[BENG – 1129]

I/IV B.Tech. DEGREE EXAMINATION.

First Semester

MATHEMATICS — I

(Common for Group-A and Group-B branches)

(Effective from the admitted batch of 2022–2023)

Time : Three hours

Maximum : 70 marks

Answer ALL question of Part A and any FOUR from Part B

All questions carry equal marks.

PART — A

1. (a) If $u = \sin(x^2 + y^2)$, where $a^2x^2 + b^2y^2 = c^2$ then find $\dfrac{du}{dt}$.

   (b) If $x = e^r \sec\theta$, $y = e^r \tan\theta$, then find $\dfrac{\partial(x, y)}{\partial(r, \theta)}$.

   (c) Find the equation of the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at (1, –1, 2).

   (d) $\int_1^2 \int_2^3 \int_3^4 xy^2x \, dx \, dy \, dz$.

(e) Find $\beta\left(\frac{5}{2}, \frac{7}{2}\right)$.

(f) Examine the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$

is even or odd Justify?

(g) Write the Parseval's formula for Fourier series.

PART — B

2. (a) If $v = x^y y^x$ then prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = v(x + y + \log v)$.

(b) If $z = x^n f_1\left(\frac{y}{x}\right) + y^{-n} f_2\left(\frac{x}{y}\right)$, then prove that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\,\partial y} + y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + n^2 z.$$

3. (a) If $u = \sqrt{1-y^2} + y\sqrt{1-x^2}$, $v = \sin^{-1} x + \sin^{-1} y$ then show that u, v are functionally related and find the relationship.

(b) Expand $f(x, y) = x^2 y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ by Taylor's theorem upto 3rd degree.

4. (a) The indicated horse power l of an engine is calculated from the formula $l = \dfrac{\text{PLAN}}{33000}$, where $A = \dfrac{\pi d^4}{4}$. Assuming that error of r percent may have been made in measuring P, L, N and d, find the greatest possible error in l.

(b) Find the minimum value of $x^2 + y^2 + z^2$ given that $x + y + z = 3a$.

5. (a) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\, dx$ by changing the order or integration.

(b) Evaluate $\iint r^3 dr\, d\theta$ over the area bounded between the circle $r = s\cos\theta$ and $r = 4\cos\theta$.

6. (a) Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ay$.

(b) Find the center of gravity of the cardinoid $r = a(1 + \cos\theta)$.

7. (a) Find the value of $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}dx$ using Beta and Gamma functions.

(b) Expand $f(x)=e^{-x}$ as a Fourier series in the interval $(-l,l)$.

8. (a) Find the Fourier series expansion of $f(x)=2x-x^2$ in (0, 3) and hence deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots-\infty=\frac{\pi}{12}$.

(b) Obtain half range cosine series for $f(x)=x^2$ in the range $0\le x\le\pi$.

[BENG – 1129]

I/IV B.Tech. DEGREE EXAMINATION.

First Semester

MATHEMATICS – I

(Common for Group-A and Group-B branches)

(Effective from the admitted batch of 2022–2023).

Time : Three hours Maximum : 70 marks

Answer question No.1 compulsorily and any FOUR questions from remaining.

All questions carry equal marks.

All parts of a question must be answered at one place only.

1. (a) If $u = x^4 y^5$, $x = t^2$, $y = t^3$ then find $\frac{du}{dt}$.

   (b) State Rolle's mean value theorem.

   (c) What do you mean by saddle point?

   (d) Find $\Gamma(-3.5)$.

(e) Change the integral $\int_{\partial}^{a}\int_{y}^{a} \frac{x}{x^2+y^2}\,dx\,dy$ into polar coordinates.

$\frac{15}{16a^2}$

(f) Write the conditions for the expansion of a function as a Fourier series.

(g) Is the function $f(x)=\begin{cases} 1-x, & -\pi < x < 0 \\ 1+x, & 0 < x < \pi \end{cases}$ is even or odd? Justify.

2. (a) If $z = \frac{x^2+y^2}{x+y}$ then show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right).$$

(b) If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$ then prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 2f = 0$.

3. (a) If $u = f(y-z, z-x, x-y)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

(b) Expand $F(x,y) = x^2y + 3y - 2$ is powers of $(x-1)$ and $(y+2)$ by Taylor's theorem.

4. (a) Determine the points where a function $x^3 + y^3 - 3axy$ has a maximum or minimum.

(b) Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2 + y^2 + z^2 = 1$.

5. (a) Change the order of integration is $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy \, dx$ and hence evaluate.

(b) Evaluate $\iint r^3 \, dr \, d\theta$ over the area between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.

6. (a) Find the volume bounded by the xy-plane, the paraboloid $2z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 4$.

(b) Find by triple integration the volume of the solid bounded by the surface $x = 0, y = 0, x + y + z = 1$ and $z = 0$.

7. (a) Find the mass of the tetrahedron plate bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, the variable density $P=\lambda\, xyz$.

(b) Express $\int_0^1 x^m\left(1-x^n\right)^p dx$ in terms of Gamma function and hence evaluate $\int_0^1 x^5\left(1-x^3\right)^{10} dx$.

8. (a) Obtain the Fourier series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$.

(b) Find the Fourier sine and cosine series for $f(x)=x$ in the interval $[0,\pi]$. Hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$.

$(e^{2\pi}-1)+\sum_{n=1}^{\infty}$

[BENG – 1122]

I/IV B.Tech. DEGREE EXAMINATION.

First Semester

MATHEMATICS – I

(Common for Group-A and Group-B branches)

(Effective from the admitted batch of 2020-2021)

Time : Three hours Maximum : 70 marks

Answer question No. 1 compulsorily and any FOUR questions from remaining.

All questions carry equal marks.

All parts of a question must be answered at one place only.

1. (a) If $x + y + z = \log z$, Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(b) Find the value of $\int_0^1 \int_0^x e^x dx\, dy$.

(c) Find the equation of the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at $(1, -1, 2)$.

(d) Compute $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$.

(e) Find the maximum value of $x^3 + y^3 - 3axy$.

(f) Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2 yz \, dx \, dy \, dz$.

(g) Write the Dirichlet conditions for the expansion of a function as a Fourier series.

2. (a) If $v = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$.

(b) If z is a homogeneous function of degree n in x and y, show that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \cdot \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

3. (a) Expand $x^2 y + 3y - 2$ in powers of $x - 1$ and $y + 2$ using Tailor's theorem.

(b) Discuss the maxima and minima of $f(x, y) = x^3 y^2 (1 - x - y)$.

4. (a) Prove that $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)}dx = \frac{\pi}{2}\log(1+a)$ where $a \geq 0$.

(b) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.

5. (a) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$ by changing the order of integration.

(b) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

6. (a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dx\, dy$ by changing to polar co-ordinates.

(b) Calculate the area included between the curve $r = a(\sec\theta + \cos\theta)$ and its asymptote.

7. (a) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

(b) Show that $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$.

8. (a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

(b) Obtain sine and cosine series for $f(x) = x$ in the interval $0 \le x \le \pi$. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$.

$f(1) = f(-1)$
$f(-1) = -f(1)$

[BENG – 1112]

I/IV B.Tech. DEGREE EXAMINATION.

First Semester

MATHEMATICS — I

(Common for all branches)

(Effective from the admitted batch of 2019–2020)

Time : Three hours Maximum : 70 marks

Answer ALL questions in Part-A and any FOUR from Part-B.

All questions carry equal marks.

PART — A

1. (a) If $u = \log xy$, where $x^3 + y^3 + 3xy = 1$ then find $\frac{du}{dx}$

(b) If $u = x(1-y), v = xy$, prove that $JJ^1 = 1$

(c) Find the equation of the normal line to the surface $2x^2 + y^2 + 2z = 3$ at the point $(2, 1, -3)$

(d) Find the integrating factor of $(1+xy)y\,dx + (1-xy)x\,dy = 0$

(e) Define orthogonal trajectory

(f) Solve $\left(4D^2-4D+1\right)y=100$.

(g) State the general form of Cauchy's linear equation.

PART — B

2\. (a) If $u=\log\left(x^3+y^3+z^3-3xyz\right)$ then show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2=\frac{-9}{(x+y+z)^2}$.

(b) If $u=\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\frac{1}{2}\cot u$.

3\. (a) Show that $e^y\log(1+x)=x+xy-\frac{x^2}{2}$ approximately.

(b) If $u=3x+2y-z$, $v=x-2y+z$ and $w=x(x+2y-z)$ then show that they are functionally related and find the relation.

4. (a) If $xyz=8$ then find the values of x, y for which $u=\frac{5xyz}{(x+2y+4z)}$ is maximum.

(b) Evaluate the integral $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$ by applying differentiation under the integral sign. $(\alpha \geq 0)$.

5. (a) Solve $xy\left(1+xy^2\right)\frac{dy}{dx}=1$.

(b) Solve $(2x\log x - x\,y)dy + 2y\,dx = 0$.

6. (a) A body kept in air with temperature 25°C cools from 140°C to 80°C in 20 minutes. At what time, the body cools down to 35°C?

(b) Find the orthogonal trajectory of the cardioids $r=a(1-\cos\theta)$.

7. (a) Solve $\left(D^2-4D+4\right)y=8x^2 e^x \sin 2x$.

(b) Solve $\left(D^2+4\right)y=\tan 2x$ by the method of variation of parameters.

8. (a) Solve $\left(x^2 D^2 + 5x D + 4\right) y = x \log x$.

(b) Solve $(D+4)x + 3y = t$ and $(D+5)y + 2x = e^t$ simultaneously.

[BENG – 1102]

I/IV B.Tech. DEGREE EXAMINATION.

First Year/First Semester

MATHEMATICS - I

(Common for all branches Including Dual Degree)

(Effective from the admitted batch of 2015–2016)

Time : Three hours | Maximum : 70 marks

Answer any FIVE questions.

First question is compulsory.

Answer any FOUR from the remaining questions.

All questions carry equal marks.

PART — I

1. (a) If $u = \cos^{-1}\left[\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right]$, find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$?

   (b) Find the equations of the tangent plane and the normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$.

(c) Find the maximum and minimum values of

$$xy + \frac{a^3}{x} + \frac{a^3}{y}.$$

(d) Form the differential equation of simple harmonic motion given by $x = A\cos(nt + \alpha)$.

(e) Solve $\cos^2 x \frac{dy}{dx} + y = \tan x$.

(f) Find the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

(g) Test the convergence of the series $\Sigma\left(\frac{n^3}{3^n}\right)$.

PART – II

2. (a) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}.$$

(b) If $x = u(1-v), y = uv$, prove that $JJ^{*} = 1$.

3. (a) Expand $f(x, y) = \sin xy$ in powers of $(x-1)$ and $(y-\pi/2)$ upto the second degree terms

(b) Prove that a rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

4. (a) Solve $(1+y^2)\,dx = (\tan^{-1} y - x)\,dy$.

(b) Solve $y(2xy + e^x)\,dx = e^x dy$.

5. (a) Find the orthogonal trajectories of the family of confacal conics $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2+\lambda} = 1$, where λ is a parameter.

(b) If the temperature of the air is 30° C and the substance cools from 100° C to 70° C in 15 minutes. Find when the temperature will be 40° C.

6. (a) Solve $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = e^{-2x}\sin 2x$.

(b) Solve $\dfrac{d^2y}{dx^2} + a^2 y = \sec ax$, by using the method of variation of parameters.

7. (a) Solve $(2x+3)^2 \dfrac{d^2y}{dx^2} - (2x+3)\dfrac{dy}{dx} - 12y = 6x$.

(b) Solve the following simultaneous equations.

$$\frac{dx}{dt} = 5x + y, \quad \frac{dy}{dt} = y - 4x.$$

8. (a) Test for the convergence of the series

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots$$

(b) Discuss the absolute convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}.$$

(b) Solve the differential equations $y''-3y'+2y=e^{3t}$, when $y(0)=1$ and $y'(0)=0$, by Laplace transform method.

7. (a) Find the Laplace transform of the triangular wave function of period 2a given by

$$f(t)=\begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$$

(b) Define unit step function and find the Laplace transform of this function. Also evaluate $L\{e^{-t}[1-u(t-2)]\}$.

8. (a) Find a Fourier series to represent the functions $x-x^2$ from $x=-\pi$ to $x=\pi$.

(b) Express $f(x)=x$ as a half – range cosine series in $0<x<2$.

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[BENG – 1113]

I/IV B.Tech. DEGREE EXAMINATION.

First Semester

Mathematics – II

(Common for all branches)

(Effective from the admitted batch of 2019–2020)

Time : Three hours Maximum : 70 marks

PART A is compulsory.

Answer any FOUR from the PART B.

PART A

1. (a) How many number of linearly independent solutions does the system have: $x+2y+3z=2, y+z=-1, 2y+2z=0$.

(b) The product of two eigen values of a matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.

(c) Prove that $A=\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & (1+i) \\ (1-i) & -1 \end{bmatrix}$ is unitary.

(d) Find the Laplace transform of $\sqrt{t}$.

(e) Find $L^{-1}\left\{\frac{s}{(2s+3)^2}\right\}$

(f) Define unit impulse function and write its Laplace transform.

(g) Write Euler's formulae for Fourier series.

PART B

2. (a) Reduce the matrix $A=\begin{bmatrix}1 & 2 & -1 & 4\\ 2 & 4 & 3 & 4\\ 1 & 2 & 3 & 4\\ -1 & -2 & 6 & -7\end{bmatrix}$ to its normal-form. Hence find the rank of A.

(b) Test for consistency and solve : $2x-3y+7z=5, 3x+y-3z=13,$ $2x+19y-47z=32$

3. (a) Find all the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$.

(b) State Cayley–Hamilton theorem and use it to find A^{-1}, where $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$. Also express $A^6-6A^5+9A^4-2A^3-12A^2+23A-9I$ as a linear polynomial in A.

4. (a) Diagonalise the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$. Hence find A^5.

(b) Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the Canomical form. Also discuss the nature of the quadratic form.

5. (a) Evaluate

(i) $L\{t^2e^{-3t}\sin 2t\}$

(ii) $L\left\{e^{-t}\int_0^t\frac{\sin t}{t}dt\right\}$.

(b) Apply Laplace transform to evaluate $\int_0^\infty e^{-t}\left(\frac{\cos at-\cos bt}{t}\right)dt$

6. (a) Find

(i) $L^{-1}\left\{\log\left(\frac{s^2+1}{s^2+s}\right)\right\}$

(ii) $L^{-1}\left\{\left(\frac{s^2}{(s-3)(s-6)}\right)\right\}$.