4:

(a) To prove:

$$P(A|BC) = \frac{P(B|AC) \cdot P(A|C)}{P(B|C)}$$

$$P(X|Y) = \frac{P(X \cdot Y)}{P(Y)}$$

RHS: $\frac{P(B|AC) \cdot P(A|C)}{P(B|C)}$

$$\Rightarrow \frac{P(BAC)}{P(AC)} \times \frac{P(AC)}{P(C)} \times \frac{P(C)}{P(BC)}$$

$$\Rightarrow \frac{P(BAC)}{P(BC)}$$

$\Rightarrow$ Since $P(ABC) = P(BAC) = P(CAB)$

$$\Rightarrow \frac{P(ABC)}{P(BC)}$$

$$\Rightarrow P(A|BC)$$

Proved

Ans 4. (b)

Bernoulli model = $(p)^n (1-p)^{n-k}$

Model M_1 for fair coin = $(.5)^n (.5)^{n-k}$

Model M_2 for double headed coin = $(1)^n (0)^{n-k}$

probability of model 1 = $\frac{F}{1+F}$

probability of model 2 = $\frac{1}{1+F}$

Let the number of consecutive heads be N.

probability of getting N heads from model 1 =

$= \left(\frac{1}{2}\right)^N$

probability of N heads from Model 2 =

$= (1)^N$

for more than even chance of conclusion:

Likelihood (Model 2) > Likelihood (Model 1)

$$\Rightarrow \quad \frac{1}{1+F} \cdot > \frac{F}{1+F} \left(\frac{1}{2}\right)^N$$

$$\Rightarrow \quad 2^N > F$$

$$\left\{ N > \log_2 F \right\}$$

Answer