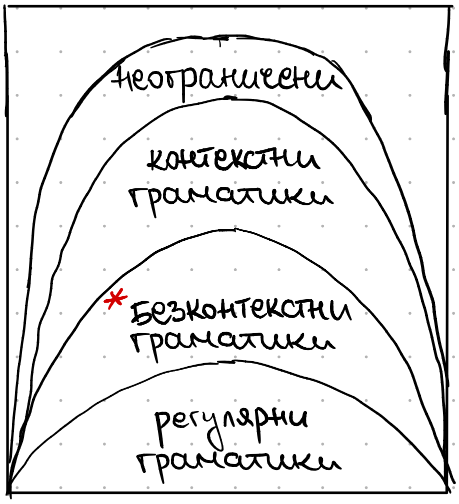
28.04.23

① ХН1  $Sort(L) = \{ a^{|w|_a} b^{|w|_b} c^{|w|_c} \mid w \in L \}$  $\Sigma = \{a, b, c\}$

Взимаме $L = \{a,b,c\}^*$ Тогава $Sort(L) = a^n b^n c^n$

Класове граматики отличаваме спрямо ограниченията над правилата им




$\alpha \to \beta \mid a \to b$

$A \to \alpha \mid \lambda A \rho \to \lambda \alpha \rho$ (изисква символи до пром.)

$A \to a$

$A \to aB$

релация $X \overset{l}{\Delta} \alpha$, където $X \in V \cup \Sigma$, $\alpha \in (V \cup \Sigma)^*$

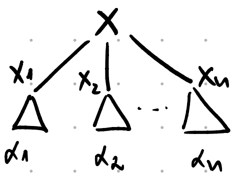
X, Y - променливи/букви

A, B, C - променливи

a, b, c - букви

0) $$\frac{}{X \overset{0}{\Delta} X}$$

1) $$\frac{X \to_G X_1 \dots X_n ,\ X_1 \overset{l_1}{\Delta} \alpha_1, \dots, X_n \overset{l_n}{\Delta} \alpha_n}{X \overset{l}{\Delta} \alpha_1 \dots \alpha_n}$$




$l = 1 + \sup\{l_1, \dots, l_n\}$

За $n = 0$: $X_1 \dots X_n = \varepsilon$  1) $$\frac{X \to_G \overbrace{X_1 \dots X_n}^{\varepsilon} ,\ X_1 \overset{l_1}{\Delta} \alpha_1, \dots, X_n \overset{l_n}{\Delta} \alpha_n}{X \overset{l}{\Delta} \underbrace{\alpha_1 \dots \alpha_n}_{\varepsilon}}$$

$$l = 1 + \sup \underbrace{\{l_1, \dots, l_n\}}_{\emptyset} = 1 + 0 = 1$$

Пример:

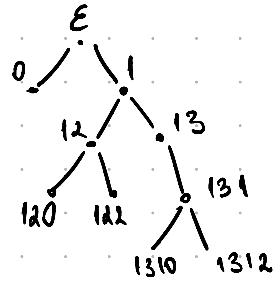
$L = \{a^n b^m c^k \mid n + m \leq k\}$

| | |
|---|---|
| $C = cC \mid c$ | $C = cC \mid \varepsilon$ |
| $S = aSC \mid bBC \mid \varepsilon$ | $S \to AC \mid \varepsilon$ |
| $B = bBC \mid \varepsilon$ | $A \to aAc \mid bBc \mid \varepsilon$ |
| | $B \to bBc \mid \varepsilon$ |

$\Sigma = \{0, 1, \dots, b-1\}$

$T \subseteq \Sigma^*$ е дърво $\overset{def}{\iff} Pref(T) = T$

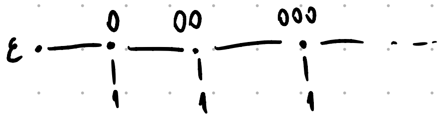
мн-во от думи затв. относно префикси




$T = \{\varepsilon, 0, 1, 12, 13, 120, 122, 131, 1310, 1312\}$

Оптимално дърво $T$: $\left.\begin{array}{l}\alpha i \in T \\ j<i\end{array}\right\} \Rightarrow \alpha j \in T$

$\{0\}^+ . \{1\} \cup \varepsilon$




$P = (T, \lambda)$ – дърво на извод, съвместимо с граматиката $G$

$T$ – оптимално дърво $\{0, \dots, b-1\}^*$ където $b = \max\{|\alpha| : A \to \alpha$ е правило в $G\}$

$\lambda$ – етикет $\lambda : T \to V \cup \Sigma$

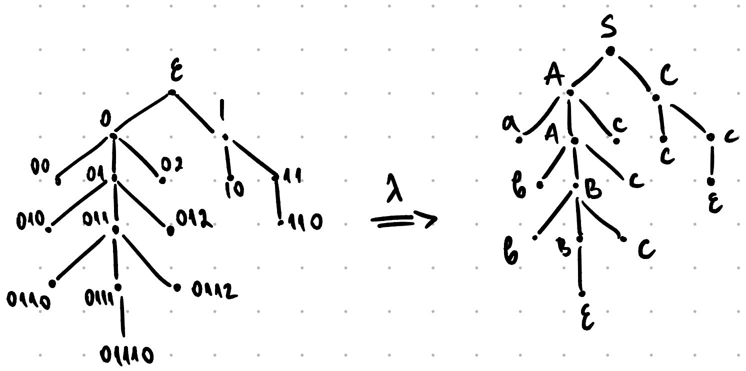
Нека $\alpha \in T$ и $k+1 = |\{\alpha.i \in T \mid i < b\}|$

Тогава $\lambda(\alpha) \Rightarrow_G \lambda(\alpha 0).\lambda(\alpha 1) \dots \lambda(\alpha k)$ е правило в $G$

$height(T) = \max\{|\alpha| : \alpha \in T\}$

$\lambda(\varepsilon) = S \qquad \lambda(011) = B$

$\lambda(01) \xrightarrow[G]{} \underbrace{\lambda(010)}_{a}\, \underbrace{\lambda(011)}_{A}\, \underbrace{\lambda(012)}_{c}$




$S \to Ac$

$C \to cC \mid \varepsilon$

$A \to aAc \mid bBc \mid \varepsilon$

$B \to bBc \mid \varepsilon$

$A \overset{\ell}{\Delta}_G \alpha \iff$ същ. дърво на извод $P = (T, \lambda)$ съвместимо с $G$

$height(T) = \ell \qquad leaves(P) = \alpha$

$root(P) = A$

$A \overset{l}{\Delta}_G \alpha \Rightarrow |\alpha| \leq b^l$, където $b = \max\{|\alpha|, A \to_G \alpha$ е правило в $G\}$

$|\alpha| > b^l \Rightarrow A \overset{\geq l+1}{\Delta} \alpha$

за $\alpha \in \mathcal{L}(G)$

$\mathcal{L}(G) = \{\alpha \in \Sigma^* \mid S \overset{*}{\Delta}_G \alpha\}$

$\mathcal{L}_A(q) = \{\alpha \in \Sigma^* \mid \delta_A^*(q, \alpha) \in F\}$ $\quad$ $\mathcal{L}_A^l(q) = \{\alpha \in \Sigma^* \mid |\alpha| \leq l \ \& \ \delta_A^*(q, \alpha) \in F\}$

$\mathcal{L}_G(X) = \{\alpha \in \Sigma^* \mid X \overset{*}{\Delta} \alpha\}$ $\quad$ $\mathcal{L}_G^l(X) = \{\alpha \in \Sigma^* \mid X \overset{\leq l}{\Delta}_G \alpha\}$

за $X \in V \cup \Sigma$

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$X \in \Sigma^*$: $\quad \mathcal{L}_G^0(X) = \emptyset$

$\mathcal{L}_G^l(X) \subseteq \mathcal{L}_G^{l+1}(X)$

$\mathcal{L}_G(X) = \bigcup_l \mathcal{L}_G^l(X)$

Тв: $\mathcal{L}_G^{l+1}(A) = \bigcup\{\mathcal{L}_G^l(x_1) \dots \mathcal{L}_G^l(x_n) \mid A \to_G x_1 \dots x_n$ е правило в $G\}$

Док: Индукция по $l$

- $l = 0$ $\quad \mathcal{L}_G^1(A) = \{\alpha \in \Sigma^* \mid A \overset{1}{\Delta}_G \alpha\}$ $\quad A \to x_1 \dots x_n$ и

$\alpha \in \mathcal{L}_G^0(x_i)$ $x_i \overset{0}{\Delta} \alpha_i$ и

$\alpha = \alpha_1 \dots \alpha_n$

$\alpha \in \mathcal{L}_G^0(x_1) \dots \mathcal{L}_G^0(x_n)$

$\mathcal{L}_G^1(A) \subseteq \bigcup\{\mathcal{L}_G^0(x_1) \dots \mathcal{L}_G^0(x_n)$

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$S \to aSb \mid \varepsilon$

$\mathcal{L}_G^{l+1}(S) = \mathcal{L}_G^l(a) . \mathcal{L}_G^l(S) . \mathcal{L}_G^s(b)$

$\{a\} . \mathcal{L}^l(S) . \{b\} \cup \{\varepsilon\}$