AI1110 - Probability and Random Variables
Assignment 11

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Outline

Question

Exercise 11.6

Show that if x(t) is a WSS process and

$$s = \frac{1}{n}\sum_{k=1}^{n} x(kT)$$

then

$$E\left\{s^2\right\} = \frac{1}{2\pi n^2}\int_{-\infty}^{\infty} S_x(w)\frac{\sin^2 n\omega T/2}{\sin^2 \omega T/2}\, d\omega$$

Solution

Solution

The process

$$y[n] = \frac{1}{n}\sum_{k=1}^{n} x(nT + kT)$$

is the output of a system with input x[n] and system function

$$H(z) = \frac{1}{n}\sum_{k=1}^{n} z^{k}$$

Solution Continued

Furthermore, s = y[0] and

$$n^2 \left|H(e^{j\omega T})\right|^2 = \left|\sum_{k=1}^{n}(e^{j\omega T})\right|^2$$

$$= \left|\frac{e^{j\omega T} - e^{j(n+1)\omega T}}{1 - e^{j\omega T}}\right|$$

$$= \frac{\sin^2 n\omega T/2}{\sin^2 \omega T/2}$$

Solution Continued

Hence,

$$E\left\{s^2\right\} = R_{[y]} = \frac{1}{2\pi n^2}\int_{-\infty}^{\infty} S_x(w)\frac{\sin^2 n\omega T/2}{\sin^2 \omega T/2}\, d\omega$$