$x = q_1 q_2 \ldots q_n$, q_i prime quanta and all distinct

$y = p_1 p_2 \ldots p_m$, p_i prime quanta and all distinct

Now for every prime quantum $q_i \mid x$ ($i = 1,2,\ldots n$) q_i has a common factor with y or does not. Also if q_i does have a common factor with y then q_i is similar to one and only one of $p_j \mid y$ (Def. 3). Now select out of $q_1, q_2, \ldots q_n$ all those prime quanta $q_1', q_2', \ldots q_r'$ such that $(q_i', y) \neq 1$. Similarly select out of $p_1, p_2, \ldots p_m$ all those $p_1', p_2', \ldots p_s'$ such that p_j' are non co-prime to x. By the above assertion $r = s$ and we can form pairs $\{ q_i', p_i' \}$ of similar prime quanta for a suitable permutation of p_i' say.

Let $d_i = (p_i', q_i')$ where $d_i = p_i'$ if $p_i' \mid q_i'$ and $d_i = q_i'$ if $q_i' \mid p'$. Obviously as p_i' and q_i' are similar in pairs, d_i exists for each $i = 1,2,\ldots r$. And it is easy to see that in each case d_i is the HCF of p_i', q_i'.

Let $d = d_1 d_2 \ldots d_r$; that d is a common factor of x and y is obvious. To prove that d is the highest common factor we have to show that every common factor d' of x and y divides d. We first note that d' is a product of prime quanta that is $d' = \pi_1 \pi_2 \ldots \pi_t$; π_i distinct prime quanta dividing x and y. That is each π_i is similar to one of $d_1, d_2, \ldots d_r$ and so divides it. And it is easy to see that $d' \mid d$ and that d is the highest common factor.

Remark 2. Many notions in the classical theory of Unique Factorization are taken as granted; for example we hardly need to state the fact that if in a UFD, x is a non unit factor of y then there exists a positive integer n such that $x^n \nmid y$. If on the other hand we need to stress this fact we content ourselves by saying that a UFD is atomic. In case of a GUFD the above mentioned property holds but needs an explanation: