the assumption that (a) is reduced and hence establishes (1). For (2) let $P_i, P_j \subseteq Q$ a prime ideal then Rad P_i , Rad $P_j \subseteq Q$. But since R is a Prüfer domain R_Q is a valuation domain and so Rad $P_i \subseteq$ Rad P_j or Rad $P_j \subseteq$ Rad P_i , this contradicts (1) and hence establishes (2). Now let

$$A = P_1' \cap P_2' \cap \ldots \cap P_m' \quad \text{-----------------}(b)$$

be another primary decomposition of A and since every primary decomposition can be reduced, suppose that (b) is reduced and let Rad $P_j' = Q_j'$ $(j = 1,2,\ldots m)$

We note that the above claim holds for (b) as well and that $(P_1 \cap P_2 \cap \ldots \cap P_n)R_{Q_i} = (P_1' \cap P_2' \cap \ldots \cap P_m')R_{Q_i}, (i=1,\ldots n)$ can be written as

$$P_1R_{Q_i} \cap P_2R_{Q_i} \ \ldots \cap P_nR_{Q_i} = P_1'R_{Q_i} \cap \ldots \cap P_m'R_{Q_i} \quad \text{----------}(c)$$

(cf [9] p 34)

In view of the above claim there exists only one primary ideal $P_i \subseteq Q_i$ in the decomposition (a) and so (c) can be written as

$$\bigcap_{k=1}^{n} P_k R_{Q_i} = P_1'R_{Q_i} \cap P_2'R_{Q_i} \cap \ldots \cap P_m'R_{Q_i} \quad \text{------}(d)$$

Now on the right hand side of (d), no two of P_j' are in Q_i and since the left hand side is a proper ideal of R_{Q_i} there must at least one of P_j' be contained in Q_i and thus

$P_iR_{Q_i} = P_j'R_{Q_i}$, but since P_i is Q_i-primary and R is a Prüfer domain (cf [28])

$$P_i = P_iR_{Q_i} \cap R = P_j'R_{Q_i} \cap R \quad \text{--------------------}(e)$$

we have $P_j' \subseteq P_i$ - --------------------(f)

Similarly considering

$$(P_1' \cap P_2' \cap \ldots \cap P_m')R_{Q_j'} = (P_1 \cap P_2 \cap \ldots \cap P_n)R_{Q_j'}$$

where $Q_j' =$ Rad P_j', we find that there exists some primary ideal P_k in the decomposition (a) such that

$$P_k \subseteq P_j' \quad \text{------------------------ --}(g)$$