(3) $(A_v)_v = A_v$

It is also known (cf (c) 32.2 [11]) that

$(AB)_v = (AB_v)_v = (A_vB_v)_v$ ---------------------------(VM)

A fractional ideal A is a v-ideal if $A = A_v$, and a v-ideal F is said to be of finite type if there exists a finitely generated ideal A such that $A_v = F$.

Definition . An integral domain R is called a Prüfer v-multiplication domain if the v-ideals of finite type in F(R) form a group under v-multiplication as (VM) above.

Note . Griffin [19] and [20] calls these integral domains, "v-multiplication rings" while in the present literature, a v-multiplication ring is an integral domain in which $(AB)_v \subset (AC)_v$ implies that $B_v \subset C_v$.

Turning our attention towards HCF domains we see that it is well known (cf e.g.[8]page 584) that each v-ideal of finite type of an HCF domain is principal. And to prove that an HCF domain is a Prüfer v-multiplication domain we only need to verify that the principal fractional ideals in F(R) form a group under multiplication which is evident. Thus an HCF domain is a Prüfer v-multiplication domain and hence according to Griffin [19] an essential domain.

We recall that an integral domain R is an essential domain if there exists a family $\Phi = \{ P_\alpha \}_{\alpha\in I}$ of prime ideals such that E_1. R_{P_α} is a valuation domain for each $\alpha \in I$

E_2. $R = \cap R_{P_\alpha}, \alpha \in I$.

~~We can assume that no two members of Φ are comparable w.r.t. inclusion and~~ We shall call $\{ P_\alpha \}_{\alpha\in I}$ the family of valued primes defining R. Clearly by E_2 above, a non zero non unit x in R is contained in at least one valued prime in