x and y have a highest common factor d say, that is $x = x_1 d$, $y = y_1 d$ and $(x_1, y_1) = 1$. Obviously x_1, y_1 are non unit factors of a quantum and so by the definition of a quantum there exist m,n such that $q \mid x^m$ and $q \mid y^n$, so that $x_1 \mid y_1^n$ and $y_1 \mid x_1^m$, which in view of the HCF property implies that $(x_1, y_1) \neq 1$ a contradiction and so for all x,y dividing q, $x \mid y$ or $y \mid x$. Further we see that if $x \mid q^n$ for some n then by the HCF property if x is a non unit then it has a non unit factor d common with q. But $q \mid d^n$ for some n because q is a quantum and it follows that $q \mid x^n$ and that $q^n \mid x^{n^2}$, that is q^n is a quantum for all n and it can be shown on the same lines as above that for each pair $u,v \mid q^n$, $u \mid v$ or $v \mid u$, which is exactly (1) of Def. 3. Moreover since an HCF domain is also Schreier every factor of q^n for each n is primal that is (2) of Def. 3, also holds and q is a prime quantum.

Lemma 9. If R is an HCF domain and S is a multiplicative set in R then R_S is an HCF domain.

Proof. It is well known that if A and B are ideals of an integral domain R and S is a multiplicative set in R then

$$(A \cap B)R_S = AR_S \cap BR_S \qquad (\text{ cf [9] p 34}) .$$

Moreover the necessary and sufficient condition for an integral domain R to be an HCF domain is that the intersection of every two principal ideals is principal(can be verified easily).

Now let $x,y \in R_S$, where R and S are as in the hypothesis. We can write $x = r_1/s_1$, $y = r_2/s_2$ where $(r_i, s_i) = 1$, and s_i are units in R_S.

Consider $xR_S \cap yR_S = (r_1/s_1)R_S \cap (r_2/s_2)R_S$, s_i being units we can write the RHS as $r_1R_S \cap r_2R_S$ but since $r_1R_S \cap r_2R_S = (r_1R \cap r_2R)R_S = [r_1, r_2]R_S$ where $[r_1, r_2]$ is the