

largest possible prime ideals for which,  $R_p^\alpha$  is a valuation domain for each  $\alpha \in I$ . Thus by the family of valued primes defining a ring  $R$  of Krull type we shall in future mean the family  $\{P^\alpha\}$  consisting of the largest valued primes of  $R$ . We recall that

Definition 4. An integral domain  $R$  is called a Krull

domain if

- (1) every non zero non unit element of  $R$  is contained in only a finite number of minimal prime ideals of  $R$
- (2)  $R_p$  is a discrete rank one valuation ring for each minimal (non zero) prime ideal  $P$  of  $R$
- (3)  $R = \bigcap R_p$  where  $P$  ranges over all the minimal prime ideals of  $R$ .

Comparing the Definitions 3 and 4, we infer that a Krull domain is a ring of Krull type with the difference that the defining family of prime ideals of a Krull domain consists only of minimal non zero prime ideals, and of course that  $R_p$  is a discrete for each  $P$  in the defining family. Similarly recalling Def. 3 of Chapter 2, we infer that a  $*GKD$  is also a ring of Krull type. Thus if  $\leq$  denotes, "Form a special case of" then

$$\text{Krull domains} \leq *GKD's \leq \text{rings of Krull type.}$$

The examples given or mentioned at the end of section 4 of this chapter ensure that the above is a chain of distinct classes of integral domains.

There may be many further generalizations of a ring of Krull type but we shall restrict our attention to essential domains and their special case to which we have given the name  $*GKD$  essential domains.