

AI1110 - Probability and Random Variables

Assignment 10

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Outline

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Question

Exercise 10.2

In a two-dimensional random walk. The coordinates $x(t)$ and $y(t)$ of a moving object are two independent random-walk processes with the same s and T . Given a $z(t) = \sqrt{x^2(t) + y^2(t)}$

Question Continued

Question Continued

Show that if $z(t) = \sqrt{x^2(t) + y^2(t)}$ is the distance of the object from the origin and $t \gg T$, then for z of the order of $\sqrt{\alpha t}$:

$$f_z(z, t) \simeq \frac{z}{\alpha t} e^{\frac{-z^2}{2\alpha t}} U(z)$$

where $\alpha = \frac{s^2}{t}$

Solution

For large t, $x(t)$ and $y(t)$ can be approximated by two independent Wiener processes as in

$$f_x(x, t) = \frac{1}{\sqrt{2\pi\alpha t}} e^{\frac{-x^2}{2\alpha t}}$$

$$f_y(y, t) = \frac{1}{\sqrt{2\pi\alpha t}} e^{\frac{-y^2}{2\alpha t}}$$

Hence $z(t)$ has a Rayleigh density. The product $f_z(z, t)dz$ equals approximately the probability $z(t)$ is between z and $z + dz$ provided that $dz \gg T$.

Solution Continued

Here $z(t)$ is a discrete type RV taking the values $\sqrt{m^2 + n^2}$ where m, n are integers.

Hence that if $z(t) = \sqrt{x^2(t) + y^2(t)}$ is the distance of the object from the origin and $t \gg T$, then z is the order of $\sqrt{\alpha t}$ and

$$f_z(z, t) \simeq \frac{z}{\alpha t} e^{-\frac{z^2}{2\alpha t}} U(z)$$

where $\alpha = \frac{s^2}{t}$