

Semirigid Domain is a generalization of a GUD. And to display another feature of Semirigid Domains we prove the following

Theorem 2. Let R be a Semirigid Domain, then there exists a family $\Phi = \{P_\alpha\} \mid (\alpha \in I \text{ an index set})$ of prime ideals of R such that

- (1) R_{P_α} is a valuation domain for each $\alpha \in I$
- (2) each non zero non unit $x \in R$ is contained in only a finite number of members of Φ
- (3) $P_{\alpha_1} \cup P_{\alpha_2}$ does not contain a non zero prime ideal if $\alpha_1 \neq \alpha_2, \alpha_i \in I$

(4) $R = \bigcap_{\alpha \in I} R_{P_\alpha}$, $\alpha \in I$.

Proof. By part(3) of Lemma 1, in an HCF domain R , corresponding to each rigid non unit r , there exists a prime ideal $P(r) = \{x \in R \mid (x, r) \neq 1\}$ associated to r , and by (4) of Lemma 1, $P(r) = P(s)$ iff s is a rigid non unit non co-prime to r .

Now let I be a set of mutually co-prime rigid non units r_α of the given Semirigid domain R , where $\alpha \in I$ an index set. According to the above observation we have a family of prime ideals $\Phi = \{P(r_\alpha) \mid r_\alpha \in I; \alpha \in I\}$, and by part (5) of Lemma 1, $R_{P_\alpha} = R_{P(r_\alpha)}$ is a valuation domain for each $\alpha \in I$, that is (1) holds for the selected family Φ .

Since R is a Semirigid Domain, each non zero non unit being a product of a finite number of mutually co-prime rigid non units is a member of at most a finite number of elements of Φ , that is (2) also holds for Φ .

Now let q be a non zero prime ideal contained in the intersection $P_{\alpha_1} \cup P_{\alpha_2} \equiv P(r_{\alpha_1}) \cup P(r_{\alpha_2})$, $(P_{\alpha_1} \neq P_{\alpha_2})$ and let $x \in q$. Then since x is semirigid