rationals i.e.

$S = \{ x^{\alpha} y^{\beta} \;;\alpha, \beta$ rationals $\geq 0 \}$ where x,y are indeterminates over the field of reals. Let R be the field of real numbers and consider the algebra $R[S] = L$ say. It is not difficult to prove that L is an integral domain. Let

$T = \{ t \in L \mid t$ is co-prime to x and y both$\}$.

The set T has elements of the type:

$$\left.\begin{array}{l} t_1 = r_1 + ax^{\alpha} \\ t_2 = r_2 + by^{\beta} \\ t_3 = ax^{\alpha} + by^{\beta} \end{array}\right\} \begin{array}{l} r_1, r_2 \in R - \{0\} \\ a,b \in R[S] \\ ;(y^{\beta},a) = 1 = (x^{\alpha}, b) \end{array}$$

$t_4 = \ell + ax^{\alpha} + by^{\beta}$ where $\alpha,\beta > 0$ $\ell \in R - \{0\}$ and $a, b \in R[S]$.

The forms of these elements show that T is a multiplicative set, and is saturated(cf Sec.3). Now in the localization, $(R[S])_T = D$, every element d can be written as

$d = ux^{\alpha}y^{\beta}$; where u is a unit and obviously this expression is unique. It can also be verified that x^{α}, y^{β} are prime quanta (α ,β rational ≥ 0).

Example 7, above ensures the existence of GUFD's and as we develop the theory further we shall see that there exists a sufficiently large class of integral domains which are GUFD's but are not UFD's.

3. Some Results analogous to Classical theorems.

First we recall that in a ring R a set S is said to be multiplicative if $a,b \in S$ implies that $ab \in S$ and S is saturated if $ab \in S$ implies that $a,b \in S$. Further it is well known that in an integral domain R a set S generated by primes is multiplicative and saturated. Analogously we prove

Proposition 3. Let R be an integral domain and H the set generated multiplicatively by units and prime quanta then H is multiplicative and saturated.

Proof. The hypothesis implies that if $x \in H$ then

$x = q_1 q_2 \ldots q_n$ where each q_i is a prime quantum or a