(2) Let r,s be any two non co-prime rigid non units of R their product rs is again a rigid non unit (obviously non co-prime to both r and s).

(3) To each rigid non unit $r \in R$, there is associated a prime ideal $P(r) = \{ x \in R \mid x \text{ is non co-prime to } r \}$.

(4) Let r,s be two rigid non units in R then $P(r) = P(s)$ iff r,s are non co-prime .

(5) If r is a rigid non unit in R and $P(r)$ is the prime ideal associated to r then the localization $R_{P(r)}$ is a valuation domain.

Proof. Let $(r,s) = d \ (\neq 1)$; $r = r_1 d$, $s = s_1 d$ where $(r_1, s_1) = 1$. If either of r_1, s_1 is a unit, (1) holds and we have nothing to prove. So we suppose on the contrary that r_1, s_1 are both non units . By the definition of a rigid element $r_1 \mid d$ or $d \mid r_1$ ------------------------------- -(a)

and $s_1 \mid d$ or $d \mid s_1$ --------------------------------(b)

Now if $r_1 \mid d$ and $d \mid s_1$; $r_1 \mid s_1$ a contradiction -----(i)

and if r_1 and s_1 divide d which being a factor of a rigid element is it self rigid and hence $r_1 \mid s_1$ or $s_1 \mid r_1$

a contradiction ----------------(ii).

Further if $d \mid r_1$ and $s_1 \mid d$ then $s_1 \mid r_1$ a contradiction ---(iii).

Finally if $d \mid r_1$ and $d \mid s_1$ then again $(r_1, s_1) \neq 1$

a contradiction ---------------(iv).

To sum up we get contradiction as a result in all the four cases which arise from the assumption that $r \nmid s$ and $s \nmid r$ and this confirms the truth of (1).

(2) Let $z = rs$, where r,s are non co-prime rigid elements. Let x,y be any pair of factors of z and suppose that $x \nmid y$ and $y \nmid x$ (in other words we suppose that z is not a rigid element). Now let $(x,y) = d$, where $x = x_1 d$, $y = y_1 d$ and