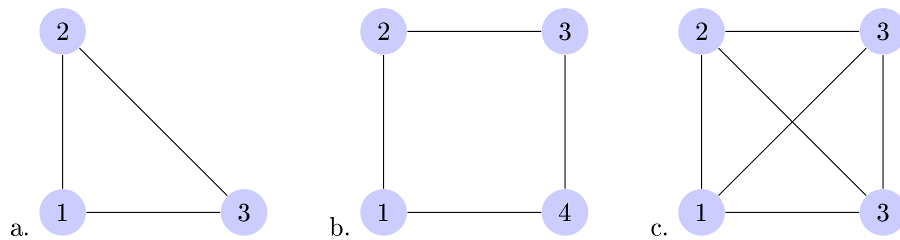
# Metric and normed spaces

**1.** Is the following function $\rho(x, y)$ a metric on $\mathbb{R}$?

1. $|x^2 - y^2|$;
2. $\sin(x - y)$;
3. $|2^x - 2^y|$?

**2.** For each of the following graph, consider the corresponding metric space $X$ (the weights of the edges are equal to 1). Does there exist a collection of points in $\mathbb{R}^3$ such that the distance between each pair of points is exactly the length of shortest path in the graph?




a. b. c.

**3.** Let $V$ be the space $\mathbb{R}^2$ with the norm $|v|_\infty$. Prove that for each graph in Problem 2 the space $V$ contains a collection of points such that the the distance between each pair of points (in terms of the metric induced by the norm in $V$) is exactly the length of shortest path in the graph.

**4.** Construct an example (or prove that this is impossible) of a metric space $M$ and two balls $B$ and $B'$ in it or radii $r = 10$ and $r'$ respectively such that $B' \subset B$ and $B' \neq B$, if

1. $r' = 15$;
2. $r' = 20$;
3. $r' = 100$?

**5.** Prove that in each normed space

$$|x| \leq \max(|x + y|, |x - y|).$$

**6.** Is the following function a norm on the set of smooth functions $C^1[a, b]$?

$$\mu(f) = \underset{a \leqslant x \leqslant b}{max} (|f(x)| + |f'(x)|).$$