$\{P_\alpha\}$, for if not: x is a unit in each R_{P_α} $\alpha \in I$

$$\text{i.e. } x^{-1} \in R_{P_\alpha} \quad \alpha \in I$$

$$\text{i.e. } x^{-1} \in \cap R_{P_\alpha} = R, \text{ that is x is a unit}$$

in R, a contradiction establishing the result.

In what follows, the family of valued primes defining an HCF domain R will be denoted by $\{P_\alpha\}_{\alpha \in I}$ and by a valued prime we shall mean a valued prime in $\{P_\alpha\}_{\alpha \in I}$ and by a subvalued prime we shall mean a prime contained in a valued prime in $\{P_\alpha\}$.

Lemma 4. A non zero non unit x in an HCF domain R is a packet iff x has a single minimal subvalued prime .

Proof. Let x be a non zero non unit in an HCF domain R and let x have a single minimal subvalued prime q. We have to show that x is a packet i.e.

(p_1) if $x = x_1x_2$, where x_i are non units then $(x_1, x_2) \neq 1$

(p_2) if $x = x_1x_2$, with x_i non units then there exists a positive integer n such that $x_1 | x_2^n$ or $x_2 | x_1^n$.

We first show that (p_1) holds for x, for if we assume on the contrary that $x = x_1x_2$, x_i non units and $(x_1, x_2) = 1$ then x_1 and x_2 cannot both belong to the same valued prime P because then $(x_1, x_2) = 1$ in R implies that $(x_1, x_2) = 1$ in R_P which in turn implies that at least one of x_1, x_2 is not contained in a given valued prime.

Let P_1 be one of the valued primes containing x_1 and P_2 be one of those containing x_2 then the minimal subvalued primes q_1, q_2 of x_1 and x_2 respectively are distinct and obviously these are minimal subvalued primes of x, a contradiction establishing that $(x_1, x_2) \neq 1$.

Before establishing that (p_2) holds for x, we prove the following lemma to make our task easier.