1. All words in which a appears tripled, if at all. This means every clump of a’s contains 3 or 6 or 9 or 12 …. a’s.

   (aaa+b)*

2. All words that contain exactly two b’s or exactly three b’s, not more.

   A*ba*ba*(b+ε)a*

3. (i) All strings that end with a double letter.
   (ii) All strings that do not end in a double letter.

   (i) (a+b)*(aa+bb)
   (ii) (a+b)*(ab+ba) + a + b + ε

4. All strings that have exactly one double letter in them.

   (b+ε)(ab)*aa(ba)*(b+ε) + (a+ε)(ba)*bb(ab)*(a+ε)

5. All strings in which the letter b is never tripled. This means that no word contains the substring bbb.

   (a+ba+bba)*(bb+b+ε)

6. All words in which a is tripled or b is tripled, but not both. This means each word contains the substring aaa or the substring bbb, but not both.

   (a+ba+bba)*(bb+b+ε)aaa(a+ba+bba)*(bb+b+ε) +
   (b+ab+aab)*(aa+a+ε)bbb(b+ab+aab)*(aa+a+ε)

7. All strings in which the total number of a’s is divisible by 3 no matter how they are distributed, such as aabaabbaba.

   (b*ab*ab*ab*)*

8. Show that the following pairs of regular expressions define the same language over the alphabet {a b}
   (i) (ab)*a and a(ba)*
   (ii) (a*+b)* and (a+b)*
   (iii) (a*+b*)* and (a+b)*

(i). $(ab)^*a$ consists of all strings $(ab)^n a$ for n = 0, 1, ……
$a(ba)^*$ consists of all strings $a(ba)^n$ for n = 0, 1, ……
Using induction, we can prove $(ab)^n a = a(ba)^n$ for all n's

1) $(ab)^0 a = a(ab)^0 = a$.
2) Assume $(ab)^{n-1}a = a(ba)^{n-1}$
   $(ab)^n a = (ab)^{n-1}aba = ((ab)^{n-1}a)ba = a(ba)^{n-1}ba = a(ba)^n$

(ii). $(a+b) \subseteq (a^*+b)$, so $(a+b)^* \subseteq (a^*+b)^*$
Because $(a+b)^*$ contains all strings on {a b}, so $(a^*+b)^* \subseteq (a+b)^*$.

(iii) $(a+b) \subseteq (a^*+b^*)$, so $(a+b)^* \subseteq (a^*+b^*)^*$
Because $(a+b)^*$ contains all strings on {a b}, so $(a^*+b^*)^* \subseteq (a+b)^*$.

9. Describe in English the languages associated with the following regular expressions.

(i). $(a(a+bb)^*)^*$

All words that do not begin with b and in which b's appear in clumps of even lengths.

(ii). $(b(bb)^*)^*(a(aa)^*b(bb)^*)^*$

all words that start with 0 or more b's followed by odd number o a's and b's.

(iii). $((a+b)a)^*$

all words with even lengths and in which each b is separated by some a's and a occupies all even positions.

10. Find a DFA accepting the language $10+(0+11)0^*1$