

can be calculated.

Now if  $s_1 < s_2$  it is easy to see that  $b_1 x^{s_1} d$  is the

highest common factor of  $y_1, y_2$ . Further if  $s_1 = s_2 = s$ , the

highest common factor of  $b_1 x^{s_1}$  and  $b_2 x^{s_2}$  (if it exists) must

be of degree  $s$  in  $x$ . If  $b_1 = c_1/d_1$  and  $b_2 = c_2/d_2$  (we can

assume  $(c_i, d_i) = 1$  because of  $R$  being HCF) it can be veri-

fied that  $((c_1, c_2)/[d_1, d_2])x^s d$  is the highest common factor

of  $y_1, y_2$ , where  $[d_1, d_2]$  denotes the least common multiple of

$d_1$  and  $d_2$ .

If the case (b) holds let  $y_1$  be of type (a) and  $y_2$  be of

type (b), that is  $y_1 = b_1 x^{s_1} (1 + \sum_{j=1}^{m_1-s_1} a_{j1} x^{j_1})$

$y_2 = r_{02} (1 + \sum_{j_2=1}^{n_2-s_2} a_{j_2} x^{j_2})$  and if  $d$  is the

HCF of the elements in the braces then  $r_{02} d$  is the HCF of

$y_1$  and  $y_2$ .

Finally if (c) holds let  $y_1 = r_{01} (1 + \sum_{i=1}^{n_1} a_{i1} x^{i_1})$

$y_2 = r_{02} (1 + \sum_{i_2=1}^{n_2} a_{i_2} x^{i_2})$

and if  $d$  is the HCF of the elements in the braces then

$(r_{01}, r_{02})d$  is the HCF of  $y_1, y_2$ .

To sum up, each pair of non units in  $S$  has the highest

common factor and this establishes the lemma.

Now let  $y$  be a general non zero non unit element in  $S$

then  $y = r_0 + \sum_{i=1}^n a_i x^{i_1}$ ;  $r_0 \in R$ ,  $a_i \in K$ , and  $y$  can be of two

types: (a)  $y = b x^s (1 + \sum_{j=1}^{m-s} a_j x^{j_1})$ ;  $b \in K$ , or

(b)  $y = r_0 (1 + \sum_{i=1}^n a_i x^{i_1})$ .