AI1110 - Probability and Random Variables Assignment 10

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Outline

Question

EXAMPLE 8-29

In a two-dimensional random walk.The coordinates x(t) and y(t) of a moving object are two independent random-walk processes with the same s and T.Given a $z(t) = \sqrt{x^2(t) + y^2(t)}$

Question Continued

Show that if $z(t) = \sqrt{x^2(t) + y^2(t)}$ is the distance of the object from the origin and $t >> T$, then for z of the order of $\sqrt{\alpha t}$:

$$f_z(z,t) \simeq \frac{z}{\alpha t} e^{\frac{-z^2}{2\alpha t}} U(z)$$

where $\alpha = \frac{S^2}{t}$

Solution

For large t, x(t) and y(t) can be approximated by two independent Wiener processes as in

$$f_x(x,t) = \frac{1}{\sqrt{2\pi\alpha t}} e^{\frac{-x^2}{2\alpha t}}$$

$$f_y(y,t) = \frac{1}{\sqrt{2\pi\alpha t}} e^{\frac{-y^2}{2\alpha t}}$$

Hence z(t) has a Rayleigh density.The product $f_z(z,t)dz$ equals approximately the probability z(t) is between z and z $+$ dz provided that dz $>>$ T.

Solution Continued

Here z(t) is a discrete type RV taking the values$\sqrt{m^2+n^2}$ where m,n are integers.
Hence that if $z(t)=\sqrt{x^2(t)+y^2(t)}$ is the distance of the object from the origin and $t >> T$, then z is the order of $\sqrt{\alpha t}$ and

$$f_z(z,t) \simeq \frac{z}{\alpha t} e^{\frac{-z^2}{2\alpha t}} U(z)$$

where $\alpha = \frac{S^2}{t}$