(p_1) $(d_1, d_2) \neq 1$

(p_2) there exists a positive integer n such that

$$d_1 | d_2^n \text{ or } d_2 | d_1^n.$$

Finally we state the

Theorem 1. In an HCF domain of Krull type R, a non zero non unit x, is expressible as the product of a finite number of mutually co-prime packets and this factorization is unique up to associates of the respective packets and up to their order.

Proof. Let x be a non zero non unit in R, let $P_1, P_2, \ldots, P_n$ be the set of all the valued primes containing x and let $q_1, q_2, \ldots, q_m$ be the set of all the distinct minimal subvalued primes of x. By Lemma 2, corresponding to each q_i there exists a $p_i | x$ such that $p_i \in q_i$ and $p_i \notin q_j$ for each $i \neq j$.

We first take up q_1; there exists a p_1 such that

$$x = p_1 x' \text{ where } p_1 \in q_1 \text{ and } p_1 \notin q_j \quad j = 2, \ldots, m.$$

And by (4) of Lemma 3 we can write

$x = x_1 x_2'$ where $(x_1, x_2') = 1$ and x_1 has q_1 as its only minimal subvalued prime i.e. $x_1 \notin q_j$ ($j = 2, \ldots, m$).

Similarly corresponding to q_2, there exists $p_2 \in q_2$ such that $p_2 | x$ and $p_2 \notin q_j$ $j \neq 2$. Being in q_2, p_2 is not in the bunch of valued primes of x containing q_1 we conclude that $x = x_1 p_2 x_2''$ and by an application of (4) of Lemma 3 again

$$x = x_1 x_2 x_3' \quad ; (x_1 x_2, x_3') = 1.$$

Repeating the above process we get

$x = x_1 x_2 \ldots x_m$; where each x_i is a packet and $(x_i, x_j) = 1$ whenever $i \neq j$.

Moreover if $x = y_1 y_2 \ldots y_s$ where y_j are mutually co-prime packets then $s = m$, because the set of the valued primes(and