

contains a minimal (non zero) prime ideal.

Proof. Immediate from Cor. 3 above.

Corollary 5. In a GUD every non zero minimal prime ideal

$P$  is associated to a prime quantum  $q$  i.e.  $P = q$ .

Proof. By Cor. 4,  $P$  contains a prime quantum  $q$  and the

result follows from Prop. 5.

#### 4. Stability Properties of GUD's.

In this section we shall establish that the property of

being a GUD remains invariant under localizations and poly-

nomial extensions. For this purpose we need to introduce the

concept of a generalized Krull Domain (GKD).

An integral domain  $R$  is called a generalized Krull

Domain if

(1) every non zero non unit  $x$  in  $R$  is contained in a

finite number of minimal prime ideals of  $R$ .

(2) For every minimal prime ideal  $P$  of  $R$ ,  $R_P$  is a

rank one valuation domain.

(3)  $R = \bigcap R_P$ , where  $P$  varies over all the minimal prime

ideals of  $R$ .

It may be noted that a Krull domain is a generalized

Krull Domain. In this section we shall use the facts that

(1) every localization of a GKD is a GKD (2) if  $x$  is an

indeterminate over a GKD  $R$  then  $R[x]$  is a GKD. For a detailed

theory of GKD's the reader is referred to [21], [29] and [9].

As our first step towards the consideration of stability

properties of GUD's we collect some useful facts.

Lemma 8. In an HCF domain a quantum is a prime quantum.

Proof. Let  $q$  be a quantum in an HCF domain  $R$  and suppose

that  $x, y | q$ . We claim that  $x | y$  or  $y | x$ . For if we suppose on

the contrary that  $x \nmid y$  and  $y \nmid x$  then  $R$  being an HCF domain