EE3900: Linear Systems and Signal Processing Assignment-1

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***Abstract*—This document contains solution to Assignment-1 [Question 3.1(b) from Discrete-Time Signal Processing by Alan V. Oppenheim and Ronald W. Schafer]**

1. Z-Transform

1 Determine the z-transform and region of convergence for the following sequence:

$$-\left(\frac{1}{2}\right)^n u[-n-1] \tag{1.1}$$

Solution:
Given

$$x(n) = -\left(\frac{1}{2}\right)^n u[-n-1] \tag{1.2}$$

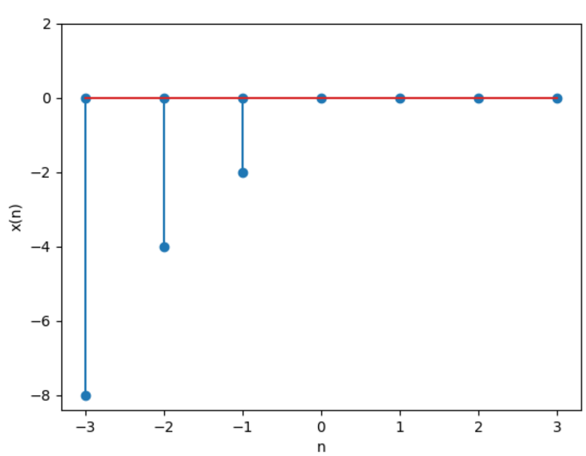

Fig. 1. $x(n)$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases} \tag{1.3}$$

So,

$$u[-n-1] = \begin{cases} 1 & n \leq -1 \\ 0 & otherwise \end{cases} \tag{1.4}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{1.5}$$

$$= \sum_{n=-\infty}^{\infty} -\left(\frac{1}{2}\right)^n u[-n-1] z^{-n} \tag{1.6}$$

$$= \sum_{n=-\infty}^{-1} -\left(\frac{1}{2}\right)^n z^{-n} \tag{1.7}$$

$$= -z\left(\frac{1}{2}\right)^{-1} - z^2\left(\frac{1}{2}\right)^{-2} - z^3\left(\frac{1}{2}\right)^{-3} - \ldots \tag{1.8}$$

$$= -\frac{2z}{1-2z} \tag{1.9}$$

For $X(z)$ to converge, $|X(z)| < \infty$.
Region of convergence:

$$|z| < \frac{1}{2} \tag{1.10}$$