

FORMAL LANGUAGES AND AUTOMATA THEORY

unit I - introduction to FLAT.

$$\begin{array}{c} \text{FA} \\ / \quad \backslash \\ \text{DFA} = \text{NFA} \end{array}$$

unit II - Regular Expression (RE).

PDA

unit III - Grammar Formalism

$$\text{PDDA} \neq \text{NDFDA}$$

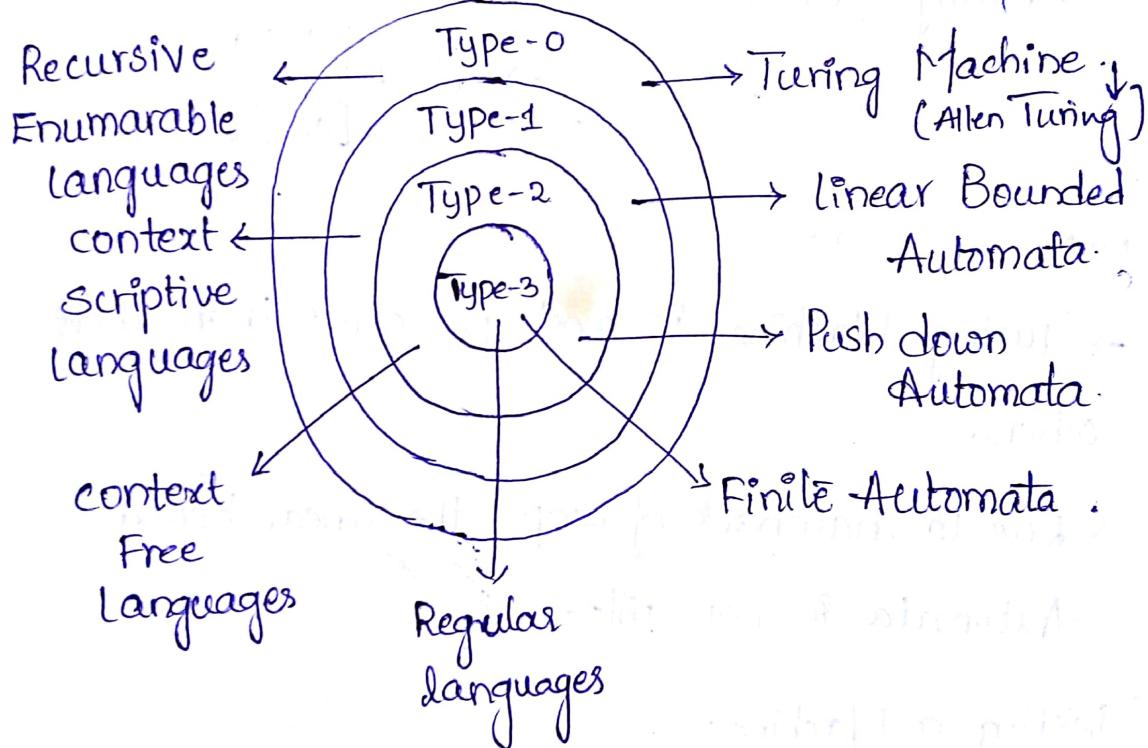
unit IV - PUSH DOWN AUTOMATA (PDA).

TM

unit V - Turing Machines (TM).

$$\text{DTM} = \text{NDTM}$$

Chomsky's classification:



(1) Finite Automata:-

$a^n, n > 0$ - a, aa, aaa, aaaa,

drawback:-

- doesn't have stacks. (0 stacks).
- no storage elements

(2) Push Down Automata:

$a^n b^n, n > 0$ - ab, aabb, aaabbb, ---

- contains one stack.

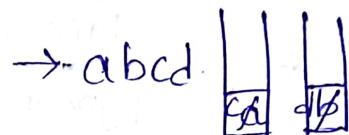
(3) Linear Bound Automata:

$a^n b^n c^n, n > 0$ - abc, aabbcc, aaabbcc, ---

- contains two stacks.

(4) $a^n b^n c^n d^n, n > 0$ - abcd, aabbccdd, ---

- Turing Machine also contains two stacks



Note:

→ Turing Machine is most powerful than above others.

→ Due to drawback of loops the linear Bound Automata is not able.

Testing a Machine:

- Taking a string which is in Machine.
- Rejecting a string which is not available in Machine.

FINITE AUTOMATA :- Mathematical representation of a machine.

→ It is a 5-Tuple Machine.

→ 5-Tuple consists of $[Q, \Sigma, S, s, F]$

Q - set of states.

Σ - set of alphabets.

S - Transitions (or) Grammer Rules.

s - starting state (First state)

F - Final state.

S is defined / derived from $Q \times \Sigma \rightarrow Q$

Representations:

○ → intermediate state.

○ → final state

→ ○ → starting state

ϵ → Epsilon length of the string = 0 (starting)

$(a, b)^*$ → a, b whole star means possible strings.

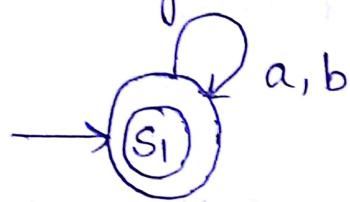
Prblm-1 :

construct a finite Automata on input alphabet $\Sigma = \{a, b\}$ having all strings formed with the given alphabet.

(1) The set of possible strings are:

$$(a, b)^* - \epsilon, a, b, ab, \dots$$

(2) State diagram:



$$(3) Q = \{s_1\}$$

$$\Sigma = \{a, b\}$$

		set of alphabets	
		a	b
States	→(s1)	s1	s1

Starting state - $s_1 \rightarrow$ final state - s_1

Final state - s_1

Note: no. of transitions in a state = no. of alphabets

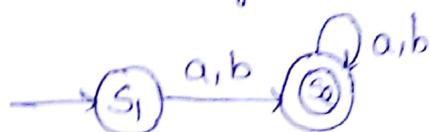
Prblm-2:

construct a finite Automata on input symbol/ alphabet $\Sigma(a,b)$ which accept all the possible strings except ϵ and $\epsilon_{a,b}(\Sigma)$.

(1) The possible strings are:

$$\Sigma(a,b)^+ = a, b, ab, ba, aa, bb, \dots$$

(2) State diagram:



$$Q = (S_1, S_2)$$

starting state - S_1

$$\Sigma = (a, b)$$

Final state - S_2

$S =$	a	b
$\rightarrow S_1$	S_2	S_2
$\circledcirc S_2$	S_2	S_2

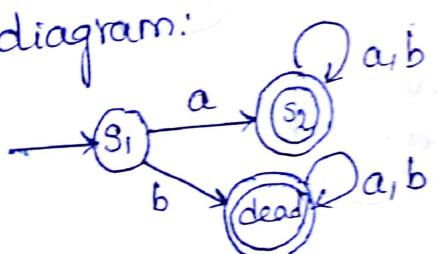
Prblm-3:

construct a finite Automata on input symbol $\Sigma(a,b)$ where each string starts with 'a'.

(1) The possible strings are:

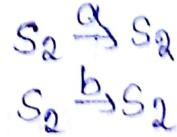
$$\Sigma(a,b)^+ = a, aa, ab, aaa, abb, aba, aab, \dots$$

(2) State diagram:



Here dead state \rightarrow intermediate path.

- s_1 with 'b' goes to dead state.
- s_1 with 'a' goes to s_2 state.



$$(3) Q = (s_1, s_2, \text{dead}) \text{ or } (s_1, s_2)$$

$$\Sigma = (a, b)$$

S =	a	b
$\xrightarrow{s_1}$	s_2	dead
$\circled{s_2}$	s_2	s_2
dead	dead	dead

Starting state = s_1 .

Final State = s_2 .

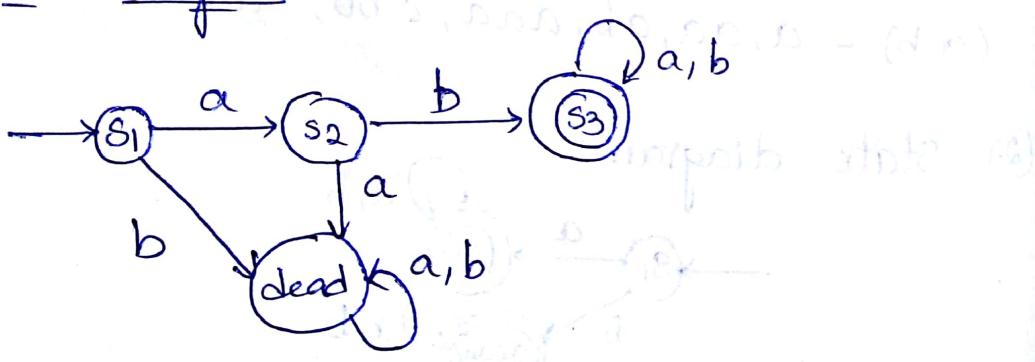
Prblm-4:

construct a finite Automata on $\Sigma(a, b)$ where each string starts with ab.

(1) The possible strings are:

(a, b) - ab, abb, aba, abaa, abab, abbb, abba,

(2) state diagram:



$$(3) \quad Q = (s_1, s_2, s_3)$$

$$\mathcal{E} = (a, b)$$

Starting state - s_1

'Final state' :- s_2

$S =$	a	b
$\rightarrow (S)$	S_2	dead
(S_2)	dead	S_3
(S_3)	S_3	S_3

Prblm-5:

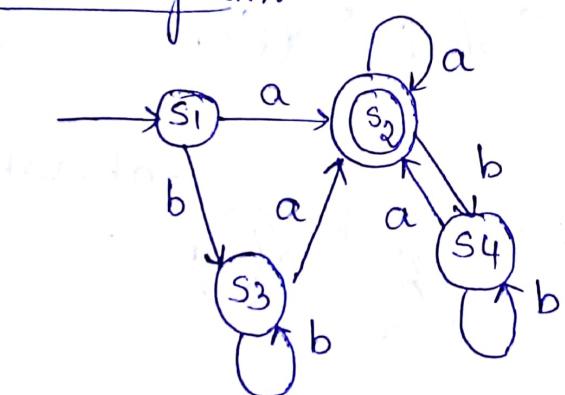
Construct a finite Automata on $\Sigma(a,b)$ where each string st ends with a .

(1.) The set of possible strings are:

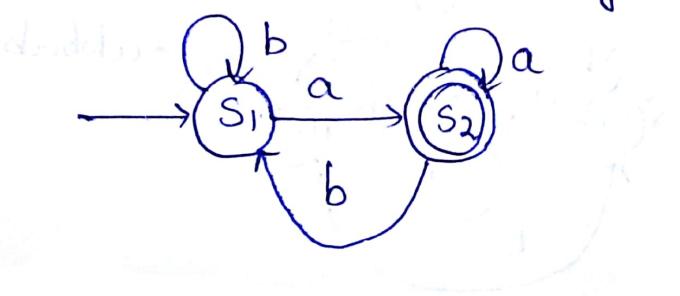
$(a, b) - @, aa, ba, aaa, aba, bba, baa, aaaa,$

abba, abaa, - - - - -

(2) state diagram:



Minimal Finite Automata - having low no. of states.



$$3.) Q = (S_1, S_2)$$

$$\Sigma = (a, b)$$

$\delta =$	a	b
	S_1	S_2
	S_2	S_1

Starting state - S_1 .

Final state - S_2 .

Prblm - 6 :-

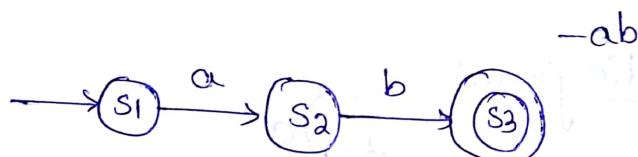
Construct a finite Automata with input alphabet $\Sigma(a, b)$ where each string ends with ab.

(1) The possible strings are :

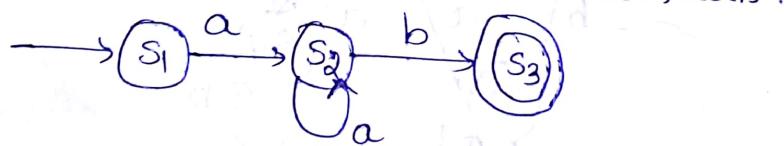
(a, b) - ab, aab, bab, abab, bbab, aaab,
baab, ...

(2) State diagram :

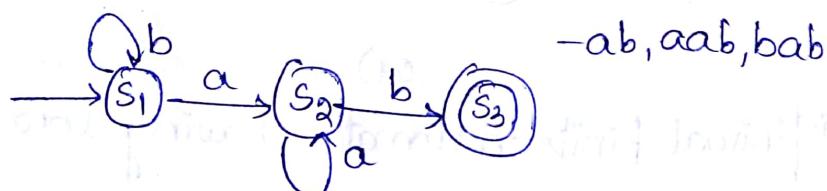
Level - I :



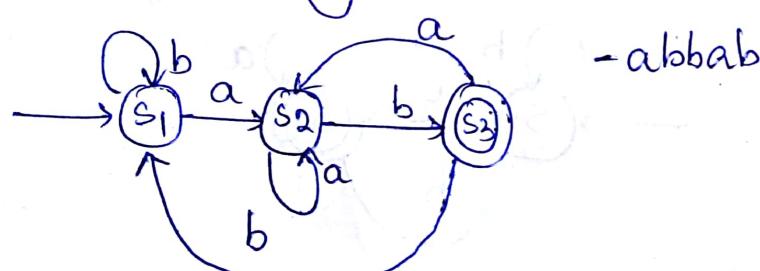
Level - II :



Level - III :



Final :



(3) $Q = (S_1, S_2, S_3)$

$\Sigma = \{a, b\}$

δ	a	b
$\rightarrow S_1$	S_2	S_1
S_2	S_2	S_3
$\circledcirc S_3$	S_2	S_1

starting state - S_1

final start - S_3

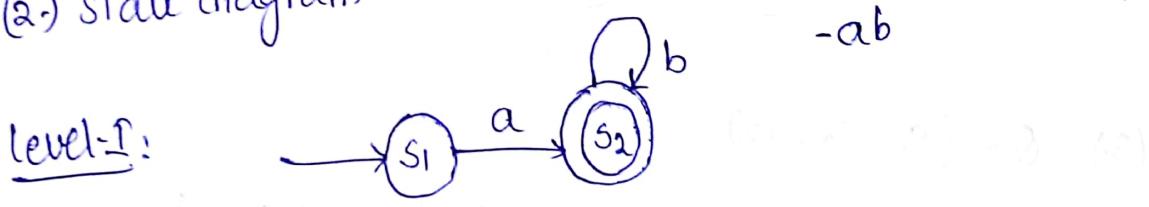
Prblm-7:

construct a finite Automata with input symbol ($\leq a, b$) each string starts with a and ends with b.

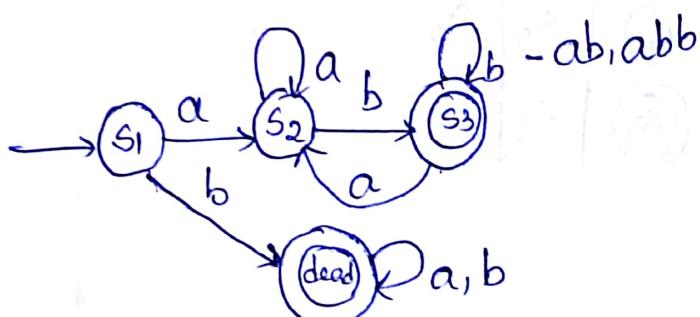
(1) The possible strings are:

$\cdot (a, b) - ab, abb, aab, abab, aaab, abbb, aabb -$

(2) state diagram:



level-II



$$(3) Q = (S_1, S_2, S_3)$$

$$\Sigma = (a, b)$$

Starting state - S_1 .

Final state - S_3 .

$S =$	a	b
$\rightarrow S_1$	S_2	dead
(S_2)	S_2	S_3
(S_3)	S_2	S_3

8. construct a finite automata on input symbol

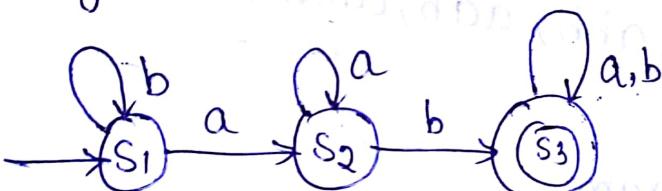
$\Sigma(a, b)$ where each string having ab as a substring

(1) The possible strings are

$(a, b) - ab, aba, bab, abb, aaab, aabb, \dots$

(2) The State diagram:

Level-I:



$$(3) Q = (S_1, S_2, S_3)$$

$$\Sigma = (a, b)$$

Starting state - S_1

Final state - S_3

$S =$	a	b
$\rightarrow S_1$	S_2	S_1
(S_2)	S_2	S_3
(S_3)	S_3	S_3

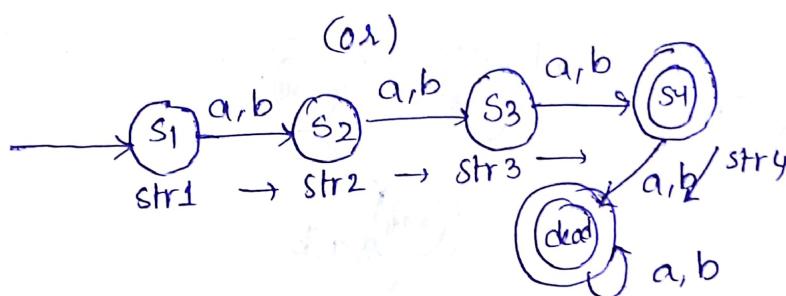
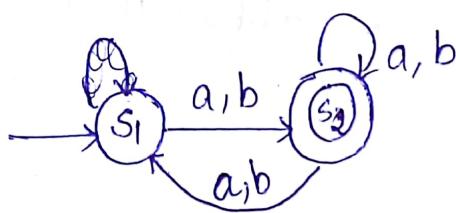
9. construct a F.A on $\Sigma(a,b)$ where each string length is equal 3.

(1) The possible strings are:

(a,b) - aaa, bbb, baa, aba, aab, aab, abb, bba

(2) The state diagram:

level-I



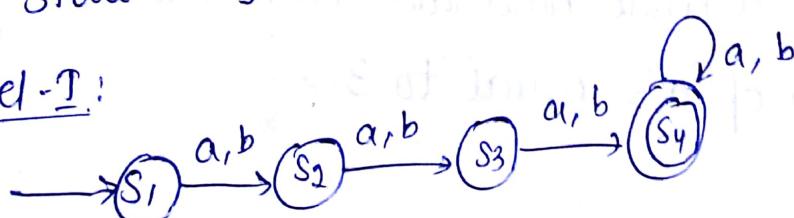
10. construct a Finite automata on $\Sigma(a,b)$ where length of each string is minimum 3. can be 3, 4, 5, ...

(1) The Minimum '3' strings possible are:

(a,b) - ~~aaa, bbb, baa, aba, aab, aab, abb, bba~~, aab, abb, bba, ---

(2) state diagram

level-I:



$$Q = (S_1, S_2, S_3, S_4)$$

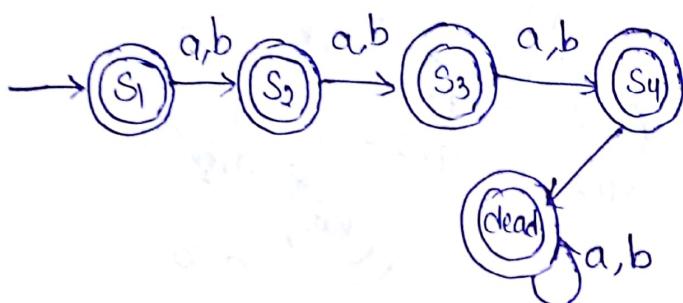
11. construct a finite Automata on input symbol

(a,b) where each string length is maximum 3.

(1) The possible strings:

(a,b) - $\epsilon, a, b, ab, ba, aa, bb, aaa, bbb, aab, aba,$
 $baa, abb, bab, bba, bbb.$

(2) State diagram:



$$(3) Q = (S_1, S_2, S_3, S_4)$$

$$\Sigma = \{a, b\}$$

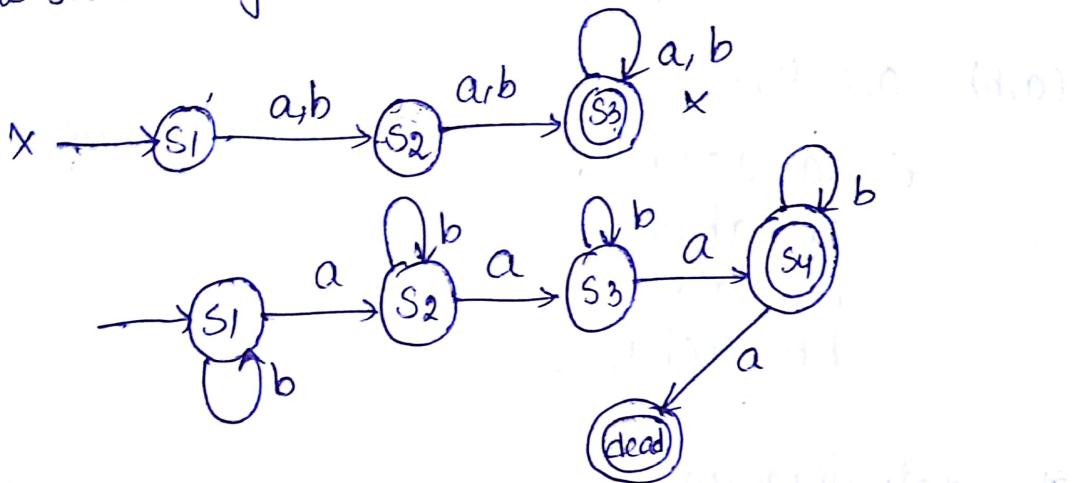
δ	a	b
S_1	S_2	S_2
S_2	S_3	S_3
S_3	S_4	S_4
S_4	dead	dead

$$S - S_1, S_2, S_3, S_4$$

12. construct a finite Automata where each string having no. of 'a's equal to 3.

(1) The possible strings are:
 $\Sigma(a,b) = \{aaa, aaba, baaa, aaab, abaa, \dots\}$

(2) The state diagram:



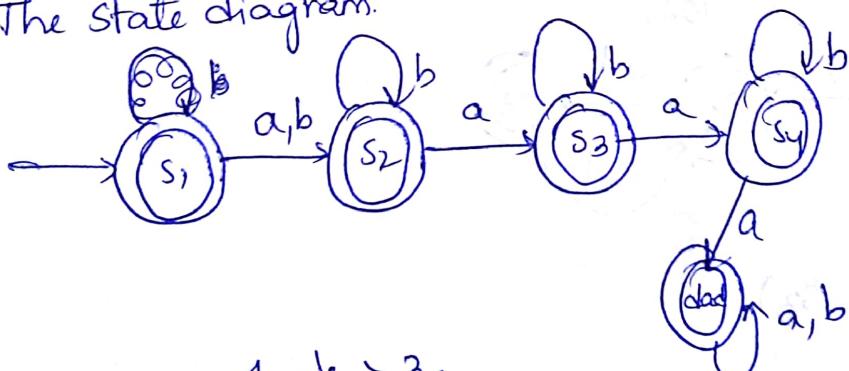
13. construct a FA on $\Sigma(a,b)$ where each string

(1) having no. of a's > 3 .

(2) having no. of a's ≤ 3 .

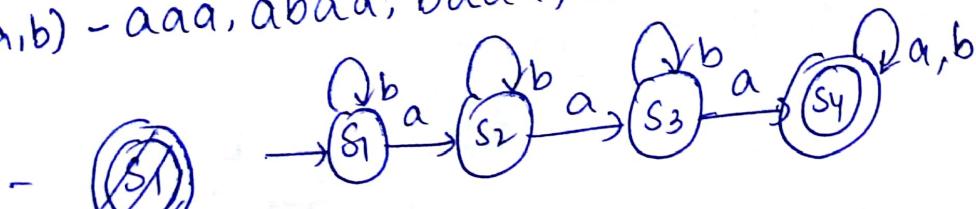
(2) $\Sigma(a,b) = \{a, ab, ba, aa, aaa, aba, baa, aab, bba, aaab, abaa, abba, \dots\}$

The state diagram:



(1) having no. of a's > 3 .

$\Sigma(a,b) = \{aaa, abaa, baaa, aaab, aaba, \dots\}$



14. construct a Finite Automata where each string length is divisible by 2. $\Sigma(a, b)$.

(1) The possible strings are:

$$(a, b) = 0, 2, 4, 6$$

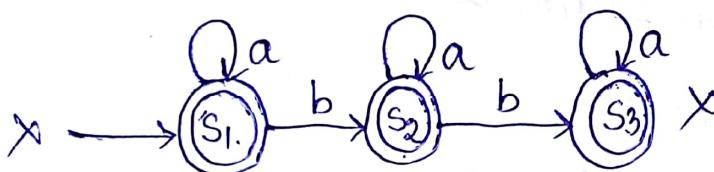
$\epsilon, aa, aaaa$

$ab, abaa$

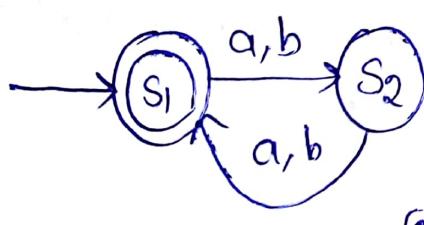
$ba, abba$

$bb, aaba, \dots$

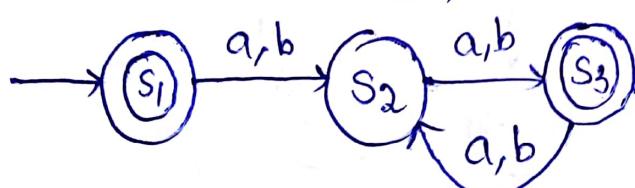
(2) The state diagram:



Minimal Finite Automata:



(Q1)



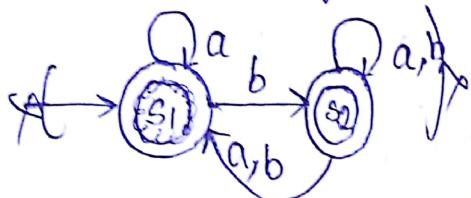
$$Q = (S_1, S_2, S_3)$$

15. On Odd length Strings: (1, 3, 5, 7, ...)

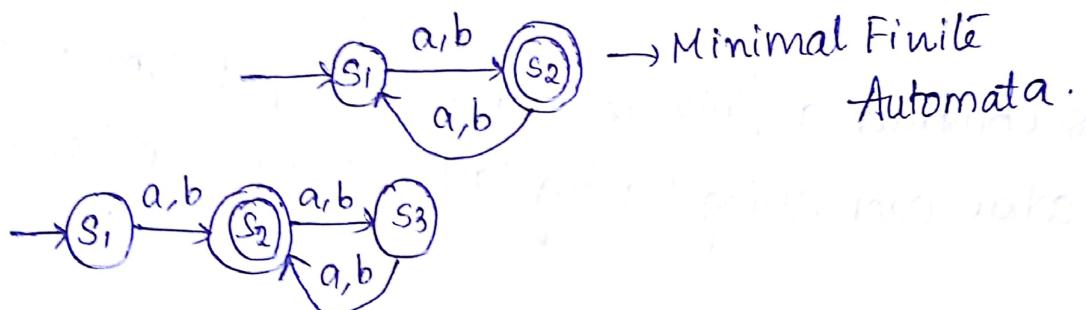
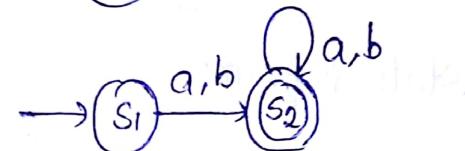
(1) The possible strings are:

(a, b) - $\epsilon, a, aaa, aba, bba, bbb, aab, \dots$

(2) The state diagram is:



(Q.E)



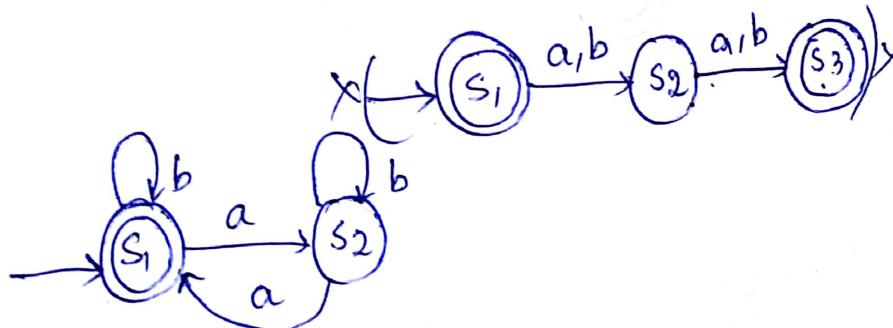
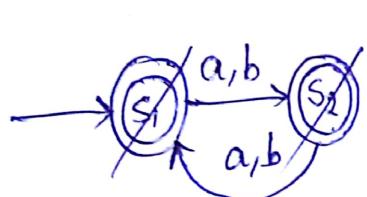
→ Minimal Finite Automata.

16. construct a FA on $\Sigma(a, b)$

where each string having even no. of a's.

(1) The possible strings are:

(a, b) - $\epsilon, aa, b, aab, baa, aba, bb, bbb$

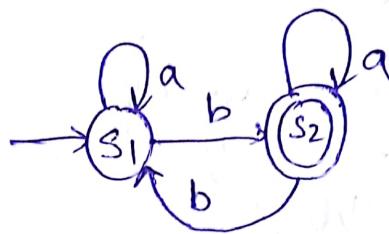
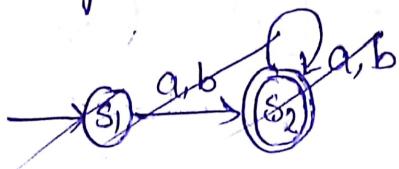


17. construct a Finite Automata where each string having odd no. of b's.

Possible strings are:

(a, b) - b, ab, ba, bbb, aab, baa, aba,

State diagram:-

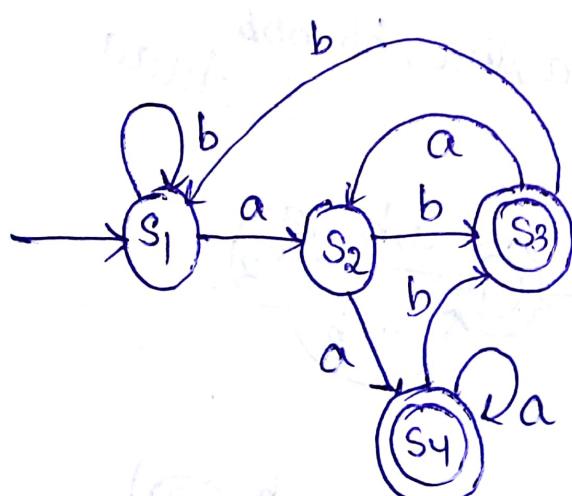
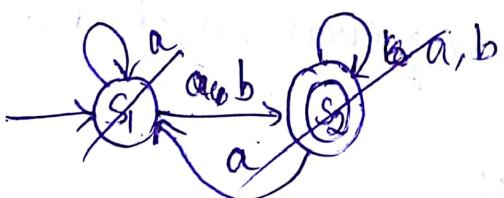


18. Construct a finite Automata on input symbol $\Sigma_{(a,b)}$ where each string having last but one symbol as 'a'

Possible strings are:

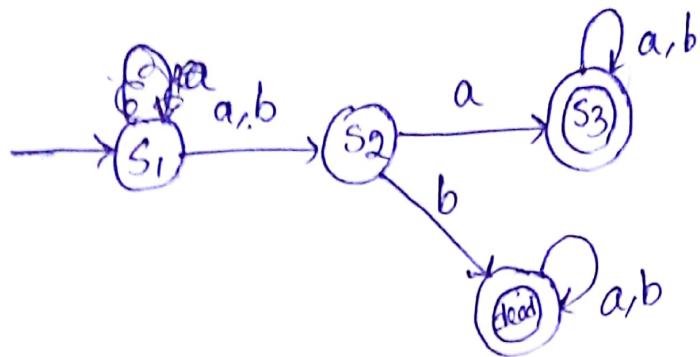
(a, b) $\frac{aa}{-}, ab, aab, bab, baa, aaaa, aaab, abab, baab,$

$bbab, abaa, \dots$



(19) Construct a finite automata on input symbols $\{a, b\}$ where each string having second symbol 'a'.

(a, b) - ba, aa, aab, bab, baa, aaaa, aaa, aaab,



(20) construct a FA on $\Sigma(a, b)$ where each string having:

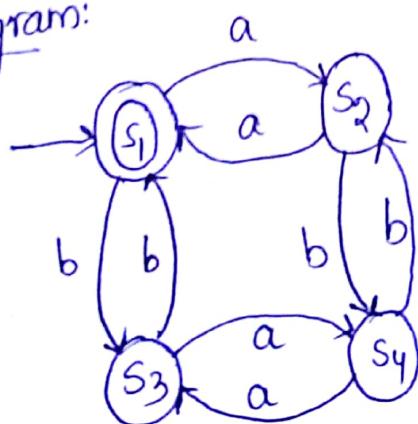
(1) Even no. of a's (or) Even no. of b's.

(2) Even no. of a's and Even no. of b's.

(2) The possible strings are:

(a, b) - ϵ , aa, bb, aaaa, aabb, abab, baab, baba, - - -

State diagram:



(1)

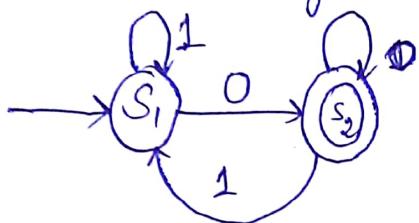
* construct a Finite Automata on $\Sigma(0,1)$ where each binary string divisible by 2.

1. The possible strings are =

$000, 0100, 0110, 1000, 1010, \dots$

0000 - 0
0001 - 1
0010 - 2 ✓
0011 - 3
0100 - 4 ✓
0101 - 5
0110 - 6 ✓
1000 - 8 ✓

2. The state diagram is.



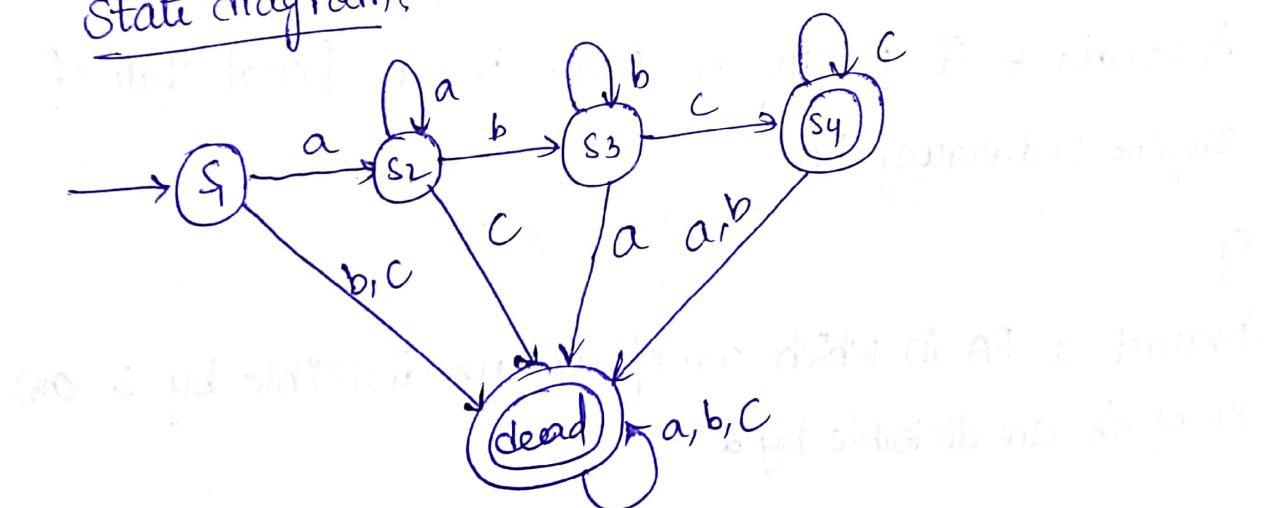
* construct a FA on $\Sigma(a,b,c)$ where language

$$l = a^l b^m c^n \mid l, m, n \geq 0$$

The possible strings are:

abc, aabc, abbc, abcc, ...

State diagram:

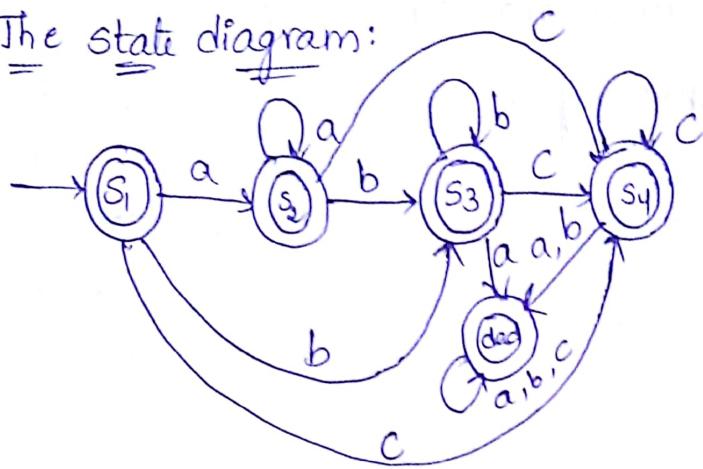


$$* l = a^l b^m c^n; l, m, n \geq 0$$

The Possible strings are:

(a,b) - e, a, b, c, ab, ac, bc, aa, bb, cc, aaa, ...

The state diagram:



Compound Finite Automata:

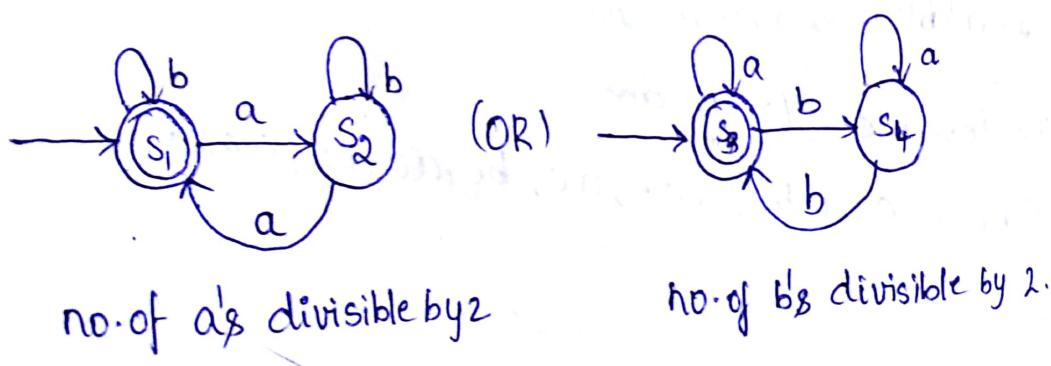
1. Union
2. Intersection.
3. Set difference.
4. Complement.

(1) Union:

If two Finite Automata's Combined with 'OR' then We have to make Final States of Compound Finite Automata if it is anyone state is a final state of anyone individual FA.

e.g:

Construct a FA in which no. of a's are divisible by 2 & no. of b's are divisible by 2.



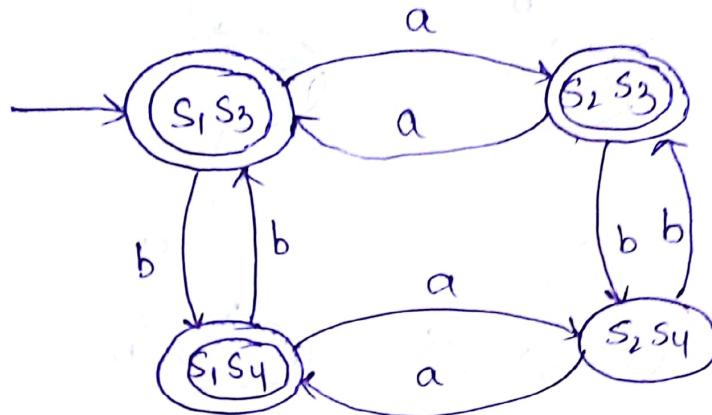
Starting state: Combined Starting State of both FA.

$S_1 S_3$
 $S_1 S_4$
 $S_2 S_3$
 $S_2 S_4$

Final state: Final states in both FA is S_1, S_3 .

so mark the S_1/S_3 states as final states.

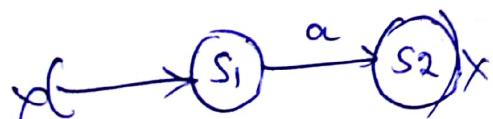
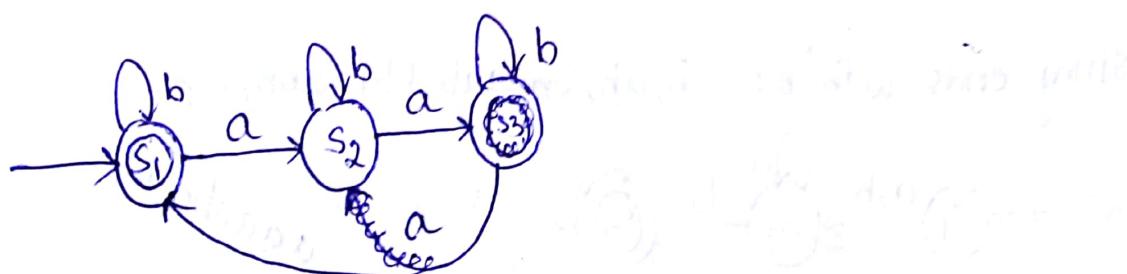
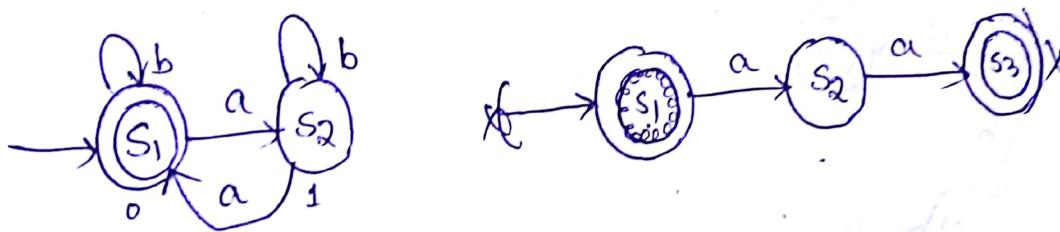
Compound FA: no. of a's even (or) no. of b's even.



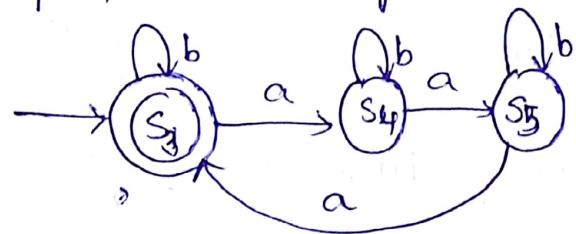
The Possible strings are:

$\epsilon, a, b, aa, bb, aab, baa, aba, bba, \dots$

(2) Construct a finite automata where no. of a's are divisible by 2 (or) 3.



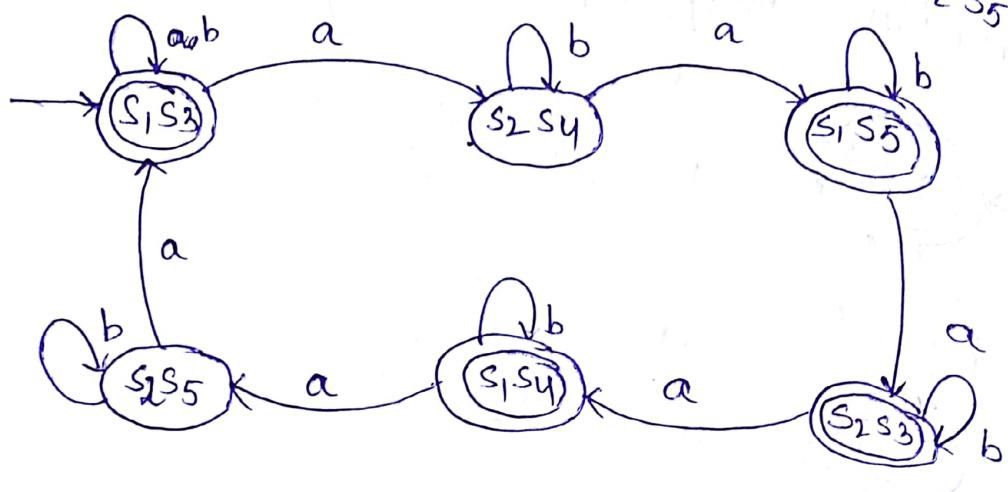
* no. of 'a's divisible by 3



$S_1 S_3$
 $S_1 S_4$
 $S_1 S_5$

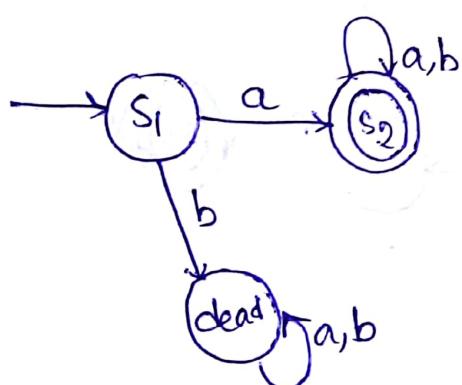
$S_2 S_3$
 $S_2 S_4$
 $S_2 S_5$

Compound Finite Automata:

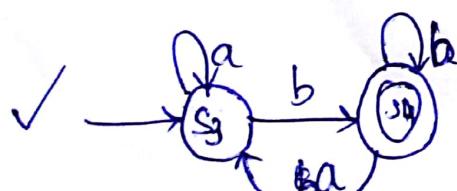
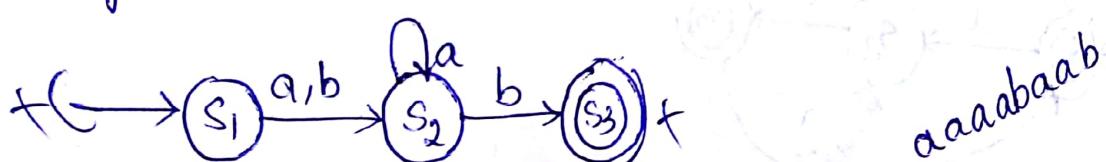


3. Construct a FA on input symbol $\Sigma(a,b)$ where
String starts with 'a' (or) ends with 'b'.

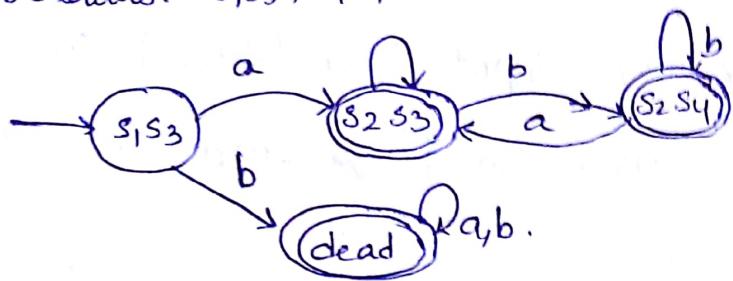
String starts with 'a': a, ab, aa, aab, aba, abb, ..



String ends with 'b': b, ab, bb, aab, bbb, abb, ..

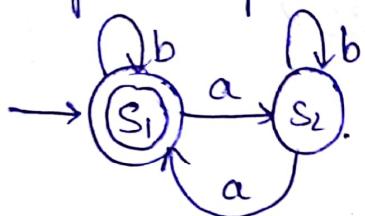


Possible States: $S_1 S_3, S_1 S_4, S_2 S_3, S_2 S_4, \text{dead}$

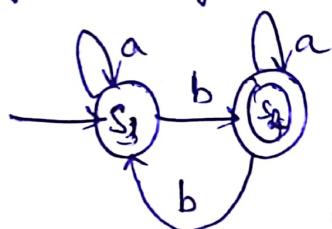


Q. Construct a FA on input symbol a,b where each string having even no. of a's (or) odd no. of b's.

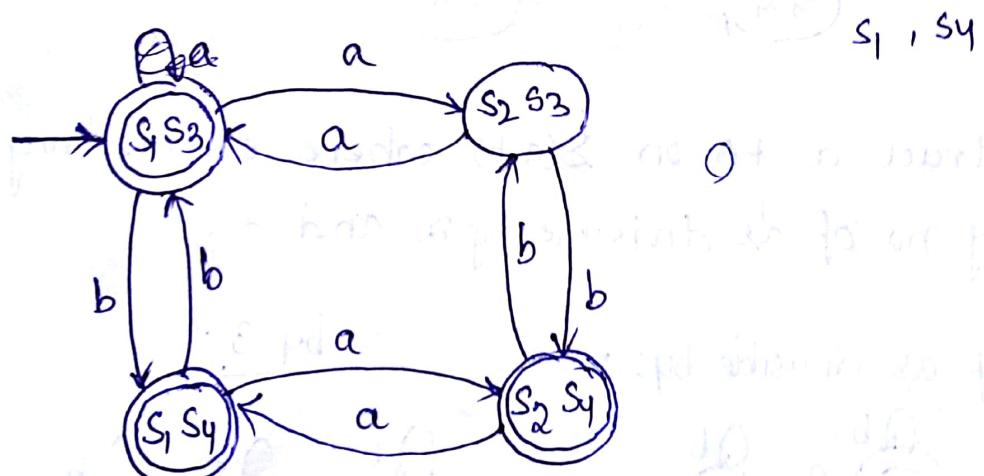
Strings having even no. of a's:



Strings having odd no. of b's: b, ab, ba



Possible States are: $S_1 S_3, S_1 S_4, S_2 S_3, S_2 S_4$.



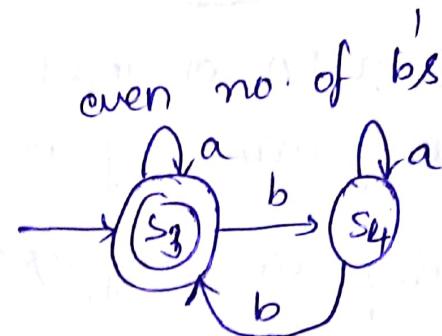
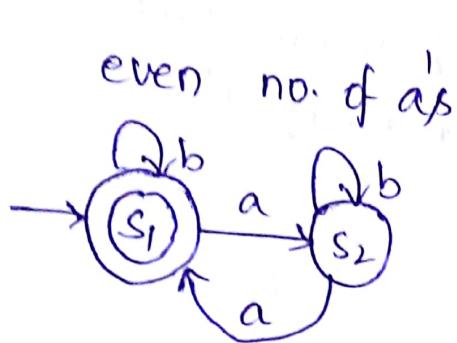
(2)

Intersection:

If any two FA are combined with 'AND' u have
2 make final states of both FA's as final states
of combined FA.

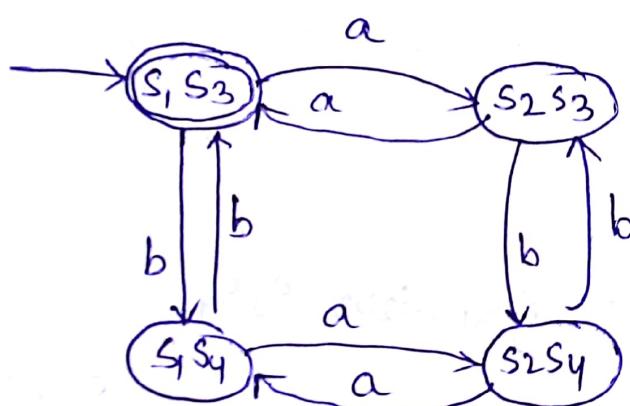
Q. Intersection:

1. Construct a FA on Input Symbols $\Sigma(a, b)$ where each string having even no. of 'a's and even no. of 'b's.



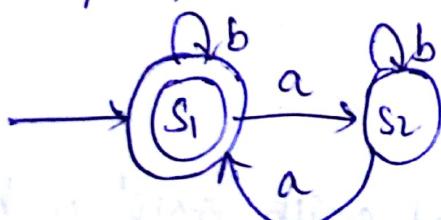
Compound FA:-

$S_1 S_3, S_1 S_4, S_2 S_3, S_2 S_4$

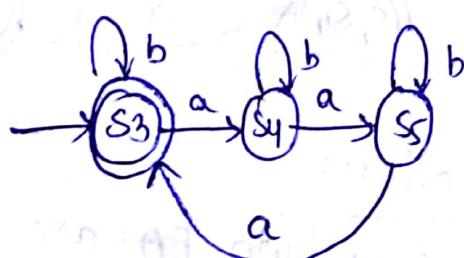


Q. Construct a FA on $\Sigma(a,b)$ where each string having no. of 'a's divisible by 2 and 3.

no. of 'a's divisible by 2:

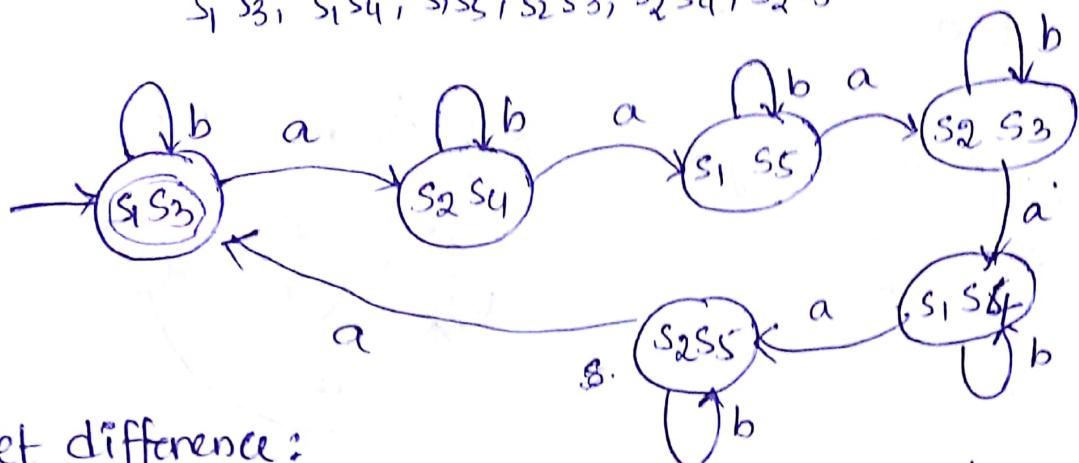


by 3:



Compound FA :-

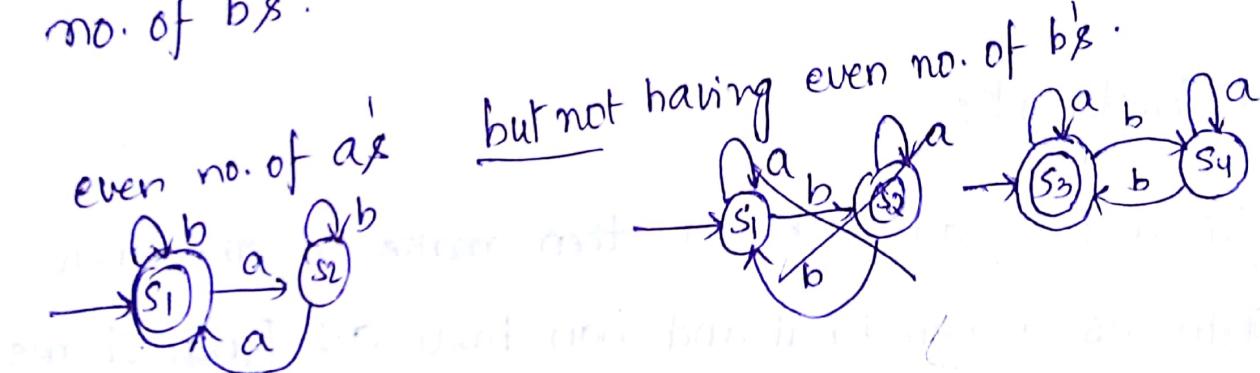
$S_1 S_3, S_1 S_4, S_1 S_5, S_2 S_3, S_2 S_4, S_2 S_5$



Set difference :-

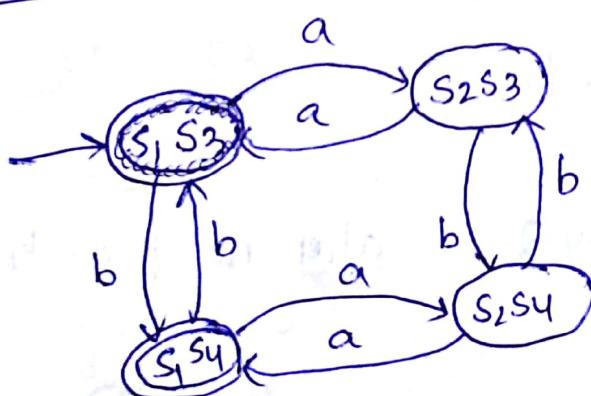
(3.) If two FA's combined with 'but not' we will make Final state as 1st FA but not having Final state of Second FA, as a Final state in Combined FA.

1. Construct a FA on Input symbol $\Sigma(a,b)$ where each string having even no. of a's but not even no. of b's.



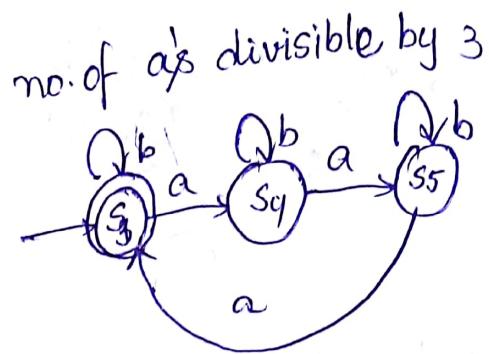
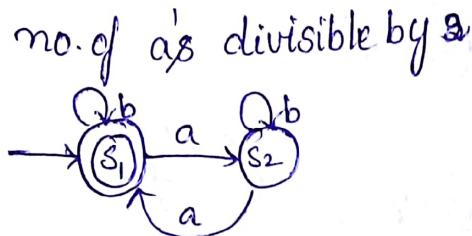
Compound FA :-

$S_1 \checkmark$
 $S_2 \times$

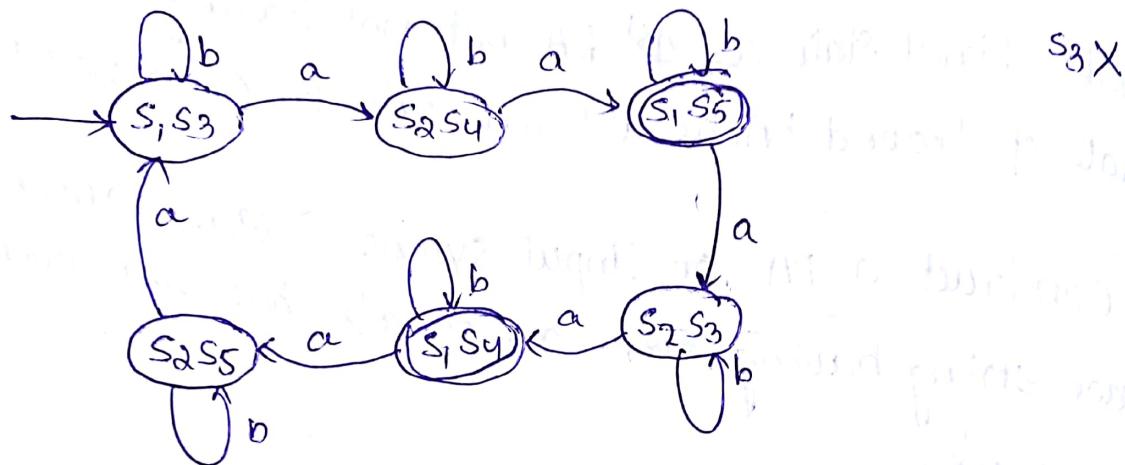


Q: Construct a FA on $\Sigma(a,b)$ no. of a 's divisible by 2

- but not 3.



compound FA:-



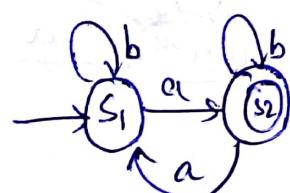
(4.) Complement:-

IF a FA starts with 'not' then make all the final States as a non-final and non-final as final states

1. Construct a FA on $\Sigma(a,b)$ where each string having no. of a 's not divisible by 2.

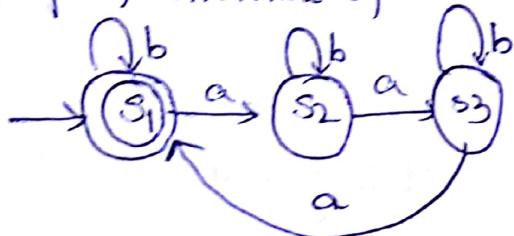


after no. of a 's not divisible by 2:

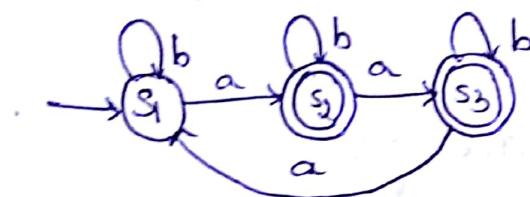


2. Construct a FA on $\Sigma(a,b)$ where each string having no. of 'a's not divisible by 3.

no. of 'a's divisible by 3:



after no. of 'a's not divisible by 3

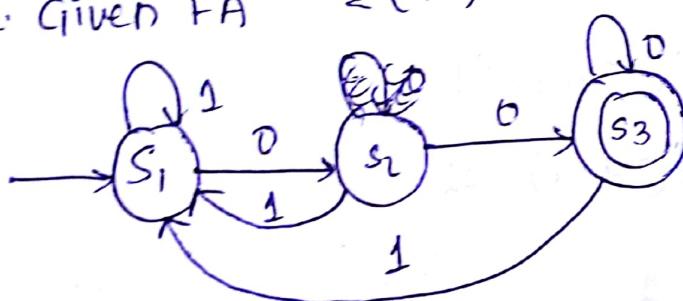


Exercise:-

1. Given FA

$$\Sigma(0,1)$$

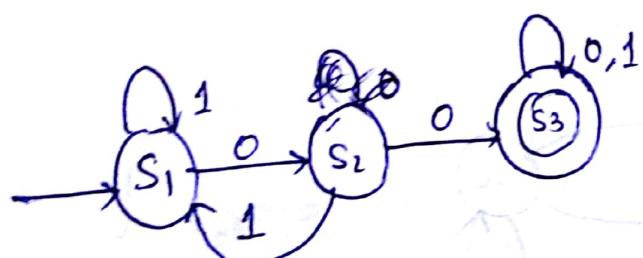
strings ends with '0'



The possible strings are:

0, 00, 000, 1000, 1000, 0100, 1100, ...

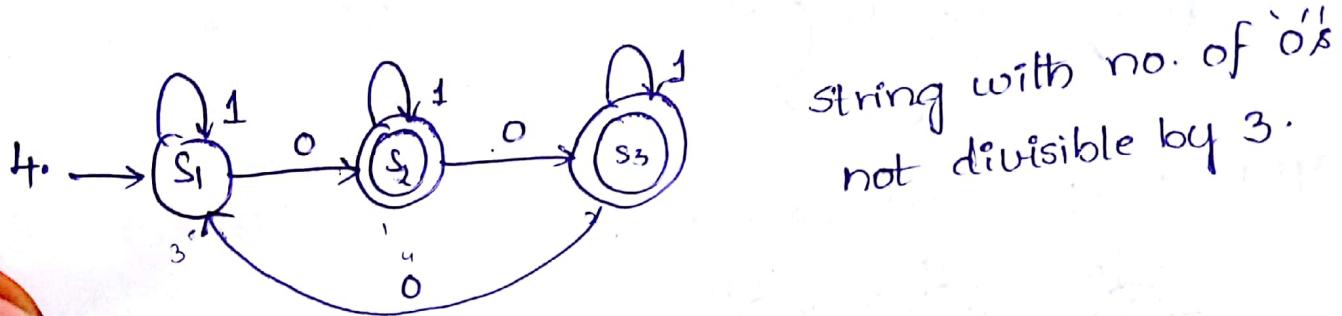
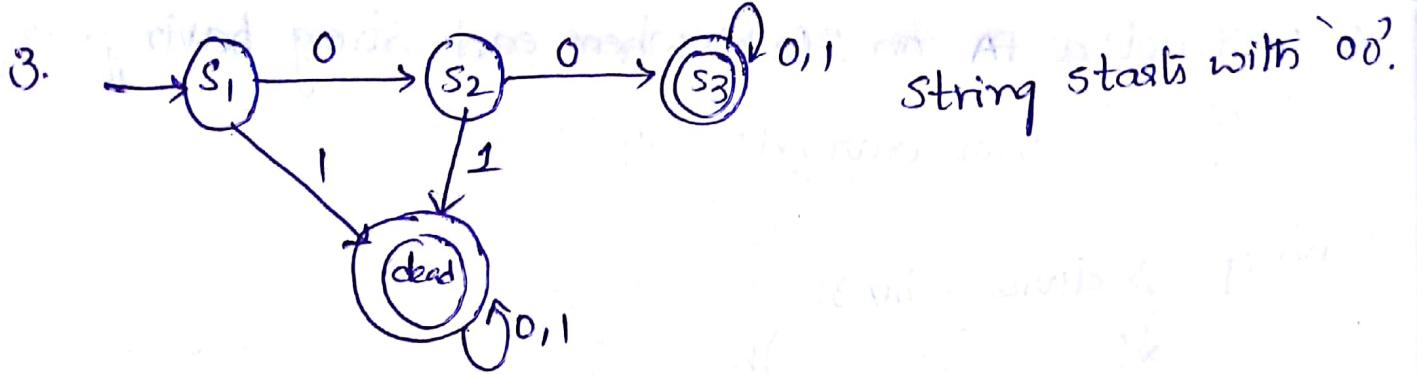
2.



The Possible Strings are:

00, 100, 001100, 0100, 1001,

String with substring



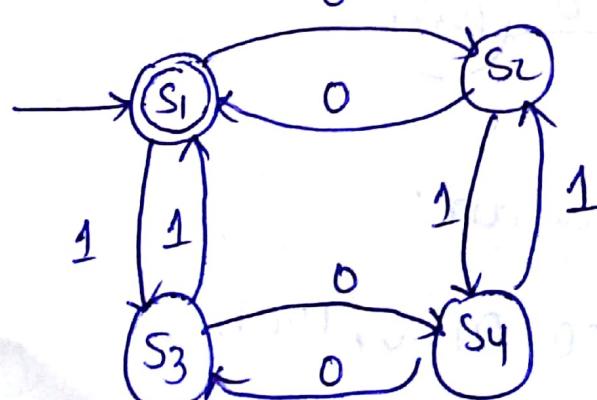
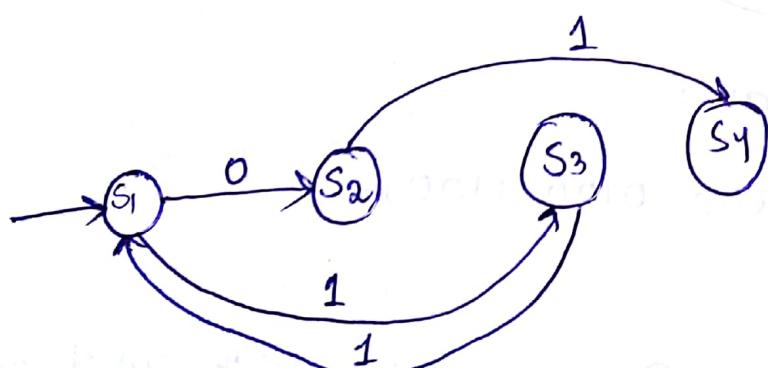
5. When set of Transitions are given:

$$\delta(s_1, 0) = s_2 \quad \delta(s_1, 1) = s_3$$

$$\delta(s_2, 0) = s_1 \quad \delta(s_2, 1) = s_4$$

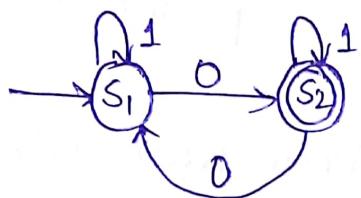
$$\delta(s_3, 0) = s_4 \quad \delta(s_3, 1) = s_1$$

$$\delta(s_4, 0) = s_3 \quad \delta(s_4, 1) = s_2$$

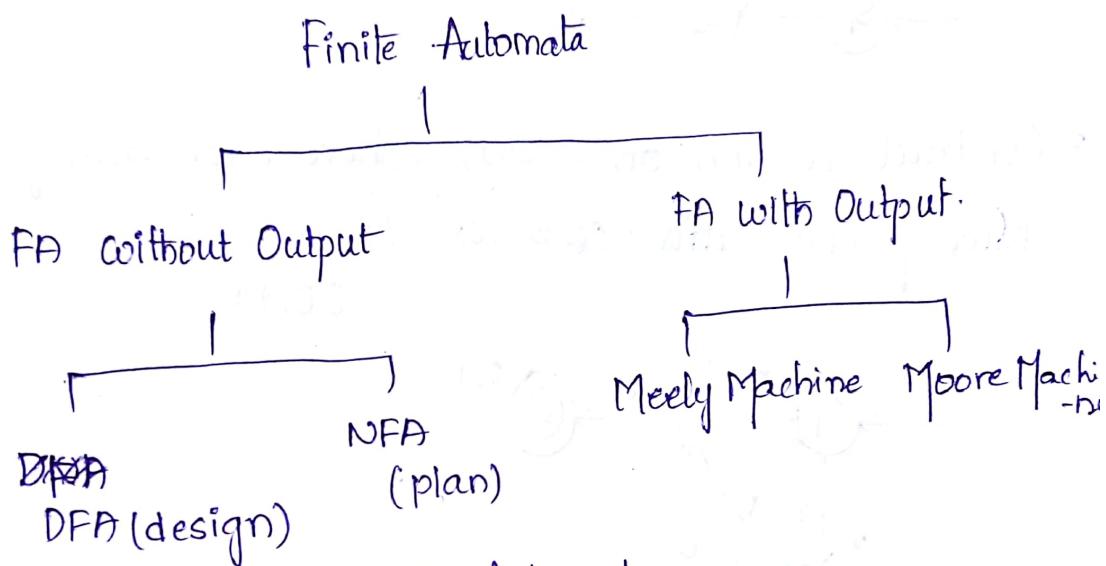


6. Construct a FA on following language

$$\mathcal{L} = 0,000,0000, \dots \text{ on } \Sigma(0,1)$$



Classification of Finite Automata:



Non-Deterministic Finite Automata:

NFA is a 5 Tuple $(Q, \Sigma, \delta, s, F)$

Q - set of states.

Σ - set of input symbols.

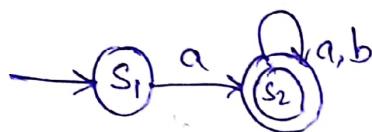
δ - Transition function $Q \times \Sigma \rightarrow 2^Q$

where 2^Q is power set of Q .

s - starting state.

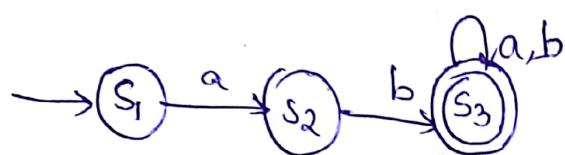
F - Final state.

1. Construct a NFA on $\Sigma(a,b)$ where each string starts with 'a'.

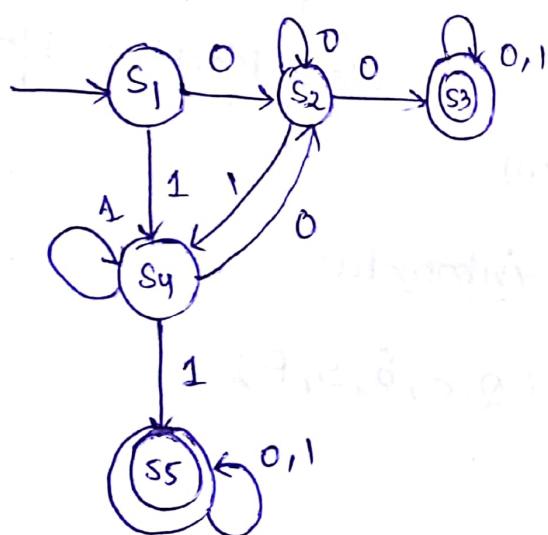


Drawing FA for possible strings avoiding not possible strings.

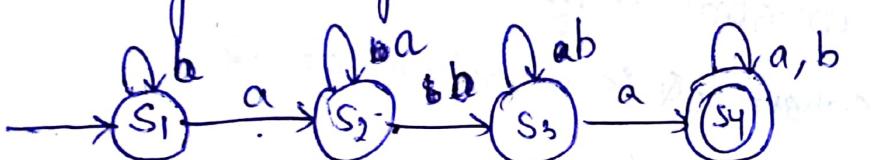
2. Construct a NFA on $\Sigma(a,b)$ where each string starts with 'ab'.



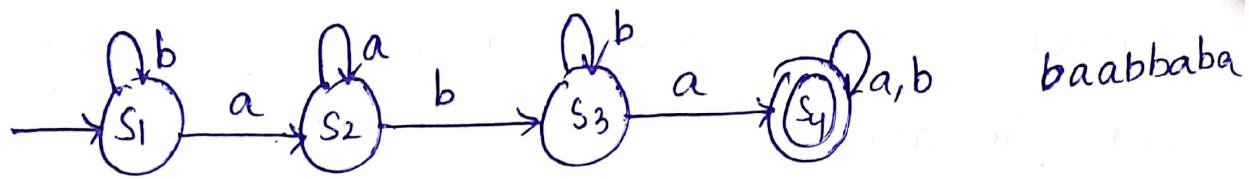
3. Construct a NFA on $\Sigma(0,1)$ where each string having consecutive '0's (00) is 00, 11,



4. construct a NFA on input symbol $\Sigma(a,b)$ where each string having aba as a substring.



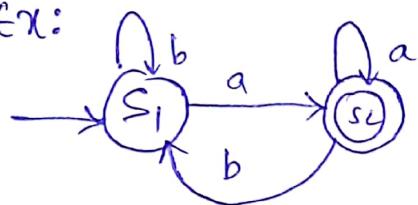
aababba



DFA

$$Q \times \Sigma \rightarrow Q$$

Ex:



$$S_1 \times a \rightarrow S_2$$

- (3) All the possibilities but not non-possibilities

(1.) Transition function.

$$Q \times \Sigma \rightarrow Q$$

NFA

$$Q \times \Sigma \rightarrow 2^Q$$

Ex:



$$2^2 = 4$$

$$S_1 \times a \rightarrow \emptyset,$$

$$\begin{matrix} \text{no. of states} \\ 2^2 = 4 \end{matrix}$$

$$S_2,$$

$$S_1, S_2$$

- (3) Here the NFA accepts all the possibilities only.

(1.) Transition function

$$Q \times \Sigma \rightarrow 2^Q$$

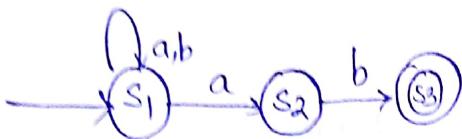
$2^Q \rightarrow$ power set of set of states.

Note: The Power of DFA and NFA are both are same.

- By default all DFA's are NFA's.

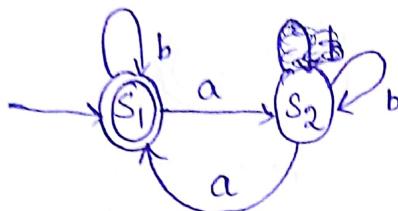
• For each NFA We can construct equivalent DFA.

1. Construct a NFA on $\Sigma(a,b)$ where each string ends with ab.

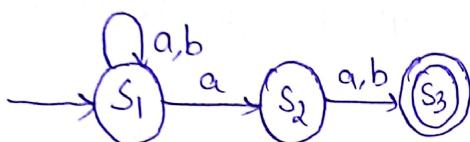


bab, ab, baab,

2. construct a NFA on $\Sigma(a,b)$ which accepts even no. of



3. construct a NFA on $\Sigma(a,b)$ where each string having last but one symbol as a

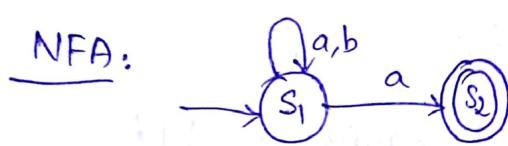


NFA to DFA Construction:-

construct a equivalent DFA for the following NFA.

(1) where each string ends with 'ab'.

Final state - s_2

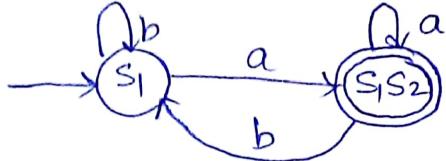


	a	b
s_1	$\{s_1, s_2\}$	s_1
s_2	$\{\}$	$\{\}$

Equivalent DFA:-

	a	b
s_1	$\{s_1, s_2\}$	s_1
$\{s_1, s_2\}$	$\{s_1, s_2\}$	\emptyset

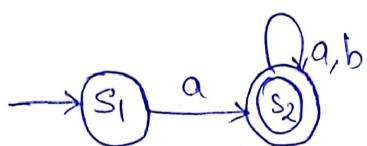
so, here consider,
FS as s_2 .



This is the Equivalent DFA.

2. Construct a Equivalent DFA for the following NFA.
where each string starts with 'a'.

NFA:-

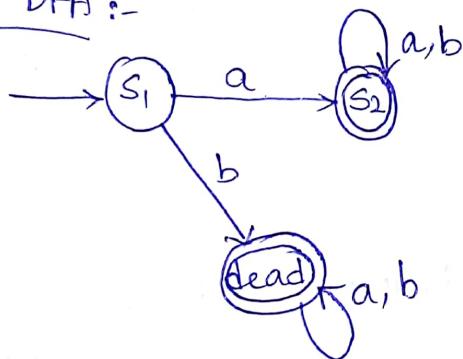


	a	b
s1	s2	{ }
s2	s2	s2

Equivalent DFA

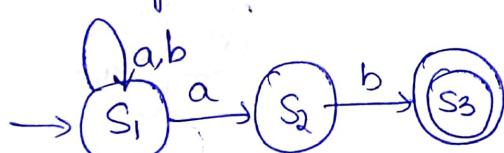
	a	b
s1	s2	dead
s2	s2	s2

Required DFA :-



3. Construct an Equivalent DFA for the following NFA.
where each string ends with 'ab'

NFA NFA:



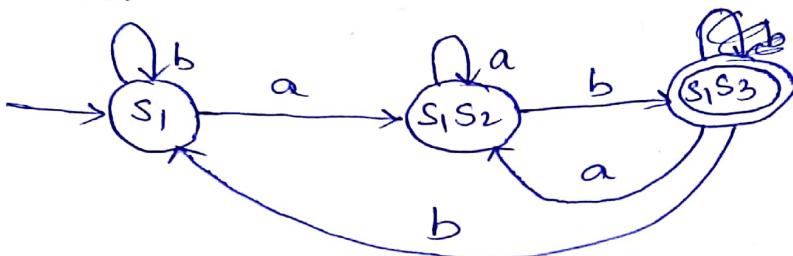
	a	b
s1	[s1, s2]	s1
s2	{ }	s3
s3	{ }	{ }

1. c Equivalent DFA :-

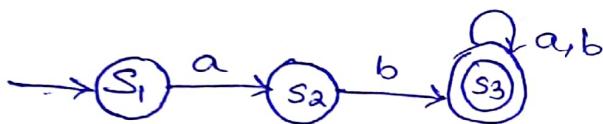
	a	b
S_1	$[S_1, S_2]$	S_1
$[S_1, S_2]$	$[S_1, S_2]$	$[S_1, S_3]$
$[S_1, S_3]$	$[S_1, S_2]$	$[S_1, S_3] - S_1$

5. Construct

2. Required DFA :-



3. If; Construct a equivalent DFA for the following NFA.

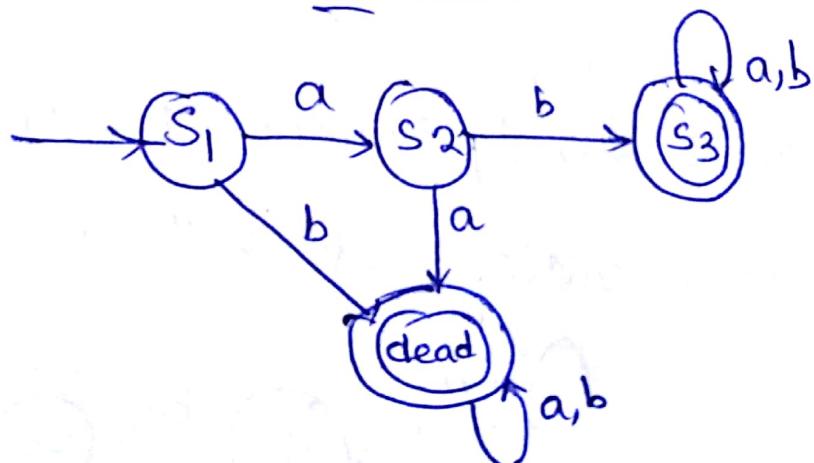


	a	b
S_1	S_2	{ } \rightarrow dead.
S_2	{ }	S_3
S_3	S_3	S_3

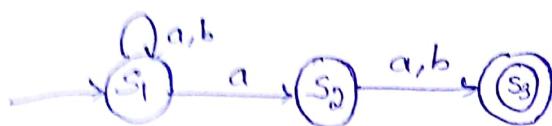
For DFA:

	a	b
S_1	S_2	dead
S_2	dead	S_3
S_3	S_3	S_3

Required DFA :-



b. Construct DFA for the following NFA.



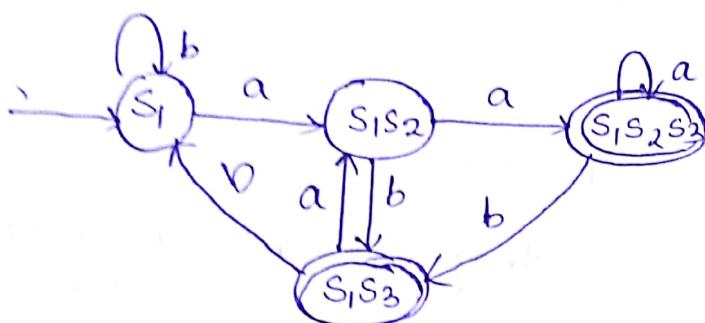
String with last but one as 'a'.

	a	b
s_1	$\{s_1 s_2\}$	s_1
s_2	s_3	s_3
s_3	$\{\}$	$\{\}$

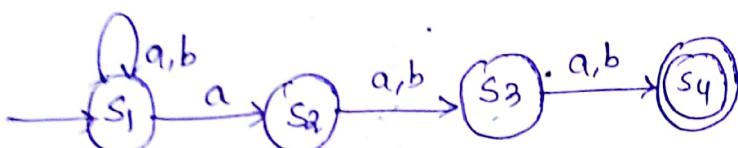
Required DFA :-

For DFA

	a	b
s_1	$\{s_1 s_2\}$	s_1
$s_1 s_2$	$\{s_1 s_2 s_3\}$	$\{s_1 s_3\}$
$s_1 s_3$	$\{s_1 s_2 s_3\}$	$s_1 s_3$
$s_2 s_3$	$\{s_1 s_2\}$	s_1



6. Construct an equivalent DFA for the following NFA.
String with left side 'a' 3rd.

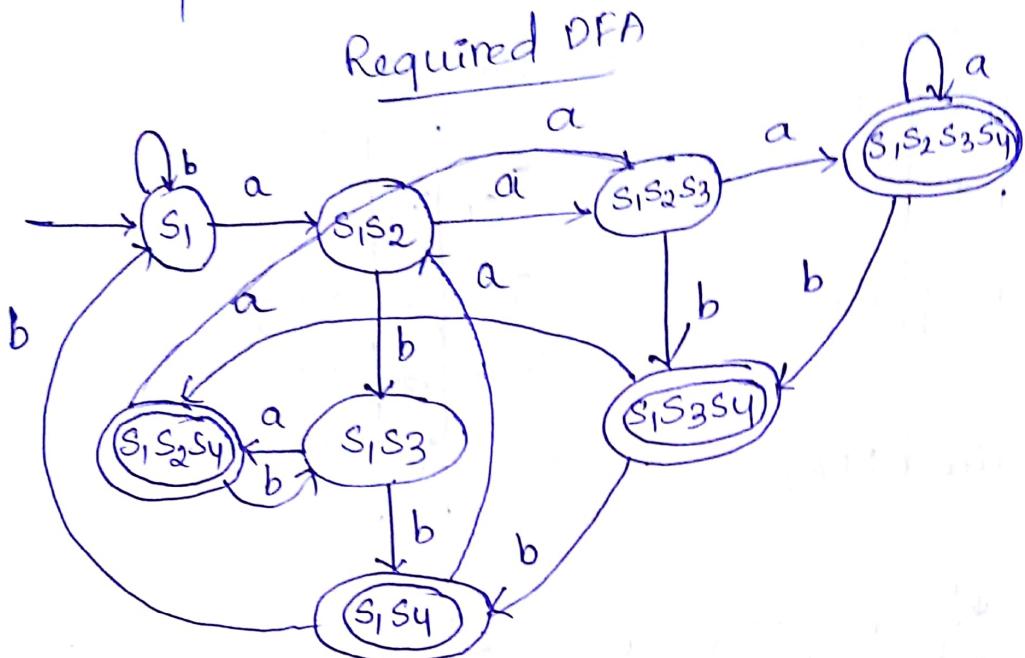


For DFA

	a	b
s_1	$\{s_1 s_2\}$	s_1
s_2	s_3	s_3
s_3	s_4	s_4
s_4	$\{\}$	$\{\}$

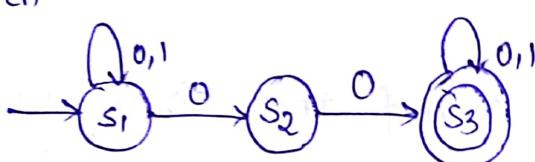
	a	b
s_1	$\{\}$	s_1
$\{s_1 s_2\}$	$\{s_1 s_2\}$	$s_1 s_3$
$\{s_1 s_2 s_3\}$	$\{s_1 s_2 s_3\}$	$s_1 s_3 s_4$
$\{s_1 s_2 s_3 s_4\}$	$\{s_1 s_2 s_3 s_4\}$	$s_1 s_3 s_4$
$\{s_1 s_3\}$	$\{s_1 s_3\}$	$s_1 s_4$
$\{s_1 s_3 s_4\}$	$\{s_1 s_3 s_4\}$	$s_1 s_4$

	a	b
$\{S_1S_2S_3\}$	S_2S_3	S_1, S_3
S_1S_4	S_1S_2	S_1



7. Construct Equivalent DFA for the following NFA.
string with two consecutive zeros

Given

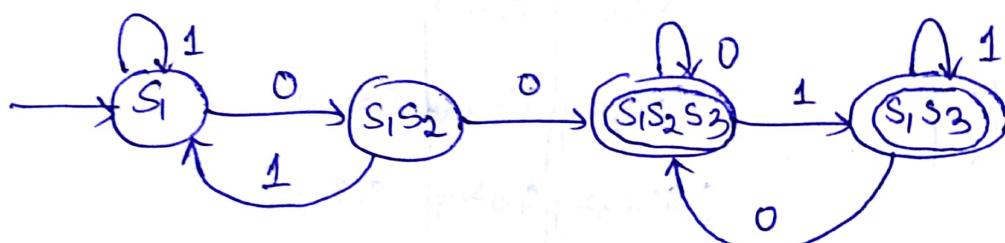


DFA

	0	1
S_1	$[S_1S_2]$	S_1
S_2	S_3	$\{\}$
S_3	S_3	S_3

	0	1
S_1	$[S_1S_2]$	S_1
S_2	$S_1S_2S_3$	S_1
S_3	$S_1S_2S_3$	S_1, S_3
S_4	$S_1S_2S_3$	S_1, S_3

Required DFA :-



E-NFA | Epsilon = 0FFA:

E-NFA is a 5-Tuple $(Q, \Sigma, \delta, S, F)$

$Q \rightarrow$ set of states.

$\Sigma \rightarrow$ set of alphabets.

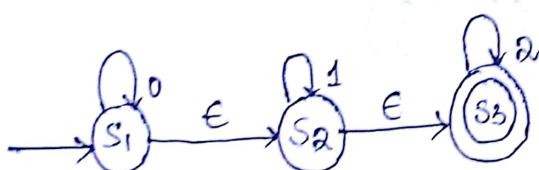
$\delta \rightarrow$ transition function on set of alphabets

$$Q \times (\Sigma \cup \epsilon) \rightarrow 2^Q$$

$S \rightarrow$ starting state.

$F \rightarrow$ final state.

1. construct a equivalent NFA for the following E-NFA



$\epsilon\text{-closer}(S_1) =$ (without reading any alphabet from S_1 , what states u can reach).

$$\epsilon\text{-closer}(S_1) = \{S_1, S_2, S_3\}$$

$S \rightarrow E\text{-NFA}$

$$\epsilon\text{-closer}(S_2) = \{S_2, S_3\}$$

$S' \rightarrow NFA$

$$\epsilon\text{-closer}(S_3) = \{S_3\}$$

$$S'(S_1, Q) = \epsilon\text{-closer}(\delta(\epsilon\text{-closure}(S_1)), Q)$$

$$S'(S_1, Q) = \epsilon\text{-closer}(\delta(S_1, S_2, S_3), Q)$$

$$= \epsilon\text{-closer}(S_1, \emptyset, \emptyset)$$

$$= \epsilon\text{-closer}(S_1)$$

$$= \{S_1, S_2, S_3\}$$

$$\delta'(s_1, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_1)), 1)$$

$$= \epsilon\text{-closure}(\delta(s_1, s_2, s_3), 1)$$

$$= \epsilon\text{-closure}(\{\}, s_2, \{\})$$

$$= \epsilon\text{-closure}(s_2)$$

$$= \{s_2, s_3\}$$

$$\delta'(s_2, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_2)), 0)$$

$$= \epsilon\text{-closure}(\delta(s_2, s_3), 0)$$

$$= \epsilon\text{-closure}(\{\}, \{\})$$

$$= \emptyset$$

$$\delta'(s_2, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_2)), 1)$$

$$= \epsilon\text{-closure}(\delta(s_2, s_3), 1)$$

$$= \epsilon\text{-closure}(s_2, \{\})$$

$$= \{s_2, s_3\}$$

$$\delta'(s_3, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_3)), 0)$$

$$= \epsilon\text{-closure}(\delta(s_3), 0)$$

$$= \epsilon\text{-closure}(\{\})$$

$$= \emptyset$$

$$\delta'(s_3, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_3)), 1)$$

$$= \epsilon\text{-closure}(\delta(s_3), 1)$$

$$= \emptyset$$

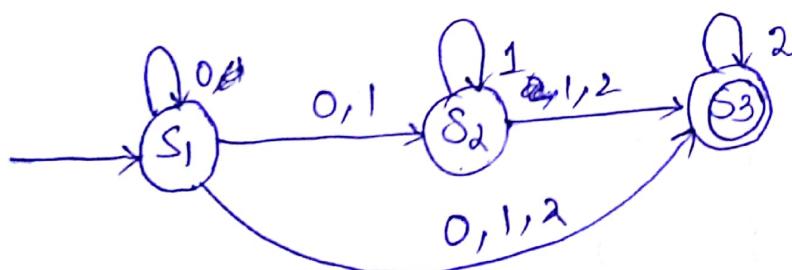
$$\begin{aligned}
 S'(s_1, 2) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_1)), 2) \\
 &= \epsilon\text{-closure}(\delta(s_1, s_2, s_3), 2) \\
 &= \epsilon\text{-closure}(\{s_3\}, \{s_3\}, s_3) \\
 &= \{s_3\}
 \end{aligned}$$

$$\begin{aligned}
 S'(s_2, 2) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_2)), 2) \\
 &= \epsilon\text{-closure}(\delta(s_2, s_3), 2) \\
 &= \{s_3\}
 \end{aligned}$$

$$\begin{aligned}
 S'(s_3, 2) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_3)), 2) \\
 &= \epsilon\text{-closure}(\delta(s_3), 2) \\
 &= \{s_3\}
 \end{aligned}$$

$$\begin{array}{lll}
 S'(s_1, 0) = \{s_1, s_2, s_3\} & S'(s_2, 0) = \{\emptyset\} & S'(s_3, 0) = \{\emptyset\} \\
 S'(s_1, 1) = \{s_2, s_3\} & S'(s_2, 1) = \{s_2, s_3\} & S'(s_3, 1) = \{s_3\} \\
 S'(s_1, 2) = \{s_3\} & S'(s_2, 2) = \{s_3\} & S'(s_3, 2) = s_3
 \end{array}$$

Equivalent NFA:

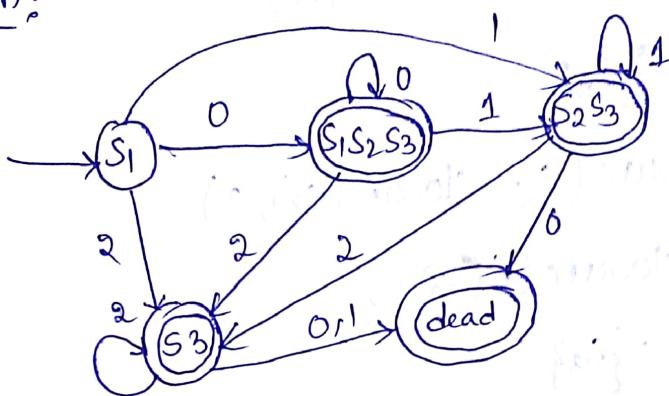


	0	1	2	NFA:
s1	{s1, s2, s3}	s2, s3	s3	
s2	∅	s2, s3	s3	
s3	∅	∅	s3	

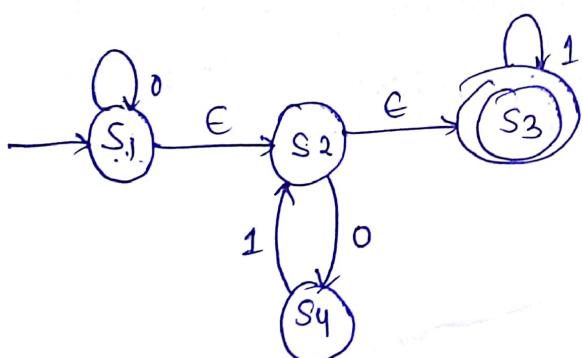
Equivalent DFA:

	0	1	2
s_1	$[s_1 s_2 s_3]$	$[s_2 s_3]$	s_3
$[s_1 s_2 s_3]$	$[s_1 s_2 s_3]$	$[s_2 s_3]$	s_3
$[s_2 s_3]$	dead	$[s_2 s_3]$	s_3
s_3	dead	dead	s_3

DFA:



2. Construct an equivalent NFA for the following ϵ -NFA.



$$\epsilon\text{-closure}(S_1) = \{S_1, S_2, S_3\}$$

$$\epsilon\text{-closure}(S_2) = \{S_2, S_3\}$$

$$\epsilon\text{-closure}(S_3) = \{S_3\}$$

$$\epsilon\text{-closure}(S_4) = \{S_4\}$$

$$s'(s_1, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_1)), 0)$$

$$= \epsilon\text{-closure}(\delta(s_1, s_2, s_3), 0)$$

$$= \epsilon\text{-closure}(s_1, s_4, \{\})$$

$$= \{s_1, s_2, s_3, s_4\}$$

$$s'(s_1, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_1)), 1)$$

$$= \epsilon\text{-closure}(\delta(s_1, s_2, s_3), 1)$$

$$= \epsilon\text{-closure}(\{\}, \{\}, s_3)$$

$$= \{s_3\}$$

$$s'(s_2, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_2)), 0)$$

$$= \epsilon\text{-closure}(\delta(s_2, s_3), 0)$$

$$= \epsilon\text{-closure}(s_4, \{\})$$

$$= \{s_4\}$$

$$s'(s_2, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_2)), 1)$$

$$= \epsilon\text{-closure}(\delta(s_2, s_3), 1)$$

$$= \epsilon\text{-closure}(\{\}, s_3)$$

$$= \{s_3\}$$

$$s'(s_3, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_3)), 0)$$

$$= \epsilon\text{-closure}(\delta(s_3), 0)$$

$$= \{\} = \emptyset$$

$$\begin{aligned}
 \delta^1(s_3, 1) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_3)), 1) \\
 &= \epsilon\text{-closure}(\delta(s_3), 1) \\
 &= \epsilon\text{-closure}(s_3) \\
 &= \{s_3\}
 \end{aligned}$$

$$\begin{aligned}
 \delta^1(s_4, 0) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_4)), 0) \\
 &= \epsilon\text{-closure}(\delta(s_4), 0) \\
 &= \{\}
 \end{aligned}$$

$$\begin{aligned}
 \delta^1(s_4, 1) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_4)), 1) \\
 &= \epsilon\text{-closure}(\delta(s_4), 1) \\
 &= \epsilon\text{-closure}(s_2) \\
 &= \{s_2, s_3\}
 \end{aligned}$$

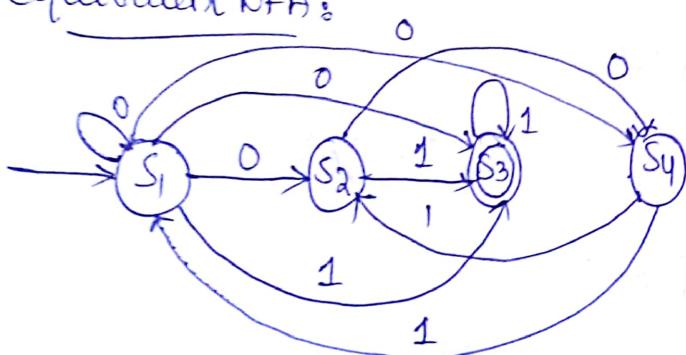
$$\delta^1(s_1, 0) = \{s_1, s_2, s_3, s_4\} \quad \delta^1(s_3, 0) = \emptyset$$

$$\delta^1(s_1, 1) = \{s_3\} \quad \delta^1(s_3, 1) = \{s_3\}$$

$$\delta^1(s_2, 0) = \{s_4\} \quad \delta^1(s_4, 0) = \{\}$$

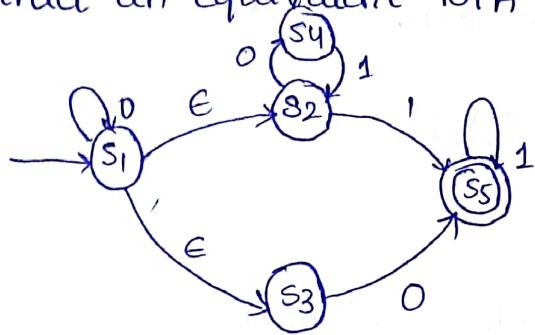
$$\delta^1(s_2, 1) = \{s_3\} \quad \delta^1(s_4, 1) = \{s_1, s_3\}$$

Equivalent NFA:



	0	1
s1	{s1, s2, s3, s4}	{s3}
s2	s4	s3
s3	{s3}	s3
s4	{s3}	{s1, s2}

3. Construct an Equivalent NFA for the following ϵ -NFA.



$$\epsilon\text{-closure}(S_1) = \{S_1, S_2, S_3\}$$

$$\epsilon\text{-closure}(S_2) = \{S_2\}$$

$$\epsilon\text{-closure}(S_3) = \{S_3\}$$

$$\epsilon\text{-closure}(S_4) = \{S_4\}$$

$$\epsilon\text{-closure}(S_5) = \{S_5\}$$

$$S'(S_1, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(S_1)), 0)$$

$$= \epsilon\text{-closure}(\delta(S_1, S_2, S_3), 0)$$

$$= \epsilon\text{-closure}(S_1, S_4, \overline{\{S_3\}})$$

$$= \{S_1, S_2, S_3, S_4\}_{S_5}$$

$$S'(S_1, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(S_1)), 1)$$

$$= \epsilon\text{-closure}(\delta(S_1, S_2, S_3), 1)$$

$$= \epsilon\text{-closure}(\{\}, S_5, \{\})$$

$$= \{S_5\}$$

$$S'(S_2, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(S_2)), 0)$$

$$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(S_2)), 0)$$

$$= \epsilon\text{-closure}(S_4)$$

$$= \{S_4\}$$

$$\delta^1(s_2, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_2)), 1)$$

$$= \epsilon\text{-closure}(\delta(s_2), 1)$$

$$= \{s_5\}$$

$$s^1(s_3, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_3)), 0)$$

$$= \epsilon\text{-closure}(\delta(s_3), 0)$$

$$= \{s_5\}$$

$$\delta^1(s_3, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_3)), 1)$$

$$= \epsilon\text{-closure}(\delta(s_3), 1)$$

$$= \{\} = \emptyset$$

$$s^1(s_4, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_4)), 0)$$

$$= \epsilon\text{-closure}(\delta(s_4), 0)$$

$$= \{\}$$

$$s^1(s_4, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_4)), 1)$$

$$= \epsilon\text{-closure}(\delta(s_4), 1)$$

$$= \{s_2\}$$

$$s^1(s_5, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_5)), 0)$$

$$= \epsilon\text{-closure}(\delta(s_5), 0)$$

$$= \{\}$$

$$s^1(s_5, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(s_5)), 1)$$

$$= \epsilon\text{-closure}(\delta(s_5), 1)$$

$$= \{s_5\}$$

$$\delta^1(s_1, 0) = \{s_1, s_2, s_3, s_4, s_5\} \quad \delta^1(s_3, 0) = \{s_5\}$$

$$\delta^1(s_2, 1) = \{s_5\}$$

$$\delta^1(s_3, 1) = \{\emptyset\}$$

$$\delta^1(s_4, 0) = \{\emptyset\}$$

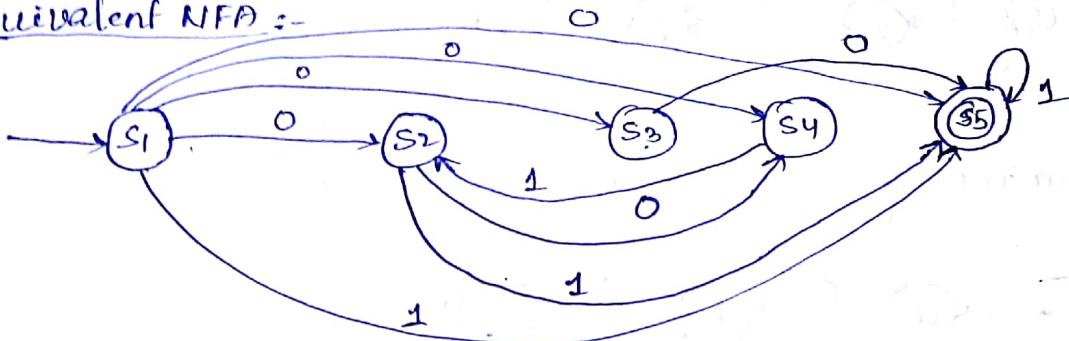
$$\delta^1(s_4, 1) = \{s_2\}$$

$$\delta^1(s_2, 1) = \{s_5\}$$

$$\delta^1(s_5, 0) = \{\}\}$$

$$\delta^1(s_5, 1) = \{s_5\}$$

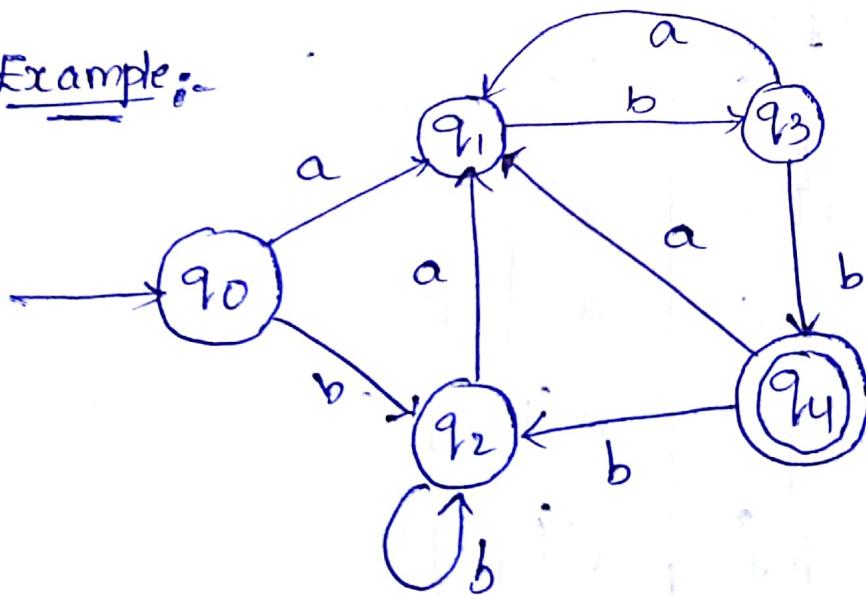
-equivalent NFA :-

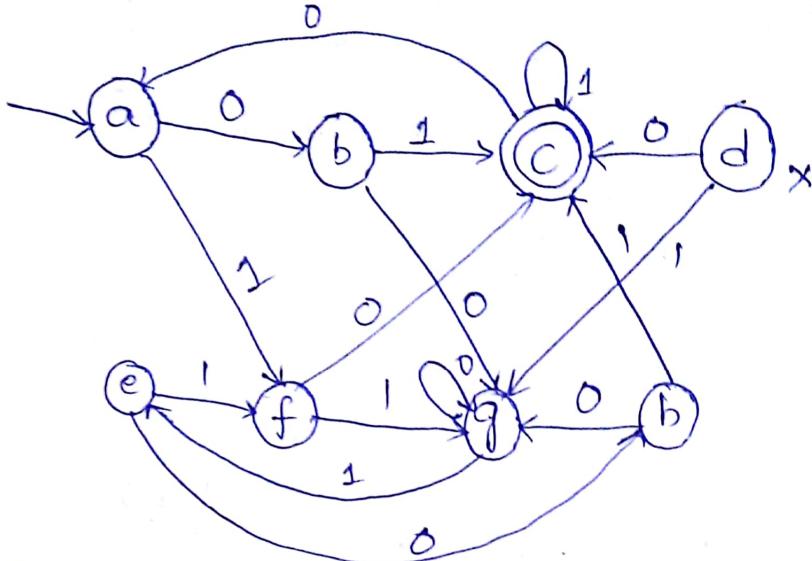


Minimisation of DFA:

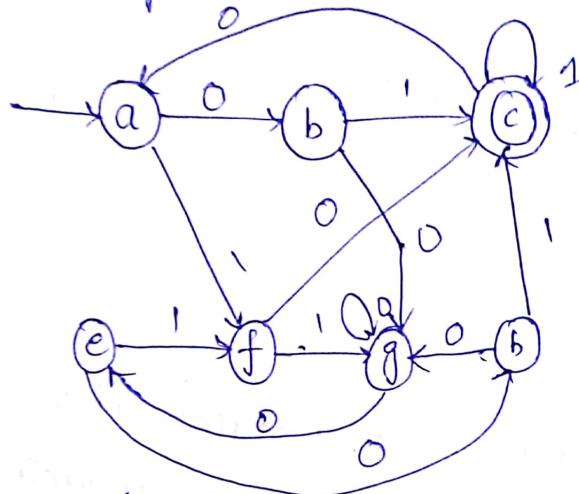
IF a DFA having more no. of states then we can construct equivalent minimal DFA by reducing no. of states

Example:-





By eliminating unreachable states like 'd' in above,



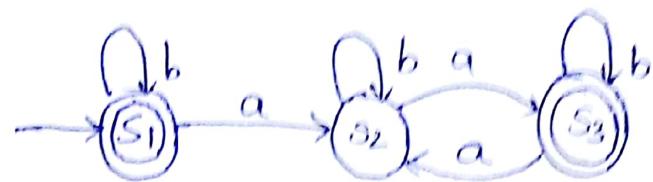
$$\Pi_0 = \left\{ \begin{array}{l} \text{non Final} \\ a, b, e, g, h, f \end{array} \right\}$$

$$\left\{ \begin{array}{l} \text{Final} \\ c \end{array} \right\}$$

$$\Pi_1 = \{a, e, g\}$$

	0	1
a	bq	fc ₁
b	qc ₁	cc ₂
c	a	c
e	hc ₁	fc ₁
f	cq ₂	qc ₁
g	g	e
h	g	c

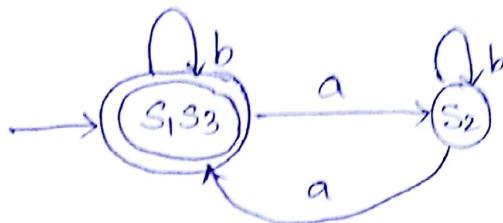
1.



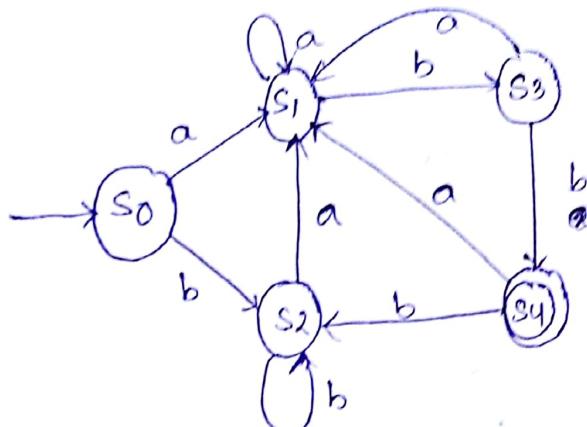
$$\Pi_0 = \{S_2\} \quad \{S_1, S_3\}$$

	a	b
S_1	S_2	S_1
S_2	S_3	S_2
S_3	S_2	S_3

$$\Pi_1 = \{S_2\} \quad \{S_1, S_3\}$$



2.



$$\Pi_B = \underbrace{\{S_0, S_1, S_2, S_3\}}_{C_1} \cup \underbrace{\{S_4\}}_{C_2}$$

	a	b	c1	c2
S_0	S_1	S_2	C_1	C_1 C_2
S_1	S_1	S_3	C_1	
S_2	S_1	S_2	C_1	
S_3	S_1	S_4	C_2	
S_4	S_1	S_2		

$$\Pi_1 = \underbrace{\{S_0, S_1, S_2\}}_{C_1} \quad \{S_3\} \quad \{S_4\}$$

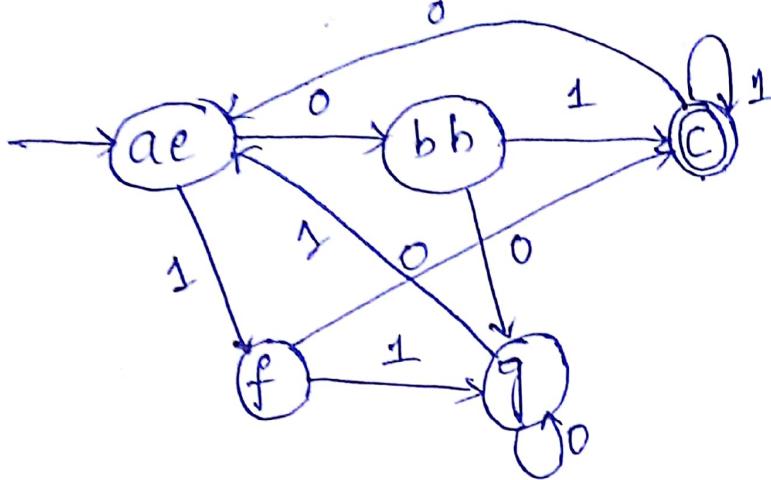
$$\Pi_2 = \underbrace{\{S_0, S_2\}}_{C_1} \quad \{S_1\} \quad \{S_3\} \quad \{S_4\}$$

$$\Pi_3 = \{S_0, S_2\} \quad \{S_1\} \quad \{S_3\} \quad \{S_4\}$$

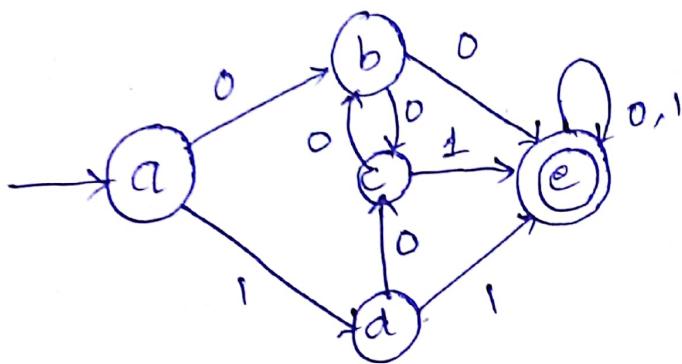
$$\therefore \Pi_2 = \Pi_B$$

Stop The Process.

Minimal Finite Automata:



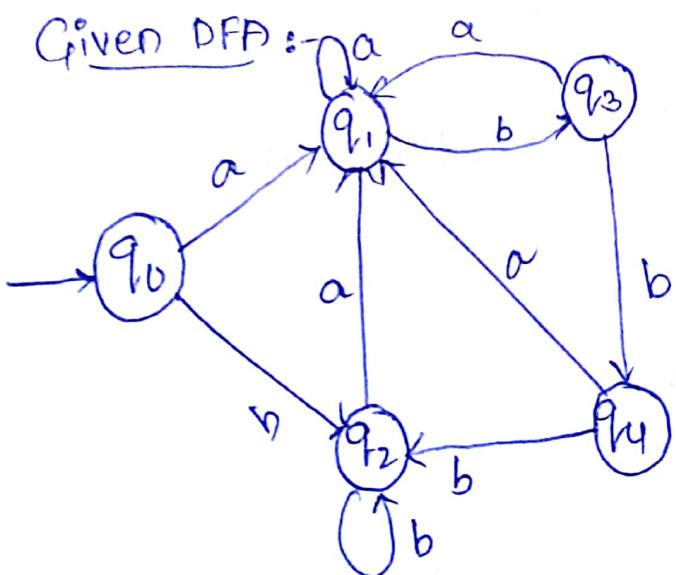
4.



Myhill Nerode theorems: (Table Filling Method).

* Construct the Equivalent Minimal DFA for the
Following DFA by using Myhill Nerode theorem.

Given DFA:



	q_0	q_1	q_2	q_3
q_0				
q_1	✓			
q_2		✓		
q_3	✓	✓	✓	
q_4	✓	✓	✓	✓

$$\begin{aligned} s(q_1, a) &= q_1 \\ s(q_0, a) &= q_1 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

$$\begin{aligned} s(q_1, b) &= q_3 \\ s(q_0, b) &= q_3 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

$$\begin{aligned} s(q_2, a) &= q_1 \\ s(q_0, a) &= q_1 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

$$\begin{aligned} s(q_2, b) &= q_2 \\ s(q_0, b) &= q_2 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

$$\begin{aligned} s(q_1, a) &= q_1 \\ s(q_2, a) &= q_1 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

$$\begin{aligned} s(q_1, b) &= q_3 \\ s(q_2, b) &= q_2 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

Compare q_3, q_0

$$\begin{aligned} s(q_3, a) &= q_1 \\ s(q_0, a) &= q_1 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

$$\begin{aligned} s(q_3, b) &= q_4 \\ s(q_0, b) &= q_2 \end{aligned} \quad \left. \begin{array}{l} \text{Mark} \\ \text{the} \end{array} \right\}$$

Compare q_3, q_1

$$\begin{aligned} s(q_3, a) &= q_1 \\ s(q_1, a) &= q_1 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

$$\begin{aligned} s(q_3, b) &= q_4 \\ s(q_1, b) &= q_3 \end{aligned} \quad \left. \begin{array}{l} \text{Mark} \\ \text{the} \end{array} \right\}$$

Compare q_2, q_3

$$\begin{aligned} s(q_3, a) &= q_1 \\ s(q_2, a) &= q_1 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

$$\begin{aligned} s(q_3, b) &= q_4 \\ s(q_2, b) &= q_2 \end{aligned} \quad \left. \begin{array}{l} \text{Mark} \\ \text{the} \end{array} \right\}$$

Compare q_1, q_0

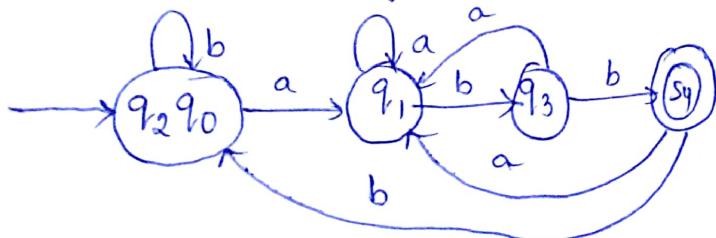
$$\begin{aligned} s(q_1, a) &= q_1 \\ s(q_0, a) &= q_1 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

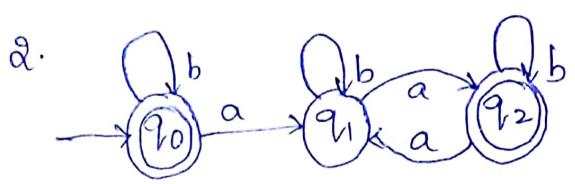
$$\begin{aligned} s(q_1, b) &= q_3 \\ s(q_0, b) &= q_3 \end{aligned}$$

Compare q_2, q_0

$$\begin{aligned} s(q_2, a) &= q_1 \\ s(q_0, a) &= q_1 \end{aligned} \quad \left. \begin{array}{l} \text{by} \\ \text{rule} \end{array} \right\}$$

\therefore The States are $\{q_2, q_0\}, \{q_1\}, \{q_3\}, \{q_4\}$





compare q_2, q_0

$$\delta(q_0, a) = q_1 \quad \boxed{}$$

$$\delta(q_2, a) = q_1 \quad \boxed{}$$

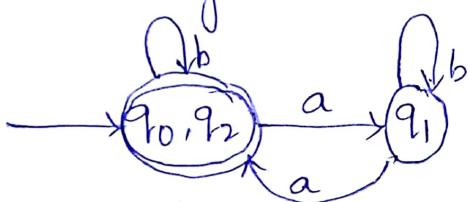
$$\delta(q_0, b) = q_2$$

$$\delta(q_2, b) = q_2$$

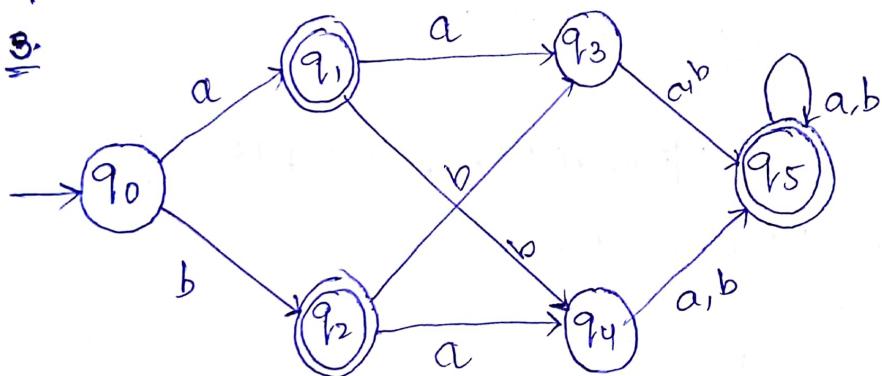
q_0	q_1	q_2
q_0		
q_1	✓	
q_2		✓

$$\therefore \{q_1\} \{q_2, q_0\} \quad \boxed{\text{F}}$$

The state diagram



* 3.



q_0	q_1	q_2	q_3	q_4	q_5
q_0					
q_1	✓				
q_2	✓				
q_3	✓	✓	✓		
q_4	✓	✓	✓	✓	
q_5					✓

Compare q_2, q_1

$$\begin{aligned}\delta(q_1, a) &= q_3 \\ \delta(q_2, a) &= q_4\end{aligned}\} \text{ No need}$$

$$\begin{aligned}\delta(q_1, b) &= q_4 \\ \delta(q_2, b) &= q_3\end{aligned}\} \text{ No need}$$

Compare q_3, q_0

$$\begin{aligned}\delta(q_0, a) &= q_1 \\ \delta(q_3, a) &= q_5\end{aligned}\} \text{ No mark}$$

$$\begin{aligned}\delta(q_0, b) &= q_2 \\ \delta(q_3, b) &= q_5\end{aligned}\} \text{ No mark}$$

Compare q_5, q_1

$$\begin{aligned}\delta(q_2, a) &= q_1 \\ \delta(q_5, a) &= q_5\end{aligned}\} \text{ No mark}$$

$$\begin{aligned}\delta(q_2, b) &= q_3 \\ \delta(q_5, b) &= q_1\end{aligned}\} \text{ No mark}$$

Compare q_4, q_0

$$\begin{aligned}\delta(q_0, a) &= q_1 \\ \delta(q_4, a) &= q_5\end{aligned}\} \text{ No mark}$$

$$\begin{aligned}\delta(q_0, b) &= q_2 \\ \delta(q_4, b) &= q_5\end{aligned}\} \text{ No mark}$$

Compare q_4, q_3

$$\begin{aligned}\delta(q_3, a) &= q_5 \\ \delta(q_4, a) &= q_5\end{aligned}\} \text{ No mark}$$

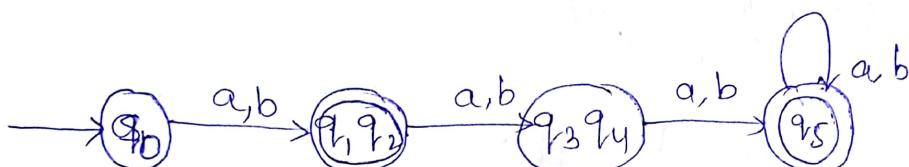
$$\begin{aligned}\delta(q_3, b) &= q_5 \\ \delta(q_4, b) &= q_5\end{aligned}\} \text{ No mark}$$

Compare q_5, q_1

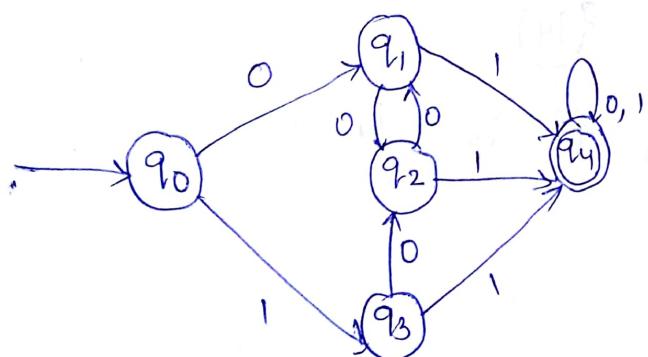
$$\begin{aligned}\delta(q_1, a) &= q_3 \\ \delta(q_5, a) &= q_5\end{aligned}\} \text{ No mark}$$

$$\begin{aligned}\delta(q_5, b) &= q_5 \\ \delta(q_1, b) &= q_4\end{aligned}\} \text{ No mark}$$

$$\therefore \{q_0, \{q_1, q_2\}, \{q_4, q_3\}, \{q_5\}\}$$



4. Construct Equivalent Minimal DFA for the following DFA by using Myhill Neard theorem.



	q_0	q_1	q_2	q_3	q_4
q_0					
q_1		✓			
q_2	✓		.	.	
q_3	✓		.	.	
q_4	✓	✓	✓	✓	✓

$S(q_{10})$

$q_4, q_1 q_3, q_2 q_3, q_2 q_1$

$q_4, q_1 q_2 q_3, q_0$

Compare q_0, q_1

$$S(q_{0,0}) = q_1 \quad \}$$

$$S(q_{1,0}) = q_2 \quad \}$$

$$S(q_{1,1}) = q_4 \quad \}$$

$$S(q_{0,1}) = q_3 \quad \}$$

Compare q_0, q_2

$$S(q_{0,0}) = q_1 \quad \}$$

$$S(q_{2,0}) = q_1 \quad \}$$

$$S(q_{2,1}) = q_4 \quad \}$$

$$S(q_{0,1}) = q_3 \quad \}$$

Compare q_1, q_2

$$S(q_{1,0}) = q_2 \quad \}$$

$$S(q_{2,0}) = q_1 \quad \}$$

$$S(q_{1,1}) = q_4 \quad \}$$

$$S(q_{2,1}) = q_4 \quad \}$$

Compare q_3, q_0

$$S(q_{0,0}) = q_1 \quad \}$$

$$S(q_{3,0}) = q_4 \quad \}$$

$$S(q_{0,1}) = q_3 \quad \}$$

$$S(q_{3,1}) = q_4 \quad \}$$

Compare q_1, q_3

$$S(q_{1,0}) = q_2 \quad \}$$

$$S(q_{3,0}) = q_4 \quad \}$$

$$S(q_{1,1}) = q_4 \quad \}$$

$$S(q_{3,1}) = q_4 \quad \}$$

Compare q_2, q_3

$$S(q_{2,0}) = q_1 \quad \}$$

$$S(q_{3,0}) = q_2 \quad \}$$

$$S(q_{2,1}) = q_4 \quad \}$$

$$S(q_{3,1}) = q_4 \quad \}$$

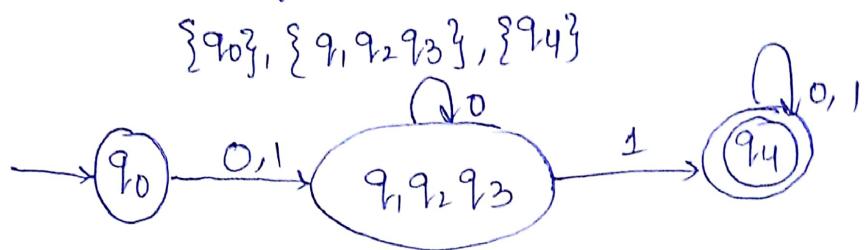
$$\therefore \{q_0\}, \{q_1, q_2\}, \{q_2, q_3\} \{q_1\}, \{q_4\}$$



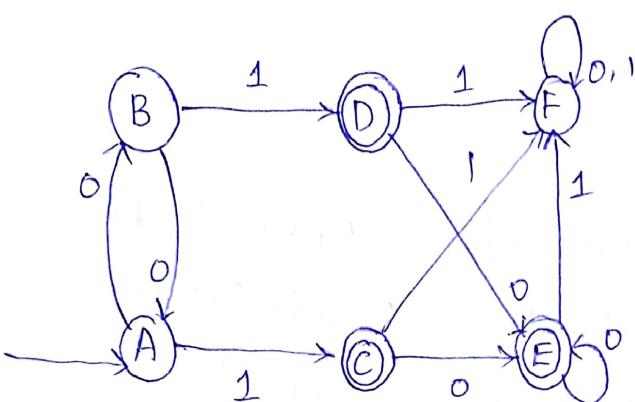
$q_1 q_2 q_3$

$$\therefore \{q_0\}, \{q_1, q_2, q_3\} \{q_4\}$$

The state diagram is:



5. Construct an equivalent Minimal DFA for the following DFA.



	A	B	C	D	E	F
A						
B						
C	✓		✓			
D	✓		✓			
E	✓		✓		✓	
F	✓		✓	✓	✓	✓

Compare A, B

$$\delta(A, 0) = B \\ \delta(B, 0) = A$$

$$\delta(A, 1) = C \\ \delta(B, 1) = D$$

Compare D, C

$$\delta(D, 0) = E \\ \delta(C, 0) = E$$

$$\delta(D, 1) = F \\ \delta(C, 1) = F$$

Compare E, C

$$\delta(E, 0) = E \\ \delta(C, 0) = E$$

$$\delta(E, 1) = F \\ \delta(C, 1) = F$$

Compare E, D

$$\delta(E, 0) = E \\ \delta(D, 0) = E$$

$$\delta(E, 1) = F \\ \delta(D, 1) = E$$

Compare A, F	Compare B, F
$s(A, 0) = B$	$s(B, 0) = F$
$s(F, 0) = F$	$s(F, 0) = F$
$s(A, 1) = C$	$s(B, 1) = D$
$s(F, 1) = F$	$s(F, 1) = F$

FINITE AUTOMATA WITH NO OUTPUTS

Moore Machine:

Purpose: Counting

Output of Moore Machine depends on present state. Moore Machine defined as 6-tuple

$$(Q, \Sigma, \Delta, \delta, \lambda, q_0)$$

$Q \rightarrow$ set of states

$\Sigma \rightarrow$ set of input alphabet

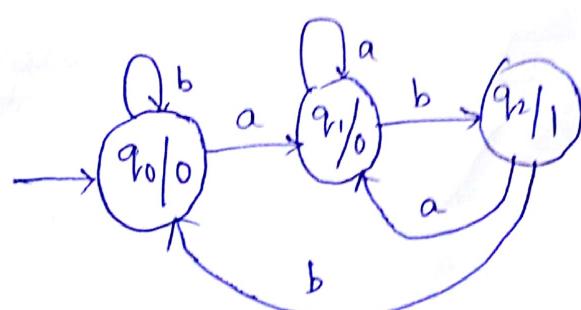
$\Delta \rightarrow$ set of Output alphabet

$\delta \rightarrow$ set of Transitions having $Q \times \Sigma \rightarrow Q$

$\lambda \rightarrow$ set of Transitions Mapping $Q \rightarrow \Delta$

$q_0 \rightarrow$ starting state.

(1) Construct a Moore Machine on $\Sigma(a, b)$ and which produces 1 for each sequence of ab otherwise produces 0.



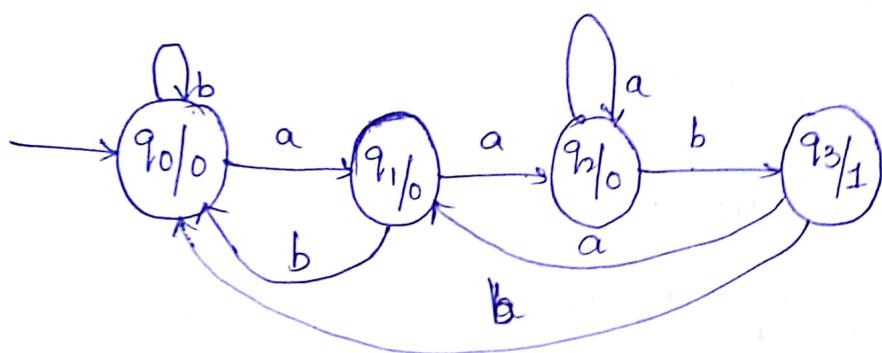
	$\Sigma(a,b)$		$\Delta(0,1)$
	a	b	
q_0	q_1	q_0	0
q_1	q_1	q_2	0
q_2	q_1	q_0	1

$Q = (q_0, q_1, q_2)$

$\Sigma = (a, b)$

q_0 - starting state.

- Q. Construct a MM which produces 1 for sequence of aab
and produces 0 Otherwise.



	$\Sigma(a,b)$		$\Delta(0,1)$
	a	b	
q_0	q_1	q_0	0
q_1	q_2	q_0	0
q_2	q_2	q_3	0
q_3	q_1	q_0	1

$Q = (q_0, q_1, q_2, q_3)$

$\Sigma = (a, b)$

q_0 - starting state.

Mealy Machine :-

Mealy Machine Produces Output based on present state and present input.

Mealy Machine is defined as 6-tuple.

$$(Q, \Sigma, \Delta, S, \lambda, q_0)$$

$Q \rightarrow$ set of states.

$\Sigma \rightarrow$ set of input alphabets.

$\Delta \rightarrow$ set of output alphabets.

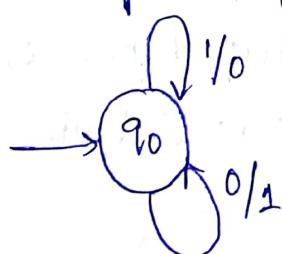
$S \rightarrow$ transition Mapping $Q \times \Sigma \rightarrow Q$

$\lambda \rightarrow$ transition Mapping $Q \times \Sigma \rightarrow \Delta$

q_0 - starting state.

1. Construct a Mealy Machine which produces Complement

of a given binary string

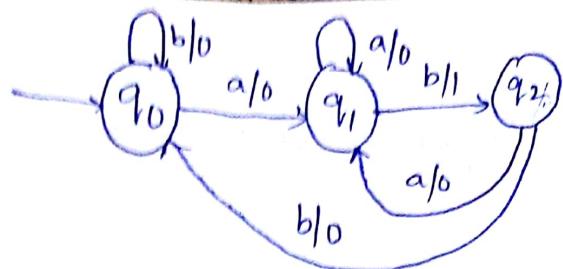


1100

0011

	$\Sigma(0,1)$		$\Delta(0,1)$	
	0	1	0	1
q_0	q_0	q_0	1	0

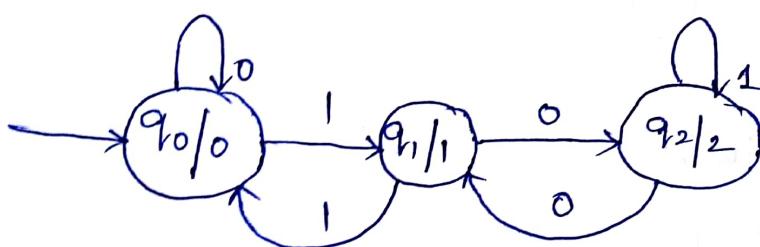
2. Construct Mealy Machine which Counts Occurrences of ab-



	$\Sigma(0,1)$	$\Delta(0,1)$	
	0 1	a b	
q_0	$q_1 \ q_0$	0 0	
q_1	$q_1 \ q_2$	0 1	
q_2	$q_1 \ q_0$	0 0	101 D11

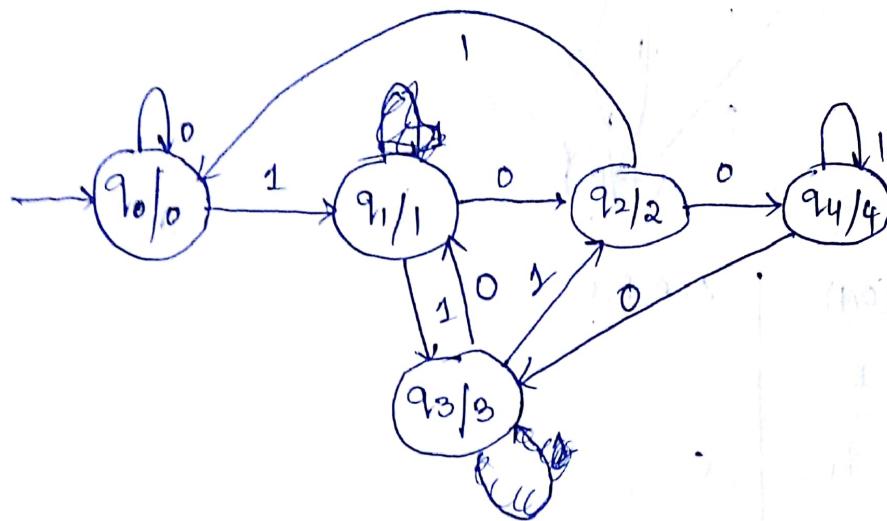
3. Construct a Mealy Machine which produces 2's complement for a given input binary string.

4. Construct a Moore Machine that takes 0,1 as input and produces residue Modulo 3 as Output



	$\Sigma(0,1)$	Output	
	0 1	$\Delta(0,1)$	$Q = \{q_0, q_1, q_2\}$
q_0	$q_0 \ q_1$	0 1	$\Sigma = (0,1)$
q_1	$q_2 \ q_0$	1 0	$\Delta = (0,1,2)$
q_2	$q_1 \ q_2$	0 2	$S - q_0$

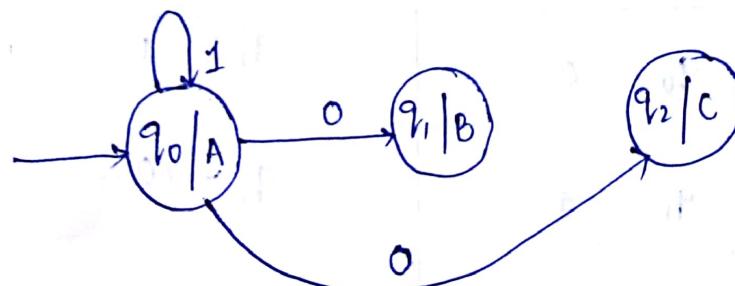
5. Construct Moore Machine that takes 0,1 as input and produces modulo 5 as Output.



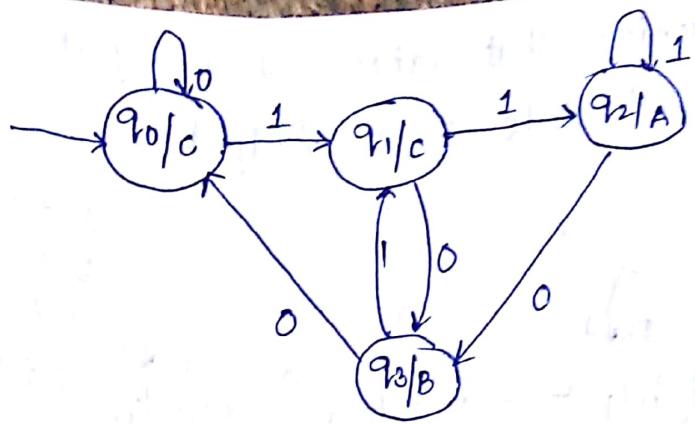
$$\begin{aligned}
 1 \div 5 &= 1 & 0001 \\
 2 \div 5 &= 2 & 0010 \\
 3 \div 5 &= 3 & 0011 \\
 4 \div 5 &= 4 & 0100 \\
 5 \div 5 &= 0 & 0000 \\
 6 \div 5 &= 1 & 0110 \\
 7 \div 5 &= 2 & 0111 \\
 8 \div 5 &= 3 & 1000 \\
 9 \div 5 &= 4 & 1001
 \end{aligned}$$

	$\Sigma = \{0, 1\}$	Output	
	0 1		
q_0	$q_0 \ q_1$	0	$Q = \{q_0, q_1, q_2, q_3, q_4\}$
q_1	$q_2 \ q_3$	1	$\Sigma = \{0, 1\}$
q_2	$q_3 \ q_0$	2	$A = \{0, 1, 2, 3, 4\}$
q_3	$q_1 \ q_2$	3	
q_4	$q_3 \ q_4$	4	

6. Construct a Moore Machine Which produces 'A' as Output for each Occurrence of 11 and produces Output 'B' for each Occurrence of 10. Otherwise produces 'C'.

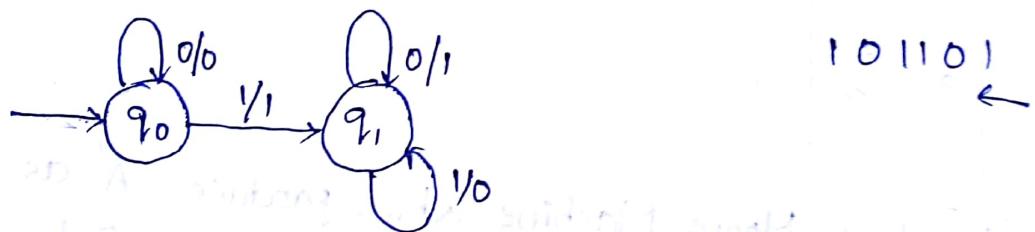


010110010
 ↓↓↓↓↓↓↓↓
 C C B C A B C C B



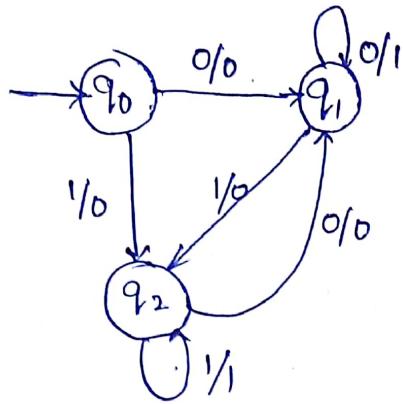
	$\Sigma(0,1)$	$\Delta(A, B, C)$
	0 1	
q_0	$q_0 \ q_1$	C
q_1	$q_3 \ q_2$	C
q_2	$q_3 \ q_2$	A
q_3	$q_0 \ q_1$	B

7. Construct Mealy Machine which takes '0,1' as input and produces 2's complement as Output.



Present state	Next state		$i=1$ Output
	$i=0$	Output	
q_0	q_0	0	$q_1 \ 1$
q_1	q_1	1	$q_1 \ 0$

8. Construct Mealy Machine that prints '1' if the last two symbols are same. Otherwise produces '0'.

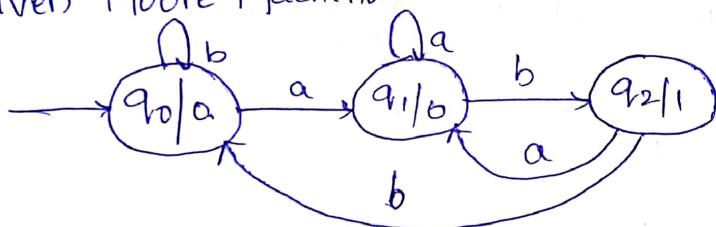


- Moore Machine to Mealy Machine Conversion:

→ The power of Mealy Machine & Moore Machine are same.

→ We can convert from one machine to another Machine.

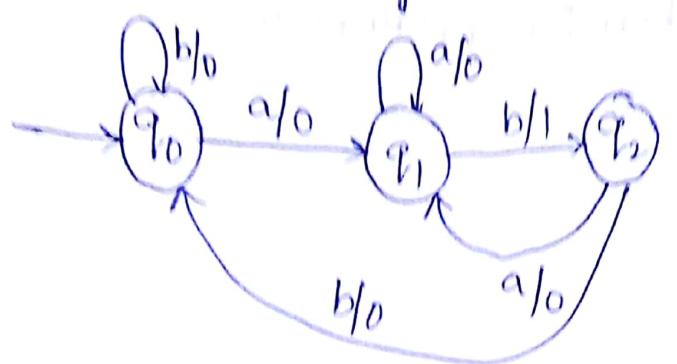
Given Moore Machine



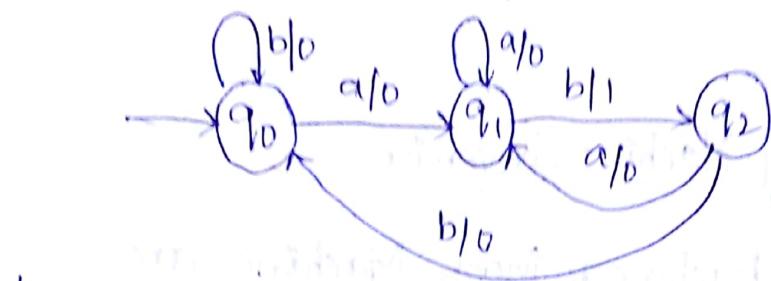
Present	Next i/p=a o/p=b		Output
q_0	q_1	q_0	0
q_1	q_1	q_2	0
q_2	q_1	q_0	1

Present State	Next state i/p=a o/p=		i/p=b o/p	
q_0	q_1	0	q_0	0
q_1	q_1	0	q_2	1
q_2	q_1	0	q_0	0

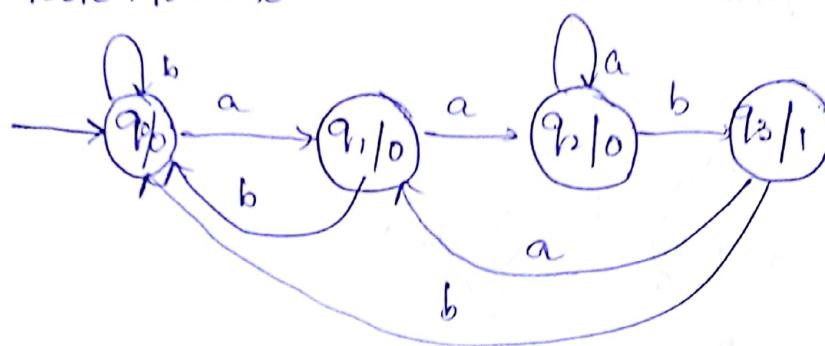
The Equivalent Mealy Machine:



-Another Method directly from Moore Machine to Mealy Machine

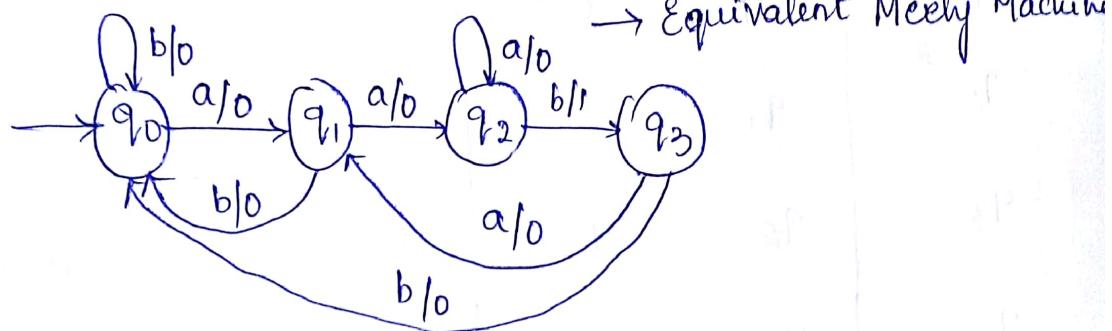


4. Construct equivalent Mealy Machine for the following Moore Machine.



Present State	i/p=a	i/p=b	Output
q_0	q_1	q_0	0
q_1	q_2	q_0	0
q_2	q_2	q_3	0
q_3	q_1	q_0	1
q_4			

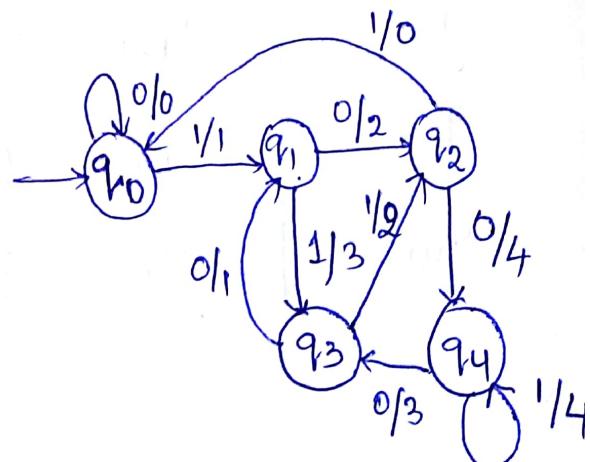
Present state	i/p=a		i/p=b		Output	
	i/p=0	i/p=1	i/p=0	i/p=1	i/p=0	i/p=1
q ₀	q ₁	0	q ₀	0		
q ₁	q ₂	0	q ₀	0		
q ₂	q ₂	0	q ₃	1		
q ₃	q ₁	0	q ₀	0		



2. construct Equivalent Mealy Machine for the following
(Residue Moduli 5)
Moore Machine.

Present state	i/p=0		i/p=1		Output
	i/p=0	i/p=1	i/p=0	i/p=1	
q ₀	q ₀	0	q ₁	1	
q ₁	q ₂	2	q ₃	3	
q ₂	q ₄	4	q ₀	0	
q ₃	q ₁	1	q ₂	2	
q ₄	q ₃	3	q ₄	4	

Present state	i/p=0		i/p=1		Output	
	i/p=0	i/p=1	i/p=0	i/p=1	i/p=0	i/p=1
q ₀	q ₀	0	q ₁	1		
q ₁	q ₂	2	q ₃	3		
q ₂	q ₄	4	q ₀	0		
q ₃	q ₁	1	q ₂	2		
q ₄	q ₃	3	q ₄	4		



Q. Construct the Equivalent Moore Machine for the following Mealy Machine.

Given Mealy Machine

n states m alphabet
Dots
 $M+n$

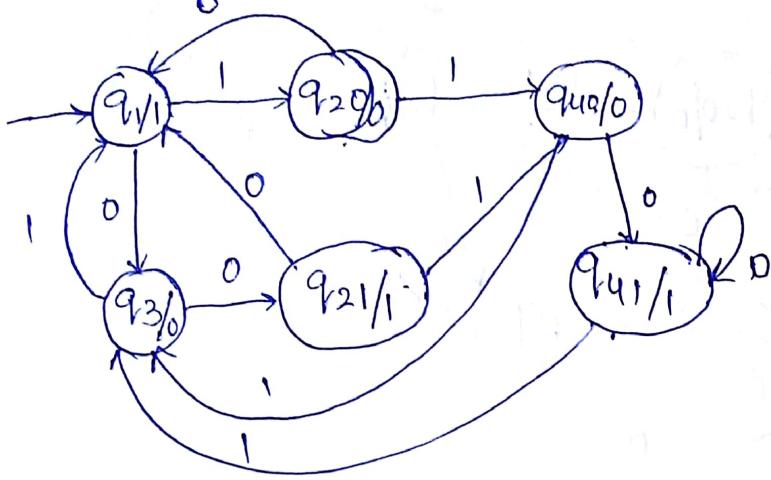
Present State	Next state		O/P	
	I/P = 0	O/P	I/P = 1	O/P
q_1	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

$q_1 \rightarrow q_1$

Since q_{20} & q_{21} have 2 outputs,
 q_{20}
 q_{21}

Moore Machine

Present State	I/P = 0	I/P = 1	Output
q_1	q_3	q_{20}	1
q_{20}	q_1	q_{40}	0
q_{21}	q_1	q_{40}	1
q_3	q_{21}	q_1	0
q_{40}	q_{41}	q_3	0
q_{41}	q_{41}	q_3	1



4. Construct the equivalent Moore Machine for the following Mealy Machine.

Mealy Machine:

Present state	i/p=0	o/p	i/p=1	o/p
q_1	q_1	1	<u>q_2</u>	0
q_2	q_4	1	q_4	1
q_3	<u>q_2</u>	1	<u>q_3</u>	1
q_4	<u>q_3</u>	0	q_4	1

Moore Machine:

Present state	i/p=0	i/p=1	Output
q_1	q_1	q_{20}	1
q_{20}	q_4	q_4	0
q_{21}	q_4	q_4	1
q_{30}	q_{21}	q_{31}	0
q_{31}	q_{21}	q_{31}	1
q_4	q_{30}	q_4	1

