either x or y is a unit in R_{P_1} i.e. either x or y is not in P_1 a contradiction.

Let$(x,y) = d$, and so $x = x_1 d$, $y = y_1 d$ where $(x_1, y_1) = 1$ and by the previous argument, x_1 and y_1 cannot both belong to P_1 . Let x_1 be such that $x_1 \notin P_1$, then $x_1 d \in P_1$ implies that $d \in P_1$. Obviously since d is a factor of y, $d \in P_1$ and being a factor of x, d belongs at most to $P_1, P_3, \ldots, P_n$. Further let $y_2 \in P_1$ such that $y_2 \notin P_3$, and repeating the above argument we get $d_1 = (d, y_2)$ where d_1 is a non unit factor of x which can belong at most to $P_1, P_4, \ldots, P_n$. And it needs a finite number of steps to reach the conclusion that x has a non unit factor q say, which is contained in P_1 and is contained in no other minimal prime ideal.

Now as $q \in P_1$ and belongs to no other minimal prime ideal, q^n is also in no minimal prime ideal other than P_1, because if we suppose on the contrary that $q^n \in P$ a minimal prime other than P_1 then $q \in P$ a contradiction.

Further let a non unit $h|q$ then since a GKD is completely integrally closed, there exists a positive integer n such that $h^n \not| q$. But R being an HCF domain h^n and q have a highest common factor d say, then $h^n = rd$, $q = q'd$ where $(r, q') = 1$.

Since $h^n \not| q$, r is not a unit, and if we assume that q' is also a non unit then either r or q' is not in P, a contradiction and hence q' is a unit. In other words, for every non unit factor h of q there exists an n such that $q|h^n$ i.e. q is a quantum and so by Lemma 8, q is a prime quantum.

Now the prime ideal Q_q associated to q is obviously contained in P, but P being minimal $Q_q = P$ (cf (2) of Prop. 5)

Finally we know that for every minimal prime P of R, $x \in P$ implies that x is in a finite number of minimal primes