$(u_1', u_2') = 1$ in R_S. Since we are approaching from a localization to the original ring, it is possible that $(u_1', u_2') \neq 1$ in R (moreover u_i' being non units in R_S are non units in R) and thus there exists a positive integer n such that $u_1' | u_2'^n$ or $u_2' | u_1'^n$(x being a packet). If we have $u_1' | u_2'^n$ then obviously $u_1' | u_2'^n$ in R_S, but since $(u_1', u_2') = 1$ in R_S which is an HCF domain, $(u_1', u_2'^n) = 1$ in R_S, which implies that u_1' is a unit in R_S a contradiction to the assumption that x_1, x_2 are both non units in R_S and hence x is a packet in R_S.

Now according to the definition

$$R_S = \{ r/s \quad r \in R ; s \in S \}.$$

If r/s is a non unit in R_S and if $r = p_1 p_2 \ldots p_n$, p_i packets and $(p_i, p_j) = 1$ if $i \neq j$ then

$$r/s = (p_1/s_1)(p_2/s_2)\ldots(p_n/s_n); \text{ where } s = s_1 s_2 \ldots s_n$$

(p_i/s_i) are packets if non unit and because of the HCF property $((p_i/s_i),(p_j/s_j)) = 1$ if $i \neq j$, that is if R is a URD then so is R_S and so we state the

Proposition 5. Let R be a URD and S be a multiplicative and saturated set in R then R_S is a URD.

The concept of a rigid non unit being simpler than that of a packet we can easily prove the

Corollary 4. If R is a Semirigid domain and S is a multiplicative and saturated set in R then R_S is again a Semirigid domain.

In Example 1, we showed that the almost integral closure of a PID is a URD, we now extend this result and state the

Theorem 6. Let R be an integral domain, K its field of fractions and x an indeterminate over R, then R is a URD iff its almost integral closure $S = R + xK[x]$ is a URD.