Несобствени интеграли

04.04.24

$$\int_a^{\infty} f(x)dx = \lim_{p\to\infty} \int_a^p f(x)dx \qquad f:[a,b)\to\mathbb{R} \quad \int_a^b f(x)dx = \lim_{p\to b^-}\int_a^p f(x)dx$$

Сравнителен критерий

$f,g:[a,+\infty)\to\mathbb{R}$, f,g интегруеми в $[a,p]$ $\forall p\in[a,+\infty)$ $\quad 0\le f(x)\le g(x) \quad x\in[a,+\infty)$

1) $\int_a^{+\infty} f(x)\,dx$ разходящ $\Rightarrow \int_a^{+\infty} g(x)\,dx$ разходящ

2) $\int_a^{+\infty} g(x)\,dx$ сходящ $\Rightarrow \int_a^{+\infty} f(x)\,dx$ сходящ

Гранична форма

$f,g:[a,+\infty)\to\mathbb{R}$, f,g интегруеми в $[a,p]$ $\forall p\in[a,+\infty)$ $\quad f(x)\ge 0 \quad g(x)>0$

$\lim_{x\to+\infty}\frac{f(x)}{g(x)} = l$

1) $l\in\mathbb{R},\ l\ne 0$ $\quad \int_a^{\infty} f(x)\,dx$ и $\int_a^{\infty} g(x)dx$ едновременно сходящи/разходящи

2) $l=0$ $\quad \int_a^{+\infty} g(x)\,dx$ сходящ $\Rightarrow \int_a^{+\infty} f(x)\,dx$ сходящ

3) $l=\infty$ $\quad \int_a^{+\infty} f(x)\,dx$ сходящ $\Rightarrow \int_a^{+\infty} g(x)\,dx$ сходящ

$\int_a^{+\infty}\frac{1}{x^\alpha}dx$ сходящ $\alpha>1$, разходящ $\alpha\le 1$ $\qquad \int_0^{1}\frac{1}{x^\alpha}dx$ разходящ $\alpha\ge 1$, сходящ $\alpha<1$

① Изсл. за сходимост в зав. от параметрите

a) $$\int_{\underline{\underline{3}}}^{\infty}\frac{x^\lambda\cdot\sin\frac{1}{x}}{\sqrt{x^2-5x+6}}dx = \underbrace{\int_{\underline{\underline{3}}}^{4}\frac{x^\lambda\cdot\sin\frac{1}{x}}{\sqrt{x^2-5x+6}}dx}_{I_2} + \underbrace{\int_4^{+\infty}\frac{x^\lambda\cdot\sin\frac{1}{x}}{\sqrt{x^2-5x+6}}dx}_{I_1}$$

$\sqrt{(x-2)(x-3)}$ $\quad \downarrow 2 \quad \downarrow \underline{\underline{3}}$

I_1: $\quad g_1(x)=\frac{1}{x^\alpha} \qquad f(x)=\frac{x^\lambda\cdot\sin\frac{1}{x}}{\sqrt{x^2-5x+6}}$

$$\boxed{I_1\sim\int_4^{+\infty}\frac{1}{x^{2-\lambda}} \quad \text{сх.}\ 2-\lambda>1 \quad \lambda<1 \quad \text{разх.}\ 2-\lambda\le 1 \quad \lambda\ge 1}$$

$$\lim_{x\to\infty}\frac{f(x)}{g_1(x)} = \lim_{x\to\infty}\frac{x^\lambda\cdot\sin\frac{1}{x}}{\sqrt{x^2-5x+6}}\cdot x^\alpha = \lim_{x\to\infty}\frac{x^\lambda\,\frac{1}{x}\cdot x^\alpha}{x\underbrace{\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}}_{\to 1}} = \lim_{x\to\infty} x^{\lambda+\alpha-2} = 1 \text{ при}$$

$\alpha+\lambda-2=0$

$\alpha=2-\lambda$

$\frac{1}{x}\xrightarrow[x\to+\infty]{}0$

$\sin\frac{1}{x}\sim\frac{1}{x}$ за $x\to+\infty$

$I_2:\quad g_2(x)=\frac{1}{(x-3)^{\alpha}} \qquad f(x)=\frac{x^{\lambda}\cdot\sin\frac{1}{x}}{\sqrt{x^2-5x+6}}$

$$\boxed{I_2\sim\int_3^4\frac{1}{(x-3)^{\frac{1}{2}}}\,dx \quad \lambda=\frac{1}{2}<1 \Rightarrow \text{сходящ } \forall\lambda}$$

$$\lim_{x\to3^+}\frac{f(x)}{g_2(x)}=\lim_{x\to3^+}\frac{x^{\lambda}\cdot\sin\frac{1}{x}}{\sqrt{(x-2)(x-3)}}\cdot(x-3)^{\alpha}=\frac{3^{\lambda}\sin\frac{1}{3}}{1}\lim_{x\to3^+}(x-3)^{\lambda-\frac{1}{2}}=3^{\lambda}\cdot\sin\frac{1}{3}\cdot1$$

ако $\alpha=\frac{1}{2}$

$I=I_1+I_2$ е сходящ за $\lambda<1$ и разходящ за $\lambda\geq1$

б) $$\int_{\underline{\underline{1}}}^{\overline{\overline{+\infty}}}\frac{\operatorname{arctg}(x-1)}{x(\ln x)^{\lambda}}\,dx=\underbrace{\int_{\underline{\underline{1}}}^{2}\frac{\operatorname{arctg}(x-1)}{x(\ln x)^{\lambda}}\,dx}_{I_2}+\underbrace{\int_{2}^{\overline{\overline{+\infty}}}\frac{\operatorname{arctg}(x-1)}{x(\ln x)^{\lambda}}\,dx}_{I_1}$$

$I_1:\quad g_1(x)=\frac{1}{x(\ln x)^{\lambda}} \qquad f(x)=\frac{\operatorname{arctg}(x-1)}{x(\ln x)^{\lambda}} \qquad I_1\sim\int_2^{+\infty}\frac{1}{x(\ln x)^{\lambda}}\,dx=\int_2^{+\infty}\frac{1}{(\ln x)^{\lambda}}\,d\ln x=\int_2^{+\infty}\frac{1}{y^{\lambda}}\,dy$

$$\lim_{x\to\infty}\frac{f(x)}{g_1(x)}=\frac{\operatorname{arctg}(x-1)}{x(\ln x)^{\lambda}}\cdot x\cdot(\ln x)^{\lambda}=\frac{\pi}{2}\cdot\lim_{x\to\infty}\frac{x(\ln x)^{\lambda}}{x(\ln x)^{\lambda}}=1$$

$$\boxed{\begin{array}{l}\text{сх } \lambda>1\\ \text{разх. } \lambda\leq1\end{array}}$$

$I_2:\quad g_2(x)=\frac{1}{(x-1)^{\alpha}} \qquad f(x)=\frac{\operatorname{arctg}(x-1)}{x(\ln x)^{\lambda}} \qquad I_2\sim\int_1^2\frac{1}{x^{\lambda-1}}\,dx$

$$\boxed{\begin{array}{lll}\text{сх} & \lambda-1<1, & \lambda<2\\ \text{разх} & \lambda-1\geq1, & \lambda\geq2\end{array}}$$

$$\lim_{x\to1^+}\frac{f(x)}{g_2(x)}=\lim_{x\to1^+}\frac{\operatorname{arctg}(x-1)}{\underset{\to1}{x}\cdot(\ln x)^{\lambda}}\cdot(x-1)^{\alpha}\overset{(\sim)}{=}\lim_{x\to1^+}\frac{(x-1)}{1.(x-1)^{\lambda}}\cdot(x-1)^{\alpha}=\lim_{x\to1^+}(x-1)^{\alpha+1-\lambda}=1$$

$\operatorname{arctg}(x-1)\sim(x-1) \quad x\to1^+$

$\ln(1+(x-1))\sim(x-1) \quad x\to1^+$

при $\alpha+1-\lambda=0$
$\alpha=\lambda-1$

$I=I_1+I_2$ сх $\lambda\in(1,2)$ разх. $\lambda\in(1,2)$

в) $$\int_{\underline{\underline{0}}}^{1}\frac{\ln x}{(1-x^2)^{\lambda}}\,dx=\underbrace{\int_{\underline{\underline{0}}}^{\frac{1}{2}}\frac{\ln x}{(1-x^2)^{\lambda}}\,dx}_{I_1}+\underbrace{\int_{1/2}^{1}\frac{\ln x}{(1-x^2)^{\lambda}}\,dx}_{I_2}$$

$I_1:\quad g_1(x)=\ln x \quad f(x)=\frac{\ln x}{(1-x^2)^{\lambda}} \qquad \lim_{x\to0^+}\frac{f(x)}{g_1(x)}=\lim_{x\to0^+}\frac{\ln x}{(1-x^2)^{\lambda}}\cdot\frac{1}{\ln x}=1$

$$I_1\sim\int_0^{1/2}\ln x\,dx=\lim_{p\to0^+}\int_p^{1/2}\ln x\,dx=\lim_{p\to0^+}x.\ln x\Big|_p^{1/2}-\int_p^{\frac{1}{2}}\cancel{x}\cdot\frac{1}{\cancel{x}}\,dx=\lim_{p\to0^+}\frac{1}{2}.\ln\frac{1}{2}-p.\ln p-x\Big|_p^{1/2}=$$

$$=\lim_{p\to0^+}\frac{1}{2}.\ln\frac{1}{2}-\underbrace{p.\ln p}_{0.\infty\to0}-\left(\frac{1}{2}-p\right)=\frac{1}{2}\ln\frac{1}{2}-\frac{1}{2}\Rightarrow\text{сходящ }\forall\lambda$$

$$\lim_{p\to0^+}p\ln p=\lim_{p\to0^+}\frac{\ln p}{\frac{1}{p}}\overset{\text{Лоп.}}{=}\lim_{p\to0^+}\frac{\frac{1}{p}}{-\frac{1}{p^2}}=\lim_{p\to0^+}-\frac{p^2}{p}=0$$

I_2: $g(x) = \frac{1}{(1-x)^\alpha}$ $\quad I_2 \sim \int_{1/2}^{1} \frac{1}{(1-x)^{\lambda-1}} dx$ сх $\lambda-1<1$ $\quad \lambda<2$
разх. $\lambda-1\geq 1$ $\quad \lambda\geq 2$

$$\lim_{x\to 1^-} \frac{\ln x}{(1-x^2)^\lambda} \cdot (1-x)^\alpha \overset{(\sim)}{=} \lim_{x\to 1^-} \frac{x-1}{(1-x)^\lambda(1+x)^\lambda} \cdot (1-x)^\alpha = -\frac{1}{2^\lambda} \lim_{x\to 1^-} (1-x)^{\alpha+1-\lambda} = -\frac{1}{2^\lambda} \text{ при } \alpha+1-\lambda=0$$
$\alpha=\lambda-1$

$\ln(1+(x-1)) \sim (x-1)$ при $x\to 1$

$I = I_1 + I_2$ сх $\lambda<2$ разх $\lambda\geq 2$

г) $\int_0^1 \frac{\ln(1+2x^2)-4(\cos x-1)}{x^\lambda} dx$ $\quad g(x) = \frac{1}{x^\alpha}$

$$\lim_{x\to 0^+} \frac{f(x)}{g(x)} = \lim_{x\to 0^+} \frac{\ln(1+2x^2)-4(\cos x-1)}{x^\lambda} \cdot x^\alpha = \lim_{x\to 0^+} \frac{2x^2-(-2x^2)+o(x^2)}{x^\lambda} \cdot x^\alpha =$$

$\ln(1+2x^2) = 2x^2 - \frac{4x^4}{2} + o(4x^4)$ $\qquad \cos x - 1 = \cancel{1} - \frac{x^2}{2} + \frac{x^4}{4} + o(x^4) - \cancel{1}$

$\ln(1+y) = y - \frac{y^2}{2} + o(y^2)$ $\qquad 4(\cos x-1) = -2x^2 + \frac{x^4}{6} + o(x^4) = -2x^2 + o(x^2)$

$$= \lim_{x\to 0^+} \frac{4x^2\left(1+\frac{o(x^2)}{4(x^2)}\right)}{x^\lambda} \cdot x^\alpha = \lim_{x\to 0^+} 4 \cdot x^{\alpha+2-\lambda} = 4$$ за $\alpha+2-\lambda=0$
$\alpha=\lambda-2$

$I \sim \int_0^1 \frac{1}{x^{\lambda-2}} dx$ сх. $\lambda-2<1$ $\lambda<3$ разх $\lambda\geq 3$

д) $\int_0^1 \frac{1-e^{\cos x-1}}{x^\lambda} dx$ $\quad g(x) = \frac{1}{x^\alpha}$

$$\lim_{x\to 0^+} \frac{f(x)}{g(x)} = \lim_{x\to 0^+} \frac{1-e^{\cos x-1}}{x^\lambda} \cdot x^\alpha = \lim_{x\to 0^+} \frac{-\frac{x^2}{2}+o(x^2)}{x^\lambda} \cdot x^\alpha = \lim_{x\to 0^+} -\frac{1}{2} \cdot x^{\alpha+2-\lambda} = -\frac{1}{2} \text{ при } \alpha=\lambda-2$$

$e^y = 1 + y + \frac{y^2}{2!} + o(y^2)$ $\qquad \cos x - 1 = \cancel{1} - \frac{x^2}{2} + \frac{x^4}{4} + o(x^4) - \cancel{1}$ $\qquad (\cos x-1)^2 = \frac{x^4}{4} + o(x^4)$

$e^{\cos x-1} = 1 + \cos x - 1 + \frac{(\cos x-1)^2}{2!} + \underset{o(x^4)}{\underset{\parallel}{o((\cos x-1)^2)}}$ $\qquad e^{\cos x-1} = 1 + \frac{x^2}{2!} - \frac{x^4}{4!} + o(x^4) + \frac{x^4}{4\cdot 2} + o(x^4)$

$\cos x - 1 \to 0$
$x\to 0$

$$1-e^{\cos x-1} = 1-\left(1-\frac{x^2}{2}-\frac{x^4}{4!}+\frac{x^4}{8}+o(x^4)\right) = -\frac{x^2}{2}+o(x^2)$$

$I \sim \int_0^1 \frac{1}{x^{\lambda-2}} dx$ сх. $\lambda-2<1$, $\lambda<3$
разх. $\lambda-2\geq 1$, $\lambda\geq 3$

e) $\int_0^{\pi/2} \frac{e^{\lambda\cos x} - \sqrt{1+2\cos x}}{\sqrt{\cos^5 x}}\,dx = -\int_{\pi/2}^{0} \frac{e^{\lambda\cos(\frac{\pi}{2}-t)} - \sqrt{1+2\cos(\frac{\pi}{2}-t)}}{\sqrt{\cos^5(\frac{\pi}{2}-t)}}\,dt = \int_0^{\pi/2} \frac{e^{\lambda\sin t} - \sqrt{1+2\sin t}}{\sqrt{\sin^5 t}}\,dt$

$t = \frac{\pi}{2} - x \qquad x=0 \quad t=\frac{\pi}{2}$

$dt = -dx \qquad x=\frac{\pi}{2} \quad t=0$

$g(x) = \frac{1}{t^\alpha}$

$\lim_{t\to 0^+} \frac{f(x)}{g(x)} = \lim_{t\to 0^+} \frac{e^{\lambda\sin t} - \sqrt{1+2\sin t}}{\sqrt{\sin^5 t}}\cdot t^\alpha = \lim_{t\to 0^+} \frac{e^{\lambda\sin t} - \sqrt{1+2\sin t}}{t^{5/2}}\cdot t^\alpha = (*)$

$e^{\lambda\sin t} = 1 + \lambda\sin t + \frac{\lambda^2\sin^2 t}{2} + o(\underbrace{\lambda^2\sin^2 t}_{t^2}) = 1+\lambda(t+o(t^2)) + \frac{\lambda^2(t^2+o(t^2))}{2} + o(t^2) =$

$= 1 + \lambda t + \frac{\lambda^2 t^2}{2} + o(t^2)$

$(1+y)^a = 1 + ay + \frac{a(a-1)y^2}{2} + o(y^2)$

$(1+2\sin t)^{\frac{1}{2}} = 1 + \frac{1}{2}\cdot 2\sin t + \frac{\frac{1}{2}\cdot(\frac{1}{2}-1)}{2}\cdot 4\sin^2 t + o(\underbrace{\sin^2 t}_{t^2}) =$

$= 1 + (t + o(t^2)) - \frac{1}{2}(t^2 + o(t^2)) + o(t^2) = 1 + t - \frac{1}{2}t^2 + o(t^2)$

$(*) = \lim_{t\to 0^+} \frac{1+\lambda t + \frac{\lambda^2 t^2}{2} + o(t^2) - (1+t+\frac{1}{2}t^2+o(t^2))}{t^{5/2}}\cdot t^\alpha = \lim_{t\to 0^+} \frac{(\lambda-1)t + \frac{1}{2}(\lambda^2+1)t^2 + o(t^2)}{t^{5/2}}\cdot t^\alpha$

1) $\lambda = 1 \quad = \lim_{t\to 0^+} \frac{\frac{1}{2}(\lambda^2+1)t^2 + o(t^2)}{t^{5/2}}\cdot t^\alpha = \frac{1}{2}(\lambda^2+1)\cdot\lim_{t\to 0^+} t^{\alpha+2-5/2} = \frac{1}{2}(\lambda^2+1)$ за $\alpha = \frac{1}{2}$

$I \sim \int_0^{\pi/2} \frac{1}{t^{1/2}}\,dt \quad \alpha = \frac{1}{2} < 1 \Rightarrow I$ е сходящ

2) $\lambda \neq 1 \quad = \lim_{t\to 0^+} \frac{(\lambda-1)t + o(t)}{t^{5/2}}\, t^\alpha = (\lambda-1)\lim_{t\to 0^+} t^{\alpha+1-\frac{5}{2}} = \lambda - 1$ за $\alpha = \frac{3}{2}$

$I \sim \int_0^{\pi/2} \frac{1}{t^{3/2}}\,dt \quad \alpha = \frac{3}{2} > 1$ разх.

$\Rightarrow I$ е сх. за $\lambda = 1$ и разх. за $\lambda \neq 1$

H1) $\int_0^{+\infty} e^{-x} x^\lambda\,dx = \underbrace{\int_0^1 e^{-x}x^\lambda dx}_{I_2} + \underbrace{\int_1^{+\infty} e^{-x}x^\lambda dx}_{I_1}$

I_1: $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \underset{\downarrow 0}{e^{-x}}\cdot x^\lambda \cdot x^\alpha = \lim_{x\to\infty} \frac{x^{\alpha+\lambda}}{e^x} = 0 \quad \forall\, \alpha$

Нека $\alpha>1$ $\alpha=2$ $\int_1^{+\infty} \frac{1}{x^\alpha} dx$ сходящ => I_1 е сходящ $\forall \lambda$

I_2: $\lim\limits_{x\to 0^+} \underset{\downarrow 1}{e^{-x}} . x^\lambda . x^\alpha = \lim\limits_{x\to 0^+} x^{\alpha+\lambda} = 1$ при $\alpha=-\lambda$

$I_2 \sim \int_0^1 \frac{1}{x^{-\lambda}} dx$ сх. $-\lambda<1$, $\lambda>-1$
разх $\lambda\le -1$

=> $I = I_1 + I_2$ сх за $\lambda>-1$ разх. за $\lambda\le-1$

$\Gamma(\lambda) = \int_0^{+\infty} e^{-x} x^{\lambda-1} dx$ $\lambda-1>-1$ $\lambda>0$ $\lambda\in\mathbb{N}$ $\Gamma(\lambda)=(\lambda-1)!$

гама ф-я

3) $\int_0^{+\infty} e^{-\mu x} x^\lambda dx = \underbrace{\int_0^1 e^{-\mu x} x^\lambda dx}_{I_2} + \underbrace{\int_1^{+\infty} e^{-\mu x} x^\lambda dx}_{I_1}$

I_2: $\lim\limits_{x\to 0^+} \underset{\downarrow 1}{e^{-\mu x}} . x^\lambda . x^\alpha = \lim\limits_{x\to 0^+} x^{\alpha+\lambda} = 1$ при $\alpha=-\lambda$

$I_2 \sim \int_0^1 \frac{1}{x^{-\lambda}} dx$ сх. $\lambda>-1$
разх. $\lambda\le-1$

I_1: $\lim\limits_{x\to\infty} e^{-\mu x} . x^\lambda . x^\alpha = \lim\limits_{x\to\infty} \frac{x^{\alpha+\lambda}}{e^{\mu x}} = \begin{cases} \infty & \mu<0 \to I_1 \text{ разх. } \forall\lambda \\ \lim\limits_{x\to+\infty} x^{\alpha+\lambda} & \mu=0 \quad I_1=\int_1^{+\infty}\frac{1}{x^{-\lambda}}dx \quad \text{сх } \lambda<-1, \text{ разх } \lambda\ge-1 \\ 0 & \mu>0 \to I_1 \text{ сх. } \forall\lambda \end{cases}$

$\mu<0$ $\lim\limits_{x\to\infty} e^{-\mu x} . x^\lambda . x^\alpha$ Нека $\alpha=1$ $\int_1^{+\infty}\frac{1}{x}dx$ разх => I_1 разх.

=> $I = I_1 + I_2$ сх $\mu>0$, $\lambda>-1$ разх. иначе

4) $\int_2^{+\infty} \frac{1}{x^\alpha (\ln x)^\beta} dx = \int_{\ln 2}^{\infty} \frac{1}{e^{\alpha t} . t^\beta} . e^t dt = \int_{\ln 2}^{+\infty} e^{-(\alpha-1)t} t^{-\beta} dt \sim \int_{\ln 2}^{+\infty} e^{-\mu t} . t^\lambda dt$

$\mu=\alpha-1$ $\lambda=-\beta$

$t=\ln x$
$x=e^t$ $dx=e^t dt$

сх. $\alpha-1>0$, $\forall\beta \longrightarrow \alpha>1, \beta\in\mathbb{R}$ разх. иначе
или
$\alpha-1=0$, $-\beta<-1 \longrightarrow \alpha=1, \beta>1$

ii) $\int_0^{+\infty} \frac{x^\lambda}{(x^3+x^6)\ln^\lambda(1+\sqrt[4]{x})}dx = \underbrace{\int_0^1 \frac{x^\lambda}{(x^3+x^6)\ln^\lambda(1+\sqrt[4]{x})}dx}_{I_2} + \underbrace{\int_1^{+\infty} \frac{x^\lambda}{(x^3+x^6)\ln^\lambda(1+\sqrt[4]{x})}dx}_{I_1}$

I_1: $g_1(x) = \frac{1}{x^\alpha (\ln x)^\beta}$

$$\lim_{x\to\infty} \frac{f(x)}{g_1(x)} = \lim_{x\to\infty} \frac{x^\lambda}{(x^3+x^6)\ln^\lambda(1+\sqrt[4]{x})} \cdot x^\alpha \cdot (\ln x)^\beta = \lim_{x\to\infty} \frac{x^{\lambda+\alpha}}{x^3+x^6} \cdot \frac{\ln x^\beta}{\ln^\lambda(1+\sqrt[4]{x})} =$$

$$\lim_{x\to\infty} \frac{x^{\lambda+\alpha}}{x^6\underbrace{(\frac{1}{x^3}+1)}_{\to 1}} \cdot \frac{\ln^\beta x}{\ln^\lambda x \cdot \underbrace{\frac{\ln^\lambda(1+\sqrt[4]{x})}{\ln^\lambda x}}_{\to c}} = \lim_{x\to\infty} c \cdot x^{\lambda+\alpha-6} \cdot (\ln x)^{\beta-\lambda} = c \quad \text{при } \alpha = 6-\lambda$$

$$\beta = \lambda$$

$$\lim_{x\to\infty} \frac{\ln x}{\ln(1+\sqrt[4]{x})} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{1+\sqrt[4]{x}} \cdot \frac{1}{4} \cdot \frac{1}{\sqrt[4]{x^3}}} = 4$$

$\Rightarrow \int_1^{+\infty} \frac{1}{x^{6-\lambda}(\ln x)^\lambda}dx$

сх. $6-\lambda>1$ и $\beta\in\mathbb{R}$
$\lambda<5$
или $6-\lambda=1$ и $\beta>1$
$\lambda=5$
разх. иначе

$$I_2: \lim_{x\to 0^+} \frac{x^\lambda}{(x^3+x^6)\ln^\lambda(1+\sqrt[4]{x})} \cdot x^\alpha \overset{(\sim)}{=} \lim_{x\to 0^+} \frac{x^\lambda}{x^3(1+x^3)(\sqrt[4]{x})^\lambda} \cdot x^\alpha = \lim_{x\to 0^+} x^{\alpha+\lambda-3-\frac{\lambda}{4}} = 1 \quad \text{при}$$

$$\alpha = 3-\tfrac{3}{4}\lambda$$

$I_2 \sim \int_0^1 \frac{1}{x^{3-\frac{3}{4}\lambda}}dx$ сх. $3-\frac{3}{4}\lambda<1$ $\lambda>\frac{8}{3}$

$I = I_1 + I_2$ сх. $\lambda\in(\frac{8}{3};5]$

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Числови редове

$a_1, \dots, a_n, \dots$ $a_n\in\mathbb{R}$

$\sum_{n=1}^{\infty} a_n = a_1+a_2+\dots+a_n+a_{n+1}+\dots$

$\sum_{n=1}^{\infty} a_n$ е сходящ, ако $\{S_n\}_{n=1}^{\infty}$ е сходяща

$S_n = a_1, \dots a_n$ - частична сума

пример 1) $a_n = n$

$\sum_{n=1}^{\infty} n = 1+2+\dots+n+\dots$

$S_n = 1+2+\dots+n = \frac{n(n+1)}{2}$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n(n+1)}{2} = +\infty \Rightarrow$ разходящ