Iterational methods

1. Prove the formula from the lecture: if $A = A^*$ and $\lambda_i(A) > 0$, then the spectral radius $\rho(P)$ of the matrix $P = I - \tau A$ is minimal (and is less then 1) for $\tau = \dfrac{2}{\lambda_{min}(A) + \lambda_{max}(A)}$.

2. Re-write the systems in a form appropriate for an iterational method. You can use either Jacobi, or Gauss–Seidel, or any other method. Implement the iterational methods and solve the systems.

1. $\begin{cases} 4x_1 + 2x_2 = 7 \\ 2x_1 - 4x_2 = -4 \end{cases}$

2. $\begin{cases} x_1 + 3x_2 + x_3 = 8 \\ 2x_1 + 2x_2 + x_3 = 9 \\ 3x_1 + x_2 + 2x_3 = 13 \end{cases}$

3. $\begin{cases} 2x_1 - 4x_2 + 5x_3 = 3 \\ 3x_1 - 3x_2 - 5x_3 = -5 \\ 4x_1 + 5x_2 + 2x_3 = 11 \end{cases}$