

there exists a valued prime \mathfrak{q} ($\neq \mathfrak{p}^p$) containing x . But then $x \in \mathfrak{q} \cap R \subseteq \mathfrak{p}$ a contradiction.

Theorem 3, gives rise to the following

Definition 5. An essential domain R with the defining

family $\{P_\alpha\}_{\alpha \in I}$ of primes will be called *-essential if

every non-zero non unit x in R has a finite number of mini-

mal subvalued primes.

Finally in view of Theorem 3, and the earlier work we

can state that a non zero non unit x in an HCF domain R is the

product of a finite number of mutually co-prime packets iff

x has a finite number of minimal primes.

4. Stability Properties of URD's.

We begin this section with results about the behaviour

of Unique Representation under the operations of adjoining

indeterminates and localization. We then go on to establish

a property of URD's which is not shared by UFD's that is if

R is a URD x an indeterminate over R and K the field of

fractions of R then the almost integral closure

$$S = R + xK[x]$$

is a URD. Finally with the help of examples we show that the

integral domains we have considered under distinct names

are in fact distinct.

Like Unique Factorization, the concept of Unique Rep-

resentation remains stable under adjoining indeterminates

and this we prove with

Proposition 4. Let R be a URD and x an indeterminate over

R then $R[x]$ is a URD.