

over  $R$  then  $R[x]$  is a GUPD.

We end this section with an application of the theory

developed in the previous sections and state the

Proposition 15. Let  $R$  be an integral domain such that for

every non zero non unit  $x$  in  $R$

$$x = q_1 \cup q_2 \cup \dots \cup q_n$$

where  $q_i$  are primary ideals such that  $\sqrt{q_i}$  is a minimal prime

ideal, then  $R$  is a GUPD if it is an HCF domain.

Proof. (1) From the hypothesis it follows that every non

zero non unit of  $R$  is contained in a finite number of mini-

mal prime ideals of  $R$ .

(2)  $R$  being an HCF ring  $R_p$  is a valuation domain for every

non zero minimal prime ideal  $P$  of  $R$ .

(3) The proof that  $R = \bigcap R_p$  follows the same lines as the

proof of Prop. 12.

From (1), (2) and (3) above it follows that  $R$  is an HCF

GKD and hence is a GUPD.

## 5. Ideal Theory.

This section includes a brief account of the behaviour

of minimal prime ideals of a GUPD. We then pass on to the

ideal theory of GKD's which are Prüfer (Bezout), the primary

decomposition being our main concern. We shall find that the

primary decomposition of every non zero ideal in a Prüfer GKD

is unique, in other words a Prüfer GKD is a  $\overline{W}$ -ring. At the

end of the section we show that the necessary and sufficient

condition for a Prüfer domain to be a Prüfer GKD is that its

non zero ideals have primary decompositions.

For the sake of reference we quote the definition and

some properties of  $\overline{W}$ -rings from [10].

Definition . A ring  $R$  is a  $\overline{W}$ -ring if each ideal of  $R$  may