$$0 = -6(240)v_2 + 4(240)^2\,\phi_2$$

$$\Rightarrow 0 = -6v_2 + 4(240)\phi_2 \Rightarrow v_2 = 160\phi_2$$

$$-2000 = 419.56\,[(12 + (2.38)\,160\phi_2 - 6(240)\phi_2]$$

$$\Rightarrow \quad \phi_2 = -0.005538 \text{ rad}$$

$$\Rightarrow \quad v_2 = 160(-0.005538) \Rightarrow v_2 = -0.886 \text{ in.}$$

Beam element

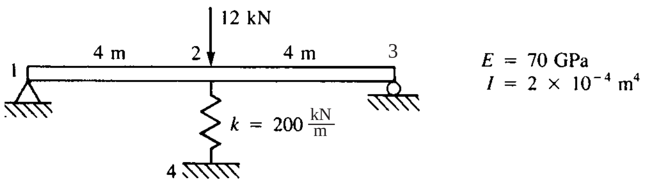
$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \end{Bmatrix} = \frac{(29\times10^6)(200)}{(20\times12)^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ -0.886 \\ -0.005538 \end{Bmatrix}$$

$$F_{1y} = 1115 \text{ lbs} \uparrow,\ M_1 = -267 \text{ kip}\cdot\text{in.}$$

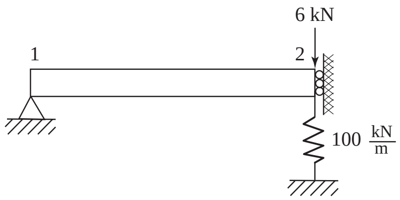
$$F_{2y} = -1115 \text{ lbs} \downarrow,\ M_2 = 0$$

The extra force at node 2 is resisted by the spring.

4.11



Applying symmetry



$$[K] = \frac{EI}{L_3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & \frac{12+KL^3}{EI} & -6L \\ \text{Symmetry} & & & 4L^2 \end{bmatrix}$$

Applying the boundary conditions $v_1 = 0$, $\phi_2 = 0$ we have

12EI/L^3+K
=>12+ K/EI/L^3

$$\begin{Bmatrix} M_1 = 0 \\ F_{2y} = -6000 \text{ N} \end{Bmatrix} = \frac{(70\times10^9)(2\times10^{-4})}{4^3}\begin{bmatrix} 4L^2 & -6L \\ -6L & \frac{12+KL^3}{EI} \end{bmatrix}\begin{Bmatrix} \phi_1 \\ v_2 \end{Bmatrix}$$

$$\Rightarrow \quad 0 = 4L^2\phi_1 - 6Lv_2 \Rightarrow \phi_1 = \frac{6}{4L}\,v_2 \Rightarrow \phi_1 = \frac{6}{16}\,v_2$$

100*1000/EI/L^3+12
=0.457+12

$$-6000 = 218750\left[-24\left(\frac{6}{16}\right)v_2 + 12.457\,v_2\right]$$

$$\Rightarrow \quad v_2 = -7.9338\times10^{-3} \text{ m}$$

$$\phi_1 = \frac{6}{16}\,(-7.9338 \times 10^{-3}) \Rightarrow \phi_1 = -2.9752 \times 10^{-3}\text{ rad}$$

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \end{Bmatrix} = \frac{(70 \times 10^9)(2 \times 10^{-4})}{4^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12.457 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 2 \\ -2.9752 \times 10^{-3} \\ -7.9336 \times 10^{-3} \\ 0 \end{Bmatrix}$$

0

$$F_{1y} = 5.208\text{ kN} \uparrow,\; M_2 = 20.83\text{ kN}\cdot\text{m} \circlearrowleft$$

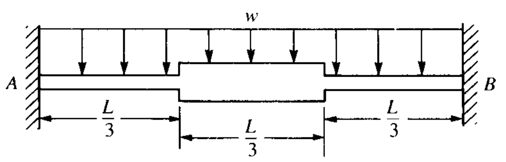
$$F_{2y} = 0\text{ kN} \downarrow$$

$$F_{\text{spring}} = (200\,\frac{\text{kN}}{\text{m}})\,(7.9338 \times 10^{-3}\text{ m})$$

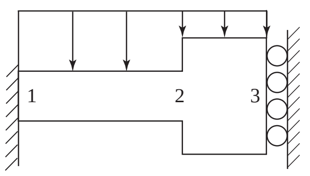
$$F_{\text{spring}} = 1.587\text{ kN}$$

From symmetry $F_{3y} = 5.208\text{ kN} \uparrow$

4.12

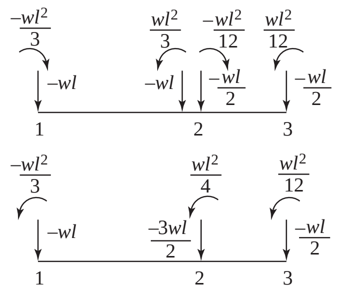


From symmetry



Let $\frac{L}{6} = l$

$\Rightarrow \frac{L}{3} = 2l$



Final nodal forces and moments

$$[k_{1-2}] = \frac{EI}{l^3} \begin{array}{c} \begin{array}{cccc} v_1 & \phi_1 & v_2 & \phi_2 \end{array} \\ \begin{bmatrix} \frac{3}{2} & \frac{3}{2}l & \frac{-3}{2} & \frac{3}{2}l \\ \frac{3}{2}l & 2l^2 & \frac{-3}{2}l & l^2 \\ \frac{-3}{2} & \frac{-3}{2}l & \frac{3}{2} & \frac{-3}{2}l \\ \frac{3}{2}l & l^2 & \frac{-3}{2}l & 2l^2 \end{bmatrix} \end{array} \begin{array}{c} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{array}$$