expressible as the product of a finite number of mutually co-prime rigid non units. For a clearer picture of factorization into rigid non units we consider the following

Example 1. Let V be a valuation domain, x an indeterminate over V and let $R = V[x]$.

Pick a general non zero non unit element

$$y = \sum_{i=0}^{n} v_i x^i ; \quad v_i \in V.$$

Since V is an HCF domain, we can calculate the HCF, d of $v_0, v_1, \ldots, v_n$ and so $y = d(\sum_0^n v_i' x^i)$; where $\{v_i'\}$ have no non unit common factor(in fact one of them is a unit).

In the factorization of $y' = \sum_0^n v_i' x^i$, every non unit element has positive degree in x and hence $\sum_0^n v_i' x^i$ is a product of atoms. Moreover since, V is an HCF domain and so is $V[x]$, every atom in $V[x]$ is a prime (cf [5]) and thus

$\sum_0^n v' x^i = p_1^{a_1} p_2^{a_2} \ldots p_s^{a_s}$; $(p_i, p_j) = 1$ for $i \neq j$. That is

$$y = d p_1^{a_1} p_2^{a_2} \ldots p_s^{a_s} \; ; \; (d, p_i) = 1 \; (i = 1, 2, \ldots, s)$$

$$(p_i, p_j) = 1 \text{ for } i \neq j \text{ -----(A)}$$

Obviously each prime power is a rigid non unit and d being a member of V is rigid and so if y is non unit, it is the product of a finite number of mutually co-prime rigid non units. It is also obvious that the factorization in the expression (A) is unique up to associates of the rigid non units. And since, y is arbitrary we conclude that every non zero non unit element in $R = V[x]$ is uniquely expressible as the product of a finite number of mutually co-prime rigid elements.

Here we note that while an atom is rigid, a quantum according to its definition, need not be . For example, in a one dimensional quasi-local domain every non zero non unit element is a quantum but a one dimensional quasi-local