

Proposition 21. In a Bezout GKD a prime ideal is either

principal or idempotent.

Finally to study the primary decomposition in Prüfer

domains we proceed as follows.

Let  $R$  be an integral domain, an ideal  $P$  is said to be an

S-ideal in  $R$  if (1)  $P$  is prime (2) the set of  $P$ -primary ideals

is linearly ordered (3) the intersection of all the  $P$ -primary

ideals is a prime ideal  $M$  (4)  $M$  contains each prime ideal

properly contained in  $P$ . An integral domain  $R$  is said to be

an S-domain if every prime ideal of  $R$  is an S-ideal (cf [13]

pp. 249-250).

According to Cor. 2.5 of [13], ( $\mathfrak{C} \equiv$  proper containment)

"If  $D$  is an S-domain and  $Q, Q_1$  are primary ideals for  $P_1, P_2$

respectively, where  $P_1 \subsetneq P_2$ , then  $Q \subsetneq Q_1$ "------(S)

It is easy to establish that a Prüfer domain is an

S-domain and that (S) can be proved for a Prüfer domain. But

for the convenience of reference we adopt (S) for Prüfer

domains and use it to prove

Theorem 22. If a non zero ideal  $A$  in a Prüfer domain  $R$  has

a reduced primary decomposition

$$A = P_1 \cap P_2 \cap \dots \cap P_n \text{ ----- (a)}$$

then (a) is unique.

Proof. Let  $\text{Rad } P_i = Q_i$  ( $i = 1, 2, \dots, n$ ), we claim that if

(a) is reduced then

(1)  $Q_i$  are incomparable under inclusion ( $i = 1, \dots, n$ )

(2) no two  $P_i, P_j$   $i \neq j$  are contained in the same prime

ideal  $Q$ .

First let  $Q_i \subsetneq Q_j$  for some  $i \neq j$ , then if  $Q_i = Q_j$ ;

$P_i \subsetneq P_j$  or  $P_j \subsetneq P_i$  because each of the  $Q_i$  is an S-ideal and

this contradicts the assumption that (a) is reduced. Further

if  $Q_i \subsetneq Q_j$  then by (S) above  $P_i \subsetneq P_j$  which again contradicts