

Fitting different models

```
In [26]: 1 import numpy as np
2 import matplotlib.pyplot as plt
3 np.set_printoptions(precision=3)
4 #generate the data
5 x = np.arange(100)
6 y = 150 + 3*x + 0.03*x**2 + 5*np.random.randn(len(x))
7
```

line fit

```
In [27]: 1
2 #Design the design matrices
3 M1 = np.vstack( (np.ones_like(x), x) ).T
4 #print(M1)
5 # Solve the equations
6 p1 = np.linalg.lstsq(M1, y)
7 #estimated parameters
8 print('The coefficients from the linear fit: {}'.format(p1[0]))
9 #line fit
10 y_pred1= p1[0][1] * x + p1[0][0]
```

The coefficients from the linear fit: [102.542 5.948]

C:\Users\jyostna\Anaconda3\lib\site-packages\ipykernel_launcher.py:6: FutureWarning: `rcond` parameter will change to the default of machine precision times ``max(M, N)`` where M and N are the input matrix dimensions.

To use the future default and silence this warning we advise to pass `rcond=None`, to keep using the old, explicitly pass `rcond=-1`.

quadratic fit

```
In [28]: 1 #Design the design matrices
2 M2 = np.vstack( (np.ones_like(x), x, x**2) ).T
3 #print(M1)
4 # Solve the equations
5 p2 = np.linalg.lstsq(M2, y)
6 #estimated parameters
7 print('The coefficients from the quadratic fit: {}'.format(p2[0]))
8
9
10 #quadratic fit
11 y_pred2= p2[0][0] + p2[0][1] * x + p2[0][2] * x * x #quadratic fit
12
```

The coefficients from the quadratic fit: [1.522e+02 2.907e+00 3.072e-02]

C:\Users\jyostna\Anaconda3\lib\site-packages\ipykernel_launcher.py:5: FutureWarning: `rcond` parameter will change to the default of machine precision times ``max(M, N)`` where M and N are the input matrix dimensions.

To use the future default and silence this warning we advise to pass `rcond=None`, to keep using the old, explicitly pass `rcond=-1`.

Cubic fit

```
In [29]: 1 #Design the design matrices
2 M3 = np.vstack( (np.ones_like(x), x, x**2, x**3) ).T
3 # Solve the equations
4 p3 = np.linalg.lstsq(M3, y)
5 #estimated parameters
6 print('The coefficients from the quadratic fit: {}'.format(p3[0]))
7 #cubic fit
8 y_pred3= p3[0][0] + p3[0][1] * x + p3[0][2] * x * x + p3[0][3] * x * x * x #c
9
10
```

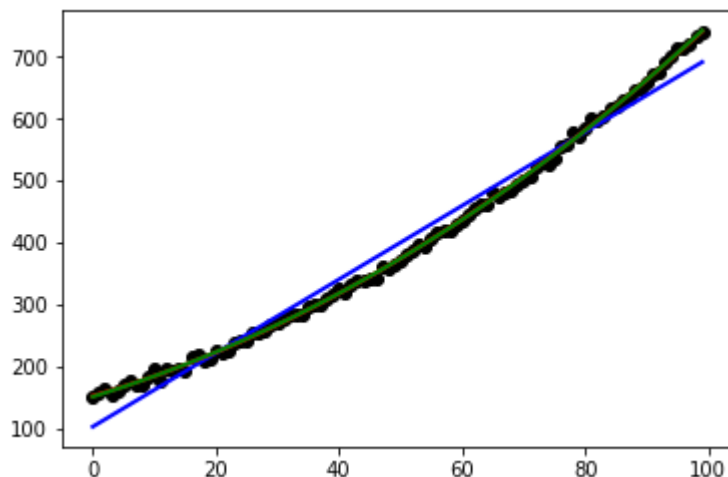
The coefficients from the quadratic fit: [1.511e+02 3.048e+00 2.714e-02 2.410e-05]

C:\Users\jyostna\Anaconda3\lib\site-packages\ipykernel_launcher.py:4: FutureWarning: `rcond` parameter will change to the default of machine precision times ``max(M, N)`` where M and N are the input matrix dimensions.

To use the future default and silence this warning we advise to pass `rcond=None`, to keep using the old, explicitly pass `rcond=-1`.
after removing the cwd from sys.path.

```
In [30]: 1 plt.scatter(x, y, color='black')
2 plt.plot(x, y_pred1, color='blue', linewidth=2)
3
4 plt.plot(x, y_pred2, color='red' , linewidth=2 )
5
6 plt.plot(x, y_pred3, color='green' , linewidth=2 )
```

Out[30]: [<matplotlib.lines.Line2D at 0x2851db92f28>]



Fit models USING STATS MODELS

```
In [36]: 1 import statsmodels.api as sm
2 Res1 = sm.OLS(y, M1).fit()
3 Res2 = sm.OLS(y, M2).fit()
4 Res3 = sm.OLS(y, M3).fit()
5 print(Res1.summary())
```

```
Model:                                OLS      Adj. R-squared:                0.9
82
Method:                            Least Squares      F-statistic:                526
4.
Date:                            Tue, 16 Apr 2019      Prob (F-statistic):        5.52e-
87
Time:                            08:59:13      Log-Likelihood:           -457.
29
No. Observations:                  100      AIC:                        91
8.6
Df Residuals:                      98      BIC:                        92
3.8
Df Model:                          1
Covariance Type:                  nonrobust
=====
==
coef      std err          t      P>|t|      [0.025      0.97
5]
-----
```

the AIC-value,

The Akaike Information Criterion, which can be used to assess the quality of the model: the lower the AIC value, the better the model. We see that the quadratic model has the lowest AIC value and therefore is the best model: it provides the same quality of fit as the cubic model, but uses fewer parameters to achieve that quality.

```
In [30]: 1 print('The AIC-value is \n {0:4.1f} for the linear fit,\n {1:4.1f} for the qua
2 Res3.aic))
```

The AIC-value is
 909.5 for the linear fit,
 647.4 for the quadratic fit, and
 648.8 for the cubic fit

The degrees of freedom (Df)

df of the model are the number of predictors, or explanatory, variables.

The Df of the residuals is the number of observations minus the degrees of freedom of the model, minus one (for the offset).

```
In [40]: 1 print( Res1.df_model, Res2.df_model, Res3.df_model)
2
```

1.0 2.0 3.0

R-Squared

R2 value indicates the proportion of variation in the y-variable that is due to variation in the x-variables.

For simple linear regression, the R2 value is the square of the sample correlation r_{xy} .

For multiple linear regression with intercept (which includes simple linear regression), the R2 value is defined as

$$R^2 = SS_{\text{mod}} / SS_{\text{tot}}$$

R^2 is close to 1 means the model is better

```
In [41]: 1 print( Res1.rsquared, Res2.rsquared, Res3.rsquared)
2
```

0.9835452518838422 0.9988266561367175 0.9988334443953123

Definitions for Regression with Intercept

The Sum-of Squares variables SS_{xx}

n is the number of observations,

and k is the number of regression parameters.

$DF_{mod} = k - 1$ is the (Corrected) Model Degrees of Freedom.

$DF_{res} = n - k$ is the Residuals Degrees of Freedom $DF_{tot} = n - 1$ is the (Corrected) Total Degrees of Freedom.

For multiple regression models with intercept, $DF_{mod} + DF_{res} = DF_{tot}$.

$MS_{mod} = SS_{mod}/DF_{mod}$: Model Mean of Squares

$MS_{res} = SS_{res}/DF_{res}$: Residuals Mean of Squares.

$MS_{tot} = SS_{tot}/DF_{tot}$: Total Mean of Squares, which is the sample variance of the y-variable.

In []:

1