CHAPTER 0

INTRODUCTION AND CONTENTS

1. Introduction . The main purpose of this work is to study Unique Factorization and its generalizations in commutative integral domains. A Unique Factorization Domain is defined to be an integral domain in which every non zero non unit element x is expressible as the product of a finite number of principal primes i.e.

$$x = p_1 p_2 \ldots p_n$$

where a principal ideal (p) is a principal prime if $p \mid ab$ implies that $p \mid a$ or $p \mid b$.

It is well known that

(1) a Unique factorization domain (UFD) is an HCF domain i.e. every two elements have a highest common factor.

(2) a UFD is a Krull domain i.e an integral domain R such that

K_1. every non zero non unit of R is contained in only a finite number of minimal non zero prime ideals of R

K_2. for every non zero minimal prime ideal P of R, R_P the localization at P is a discrete rank one valuation ring.

K_3. $R = \cap R_P$ where P ranges over all minimal non zero primes of R .

(3) every non zero non unit x of a UFD can be written as $x = up_1^{a_1}p_2^{a_2}\ldots p_n^{a_n}$; where u is a unit $a_i > 0$ and p_i, p_j are are co-prime if $i \neq j$ (cf [30] Theorem 5.3 (g)).

We observe that if $x = up_1^{a_1}p_2^{a_2}\ldots p_n^{a_n}$ as in (3) above it is expressible as a product of a finite number of mutually co-prime elements $u_i p_i^{a_i}$ $(i = 1,2,\ldots,n)$ where $u_i p_i^{a_i}$ are such that (1) for every non unit $x_i \mid u_i p_i^{a_i}$ there exists a positive integer n_i such that $u_i p_i^{a_i} \mid x_i^{n_i}$.