AI1110 - Probability and Random Variables
Assignment 6

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Outline

Question

EXAMPLE 5-11

Suppose that the resistance r is uniform between 900 and 1100 Ω.We shall determine the density function of the corresponding conductance

$$g = \frac{1}{r}$$

$f_r(r) = \frac{1}{200}$ S for r between 900 and 1100.
$f_g(g) = \frac{1}{200g^2}$ for $\frac{1}{1100} < g < \frac{1}{900}$ ‘and 0 else where
$y = ax^2$ is the relation between r,g

Solution

Solution

$y = ax^2$ a > 0 $g'(x) = 2ax$

If y ≤ 0, then the equation y $= ax^2$ has no real solutions; hence $f_y(y) = 0$.

If y > 0, then it has two solutions

$$x_1 = \sqrt{\frac{y}{a}}, x_2 = -\sqrt{\frac{y}{a}}$$

Solution Continued

By using the fundamental theorem

$$f_y(y) = \frac{f_x(x_1)}{g'(x_1)} + ... + \frac{f_x(x_n)}{g'(x_n)} + ...$$

we get,

$$f_y(y) = \frac{f_x(x_1)}{g'(x_1)} + \frac{f_x(x_2)}{g'(x_2)}$$

Solution Continued

By using the fundamental theorem

for $y > 0$

$$f_y(y) = \frac{1}{2a\sqrt{\frac{y}{a}}}[f_x(\sqrt{\frac{y}{a}}) + f_x(-\sqrt{\frac{y}{a}})]$$

for $y < 0$

$$f_y(y) = 0$$

Solution Continued

From above we note that

for $y > 0$

$$\mathrm{F}_y(y) = P\left\{-\sqrt{\frac{y}{a}} \leq x \leq \sqrt{\frac{y}{a}}\right\} = \mathrm{F}_x(\sqrt{\frac{y}{a}}) - \mathrm{F}_x(-\sqrt{\frac{y}{a}})$$

for $y < 0$

$$\mathrm{F}_y(y) = 0$$