AI1110 - Probability and Random Variables Assignment 7

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June 2, 2022

Outline

Question

EXAMPLE 7-6

If x is a random variable with distribution F(x), then the given random variable y = F(x) is uniform in the interval (0, 1).
The following is a generalization.
Given n arbitrary random variables x_i we form the random variables
$y_1 = F(x_1)$ $y_2 = F(x_2|x_1)$,...., $y_n = F(x_n|x_n - 1, ..., x_1)$
We shall show that these random variables are independent and each is uniform in the interval (0,1).

Solution

Solution

The random variables y_i are functions of the random variables Xi obtained with the transformation. For $0 \leq y_i \leq 1$, the system
$y_1 = F(x_1)$ $y_2 = F(x_2|x_1)$,...., $y_n = F(x_n|x_n - 1, ..., x_1)$
has a unique solution x_1,..,x_n and its jacobian equals

Solution Continued

$$J = \begin{vmatrix} \frac{\partial y_1}{\partial x_1} & 0 & 0 & . & . & . & 0 \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & 0 & . & . & . & 0 \\ . & . & . & . & . & . & \\ \frac{\partial y_n}{\partial x_1} & \frac{\partial y_n}{\partial x_2} & . & . & . & . & \frac{\partial y_n}{\partial x_n} \end{vmatrix}$$

This determinant is triangular; hence it equals the product of its diagonal elements.

Solution Continued

$$\frac{\partial y_n}{\partial x_n} = f(x_k|x_k - 1, ..., x_1)$$

After transforming, we obtain

$$f(y_1, y_2, ..y_n) = \frac{f(x_1, .., x_n)}{f(x_1)f(x_2|x_1)...f(x_n|x_n - 1, .., x_1)} = 1$$

in the n-dimensional cube $0 < y_i < 1$, and 0 otherwise

Solution Continued

It follows that

$$f(x_1|x_3) = \int_{-\infty}^{\infty} f(x_1)$$

$$f(x_1|x_4) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1|x_2,x_3,x_4)f(x_2,x_3|x_4)dx_2dx_3$$

Generalizing. we obtain the following rule for removing variables on the left or on the right of the conditional line: To remove any number of variables on the left of the,conditional line, we integrate with respect to them. To remove any number of variables,to the right of the line, we multiply by their conditional density with respect to the remaining variables on the right, and we integrate the product

Solution Continued

$$E(x_1|x_2, x_3) = E(E(x_2, x_3|x_4)) = \int_{-\infty}^{\infty} E(x_1|x_2, x_3, x_4) f(x_4|x_2, x_3) dx_4$$

This leads to the following generalization: To remove any number of variables on the right of the conditional expected value line, we multiply by their conditional density with respect to the remaining variables on the right and we integrate the product.So we can say that these random variables are independent and each is uniform in the interval (0,1)