domain need not be a valuation domain and to show that there does exist at least one, one dimensional quasi-local domain which is not a valuation domain we take up the following

Example 2.(cf [5] p. 262). Let G be the additive semigroup of all rationals ≥ 0 and reals ≥ 1, form the semigroup algebra F[G] and let F(G) be the ring obtained by adjoining inverses of all elements with non zero constant term. We can write

$$F(G) = \{ \Sigma u_i x^{\alpha_i} \mid \alpha_i \geq 0 \text{ if rational and } \alpha_i \geq 1 \text{ if real and } u_i \text{ are units} \}$$

No two elements of F(G) are co-prime and it can be verified that one divides a power of the other and that F(G) is a one dimensional quasi-local domain, because if $(\alpha), (\beta) \in G$ where $\beta > \alpha$ then there exists a positive integer n such that $n\alpha > \beta + 1$ (α, β being real numbers). But F(G) is not a valuation domain, since $x^{1/2} \nmid x^{1+\gamma}$, where γ is an irrational number less than 1/2.

Further it can be verified that a prime quantum is a rigid non unit while a rigid non unit may not be a prime quantum, for example every non zero non unit in a rank two valuation domain R is rigid, while if P is the maximal ideal of R and Q is the minimal non zero prime ideal then every integral power of $x \in P - Q$ will divide every element of Q, that is elements of Q do not satisfy the condition of being a quantum and hence are not prime quanta.

In the case of prime quanta it was easy to develop a theory of factorization on classical lines, as we did in the previous chapter, but in the case of rigid elements it looks not only difficult but also unnecessary to go through all those details. So we shall consider the properties of rigid