$x = x_1 x_2 \ldots x_s$, where x_i are mutually co-prime rigid non units. Since $x \in P(r_{\alpha_1})$; one of the x_i $(i = 1,2,\ldots s)$ say x_1 is non co-prime to r_{α_1}. Also since $x \in P(r_{\alpha_2})$ one of the x_i $(i = 2,3,\ldots,s)$ say x_2 is non co-prime to r_{α_2} so that

$x = x_1 x_2 a$; where $a \notin P(r_{\alpha_i})$ $i = 1,2$ (because $(a,x_i) = 1$ which is equivalent to saying that $(a, r_{\alpha_i}) = 1$).

Since we assume that Q is prime and since $a \notin P(r_{\alpha_i})$ $a \notin Q$, and so $x_1 x_2 a = x \in Q$ implies that $x_1 x_2 \in Q$, that is $x_1 \in Q$ or $x_2 \in Q$. In other words $x_1 \in P(r_{\alpha_1}) \cap P(r_{\alpha_2})$ or $x_2 \in P(r_{\alpha_1}) \cap P(r_{\alpha_2})$ that is x_1 or x_2 is a rigid non unit non co-prime to two co-prime rigid non units (since $\alpha_1 \neq \alpha_2$) a contradiction that confirms that (3) holds for Φ.

To prove (4) for Φ let $R' = \cap R_{P_\alpha}$, $\alpha \in I$, and suppose that $x = u/v \in R'$, then since R is an HCF domain we can assume that $(u,v) = 1$, but this implies that v is a unit in each R_{P_α}, that is v cannot be expressed as a product of rigid non units and we are forced to conclude that v is a unit and $x \in R$ which confirms that

$$R = \cap R_{P_\alpha} \; ;\alpha \in I.$$

The above theorem, in view of Cor 1 p61, ~~obviously~~ is a local characterization of Semirigid Domains, and, gives us another generalization of Krull domains. Being short of a suitable name for these integral domains, we call them <u>*GKD's</u>.

Definition 3. An integral domain R will be called a *GKD if there exists a family $\Phi = \{ P_\alpha \}_{\alpha \in I}$ of prime ideals of R such that

*1- every non zero non unit element of R is contained in only a finite number of members of Φ.

* 2- for each P_α ;$\alpha \in I$, R_{P_α} is a valuation domain