unit for each $i = 1,2,\ldots n$. From the fact that the product of two similar prime quanta is a prime quantum similar to them we deduce that if x is a non unit we can write

$$x = p_1 p_2 \ldots p_m \quad , \; p_i , p_j \text{ dissimilar if } i \neq j.$$

That H is multiplicative is quite obvious. To prove that H is saturated let $ab \in H$,.

First suppose that $ab = q$ a single prime quantum. Either, one of them is a unit or both are similar prime quanta, and in both cases $a,b \in H$.

Further let $ab = q_1 q_2$ where q_1, q_2 are distinct prime quanta. Now as q_1, q_2 are distinct $q_1 | ab$ completely and so $q_1 = q_{1r} q_{1s}$ where $q_{1r} | a$, $q_{1s} | b$ such that $a = a_1 q_{1r}$, $b = b_1 q_{1s}$ and $(a_1, q_1) = 1 = (b_1, q_1)$ (cf (4) of Lemma 1). Consequently $q_2 = a_1 b_1$ implying that $a_1 | b_1$ or $b_1 | a_1$ i.e. one of them is a unit or both are prime quanta. In other words a and b both are products of prime quanta and hence are in H.

Applying induction on the number of distinct prime quanta involved we can prove that if

$$ab = q_1 q_2 \ldots q_n \;\; ; \;\; q_i , q_j \text{ distinct for } i \neq j,$$

then a,b are products of prime quanta and hence are in H i.e. H is saturated.

An integral domain in which every two elements a,b have the highest common factor is called an <u>HCF domain</u>. It is well known that a UFD is an HCF domain and in analogy to this we state the

Proposition 4. A GUFD is an HCF domain.

Proof. Let R be a GUFD and let $x,y \in R$ if one of them is a unit then obviously they have a highest common factor; a unit. If one of them say y is zero then x is the highest common factor. thus we can assume x and y to be non zero non units. Now let