CHAPTER 2

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0. Introduction.

In the theory of Unique Factorization the concept of a prime element is basic. Similarly it is clear that a discrete rank one valuation domain is the simplest UFD (in the sense that it has only one prime and its associates). In the previous chapter we replaced the concept of prime element by a more general concept ,prime quantum which resulted in the replacement of a discrete rank one valuation domain by a rank one valuation domain as the simplest GUFD(every non zero non unit in a rank one valuation domain is a prime quantum similar to any other). But the generalization of Unique Factorization in the above mentioned fashion gives rise to the following

Question . Is it possible to work out a theory of Unique Factorization in which a general valuation domain replaces a rank one valuation domain ?

We note that in a general valuation domain R; no non zero non unit x can be expressed as a product of two co-prime non units. Moreover for all $v,u|x$ in R, $u|v$ or $v|u$. In other words the lattice $L(xR,R)$ is a chain for each non zero element x in a valuation domain R. According to [6] p. 129 an element x in an integral domain R is called rigid if $L(xR,R)$ is a chain, and an integral domain R with all non zero elements rigid is called a rigid domain (cf [6] p 129). It can be easily seen that a commutative valuation domain is a rigid domain.

An obvious programme is, that we should consider an integral domain in which every non zero non unit element is