AI1110 - Probability and Random Variables Assignment 4

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Example 10

A black and a red dice are rolled.
(a) Find the conditional probability of obtaining a sum greater than 9,given that the black die resulted in a 5
(b) Find the conditional probability of obtaining the sum 8 , given that the red die resulted in a number less than 4

Solution

(a) Let $X \in \{0, 1\}$is random variable that denote whether the sum is greater than 9 or not
Let X = 0 denotes the sum is less than or equal to 9 and Y = 1 denotes that sum is greater than 9.
Events when sum is greater than 9 are (4,6),(5,5),(5,6),(6,4),(6,5),(6,6)
Total events are $6^2 = 36$
$\Pr(X = 1) = 6/36$

Solution Continued

Let $Y \in \{0,1\}$is random variable that denote whether the number on black die is 5 or not
Let Y = 0 denotes the number on black die is not 5 and X = 1 denotes that number on black die is 5.
Events that satisfy Y are (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) Events that satisfy both X,Y are (5,5),(5,6)
$\Pr(Y = 1, X = 1) = \frac{2}{36}$

Solution Continued...

The desired probability is

$$\Pr\left(Y = 1 | X = 1\right) = \frac{\Pr\left(Y = 1, X = 1\right)}{\Pr\left(X = 1\right)} \text{ (Bayes' Theorem)} \tag{1}$$

$$\Pr\left(Y = 1 | X = 1\right) = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3} \tag{2}$$

Solution Continued

(b) Let $X \in \{0, 1\}$is random variable that denote whether the sum is 8 or not

Let X $=$ 0 denotes the sum is not 8 and X $=$ 1 denotes that sum is 8.

Events when sum is 8 are (2,6),(3,5),(4,4),(5,3),(6,2). Total events are 6^2 $=$ 36

$\Pr(X = 1) = \frac{5}{36}$

Solution Continued

Let $Y \in \{0,1\}$is random variable that denote whether the number on red die is less than 4 or not

Let Y = 0 denotes the number on red die is not less than 4 and X = 1 denotes that number on red die less than 4.

Events that satisfy Y are

(1,1),(2,1),(3,1),(4,1),(5,1),(6,1)

(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)

(1,3),(2,3),(3,3),(4,3),(5,3),(6,3)

Events that satisfy both X,Y are (5,3),(6,2)

$\Pr\left(Y = 1, X = 1\right) = \frac{2}{36}$

$\Pr\left(Y = 1\right) = \frac{18}{36}$

Solution Continued...

The desired probability is

$$\Pr\left(X = 1 | Y = 1\right) = \frac{\Pr\left(Y = 1, X = 1\right)}{\Pr\left(Y = 1\right)} \text{ (Bayes' Theorem)} \tag{3}$$

$$\Pr\left(Y = 1 | X = 1\right) = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9} \tag{4}$$