and so is a product of powers of primes and similarly as in the first case $(1 + \Sigma\, a_i' x^i)$ is a product of powers of primes and combining these observations $r\,(1 + \sum_{i=1}^{n} a_i' x^i)$ is the product of a finite number of mutually co-prime packets. And thus we have established that $S = R + xK[x]$ is a URD. But S is not necessarily a ring of Krull type, follows from the fact that $x \in pS$ for each prime p in R and if the number of prime ideals in R is infinite, S is not a ring of Krull type.

The above example gives rise to the question of characterization of a URD. We note that a URD by definition is an HCF domain and so, part of our task would be done if we explain the structure of an HCF domain in terms of its valuation overrings. For this purpose we prove that an HCF domain is an essential domain. To achieve this result we need to introduce some concepts which are to serve as tools.

Let R be an integral domain and K be its field of fractions and let F(R) be the set of non zero fractional ideals of R. If $A \in F(R)$, by A^{-1} we mean the set

$\{ x \in K \mid xA \subseteq R \}$ and this again is a fractional ideal. We denote by A_v the fractional ideal $(A^{-1})^{-1}$. The operation of associating A_v with each fractional ideal $A \in F(R)$ is called the v-operation (cf [11] page 416)

It is well known (cf 32.1 [11]) that if $a \in K$ and $A, B \in F(R)$

(1) $(a)_v = (a)$; $(aA)_v = aA_v$

(2) $A \subseteq A_v$; if $A \subseteq B$ then $A_v \subseteq B_v$