$\boxed{\text{Автоматен език}} \Rightarrow \boxed{\text{регулярен език}}$ 31.03.23

Взимаме един език без да гледаме дали е регулярен или не за да видим как можем да построим автомат по него

$a^{-1}(L) = \{u \in \Sigma^* \mid aw \in L\}$

$$L \xrightarrow{a} a^{-1}(L) \qquad L \xrightarrow{b} b^{-1}(L)$$

Производна на Дяковски

- $a^{-1}(\emptyset) = \emptyset$
- $a^{-1}(\{x\}) = \begin{cases} \{\varepsilon\}, & x = a \\ \emptyset, & x \neq a \end{cases}$

$$\varepsilon(L) = \begin{cases} \emptyset, & \varepsilon \notin L \\ \{\varepsilon\}, & \varepsilon \in L \end{cases}$$

- $a^{-1}(\{\varepsilon\}) = \emptyset$
- $a^{-1}(L_1 \cup L_2) = a^{-1}(L_1) \cup a^{-1}(L_2)$
- $a^{-1}(L_1 . L_2) = a^{-1}(L_1) . L_2 \cup \varepsilon(L_1) . a^{-1}(L_2)$
- $a^{-1}(L^*) = a^{-1}(L) . L^*$ $\qquad L^* = \bigcup_{n \geq 0} L^n$

$$a^{-1}(L^*) = a^{-1}\Big(\bigcup_{n\geq 0} L^n\Big) = \bigcup_{n \geq 1} a^{-1}(L^n) = \bigcup_{n\geq 0} a^{n-1}(L.L^n) = \bigcup_{n \geq 0} \left(a^{-1}(L) . L^n \cup a^{-1}(L^n)\right)$$

$a^{-1}(L^n) \subseteq a^{-1}(L) . L^{n-1}$ $\qquad a^{-1}(L^*) = a^{-1}(L) L^*$

$a^{-1}(L) . L^* \subseteq a^{-1}(L^*)$ $\qquad a^{-1}(L^2) = a^{-1}(L) . L$

- Ако $\varepsilon \notin L$: $a^{-1}(L).L^* = a^{-1}(L^*)$ $\qquad a^{-1}(L^{n+1}) = a^{-1}(L).L^n$
- Ако $\varepsilon \in L$: $a^{-1}(L).L^* = a^{-1}(L^*)$

$\alpha^{-1}(L) = \{w \in \Sigma^* \mid \alpha w \in L\}$ $\qquad \alpha \in L \Leftrightarrow \varepsilon \in \alpha^{-1}(L)$

$Q_L = \{w^{-1}(L) \mid w \in \Sigma^*\}$ $\qquad q^L_{start} = L$

$\delta_L(M, a) = a^{-1}(M)$ $\qquad F_L = \{M \in Q_L \mid \varepsilon \in M\}$ $\qquad B_L = (\Sigma, Q_L, \delta_L, q^L_{start}, F_L)$ $\qquad \mathcal{L}(B_L) = L$

$\alpha \in L \Leftrightarrow \varepsilon \in \alpha^{-1}(L)$

$$(\alpha\beta)^{-1}(L) = \{w\in\Sigma^* \mid \alpha\beta w \in L\} = \{w\in\Sigma^* \mid \beta w \in \alpha^{-1}(L)\} =$$
$$= \{w\in\Sigma^* \mid w \in \beta^{-1}(\alpha^{-1}(L))\} = \beta^{-1}(\alpha^{-1}(L))$$

$\delta_L^*(M,w) = w^{-1}(M)$ индукция по $|w|$

База: • $w=\varepsilon$: $\delta_L^*(M,\varepsilon) = \underbrace{M}_{\varepsilon^{-1}(M)}$ по дефиниция на δ_2^*

Индукционно предположение за $|w|=n$: $\delta_L^*(M,w) = w^{-1}(M)$

Индукционна стъпка: • нека $|w|=n+1$, т.е. $w=\alpha b$ и $|\alpha|=n$.

$$\delta_L^*(M,\underbrace{\alpha b}_{w}) = \delta_L(\underbrace{\delta_L^*(M,\alpha)}_{\text{ИП}}, b) = \delta_L(\alpha^{-1}(M), b) = \underbrace{b^{-1}(\alpha^{-1}(M))}_{\alpha b^{-1}(M)}$$

$\boxed{\mathcal{L}(B_L)=L}$ $\quad w\in\mathcal{L}(B_L) \iff \delta_L^*(\underset{\text{начално}}{L}, w)\in F_L$

Използвахме свойството: $\alpha\in L \iff \varepsilon\in\alpha^{-1}(L)$

Следваща стъпка със свойството: $\delta_L^*(M,w) = w^{-1}(M)$

$$\alpha\in L \iff \varepsilon\in\alpha^{-1}(L) \iff \boxed{w^{-1}(L)\in F_L \iff \varepsilon\in w^{-1}(L) \iff w\in L}$$

$L = \{w\in\{a,b\}^* \mid |w|_a \equiv 0 \bmod 2 \ \& \ |w|_b \equiv 1 \bmod 2\}$

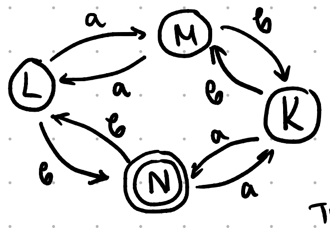
$\underbrace{a^{-1}(L)}_{M} = \{w\in\{a,b\}^* \mid |w|_a \equiv 1 \bmod 2 \ \& \ |w|_b \equiv 1 \bmod 2\}$

$\underbrace{b^{-1}(L)}_{N} = \{w\in\{a,b\}^* \mid |w|_a \equiv 0 \bmod 2 \ \& \ |w|_b \equiv 0 \bmod 2\}$

$a^{-1}(M) = L$

$\underbrace{b^{-1}(M)}_{K} = \{w\in\{a,b\}^* \mid |w|_a \equiv 1 \bmod 2 \ \& \ |w|_b \equiv 0 \bmod 2\}$

$a^{-1}(N)=K$
$b^{-1}(N)=L$
$a^{-1}(K)=N$
$b^{-1}(K)=M$



крайни състояния са езиците, в които има ε заради $\alpha\in L \iff \varepsilon\in\alpha^{-1}(L)$

Това са езиците с четен брой букви, защото ако броят е нечетен има поне една буква $\Rightarrow N$

пример: $L = \{wab \mid w \in \{a,b\}^*\}$

$a^{-1}(L)$? искаме да представим L като обединение на два езика – думите, които започват с a и думите - с b.

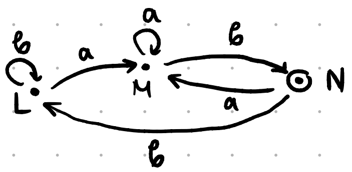
$L = \{awab \mid w \in \{a,b\}^*\} \cup \{ab\} \cup \{bwab \mid w \in \{a,b\}^*\}$

$a^{-1}(L) = \underbrace{L \cup \{b\}}_{M}$ $\quad a^{-1}(M) = a^{-1}(L) \cup a^{-1}(\{b\}) = M \cup \varnothing = M$

$b^{-1}(L) = L$ $\quad b^{-1}(M) = \underbrace{L \cup \{\varepsilon\}}_{N}$

$a^{-1}(N) = a^{-1}(L) = M$

$b^{-1}(N) = b^{-1}(L) = L$



$L = \{a^n b^n \mid n \in \mathbb{N}\}$ $\qquad L_k\{a^n b^{n+k} \mid n \in \mathbb{N}\}$

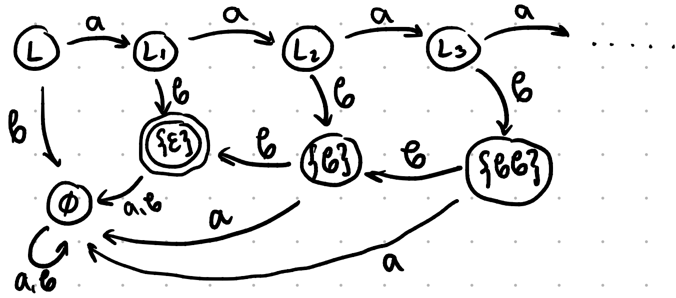
$a^{-1}(L) = \{a^n b^{n+1} \mid n \in \mathbb{N}\} = L_1$, $b^{-1}(L) = \varnothing$ $\qquad$ Автоматът е безкраен

$a^{-1}(L_1) = L_2$ $\qquad b^{-1}(L_1) = \{\varepsilon\}$

$a^{-1}(L_2) = L_3$ $\qquad b^{-1}(L_2) = \{b\}$

$\vdots \qquad \vdots$

$a^{-1}(L_k) = L_{k+1}$ $\qquad b^{-1}(L_k) = \{b^{k-1}\}$



Твърдение: Ако L е регулярен, то Q_L е крайно

$= \{w^{-1}(L) \mid w \in \Sigma^*\}$

- $L = \varnothing$ $\quad Q_L = \{\varnothing\}$
- $L = \{a\}$ $\quad Q_L = \{\varnothing, \{\varepsilon\}, L\}$
- $L = \{\varepsilon\}$ $\quad Q_L = \{\varnothing, L\}$
- $L = L_1 \cup L_2$

$w^{-1}(L) = w^{-1}(L_1) \cup w^{-1}(L_2)$ $\quad \forall N \in Q_L$ $\Rightarrow Q_L \subseteq \{M_1 \cup M_2 \mid M_1 \in Q_L \ \& \ M_2 \in Q_L\} = X$

$\exists M_1 \in Q_{L_1}\ \exists M_2 \in Q_{L_2}$ $\quad N = M_1 \cup M_2$ $\qquad |Q_L| \le |Q_{L_1} \times Q_{L_2}| = |Q_{L_1}| . |Q_{L_2}|$

$|Q_L| \le X \le |Q_{L_1}| . |Q_{L_2}|$

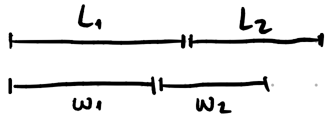
$|Q_L| \le |Q_{L_1}| . |Q_{L_2}|$

$f: Q_{L_1} \times Q_{L_2} \longrightarrow X$ е сюрективно $\Rightarrow |X| \le |Q_{L_1} \times Q_{L_2}| = |Q_{L_1}||Q_{L_2}|$

$f(M_1, M_2) = M_1 \cup M_2$

• $L = L_1 . L_2$

$$w^{-1}(L_1.L_2) = w^{-1}(L_1).L_2 \cup \bigcup_{w=w_1w_2} \varepsilon.(w_1^{-1}(L_1)).w_2^{-1}(L_2)$$



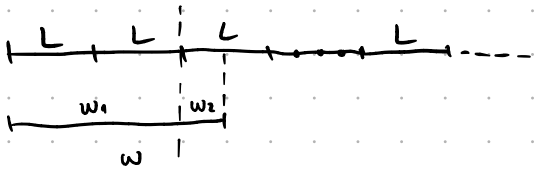
$w = w_1.w_2$

$M \in Q_{L_1.L_2}$, то същ. $M_1 \in Q_{L_1}$ и същ $X \subseteq Q_{L_2}$

$M = M_1.L_2 \cup \bigcup X$

$$|Q_{L_1.L_2}| \le |Q_{L_1}|.|P(Q_{L_2})| = |Q_{L_1}|.2^{|Q_{L_2}|}$$

$$w^{-1}(L^{n+1}) = \bigcup_{n=l+r}\bigcup_{w=w_1w_2} \varepsilon(w_1^{-1}(L^l)).w_2^{-1}(L).L^r$$



$$w^{-1}(L^*) = \bigcup_l \bigcup_r \bigcup_{w=w_1w_2} \varepsilon(w_1^{-1}(L^l)).w_2^{-1}(L).L^r = \bigcup_l \bigcup_{w=w_1w_2} \varepsilon(w_1^{-1}(L^l))w_2^{-1}(L).L^*$$

$$\underbrace{(\overset{0/1}{a_1}x_1 + a_2x_2 + \dots + a_nx_n).L^*}_{\text{един елемент на } Q_L^*}$$

x - елемент на Q_L

в най-лошия случай всички полиноми са $2^{|Q_L|}$

Заключение: $|Q_{L^*}| \le 2^{|Q_L|}$

крайно <= ако това е крайно