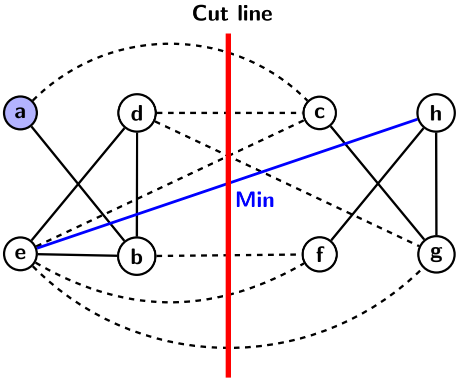
Correctness of Prim's Algorithm



- Let V_T be the subset of vertices forming current T.
- Each edge joining one end vertex in V_T and another in $V - V_T$ is known as a cut edge.
- So, the edges across the cut line are cut-edges and denoted by set C.

$$C = \{(u, v) | u \in V_T \text{ and } v \in V - V_T\}.$$

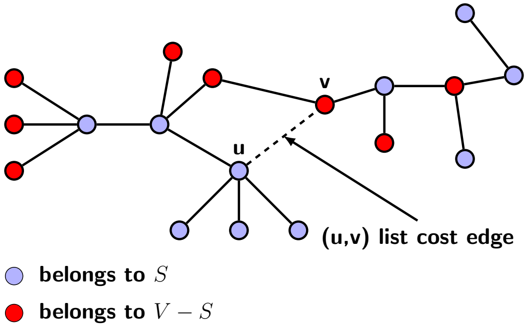
Correctness of Prim's Algorithm

- Suppose T is the spanning tree found by Prim's algorithm.
- Let T_{min} be the MST of G.
- Assume $T_{min} \neq T$, but we have $V_T = V_{T_{min}} = V$
- Consider one edge $(u, v) \in T - T_{min}$.
- When (u, v) is added to T it was the minimum cost edge between $(V_T, V - V_T)$
- Since $(u, v) \notin T_{min}$ there must be a path $P : u \rightsquigarrow v$ in T_{min}.

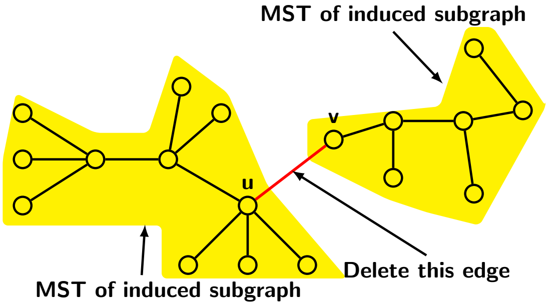
Correctness of Prim's Algorithm

- P begins in $u \in V_T$ and ends in $V - V_T$.
- So, there must be an edge (x, y) connecting a vertex $x \in V_T$ and $y \in V - V_T$.
- Let $T'_{min} = T_{min} \cup \{(u, v)\} - \{(x, y)\}$.
- Clearly, cycle formed by addition of (u, v) to T_{min} is removed by deleting (x, y).
- So, T'_{min} is a tree and its cost is lower than T_{min}.

Correctness of Prim's Algorithm



Greedy Algorithm



- Deleting edge (u, v) results in two MSTs T_1 and T_2 of induced subgraphs G_1 and G_2 respectively.
- If lower cost MSTs T_1' or T_2' were there then
  - $w(T) > w(T_1') + w(u, v) + w(T_2)$ and
  - Similarly, $w(T) > w(T_1) + w(u, v) + w(T_2')$.