2. Factorization in an HCF ring of Krull type.

In this section we first take up a non zero non unit element in an HCF ring of Krull type and prove a sequence of lemmas to establish the notions in terms of which we can describe its factorization. In brief we shall first derive the notion of a packet as we mentioned before and then prove that in an HCF ring of Krull type a non zero non unit is expressible as the product of a finite number of mutually coprime packets.

Let R be a ring of Krull type and let $\Phi = \{ P_\alpha \}_{\alpha \in I}$ be the family of valued primes defining R. We start by showing that if $0 \neq x \in P$ ($\in \Phi$) then there exists a unique prime ideal Q, minimal subject to the property $x \in Q \subseteq P$.

To achieve the above mentioned result we proceed a bit more generally as follows.

Let P be a prime ideal in an integral domain R and denote the set $\{ Q \mid Q$ is a prime ideal contained in P $\}$ by $\mathcal{E}(P)$. We note that if P is a valued prime then $\mathcal{E}(P)$ is totally ordered under inclusion and keeping in view the fact that every prime ideal contains a minimal (rank zero) prime ideal we state the

Lemma 1. Let P be a prime ideal in an integral domain R such that $\mathcal{E}(P)$ is totally ordered under inclusion, then for each non zero $x \in P$, there exists a unique prime ideal Q in P which is minimal subject to the property $x \in Q \subseteq P$.

Proof. P/xR is a prime ideal in R/xR and so contains a minimal prime ideal $Q' = Q/xR$ for some $Q \subseteq P$, but since $\mathcal{E}(P)$ is totally ordered, Q is unique and hence the lemma.

And as a result of the above lemma we can state that,