and hence q_1 is an atom.

Now since a GUFD is an HCF domain and it is well known that an atom in an HCF domain is a prime(cf.e.g. [5]), $q_1 R$ is a prime ideal contained in P, that is $q_1 R = P$ ((2) of Prop. 5)

To study another feature of GUFD's, let q be a prime quantum and let $ab \in qR$, that is $q|ab$. By Definition 3, $q = q_1 q_2$ such that $q_1|a$ and $q_2|b$, that is $a = a_1 q_1, b = b_1 q_2$ say. Obviously if $b \notin qR$, q_1 is a non unit and so there is a positive integer m(say) such that $q|q_1^m$ i.e. $q|a_1^m q_1^m = a^m$, that is if $b \notin qR$, $a^m \in qR$. In other words qR is primary. Further we note that

$$\sqrt{qR} = \{ x \mid (x,q) \neq 1 \} = Q_q$$, which in a GUFD, is the minimal prime ideal associated to q.

Now let x be a non zero non unit in a GUFD R then

$x = q_1 q_2 \ldots q_n$, where q_i are distinct prime quanta can be written as $xR = q_1 q_2 \cdots q_n R = q_1 R \cap q_2 R \cap \ldots \cap q_n R$ and a consideration of $\sqrt{q_i} R$ shows that xR has a unique primary decomposition. And so we have proved the

Theorem 17. In a GUFD, every non zero principal ideal has a primary decomposition $xR = P_1 \cap P_2 \cap \ldots \cap P_n$ where each P_i is primary to a minimal non zero prime ideal and is principal.

It may be pointed out that the above theorem is closely related to Prop. 15. In connection to these and specially as a corollary to Prop. 15, we state

Corollary 7. If in an HCF domain R every principal ideal is primary then R is a rank one valuation ring.

Proof. Let x,y be any two non zero non units of R. According to the hypothesis, xR, yR and xyR are primary. Obviously since x and y are non units, $x,y \notin xyR$ and consequently there exist m and n such that $x^m, y^n \in xyR$ i.e. $xy|x^m, y^n$. Now