Метрични канонични уравнения на кривите от $2^{ра}$ степен 28.04.23

Тв. Основна теорема за канонизация

Всяка крива от 2ра степен c:

c: $F(M) = a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0$ K. Oxy ортонорм.

може да се доведе чрез 1 ротация и 1 транслация на координатната система до:

I. $s_1 x''^2 + s_2 y''^2 = a''_{33}$ спрямо $K''\ O''x''y''$ ортонорм.

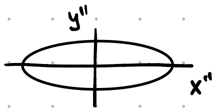
II. $s_2 y''^2 = 2a''_{13} x''$ спрямо $K''\ O''x''y''$ ортонорм.

III. $s_2 y''^2 = a''_{33}$ спрямо $K''\ O''x''y''$ ортонорм.

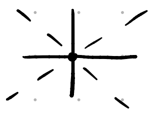
I. $s_1 x''^2 + s_2 y''^2 = a''_{33}$ спрямо $K''\ O''x''y''$ ортонорм.

$s_1 > 0 \quad s_2 > 0 \quad , \quad a''_{33} > 0$

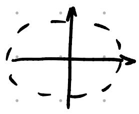
1. $\dfrac{x''^2}{a^2} + \dfrac{y''^2}{b^2} = 1$ елипса



2. $\dfrac{x''^2}{a^2} + \dfrac{y''^2}{b^2} = 0$ точка

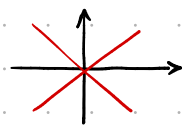


3. $\dfrac{x''^2}{a^2} + \dfrac{y''^2}{b^2} = -1$ комплексна елипса/окръжност



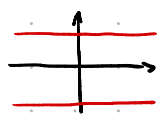
4. $\dfrac{x''^2}{a^2} - \dfrac{y''^2}{b^2} = 1$ хипербола

5. $\dfrac{x''^2}{a^2} - \dfrac{y''^2}{b^2} = 0$ пресичащи се прави, изродена крива



6. $\dfrac{x''^2}{a^2} - \dfrac{y''^2}{b^2} = -1$

7. $\dfrac{y''^2}{b^2} = 1$ успоредни прави



8. $\dfrac{y''^2}{b^2} = 0$ една права (броим 2 пъти)

9. $\dfrac{y''^2}{b^2} = -1$ комплексни прави

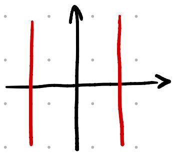
чрез ротация от Y):

$x''^2 - a^2 = 0$

$(x''-a)(x''+a) = 0$

$l_1 : x'' = a$

$l_2 : x'' = -a$



Пример:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$F_{1,2}(\pm\sqrt{a^2-b^2}, 0)$

$4x^2 + y^2 = 8 \qquad \frac{x^2}{2} + \frac{y^2}{8} = 1$

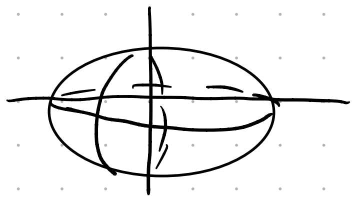
$F_{1,2} = (\pm\sqrt{2-8}, 0)$

$F_1(0, \sqrt{8-2})$

Метрични канонични уравнения на повърхнини от 2ра степен

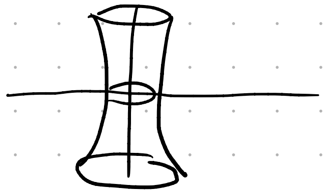
K Oxyz ортонормирана

1. $S: \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



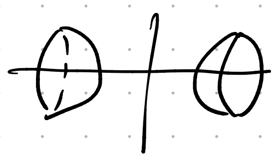
елипсоида

2. $S: \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



прав хиперболоид

3. $S \ \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



двоен хиперболоид

4. $S \ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2z$ елиптичен параболоид

5. $S \ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$ хиперболичен параболид

Повърхнини от 2ра степен съдържащи прави

1. $S \ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$

$S = \left(\frac{x}{a} - \frac{y}{b}\right)\left(\frac{x}{a} + \frac{y}{b}\right) = 2z$

$l_{\lambda,\mu} \begin{cases} \lambda\left(\frac{x}{a} - \frac{y}{b}\right) = \mu 2 \\ \mu\left(\frac{x}{a} + \frac{y}{b}\right) = \lambda z \end{cases} \qquad l_{\lambda',\mu'} \begin{cases} \lambda'\left(\frac{x}{a} - \frac{y}{b}\right) = \mu' z \\ \mu'\left(\frac{x}{a} + \frac{y}{b}\right) = \lambda' 2 \end{cases}$

$\lambda, \mu \neq (0,0)$

2\. $S: \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

$$\left(\frac{x}{a} - \frac{z}{c}\right)\left(\frac{x}{a} + \frac{z}{c}\right) = \left(1 - \frac{y}{b}\right)\left(1 + \frac{y}{b}\right)$$

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1 - \frac{y^2}{b^2}$$

$$l_{\lambda,\mu} \left| \begin{array}{l} \lambda\left(\frac{x}{a} - \frac{z}{c}\right) = \mu\left(1 - \frac{y}{b}\right) \\ \mu\left(\frac{x}{a} + \frac{z}{c}\right) = \lambda\left(1 + \frac{y}{b}\right) \end{array} \right. \qquad l'_{\lambda',\mu'} \left| \begin{array}{l} \lambda'\left(\frac{x}{a} - \frac{z}{c}\right) = \mu'\left(1 + \frac{y}{b}\right) \\ \mu'\left(\frac{x}{a} + \frac{z}{c}\right) = \lambda'\left(1 - \frac{y}{b}\right) \end{array} \right.$$

Задаване на криви в равнината

к Oxy

1. $y = f(x)$
2. $F(x,y) = 0$
3. $x = x(s)$

   $y = y(s)$

Задаване на повърхнини в пространството

к $Oxyz$

1. $S: z = f(x,y)$
2. $S: f(x,y,z) = 0$
3. $\begin{cases} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{cases} \quad u,v \in \mathbb{R}$

Задаване на крива

1. крива c $\left| \begin{array}{l} F(x,y,z) = 0 \\ G(x,y,z) = 0 \end{array} \right.$

2. $c = \begin{cases} x = x(s) \\ y = y(s) \\ z = z(s) \end{cases}$