an equivalence relation.

Remark 1. Statements (1) - (3) can be equivalently replaced by the following comprehensive statement:

" The prime quanta in an integral domain similar to a given one, with units form a multiplicative set which is saturated and totally ordered by divisibility."

Proof. (1) Let q be a prime quantum and q_1 be a non unit factor of q. To prove that q_1 is a prime quantum we have to show that q_1 satisfies (1) and (2) of Definition 3, (obviously q_1 is a quantum). Now for some n

$q_r, q_s | q_1^n$ then $q_r, q_s | q^n$ and so $q_r | q_s$ or $q_s | q_r$ i.e. (1) of Definition 3, is satisfied.

Further if q_1 is non co-prime to ab then so is q, and every factor q_t of q_1^n which divides ab, being also a factor of q^n can be written as $q_t = q_u q_v$ where $q_u | a$ and $q_v | b$, which is (2) of Definition (3).

(2) If q_1, q_2 are similar prime quanta then let q_3 be a non unit common factor of q_1, q_2. By (1) above q_3 is a prime quantum. So there exist m,n such that $q_1 | q_3^n$, $q_2 | q_3^m$ and thus $q_1 q_2 | q_3^{n+m}$ and by (1) of Definition 3, $q_1 | q_2$ or $q_2 | q_3$.

(3) We establish that if q is a prime quantum then q^m is again a prime quantum (for every positive integral m). By (1) of Def. 3, if $x, y | q^m$ then $x|y$ or $y|x$. So if a non unit $h | q^m$, $h|q$ or $q|h$. If $h|q$ then there is n such that $q | h^n$ and so $q^m | h^{nm}$, and if $q|h$ then $q^m | h^m$. Hence q^m is a quantum. Further if h_1, h_2 are factors of an integral power of q^m, h_1, h_2 are factors of a power of q and so $h_1 | h_2$ or $h_2 | h_1$. Similarly if q^m is non co-prime to ab then so is q and it is easy to see that q^m satisfies (2) of Def. 3 .

Finally if q_1, q_2 are similar prime quanta and if q_3 is a non unit common factor then there exists an integer m