$P, P_1', \ldots, P_m'$ and so by the above process we can show that $P = Q_{q'}$, where q' is the prime quantum emerging from the above process.

It is well known that in a GKD every non zro prime ideal contains a minimal prime ideal and so we have proved that in an HCF-GKD every prime ideal contains a minimal prime ideal associated to a prime quantum which by Proposition 7 is equivalent to say that R is a GUFD.

Remark 3. The above proof does not demonstrate as to how we can write a non zero non unit x in an HCF-GKD R. This end may be achieved as follows:

Let $\{ P_1, P_2, \ldots, P_r \}$ be the set of all non zero minimal prime ideals containing x. We have shown that $x \in P_1$ implies that there exists a prime quantum q_1 in P such that $q_1 | x$. Suppose that q_1 does not divide x completely (cf Def. 2), then R being a GKD, is completely integrally closed and so there is an n such that $q^n \nmid x$. Now by the HCF property $q_1' = (q_1^n, x)$ divides x completely. Similarly proceeding for $P_2, \ldots P_r$ we conclude that $x = q_1' q_2' \ldots q_r'$ and this factorization is obviously unique.

It is well known that if R is a GKD and S is a multiplicative set in R then R_S is a GKD (cf[9] p 513). Further by Lemma 9, if R is an HCF domain and S in R is multiplicative then R_S is an HCF domain and so using the above theorem we can prove the

Proposition 13. If R is a GUFD and S is a multiplicative set in R then R_S is a GUFD.

Further if R is a GKD and x is an indeterminate over R then R[x] is a GKD ([9] p. 517)and it is well known that if R is an HCF domain then so is R[x]. Hence follows the

Proposition 14. If R is a GUFD and x is an indeterminate