

be uniquely represented as an intersection of finitely many

primary ideals. A UFD is an integral domain and it is well known

that a W -ring R is called a W^* -ring if each ideal of R con-

tains a power of its radical. (2) of Prop. 5)

Theorem A ([10] Th. 1). A ring is a W -ring iff it is a

finite direct sum of primary rings and one dimensional integ-

ral domains in which every non zero ideal is contained in

only finitely many maximal ideals. (3) of Prop. 5)

Theorem B ([10] Th. 2). A W -ring is a W^* -ring iff each non

zero ideal of R contains a product of non zero prime ideals.

Theorem C ([10] Th. 4). If a W^* -domain is strongly (

completely) integrally closed then it is a Dedekind domain.

First we take up the behaviour of minimal prime ideals

in GUD's. We note that in the case of UFD's it is well known

that an integral domain R is a UFD iff every non zero prime

ideal of R contains a principal (non zero) prime, and that an

analogue of this result appears in this chapter as Prop. 7.

And to clarify the structure of minimal prime ideals of GUD's

still further we prove the

Theorem 16. If P is a minimal prime ideal in a GUD R , then

P is either principal or idempotent. (The ideal P is principal

Proof. Let P be a minimal prime ideal in a GUD R then by

(2) of Proposition 5, $P = Q$ for a prime quantum q . (3) of Prop. 5)

Suppose that $P^2 \neq P$ and let $x \in P - P^2$. Since $P = Q$

$(x, q) \neq 1$, obviously $q_1 = (x, q)$ is contained in P and no

other minimal prime ideal. We claim that q_1 is an atom. For

supposing on the contrary that $q_1 = q_2 q_3$, where q_2, q_3 are both

non units. Since $q_1 \in P$ and is in no other minimal prime ideal

every non unit factor of q_1 is in P . This implies that

$q_2, q_3 \in P$ and so $q_1 = q_2 q_3 \in P^2$ i.e. $x = x_1 q_1 \in P^2$, a contradiction