I can produce the star operation version of this conclusion. This I would do when I can find time and finally a word about Lemma A. It can be stated for any collection of mutually $\ast -$comaximal set of integral ideals $A_{1},A_{2},...,A_{n}$. That is if $C^{\ast }\supseteq A_{1}A_{2}...A_{n}$ then $C^{\ast }=(\dprod (C+A_{i}))^{\ast }.$ I tend to think of it as Multiplicative ideal theory's Chinese remainder theorem.

Now using this for $\ast =d$ and the tacit assumption that, in $D,$ every two generated invertible ideal is principal, it is easy to see that in such a D for $x\mid a_{1}a_{2}...a_{n},$ where a_{i} are mutually comaximal we have $x=r_{1}r_{2}...r_{n}$ such that $r_{i}\mid a_{i}.$ Now, r_{i} are mutually comaximal yet, even if D is a CFD, r_{i} do not have to be pseudo irreducible. Actually, each r_{i} would have to be a product of mutually comaximal pseudo irreducibles. Boy that is hard (a factor having worse factorization than the factored!) and now I know why I could not find time to go back to my unique representation domains. I see your UCFD's as a generalized d-version of URD's and I know the nooks and crannies of what I created. (By the way, in another paper I showed that a finite intersection of valuations of a field is a URD, but your result that a semilocal domain is a UCFD is far superior.)

You are deciding to branch out, well it is your choice. I can give you my experience, I tried to go into differential equations and then to coding theory. Did not seem to work out, in that every time I seemed to make some progress in branching out I would start having showers of new ideas. Now this semester I decided to read some extra Statistics, while teaching an introductory course on Statistics and I am inundated with ideas in ideal theory! I might keep on trying though and that is what you can do too.

I am sending a copy of this letter to Dan. He can often see some sense in my madness. Hopefully with Dan's help, I would like to produce at least a "t-version" of your paper, and of course we would like to cite your monumental work and of course I would try to keep you informed of what we produce.

Sincerely,

Muhammad

----- Original Message -----
From: "Steve McAdam" <mcadam@mail.ma.utexas.edu>
To: "Muhammad Zafrullah" <mzafrullah@usa.net>
Sent: Monday, November 25, 2002 4:54 PM
Subject: Re: paper#2

> >Dear Muhammad,
>
> I am delighted you found things of interest in the paper.
>
> Distinguished domains? Frankly, I had no idea there was a connection.
> (Distinguished domains are from my far distant past.)
>
> That pair of papers with Swan took a large chunk out of 1 + 1/2 years of
my
> life. To be entirely honest, I am exhausted, and a bit sick at the sight
> of them. I plan now to finish up an expository paper I am writting, and
> then set aside all further research (for awhile at least--maybe a long
> while). I am trying to branch out, and learn some new things. (I signed
> up to teach a course in cryptography, and have been sitting in a course
> best described as an introduction to mathematical biology.)
>
> Yet despite the fact that I will not be thinking about research for
awhile,
> I would still like to be in the loop, so please do send me copies of
> anything you do pertinant to UCFD's. Sounds like you have lots of ideas.
>
> Concerning your question,
>
> could it be that x is pseudo-prime element if and only if x=ab, a, b
> nonunits implies that a belongs to every prime ideal containing b
> or b
> belongs to every prime ideal containg a?
>
> the answer is no, since every nonzero non-unit in a quasi-local domain is
> pseudo-prime. Thus it fails in Z[X] localized at any height 2 prime.
(But
> you would have seen that soon enough.)
>
> By The way, Dan sent me a copy of "Factorizations of certain sets of
> polynomials in an integral domain". It looks nice.   Boy, you guys sure

> keep busy. Where do you find the time and energy?
>
> Well, back to making up final exams.
>
> Best regards,
> Steve
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>   Dear Steve,
> >   I have skimmed through your paper. You have done a lot of good,
> >thoughtful and ingenious work and I must congratulate you for it. I have
> >learned quite a few things from your paper such as:
> >   (i) How you got to Distinguished domains (using S-ideals and some
> >inverting) and thanks to your paper now I can say precisely when a
> >distinguished domain is a PVMD. If you are interested I will send you my
> >findings, but it is rough and is mostly in the star operations lingo.
> >   (ii) I had introduced a generalization of UFD's in my thesis and
> >called it "unique representation". The idea was: Impose on a GCD-domain
D
> >the condition that each nonzero nonunit of D has finitely many minimal
> >primes and call it a unique representation domain (URD). The result was
> >that a GCD domain D is a URD if and only if every nonzero nonunit x of D
> >is expressible as a product of finitely many packets. A packet was the
> >name I chose for a nonzero element with a unique minimal prime. Of
course
> >I also showed that a product of finitely many packets in a GCD domain
can
> >be expressed uniquely as a product of mutually coprime packets. I also
> >showed in my thesis that if D is a URD and K the quotient field of D
then
> >D+XK[X] is again a URD. Later, I published these results in the form of
a
> >paper: [Unique representation domains,J. Natur. Sci. Math. 18 (1978), no.
> >2, 19--29. MR 82c:13025]. In that paper I had also included the result
> >that if S is a multiplicative set in a URD such that D+XD[1/S][X] is a
> >GCD domain then D+XD[1/S][X] is a URD.

> > I was in Libya when I published it, then I got busy teaching and
> >running the department, believe me running a department as a foreign
> >worker in Libya, was only slightly easier than getting a job in the US
> >without any friends. In any case I never got round to seeing if the
paper
> >was all OK. Now your paper tells me that not only was the theory OK but
> >also that a URD is another example of a UCFD. Thank you. Then there is
> >another something to note: D+XK[X] and D+XD[1/S][X] are examples of
> >pullbacks of a slightly different kind than you have looked into. Of
> >course I had used coprime and not comaximal and I had worked in a GCD
> >domains, but I think this example will hopefully inform you that there
> >were efforts at generalizing UFD's to non atomic set up as far back as
> >1973. If you would like to read the paper mentioned above I can scan and
> >e-mail you a copy (the paper had a lot of typos and I have kept a
> >corrected copy). In fact there is now quite a bit of literature on
> >extensions of unique factorization. A comprehensive paper to read in
this
> >connection is by Dan Anderson: [Extensions of unique factorization: a
> >survey. Advances in commutative ring theory (Fez, 1997), 31--53, Lecture
> >Notes in Pure and Appl. Math., 205, Dekker, New York, 1999. MR
> >2001h:13026].
> > (iii) My memory is quite good! Even afer an "epileptic seizure"
under
> >some very mysterious circumstances in 1990. I was in Rock Hill, South
> >Carolina those days. Some people say it is impossible to do any
> >intellectual work after an epileptic seizure, I have published around
> >thirty papers after that. (I thank God for it.)
> > Now here are my comments:
> > Personally, I would like to see the properties of pseudo-prime
> >elements and use them as building blocks, but then the idea occurred to
> >you and not to me. I am just rambling but could it be that x is
> >pseudo-prime element if and only if x=ab, a, b nonunits implies that a
> >belongs to every prime ideal containing b or b belongs to every prime
> >ideal containg a?
> > I am not too deeply interested in this direction (I might get
> >interested later!) as I am fretting about, among other things,
> >characterizing domains that are locally finite intersections of
> >localizations at primes using tricks similar to the ones developed in
the
> >paper I told you about in my last message.
> > Sincerely,
> > Muhammad

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