$(x_1, y_1) = 1$ and obviously x_1, y_1 are both non units . We note that $x_1 | x$ and $x | z = rs$, therefore $x_1 | rs$, and by the HCF property of R,

$$x = x'x'' \text{ where } x'|r \text{ and } x''|s \text{ ----------------------(c)}$$

Similarly $y = y'y''$, where $y'|r$ and $y''|s$ -------------(d)

Further $y'|y_1, x'|x_1$ and $(x_1, y_1) = 1$ implies that $(x', y') = 1$. But since r is a rigid element $x'|y'$ or $y'|x'$ which is possible only if one of x', y' is a unit --------------(e) . Similarly we conclude that either of x'', y'' is a unit--(f).

Let x' be a unit , then since x_1 is a non unit and $x = x'x''$, x'' is a non unit and is an associate of x_1 but then y'' is a unit (by (f)). Again since y_1 is a non unit y' is a unit and so we conclude that

$y'|r$ where y' is an associate of y_1 and

$x''|s$ where x'' is an associate of x_1.

I.e. there exist two co-prime elements x_1, y_1 such that $y_1|r$ and $x_1|s$. But since r and s are non co-prime rigid elements $r|s$ or $s|r$ by (1) above. And in both cases x_1 and y_1 become factors of a rigid non unit (e.g. x_1, y_1 are factors of s if $r|s$ because $y_1|r$ and $r|s$ i.e. $y_1|s$ while $x_1|s$ is assumed)but this being in contradiction with $(x_1, y_1) = 1$ implies that the assumption $x \nmid y$ and $y \nmid x$ is wrong and z is a rigid non unit.

(3) Let $P(r) = \{ x \in R \quad (x,r) \neq 1 \}$.

Because of (1) above, if x and y are non co-prime to r and if $(x,r) = d$, $(y,r) = d_1$ then, being factors of a rigid non unit $d|d_1$ or $d_1|d$. Consequently if $d_1|d$ then $d_1|x,y$ and so $d_1|x+y$, similarly if $d|d_1$, $d|x+y$. In other words if $x,y \in P(r)$ then $x+y \in P(r)$. Moreover if $x \in P(r)$ then $ax \in P(r)$ for all $a \in R$, that is $P(r)$ is an ideal. Finally because of the HCF property $(xy,r) \neq 1$ iff $(x,r) \neq 1$ or