such that $q_1 q_2 \mid q_3^m$ i.e. $q_1 q_2$ is a factor of a prime quantum and hence is a prime quantum.

(4) Let q be a prime quantum such that $q \mid ab$ completely,. By (2) of Def.3, $q = q_1 q_2$ such that $q_1 \mid a$ and $q_2 \mid b$, so that $ab = a_1 b_1 q_1 q_2$. Suppose that $(a_1, q) \neq 1$, and let q_3 be a non unit common factor i.e. $a_1 = a_2 q_3$. Thus

$ab = a_2 b_1 q_1 q_2 q_3$, but then $q_1 q_2 q_3 = q q_3$ is a prime quantum higher than q with respect to ab, a contradiction and hence $(a_1, q) = 1$. Similarly $(b_1, q) = 1$.

(5) Reflexivity and symmetry are obvious. For transitivity let q_1, q_3 and q_2 be prime quanta such that (a) q_1 is similar to q_2 and (b) q_2 is similar to q_3.

Here (a) implies that q_1 and q_2 have a non unit common factor q_{12} say. Now q_2 and q_3 are similar and so by (3) above $q_2 \mid q_3$ or $q_3 \mid q_2$. If $q_2 \mid q_3$ then $q_{12} \mid q_3$ and so q_1 and q_3 are similar. Further if $q_3 \mid q_2$ then since q_{12} and q_3 both divide a prime quantum q_2, $q_{12} \mid q_3$ or $q_3 \mid q_{12}$, that is q_1 and q_3 are similar.

Corollary 1. A quantum is a prime quantum iff it has a prime quantum as a factor.

Proof. If q is a quantum and q_1 is a prime quantum dividing it then there exists a positive n such that $q \mid q_1^n$. Now q_1 being a prime quantum the result follows from (1) and (3) of the above lemma. The converse is obvious.

Corollary 2. If a prime quantum $q \mid ab$ and $(q,a) = 1$ then $q \mid b$.

Proof. By (2) of Def. 3, if $q \mid ab$ then $q = q_1 q_2$ such that $q_1 \mid a$ and $q_2 \mid b$, but since $(q,a) = 1$, q_1 is a unit and hence $q \mid b$.

Proposition 2. If an element in an integral domain R is expressible as the product of a finite number of distinct dissimilar prime quanta then the expression is unique up to