Random Numbers

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Contents

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution:
Download the following files and execute the C program.

```
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −1/1.1/exrand.c
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −1/1.1/coeffs.h
```

Download the above files and execute the following commands

```
gcc exrand.c −lm
./a.out
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \le x) \tag{1.1}$$

Solution:
The following code plots Fig. 1.2

```
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −1/1.2/cdf_plot.py
```

Download the above files and execute the following commands to produce Fig.1.2

```
python3 cdf_plot.py
```

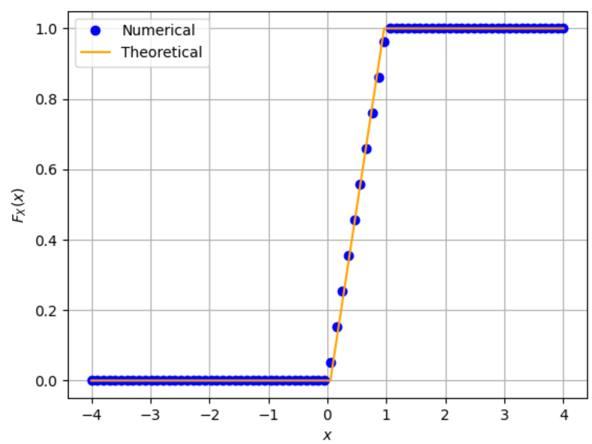

Fig. 1.2: The CDF of U

1.3 Find a theoretical expression for $F_U(x)$.
Solution:
Given U is a uniform Random Variable

$$p_U(x) = 1 \text{ for}$$
$$F_U(x) = \int_{-\infty}^{\infty} p_U(x)dx$$
$$\implies F_U(x) = \begin{cases} 0 & x \le 0 \\ x & 0 < x < 1 \\ 1 & x \ge 1 \end{cases}$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.2}$$

and its variance as $\text{var}[U] = E[U - E[U]]^2$ Write a C program to find the mean and variance of U.
Solution:
Download the following files and execute the C program.

```
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −1/1.4/m.c
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −1/1.4/coeffs.h
```

Download the above files and execute the following commands

```
gcc m.c −lm
./a.out
```

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x)$$

Solution:

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x)$$
$$E[U] = \int_{0}^{1} x$$
$$\implies E[U] = \frac{1}{2}$$
$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x)$$
$$E\left[U^2\right] = \int_{0}^{1} x^2 dF_U(x)$$
$$\implies E\left[U^2\right] = \frac{1}{3}$$
$$\text{var}[U] = E[U - E[U]]^2$$
$$\implies \text{var}[U] = E\left[U^2\right] - E[U]^2$$
$$\boxed{\text{var}[U] = \frac{1}{12} = 0.0833}$$

2 Central Limit Theorem

2.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{2.1}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat
Solution: Download the following files and execute the C program.

```
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −2/2.1/exrand.c
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −2/2.1/coeffs.h
```

Download the above files and execute the following commands

```
gcc exrand.c −lm
./a.out
```

2.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?
Solution:

- $F_X(x) = P(X \le x)$
- $Q_X(x) = P(X > x)$
- $Q_X(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du$
- $F_X(x) = 1 - Q_X(x)$ This can be used to calculate F (x).

The CDF of X is plotted in Fig. 2.2
using the code below

```
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −2/2.2/2.2.py
```

Download the above files and execute the following commands to produce Fig.2.2

```
python3 2.2.py
```

Some of the properties of CDF
1)

$$\lim_{x \to \infty} F_X(x) = 1$$

2) $F_X(x)$ is non decreasing function.
3) Symmetric about one point.

2.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{2.2}$$

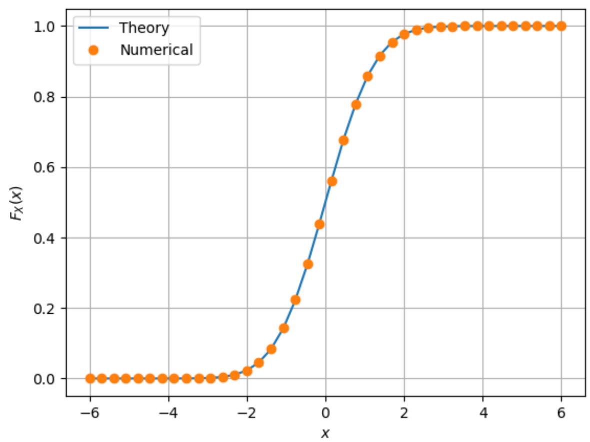

Fig. 2.2: The CDF of X

What properties does the PDF have?
Solution: The PDF of X is plotted in Fig. 2.3 using the code below

```
wget https://github.com/kamujuaakash/
     Assignment1/blob/main/Exercise/Exercise
     −2/2.3/2.3.py
```

Download the above files and execute the following commands to produce Fig.2.3

```
python3 2.3.py
```

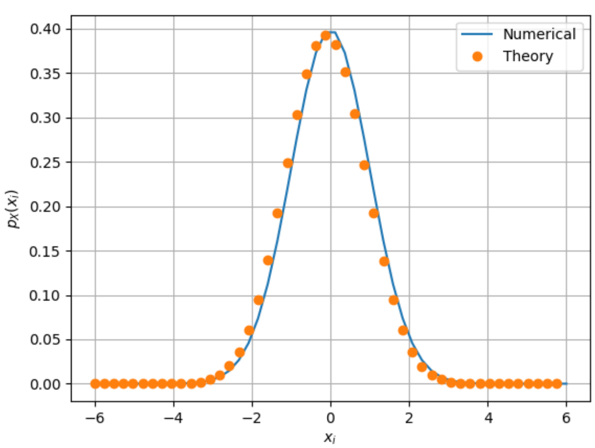

Fig. 2.3: The PDF of X

Let $\mu \approx 0$

Some of the properties of the PDF:

a) Symmetric about $x = \mu$ in this case
b) Decreasing function for $x > \mu$ and increasing for $x < \mu$ and attains maximum at $x = \mu$
c) Area under the curve is unity.

2.4 Find the mean and variance of X by writing a C program.
Solution: Download the following files and execute the C program.

```
wget https://github.com/kamujuaakash/
     Assignment1/blob/main/Exercise/Exercise
     −2/2.4/m.c
wget https://github.com/kamujuaakash/
     Assignment1/blob/main/Exercise/Exercise
     −2/2.4/coeffs.h
```

Download the above files and execute the following commands

```
gcc m.c −lm
./a.out
```

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \quad (2.3)$$

repeat the above exercise theoretically.

Solution:

1) CDF is given by

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x) dx \quad (2.4)$$

$$\boxed{F_X(x) = 1} \quad (2.5)$$

2) Mean is given by

$$E(x) = \int_{-\infty}^{\infty} x p_X(x) dx$$

$$\int_{-\infty}^{\infty} \left(xexp\left(-\frac{x^2}{2}\right) dx \right)$$

Since the above function is odd

$$\implies \boxed{E(x) = 0}$$

3) Variance is given by

$$\operatorname{var}[U] = E(U^2) - (E(U))^2$$

$$E\left(x^2\right) = \int_{-\infty}^{\infty} x^2 p_X(x) dx$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^2 exp\left(-\frac{x^2}{2}\right) dx$$
$$= \frac{1}{\sqrt{2\pi}} \left(x \int_{-\infty}^{\infty} xexp\left(-\frac{x^2}{2}\right) dx\right)$$
$$- \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(\int \left(xexp\left(-\frac{x^2}{2}\right) dx\right) .dx\right)$$

$$= \left[-x \frac{1}{\sqrt{2\pi}} exp \frac{-x^2}{2}\right]_{-\infty}^{\infty} +$$
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} exp\left(-\frac{x^2}{2}\right) dx$$
$$= \frac{\sqrt{2\pi}}{\sqrt{2\pi}}$$
$$= 1 \implies \boxed{\operatorname{var}[U] = 1}$$

3 From Uniform to Other

3.1 Generate samples of

$$V = -2\ln(1 - U) \tag{3.1}$$

and plot its CDF.
Solution: Running the below code generates samples of V from file uni.dat(U).

```
https://github.com/kamujuaakash/Assignment1
    /blob/main/Exercise/Exercise−3/3.1/3.1.py
```

Use the below command in the terminal to run the code:

```
python3 3.1.py
```

Now these samples are used to plot by running the below code,

```
wge https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −3/3.1/cdf.py
```

Use the below command to run the code:

```
python3 cdf.py
```

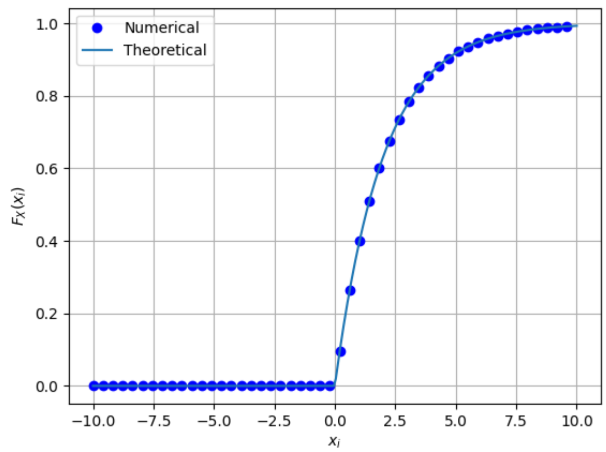

Fig. 3.1: CDF for (3)

3.2 Theorotical expression for $F_V(x)$

$$F_V(x) = P\{V \le x\}$$
$$= P\{-2 \times \ln(1 - U) \le x\}$$
$$= P\{U \le 1 - e^{(-\frac{x}{2})}\}$$
$$= F_U\{1 - e^{(-\frac{x}{2})}\}$$
$$= \begin{cases} 1 - e^{(-\frac{x}{2})} & 0 \le x < \infty \\ 0 & x < 0 \end{cases}$$

4 Triangular Distribution

4.1 Generate

$$T = U_1 + U_2 \tag{4.1}$$

Solution: Download the following files and execute the C program.

```
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −4/4.1/4.1.c
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −4/4.1/coeffs.h
```

Download the above files and execute the following commands

```
gcc 4.1.c
./a.out
```

4.2 Find the CDF of T.
Solution:
The CDF of T is plotted in figure using the code below

```
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −4/4.2/4.2.py
```

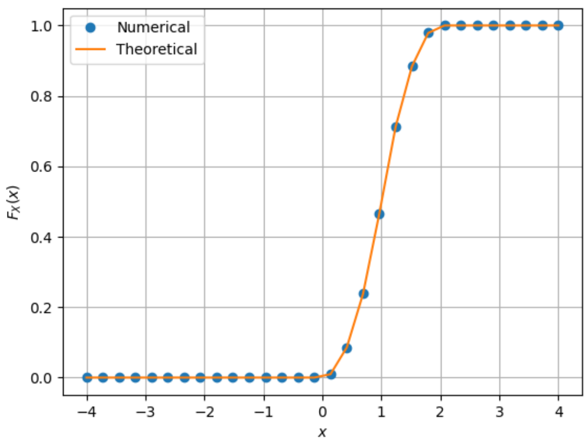

Fig. 4.2: The CDF of T

Download the above files and execute the following commands to produce Fig.4.2

```
python3 4.2.py
```

4.3 Find the PDF of T.
Solution:
The PDF of T is plotted in Fig. 4.2 using the code below

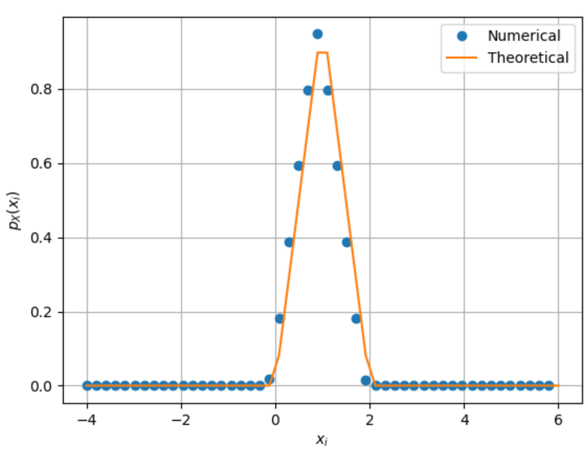

Fig. 4.3: The PDF of T

```
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/Exercise
    −4/4.3/4.3.py
```

Download the above files and execute the following commands to produce Fig.4.2

```
python3 4.3.py
```

4.4 Find the Theoretical Expression for the PDF and CDF of T
Solution:

$$T = U_1 + U_2 \tag{4.2}$$

$$\implies p_T(t) = \int_{-\infty}^{t} p_{U1}(x)p_{U2}(y)dx \tag{4.3}$$

$$\text{As,} p_{U1}(x) = p_{U1}(y) = p_U(u) \tag{4.4}$$

$$\implies p_T(t) = \int_{-\infty}^{t} p_U(u)p_U(t-u)du \tag{4.5}$$

a) Theoretical PDF
   i) $t \le 1$

$$p_T(t) = \int_0^t p_U(t-u)du \tag{4.6}$$

$$\implies p_T(t) = \int_0^t du = t \tag{4.7}$$

   ii) $t > 1$

$$p_T(t) = \int_0^1 p_U(t-u)du \tag{4.8}$$

$$\implies p_T(t) = \int_{t-1}^1 du = 2 - t \tag{4.9}$$

$$\implies \boxed{P_T(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 < t \le 2 \\ 0 & t < 0 \text{ or } t > 2 \end{cases}}$$

b) Theoretical CDF

$$F_T(t) = \int_{-\infty}^{t} p_T(u)du \tag{4.10}$$

$$\implies \boxed{F_T(t) = \begin{cases} 0 & t < 0 \\ \frac{t^2}{2} & 0 \le t \le 1 \\ 2t - 1 - \frac{t^2}{2} & 1 < t \le 2 \\ 1 & t > 2 \end{cases}}$$

4.5 Verify your results through a plot
Solution: The Results are verified in the plots in Fig.4.2and Fig.4.3

5 Maximum Likelihood

5.1 Generate equiprobable $X \in \{1, -1\}$.
Solution: The generating X or bernoulie random variable (X) is done by using uni.dat file. Download the below files

```
wget https://github.com/
    kamujuaakash/Assignment1/
    blob/main/Exercise/Exercise
    -5/coeffs.h
wget https://github.com/
    kamujuaakash/Assignment1/
    blob/main/Exercise/Exercise
    -5/5.1.c
```

Run the following command

```
gcc 5.5.c -lm
./a.out
```

5.2 Generate $Y = AX + N$, where $A = 5$ dB, and $N \sim 01$.
Solution: To generate distribution of Y random variable we will need previously generated bernoulie distribution and gaussian distribution.Download the below files

```
wget https://github.com/kamujuaakash
    /Assignment1/blob/main/Exercise/
    Exercise-5/coeffs.h
wget https://github.com/kamujuaakash
    /Assignment1/blob/main/Exercise/
    Exercise-5/5.2.c
```

Then run the following command,

```
gcc 5.2.c -lm
./a.out
```

5.3 Plot Y using a scatter plot.
Solution: Download the below files

```
wget https://github.com/
    kamujuaakash/Assignment1/blob/
    main/Exercise/Exercise-5/5.3.py
```

Then run the following command,

```
python3 5.3.py
```

5.4 Guess how to estimate X from Y.
Solution: When $Y > 0$, we can more probably say that $X = 1$ as X can take values from $[-1, 1]$.As A increases the signal contribution will increase compared to noise.The scatter plot will not be intermixed as A increases. So in this case, the scatter plot of Y is seperated with decision boundary as 0. So we can more probably say that,

$$X = \begin{cases} 1 & , Y > 0 \\ -1 & , Y < 0 \end{cases} \tag{5.1}$$

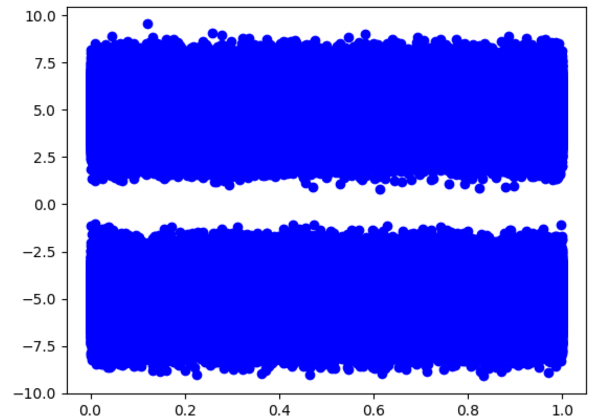

Fig. 5.5: The scatter plot of Y

5.5 Find

$$P_{e|0} = \Pr\left(\hat{X} = -1 | X = 1\right) \tag{5.2}$$

and

$$P_{e|1} = \Pr\left(\hat{X} = 1 | X = -1\right) \tag{5.3}$$

Solution: The $\hat{X}$ is defined as,

$$\hat{X} = \begin{cases} 1 & , Y > 0 \\ 0 & , Y \leq 0 \end{cases} \tag{5.4}$$

The error probability, when the actual signal is $X = 1$ but transmitted as $\hat{X} = -1$ is,

$$\begin{aligned} P_{e|0} &= \Pr\left(\hat{X} = -1 | X = 1\right) && (5.5) \\ &= \Pr\left(Y \leq 0 | X = 1\right) && (5.6) \\ &= \Pr\left(AX + N \leq 0 | X = 1\right) && (5.7) \\ &= \Pr\left(A + N \leq 0\right) && (5.8) \\ &= \Pr\left(N \leq -A\right) && (5.9) \\ &= F_N\left(-A\right) && (5.10) \\ &= 1 - Q\left(-A\right) && (5.11) \\ &= 2.866515718791946e - 07 && (5.12) \end{aligned}$$

And for the case when actual signal is $X = -1$ but transmitted as $\hat{X} = 1$ the error probability will be,

$$P_{e|1} = \Pr\left(\hat{X} = 1|X = -1\right) \tag{5.13}$$
$$= \Pr(Y > 0|X = -1) \tag{5.14}$$
$$= \Pr(AX + N > 0|X = 1) \tag{5.15}$$
$$= \Pr(N - A > 0) \tag{5.16}$$
$$= \Pr(N > A) \tag{5.17}$$
$$= 1 - F_N(A) \tag{5.18}$$
$$= Q(A) \tag{5.19}$$
$$= 2.866515719235352e - 07 \tag{5.20}$$

The above calculations are coded in below python file,

```
wget https://github.com/kamujuaakash/
    Assignment1/blob/main/Exercise/
    Exercise−5/5.5.py
```

Run the following command

```
python3 5.5.py
```

5.6 Find P_e assuming that X has equiprobable symbols.
Solution: Given that X has equiprobable symbols so,

$$\Pr(X = 1) = \frac{1}{2} \tag{5.21}$$
$$\Pr(X = -1) = \frac{1}{2} \tag{5.22}$$

From total probability theorem,

$$P_e = \Pr(e|1)\Pr(X = -1) + \Pr(e|0)\Pr(X = 1) \tag{5.23}$$
$$= \frac{1}{2}(\Pr(e|1) + \Pr(e|0)) \tag{5.24}$$

From (5.12),(5.20)

$$\Pr(e) = 2.866515719013649e - 07 \tag{5.25}$$

5.7 Verify by plotting the theoretical P_e with respect to A from 0 to 10 dB.
Solution: We know,

$$P_e = P_{e|1}\Pr(X = -1) + P_{e|0}\Pr(X = 1) \tag{5.26}$$
$$= \frac{1}{2}(1 - Q(-A)) + \frac{1}{2}(Q(A)) \tag{5.27}$$

The above mentioned is the theoritical expression of P_e w.r.t to A, it is plotted in

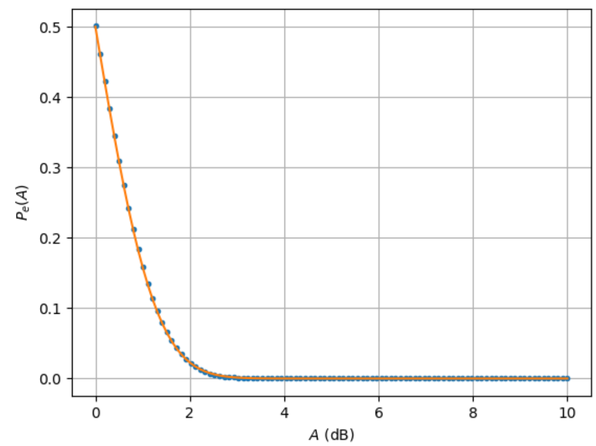

Fig. 5.5: P_e vs A

rectangular axes and semi-log y axes using the below python codes,

```
wget https://github.com/kamujuaakash
    /Assignment1/blob/main/Exercise/
    Exercise−5/5.7.py
```

Then the following commands

```
python3 5.7.py
```

5.8 Now, consider a threshold δ while estimating X from Y. Find the value of δ that maximizes the theoretical P_e.
Solution:

$$P_{e|0} = \Pr\left(\hat{X} = -1|X = 1\right) \tag{5.28}$$
$$= \Pr(Y \leq \delta|X = 1) \tag{5.29}$$
$$= \Pr(AX + N \leq \delta|X = 1) \tag{5.30}$$
$$= \Pr(A + N \leq \delta) \tag{5.31}$$
$$= \Pr(N \leq \delta - A) \tag{5.32}$$
$$= F_N(\delta - A) \tag{5.33}$$
$$= 1 - Q(\delta - A) \tag{5.34}$$

And,

$$P_{e|1} = \Pr\left(\hat{X} = 1|X = -1\right) \tag{5.35}$$
$$= \Pr(Y > \delta|X = -1) \tag{5.36}$$
$$= \Pr(AX + N > \delta|X = -1) \tag{5.37}$$
$$= \Pr(N - A > \delta) \tag{5.38}$$
$$= \Pr(N > \delta + A) \tag{5.39}$$
$$= Q(\delta + A) \tag{5.40}$$

So we can write,

$$P_e = P_{e|1}\Pr(X=-1) + P_{e|0}\Pr(X=1) \tag{5.41}$$

$$= \frac{1}{2}\left(1 - Q(\delta - A) + Q(\delta + A)\right) \tag{5.42}$$

Now to maximize the P_e, we will differentiate the expression w.r.t δ and equate it to 0.

$$\frac{dP_e}{d\delta} = \frac{d}{d\delta}\left(1 - Q(\delta - A) + Q(\delta + A)\right) = 0 \tag{5.43}$$

$$\implies \frac{d}{d\delta}\left(F_N(\delta - A) + 1 - F_N(\delta + A)\right) = 0 \tag{5.44}$$

$$\implies p_N(\delta - A) - p_N(\delta + A) = 0 \tag{5.45}$$

$$\exp\left(-\frac{(\delta - A)^2}{2}\right) - \exp\left(-\frac{(\delta + A)^2}{2}\right) = 0$$

Since e^x is one - one function, we can write,

$$-\frac{(\delta - A)^2}{2} = -\frac{(\delta + A)^2}{2} \tag{5.46}$$

$$(\delta - A)^2 = (\delta + A)^2 \tag{5.47}$$

$$\delta = 0 \tag{5.48}$$

Now we will find whether P_e attains maxima or minima at $\delta = 0$

$$\frac{d^2P_e}{d\delta^2}|_{\delta=0} = 2A\exp\left(\frac{-A^2}{2}\right) > 0 \tag{5.49}$$

5.9 Repeat the above exercise when

$$p_X(0) = p \tag{5.50}$$

Solution: Given that,

$$p_X(0) = p \tag{5.51}$$

So,

$$\Pr(X=1) = p_X(0) = p \tag{5.52}$$

$$\Pr(X=-1) = 1 - p \tag{5.53}$$

From (5.41) we can write

$$P_e = P_{e|1}\Pr(X=-1) + P_{e|0}\Pr(X=1) \tag{5.54}$$

$$= (1-p)\,Q(\delta + A) + p\left(1 - Q(\delta - A)\right) \tag{5.55}$$

$$= (1-p)\,Q(\delta + A) + pQ(A - \delta) \tag{5.56}$$

Now to maximize the P_e, we will differentiate the expression w.r.t δ and equate it to 0.

$$\frac{d}{d\delta}P_e = p\frac{d}{d\delta}Q(A - \delta) + (1-p)\frac{d}{d\delta}Q_N(A + \delta) = 0 \tag{5.57}$$

$$p\frac{d}{d\delta}F_N(-A + \delta) + (1-p)\frac{d}{d\delta}\left(1 - F_N(A + \delta)\right) = 0 \tag{5.58}$$

$$\implies p \times p_N(-A + \delta) \tag{5.59}$$

$$-(1-p)\,p_N(A + \delta) = 0 \tag{5.60}$$

From the PDF of gaussian, we will get

$$\delta = \frac{\ln\left(\frac{1}{p} - 1\right)}{2A} \tag{5.61}$$

5.10 Repeat the above exercise using the MAP criterion.

Solution: From the Bayes theorem, we can write

$$\Pr(X=1|Y=y) \tag{5.62}$$

$$= \frac{\Pr(X=1, Y=y)}{\Pr(Y=y)} \tag{5.63}$$

$$= \frac{p\Pr(N = y - A)}{pP_{Y|X}(y|1) + (1-p)\,P_{Y|X}(y|-1)} \tag{5.64}$$

$$= \frac{pp_N(y - A)}{pp_N(y - A) + (1-p)\,p_N(y + A)} \tag{5.65}$$

$$= \frac{p}{p + (1-p)\exp(-2yA)} \tag{5.66}$$

And similarly for,

$$\Pr(X = -1|Y = y) \tag{5.67}$$
$$= \frac{\Pr(X = -1, Y = y)}{\Pr(Y = y)} \tag{5.68}$$
$$= \frac{(1-p)\Pr(N = y + A)}{pP_{Y|X}(y|1) + (1-p)P_{Y|X}(y|-1)} \tag{5.69}$$
$$= \frac{(1-p)p_N(y+A)}{pp_N(y-A) + (1-p)p_N(y+A)} \tag{5.70}$$
$$= \frac{1-p}{1-p+p\exp 2yA} \tag{5.71}$$

Now for a particular y, to make $X = 1$ more likely than $X = -1$,

$$\Pr(X = 1|Y = y) > \Pr(X = -1|Y = y) \tag{5.72}$$

$$\frac{p}{p + (1-p)\exp(-2yA)} > \frac{1-p}{1-p+p\exp(2yA)} \tag{5.73}$$

$$p^2e^{2yA} > (1-p)^2 e^{-2yA} \tag{5.74}$$
$$e^{2yA} > \frac{1-p}{p} \tag{5.75}$$
$$y > \frac{\ln\left(\frac{1-p}{p}\right)}{2A} \tag{5.76}$$

And similarly for a particular y, to make $X = -1$ more likely than $X = 1$, we need

$$y < \frac{\ln\left(\frac{1-p}{p}\right)}{2A} \tag{5.77}$$

So to minimise the P_e we need a threshold of

$$\delta = \frac{\ln\left(\frac{1-p}{p}\right)}{2A} \tag{5.78}$$

6 Gaussian to Other

6.1

$$V = X_1^2 + X_2^2 \tag{6.1}$$

Solution: Download the below files to generate the random variable V,

Then run the following command,

For CDF download the below python file,

Then run the following command,

For PDF download the below pyhon file,

Then run the follwing terminal in terminal,

6.2 If

$$F_V(x) = \begin{cases} 1 - e^{-\alpha x} & x \geq 0 \\ 0 & x < 0, \end{cases} \tag{6.2}$$

find α.

6.3 Plot the CDF and PDf of

$$A = \sqrt{V} \tag{6.3}$$

7 Conditional Probability

7.1 Plot

$$P_e = \Pr\left(\hat{X} = -1|X = 1\right) \tag{7.1}$$

for

$$Y = AX + N, \tag{7.2}$$

7.2 Assuming that N is a constant, find an expression for P_e. Call this $P_e(N)$

7.3 For a function g,

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)p_X(x)\,dx \tag{7.3}$$

Find $P_e = E[P_e(N)]$.

7.4 Plot P_e in problems 7.7.5 and 7.7.5 on the same graph w.r.t γ. Comment.

8 Two Dimensions

Let

$$\mathbf{y} = A\mathbf{x} + \mathbf{n}, \tag{8.1}$$

where

$$x \in (\mathbf{s}_0, \mathbf{s}_1), \mathbf{s}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{s}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{8.2}$$

$$\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}, n_1, n_2 \sim 01. \tag{8.3}$$

8.1 Plot

$$\mathbf{y}|\mathbf{s}_0 \text{ and } \mathbf{y}|\mathbf{s}_1 \tag{8.4}$$

on the same graph using a scatter plot.

8.2 For the above problem, find a decision rule for detecting the symbols $\mathbf{s}_0$ and $\mathbf{s}_1$.

8.3 Plot

$$P_e = \Pr(\hat{\mathbf{x}} = \mathbf{s}_1|\mathbf{x} = \mathbf{s}_0) \tag{8.5}$$

with respect to the SNR from 0 to 10 dB.

8.4 Obtain an expression for P_e. Verify this by comparing the theory and simulation plots on the same graph.