Task Sheet 4 - An Ising model and the El Farol bar problem

In this exercise, you will examine an Ising model and analyze its connection to the El Farol bar problem in game theory.

Please save all plots to files, so they are available without executing the code. All results should be bundled in an zip file with your name as the file name.

Deliverables:

- A set of plots (saved to file)
- Code

The submission should be bundled in a single zip file. The file should be named `yourname_sos_ex4.zip`. Please check the LEA course page for your submission deadline.

1 The Ising model

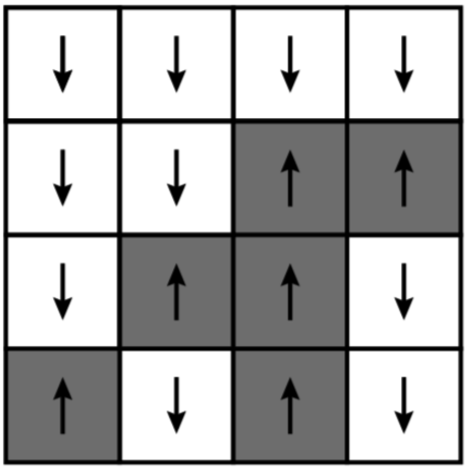

Figure 1: A very small Ising model

The Ising model is a popular grid world model that was originally developed to explain ferromagnetism. Each node i on the grid has one of two spins $s_i = \pm 1$, which correspond to the states of magnetization. To minimize energy, neighboring cells strive to have the same spin. Cells switch spontaneously based on the temperature.

The degree of consensus between the grid cells is represented by magnetization $M = \sum_i s_i$ and each grid configuration has energy $E = -\sum_{\langle ij \rangle} s_i s_j$, where $\langle ij \rangle$ denotes two neighboring grid nodes.

One way to model the dynamics of the Ising model is the Metropolis algorithm. This algorithm tries to minimize the overall energy by flipping a single spin each time step:

A. Choose node i from uniform random distribution

B. Calculate change in energy ΔE if flipping spin s_i

C. Accept the flip for:

   a) $\Delta E \leq 0$
   b) $\Delta E > 0$ with probability $A = exp(-\frac{\Delta E}{T})$

a) Implement the Ising model with $N \times N = 100 \times 100$ cells and periodic boundary conditions (i.e., cell $(1, 1)$ is neighbor of cells $(N, 1)$, $(1, 2)$, $(2, 1)$ and $(1, N)$) as well as the Metropolis algorithm. Plot the grid world, showing each node's spin. Show the grid world at different stages of the Metropolis algorithm. For initialization, use a random spin configuration and T of your choosing.

   You are free to use a programming language of your choice, but please include a guide how to execute your code.

b) Run the Metropolis algorithm for 1.000.000 time steps with different T. Roughly determine T_C, which is the largest temperature at which distant (not neighboring) spins are on average parallel. Is the transition above T_C an abrupt jump or does the behavior change smoothly? Plot the grid world, energy and magnetization over time for $T \ll T_C$, $T \approx T_C$ and $T \gg T_C$.

2 The El Farol bar problem

Figure 2: The El Farol bar in Santa Fe, New Mexico. Source: John Phelan / CC-BY-SA-3.0

The El Farol is a popular bar in Santa Fe, New Mexico. Every Thursday night, the whole neighborhood wants to meet up there. But since the bar is quite small, it quickly gets crowded and the evening is ruined for everyone. Over time, a threshold has been found:

- If **less than** 60% of the neighborhood go to the bar, it is not crowded and everyone has a good time
- If **at least** 60% of the neighborhood go to the bar, it is crowded and people would enjoy staying at home more

People have to decide quickly and at the same time, so they can't count patrons at the bar or talk to everybody in the neighborhood to see who wants to go and who doesn't. People talk to their direct neighbors about last week, so they have five pieces of information available for their decision: their own experience and the experiences of four neighbors. There are three types of experience:

- I didn't go.
- I went and had a bad time.
- I went and had a good time.

If neither you nor your neighbors went last week, the decision is made with a coin toss. If there is information available, people exhibit one of two behaviors. The **naive** people don't go to the bar when it was crowded last week, otherwise they do. The **anti-cyclical** people don't go if people had a good time in the previous week, but they go if it was crowded. To maximize happiness, we want marginally less than 60% of the neighborhood to go to the bar.

a) We want to ensure maximum happiness, so we employ a psychologist. The psychologist represents both observer and controller of the observer-controller (OC) architecture and changes the behavior of people. The psychologist knows the ratio of naive to anti-cyclical people. Each time step, he selects a person. If it is a naive person, he makes them anti-cyclical and vice versa.

   Initialize a neighborhood grid with size 50×50, a psychologist and a random ratio and distribution of behaviors. Plot the average ratio of behaviors and the average ratio of bar visitors over time. Do 100 independent runs. Please implement a clear OC architecture, redit may be reused in future tasks. You may choose the psychologist's method of selecting people. The psychologist should select people in a way that the threshold of 60% for the bar's occupancy is approached from below, but rarely crossed.

b) Repeat a), but add at least one more behavior. Please don't use trivial behaviors ("I always go", "I never go"). How does the convergence process change?