$$b \in Q \text{ and } b \notin P \quad \text{-------------------(1)}$$

$$h \in P \text{ and } h \notin Q \quad \text{-------------------(2)}$$

but since x is a packet there exists an n such that $b|h^n$ or $h|b^n$. Now if $b|h^n$ then $h^n \in Q$ i.e. $h \in Q$ which contradicts(2) and if $h|b^n$; $b \in P$ in contradiction to (1) and this establishes that a packet x in an HCF domain R cannot have more than one minimal subvalued primes.

Now going from packets to products of mutually co-prime packets, we prove the following

Theorem 3. An HCF domain R is a URD iff every non zero non unit x in R has a finite number of minimal primes.

Proof. Let R be a URD and let x be a non zero non unit in R. We can write

$$x = x_1 x_2 \dots x_n \ ; \ (x_i, x_j) = 1 \text{ if } i \neq j$$

where each of the x_i is a packet. Being mutually co-prime, no two of the x_i have a valued prime common to them and hence no subvalued prime, while each of the x_i has a single minimal subvalued prime(being a packet) and consequently x has a finite number of minimal subvalued primes.

Conversely let x be a non zero non unit in an HCF domain R and let $q_1, q_2, \dots, q_n$ be all the minimal subvalued primes containing x then following exactly the same lines as in the proof of Theorem 1, of this chapter we can show that

$$x = x_1 x_2 \dots x_n \ ; \text{ where each of the } x_i \text{ is a packet}$$

such that $(x_i, x_j) = 1$ if $i \neq j$. And to conclude the proof we mention that a minimal prime P of a principal ideal is a minimal subvalued prime. For if not let R_P be not a valuation domain. Then since R_P is an HCF domain and thus is essential