"If x is a non zero non unit in a ring of Krull type R, then each valued prime P of R with $x \in P$; contains a unique minimal subvalued prime satisfying $x \in Q \subsetneq P$." We shall call Q, the <u>minimal subvalued prime of x in P</u>.

Now let x be a non zero non unit in an HCF ring of Krull type and let $P_1, P_2, \ldots, P_n$ be the only valued primes containing x. By the above lemma, each valued prime P_i contains a unique minimal subvalued prime Q_i containing x ($i = 1,2,\ldots,n$). Here we note that unlike a *GKD, a ring of Krull type admits valued primes $P_\alpha, P_\beta \in \{ P_\alpha \}$ (the family defining the ring of Krull type) such that $P_\alpha \cap P_\beta$ contains non zero prime ideals. And so the minimal subvalued primes Q_i ($\subseteq P_i$) of x may not all be distinct. The case where $Q_i \subsetneq Q_j$; $i \neq j$ does not arise, because then Q_i becomes the minimal subvalued prime of x in P_i and P_j both.

Striking repetitions out of $\{ Q_i \}_{i=1}^{n}$ and denoting the set of distinct minimal subvalued primes of x by $\{ q_j \}_{j=1}^{r}$ we can regroup $\{P_i\}_{i=1}^{n}$ after a suitable permutation of $\{ P_i \}$ as

$$\{ P_i \}_{i=1}^{n} = \bigcup_{j=1}^{r} \Pi_j \quad \text{where} \quad \Pi_j = \{ P_k \in \{ P_i \}_{i=1}^{n} \mid q_j \subset P_k \}$$

We shall call the set Π_j , the bunch of valued primes of x containing q_j only(among all q_j of course).

Now let y be such that $y \in q_1$ but $y \notin q_2$ (since q_1, q_2 are distinct we can have such a y), then since R is an HCF domain and R_{q_1} is a valuation domain, $(y,x) = d_1 \in q_1 - q_2$, because $y = y'd_1$, $x = x'd_1$, $(x',y') = 1$ (since d_1 is the HCF) and because of the HCF property $(x',y') = 1$ in R_{q_1} that is at least one of x',y' is not in q_1 but since $x,y \in q_1$ $d_1 \in q_1$, further since $y \notin q_2$ and $d_1 | y$, $d_1 \notin q_2$. Further let $y_1 \in q_1 - q_3$, then as before $(y_1, d_1) = d_2 \in q_1 - q_3$(and also