

Lecture 11 Scribe

CSE400 — Fundamentals of Probability in Computing

Lecture 11: Transformation of Random Variables
(February 10, 2026)

****Instructor:**** Dhaval Patel, PhD (Associate Professor),
SEAS—Ahmedabad University, Ahmedabad, Gujarat,
India.

1. Outline and Learning Objective

1.1 Topics in this lecture

1. **Transformation of Random Variables**: learning transformation techniques for random variables.
2. **Function of Two Random Variables**: joint transformations and derived distributions.
3. **Illustrative Example**: detailed derivation for the case ($Z = X + Y$).

1.2 Assumption / setup stated in the outline

* **Assumption:** the **PDF of the original RV** (e.g., $(f_X(x))$) is **known a priori**; goal is **how to find the PDF of the new transformed RV** (e.g., $(f_Y(y))$).

1.3 Noted example transformations (as written on the outline slide)

- * ($Z_1 = X + Y$)
- * ($Z_2 = X - Y$)
- * ($Z_3 = X / Y$)
- * ($Z_4 = \sqrt{X^2 + Y^2}$)

These appear as representative transformation forms on the outline page.

2. Transformation of One Random Variable: ($Y = g(X)$)

2.1 Definitions and notation (as used on the slides)

- * A **transformation** is defined by

$$[Y = g(X).]$$

- * **CDF notation**: ($F_Y(y)$) and ($F_X(x)$).

- * **PDF notation**: ($f_Y(y)$) and ($f_X(x)$).

- * The slides explicitly proceed via **Step S1 (CDF)** then **Step S2 (differentiate w.r.t. (y))**.

2.2 Assumption / condition: monotonicity

The slide distinguishes monotonic cases (illustrated via increasing and decreasing sketches):

- * **Monotonically increasing** case

- * **Monotonically decreasing** case

2.3 Case A: ($g(\cdot)$) is monotonically increasing

Step S1 (CDF method)

$$[F_Y(y) := \Pr(Y \leq y) \\ := \Pr(g(X) \leq y) \\ := \Pr(X \leq g^{-1}(y)) \\ := F_X(g^{-1}(y)).]$$

This is written on the slide in the "S1" sequence.

Step S2 (Differentiate w.r.t. (y))

$$[f_Y(y) := \frac{d}{dy} [F_X(g^{-1}(y))] \\ := f_X(g^{-1}(y)) \cdot \frac{d}{dy} [g^{-1}(y)]]$$

$(y)\backslash big).$

]

The slide then rewrites this using $(x=g^{-1}(y))$:

[

$f_Y(y) := f_X(x), \frac{dx}{dy} \Big|_{x=g^{-1}(y)}.$

]

These steps are shown as the CDF \rightarrow PDF derivation chain for the increasing case.

2.4 Case B: $(g(\cdot))$ is monotonically decreasing

Step S1 (CDF method)

[

$F_Y(y) = \Pr(Y \leq y)$

$= \Pr(X \geq g^{-1}(y))$

$= 1 - F_X(g^{-1}(y)).$

]

This "S1" chain is displayed on the decreasing-case slide.

Step S2 (Differentiate w.r.t. (y))

The slide then differentiates and rearranges to the displayed boxed relationship:

[

$f_Y(y) := \frac{f_X(x)}{\left| \frac{dx}{dy} \right|} \Big|_{x=g^{-1}(y)}.$

]

The slide also annotates "S3: Change the limits for (y) " (i.e., determine the valid (y) -range from the mapping).

3. Worked Example: $(X \sim \text{Uniform}(-1,1)),$
 $(Y = \sin(\pi x/2))$

3.1 Given (from the example slide)

* (X) is a continuous RV, **uniformly distributed** on $((-1,1)).$

* The transformation is

$$[
Y = g(x) = \sin\left(\frac{\pi x}{2}\right).
]$$

* The PDF of (X) is written explicitly as

$$[
f_X(x) =
\begin{cases} \frac{1}{2}, & -1 < x < 1, \\ 0, & \text{otherwise}. \end{cases}
\end{cases}
]$$

* **Objective on the slide:** "($f_Y(y)=?$)"

3.2 Step-by-step solution (as shown)

Step 1: Invert the transformation

Starting from

$$[
y = \sin\left(\frac{\pi x}{2}\right),
]$$

the slide gives

$$[
x = \frac{2}{\pi} \arcsin(y).
]$$

Step 2: Differentiate (x) w.r.t. (y)

$$[
\frac{dx}{dy} = \frac{2}{\pi} \cdot \frac{1}{\sqrt{1-y^2}}.
]$$

Step 3: Apply the transformation formula (using the slide's substitution)

The slide applies (in the same structure as earlier derivation):

```
[
f_Y(y)= f_X(x)\cdot|\left|\frac{dx}{dy}\right|.

]

Substitute ( $f_X(x)=\frac{1}{2}$ ) and ( $|\frac{dx}{dy}|$ ) above:
[
f_Y(y)=\frac{1}{2}\cdot\frac{2}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}
=\frac{1}{\pi}\sqrt{1-y^2}.
]
```

Step 4: Determine the valid range of (y) from endpoints (as written)

The slide checks endpoints:

- * At ($x=-1$): ($y=\sin!\left(-\frac{\pi}{2}\right) = -1$)
 - * At ($x=1$): ($y=\sin!\left(\frac{\pi}{2}\right) = 1$)
- So the slide states the support as ($-1 < y < 1$), and "otherwise 0".

3.3 Final answer (as presented)

```
[
f_Y(y)=
\begin{cases}
\frac{1}{\pi}\sqrt{1-y^2}, & -1 < y < 1, [6pt] \\
0, & \text{otherwise}. \\
\end{cases}
]
```

4. Function of Two Random Variables: Example Setup
($Z = X + Y$)

4.1 Problem statements listed on the slide

For ($Z = X + Y$), the slide lists tasks:

1. **Find the PDF of (Z): ($f_Z(z)$).

2. **Find $(f_Z(z))$ if (X) and (Y) are independent.**
3. **Let $(X \sim N(0,1))$ and $(Y \sim N(0,1))$; prove that $(Z \sim N(0,2))$.** (Statement appears as an objective on the slide; no proof is included on the slide.)
4. **If (X) and (Y) are exponential distribution RVs with parameter (λ) , find $(f_Z(z))$.**

(Only these tasks are stated; the slide does not provide completed proofs for the normal/exponential claims.)

5. Detailed derivation shown: CDF setup for $(Z=X+Y)$ via region integration

5.1 Definition of (Z) and CDF start

The slide sets

```
[  
Z=X+Y,  
]
```

and defines

```
[  
F_Z(z)=Pr(Z≤ z)  
=Pr(X+Y≤ z).  
]
```

5.2 Region description and the “strip” integrals

The slide illustrates the line $(x+y=z)$ (equivalently $(x=z-y)$ or $(y=z-x)$) and writes the double-integral form using the joint pdf $(f_{XY}(x,y))$, showing two equivalent “strip” integrations:

(A) Horizontal strip (“H-strip”) form

```
[  
F_Z(z)  
=====
```

$\int_{-\infty}^{\infty} \int_{y=-\infty}^{\infty}$

```
\int_{x=-\infty}^{\infty} f_{XY}(x,y) dx dy.  
]  
The slide labels this as **H-strip**.
```

(B) Vertical strip ("Vertical strip") form

```
[  
F_Z(z)  
=====
```

```
\int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} f_{XY}(x,y) dy dx.  
]
```

The slide labels this as **Vertical strip**.

(The slide provides these CDF integral setups and the geometric interpretation; it does not proceed further to a final closed-form ($F_Z(z)$) on the provided pages.)
make a structures latex name lecture 11 scribe , Niharika Ashar with au id AU2440175

latex

```
\documentclass[11pt]{article}

% ====== PACKAGES ======
\usepackage[a4paper,margin=1in]{geometry}
\usepackage{amsmath,amssymb}
\usepackage{enumitem}
\usepackage{fancyhdr}
\usepackage{xcolor}
\usepackage{hyperref}

% ====== HEADER & FOOTER ======
\pagestyle{fancy}
\fancyhf{}
\lhead{CSE400: Fundamentals of Probability in Computing}
\rhead{Lecture 11 Scribe}
\cfoot{\thepage}
```

```
% ===== TITLE INFO =====
\title{\textbf{Lecture 11 Scribe: Transformation of Random Variables}\\"
\vspace{0.15cm}\large February 10, 2026}
\author{\textbf{Niharika Ashar}\\" AU ID: \textbf{AU2440175}\\"
\vspace{0.15cm}\small Instructor: Dhaval Patel, PhD (Associate
Professor)\\"
SEAS -- Ahmedabad University, Ahmedabad, Gujarat, India}
\date{}


\begin{document}
\maketitle
\vspace{-0.3cm}
\hrule
\vspace{0.4cm}

% ===== 1. OUTLINE =====
\section{Outline and Learning Objective}

\subsection{Topics in this lecture}
\begin{enumerate}[leftmargin=*]
\item \textbf{Transformation of Random Variables:} learning transformation techniques for random variables.
\item \textbf{Function of Two Random Variables:} joint transformations and derived distributions.
\item \textbf{Illustrative Example:} detailed derivation for the case  $Z=X+Y$ .
\end{enumerate}

\subsection{Assumption / setup stated in the outline}
\textbf{Assumption:} the PDF of the original random variable (e.g.,  $f_X(x)$ ) is known \textbf{a priori}; the goal is to find the PDF of the transformed random variable (e.g.,  $f_Y(y)$ ).

\subsection{Noted example transformations (as written on the outline slide)}
\begin{bmatrix}
Z_1=X+Y, \quad \\
Z_2=X-Y, \quad \\
Z_3=\frac{X}{Y}, \quad \\
Z_4=\sqrt{X^2+Y^2}.
\end{bmatrix}
```

% ====== 2. ONE RV TRANSFORMATION ======

\section{Transformation of One Random Variable: $Y=g(X)$ }

\subsection{Definitions and notation (as used on the slides)}

A transformation is defined by

```
\[
Y=g(X).
\]
```

CDF notation: $F_Y(y)$ and $F_X(x)$. \\

PDF notation: $f_Y(y)$ and $f_X(x)$. \\

Slides proceed via \textbf{Step S1 (CDF)} then \textbf{Step S2 (differentiate w.r.t. y)}.

\subsection{Assumption / condition: monotonicity}

The slide distinguishes:

```
\begin{itemize}[leftmargin=*]
    \item \textbf{Monotonically increasing} case
    \item \textbf{Monotonically decreasing} case
\end{itemize}
```

% ----- 2.3 Increasing -----

\subsection{Case A: $g(\cdot)$ is monotonically increasing}

\subsubsection*{Step S1 (CDF method)}

```
\[
F_Y(y)=\Pr(Y\leq y)
=\Pr(g(X)\leq y)
=\Pr!\big(X\leq g^{-1}(y)\big)
=F_X!\big(g^{-1}(y)\big).
\]
```

\subsubsection*{Step S2 (Differentiate w.r.t. y)}

```
\[
f_Y(y)=\frac{d}{dy}\Big[F_X!\big(g^{-1}(y)\big)\Big]
=f_X!\big(g^{-1}(y)\big)\cdot\frac{d}{dy}\big(g^{-1}(y)\big).
\]
```

Rewriting with $x=g^{-1}(y)$:

```
\[
f_Y(y)= f_X(x),\frac{dx}{dy}\Big|_{x=g^{-1}(y)}.
\]
```

% ----- 2.4 Decreasing -----

```
\subsection{Case B:  $g(\cdot)$  is monotonically decreasing}
```

```
\subsubsection*{Step S1 (CDF method)}
```

```
\[
```

$$\begin{aligned} F_Y(y) &= \Pr(Y \leq y) \\ &= \Pr!\big(X \geq g^{-1}(y)\big) \\ &= 1 - F_X\big(g^{-1}(y)\big). \end{aligned}$$

```
\]
```

```
\subsubsection*{Step S2 (Differentiate w.r.t.  $y$ )}
```

The slide's boxed relationship:

```
\[
```

$$f_Y(y) = \frac{f_X(x)}{\left| \frac{dy}{dx} \right|} \Big|_{x=g^{-1}(y)}.$$

```
\]
```

S3 note: Change the limits for y (determine the valid y -range from the mapping).

```
% ===== 3. WORKED EXAMPLE =====
```

```
\section{Worked Example:  $X \sim \text{Uniform}(-1, 1)$ ,  $Y = \sin(\pi x/2)$ }
```

```
$Y=\sin\left(\frac{\pi x}{2}\right)$
```

```
\subsection{Given (from the example slide)}
```

```
\begin{itemize}[leftmargin=*]
```

Item X is continuous and uniformly distributed on $(-1, 1)$.

Item Transformation:

```
\[
```

$$Y = g(x) = \sin\left(\frac{\pi x}{2}\right).$$

```
\]
```

Item PDF of X :

```
\[
```

$$f_X(x) =$$

```
\begin{cases}
```

$$\frac{1}{2}, & -1 < x < 1, \\ 0, & \text{otherwise}. \end{cases}$$

```
\end{cases}
```

```
\]
```

Item Objective: find $f_Y(y)$.

```
\end{itemize}
```

```
\subsection{Step-by-step solution (as shown)}
```

```
\subsubsection*{Step 1: Invert the transformation}
```

```

\[

y=\sin!\left(\frac{\pi}{2}\right)
\quad\Rightarrow\quad
x=\frac{2}{\pi}\sin^{-1}(y).

\]

\subsubsection*{Step 2: Differentiate  $x$  w.r.t.  $y$ }
\[

\frac{dx}{dy}=\frac{2}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}.

\]

\subsubsection*{Step 3: Apply the transformation formula}
\[

f_Y(y)= f_X(x)\cdot\left|\frac{dx}{dy}\right|.

\]

Substitute  $f_X(x)=\frac{1}{2}$ :
\[

f_Y(y)=\frac{1}{2}\cdot\frac{2}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}
= \frac{1}{\pi\sqrt{1-y^2}}.

\]

\subsubsection*{Step 4: Determine the valid range of  $y$  from endpoints}
\begin{itemize}[leftmargin=*]
    \item At  $x=-1$ :  $y=\sin\left(-\frac{\pi}{2}\right)=-1$ 
    \item At  $x=1$ :  $y=\sin\left(\frac{\pi}{2}\right)=1$ 
\end{itemize}
\end{itemize}
So the support is  $-1 < y < 1$  (otherwise  $0$ ).

\subsection*{Final answer (as presented)}
\[

f_Y(y)=
\begin{cases}
\frac{1}{\pi\sqrt{1-y^2}}, & -1 < y < 1, \\[6pt]
0, & \text{otherwise}.
\end{cases}

\]

% ===== 4. TWO RV FUNCTION =====
\section*{Function of Two Random Variables: Example Setup ( $Z=X+Y$ )}

\subsection*{Problem statements listed on the slide}
For  $Z=X+Y$ , the slide lists:

```

```
\begin{enumerate}[leftmargin=*]
    \item Find the PDF of  $Z$ :  $f_Z(z)$ .
    \item Find  $f_Z(z)$  if  $X$  and  $Y$  are independent.
    \item Let  $X \sim N(0,1)$  and  $Y \sim N(0,1)$ ; prove that  $Z \sim N(0,2)$ .
    \item If  $X$  and  $Y$  are exponential RVs with parameter  $\lambda$ , find  $f_Z(z)$ .
\end{enumerate}
```

% ====== 5. CDF DERIVATION SETUP ======
\section{Detailed Derivation Shown: CDF setup for $Z=X+Y$ via Region Integration}

```
\subsection{Definition of  $Z$  and CDF start}
\[
Z=X+Y,
\qquad
F_Z(z)=\Pr(Z \leq z)=\Pr(X+Y \leq z).
\]
```

\subsection{Region description and the “strip” integrals}
The slide illustrates the line $x+y=z$ (equivalently $x=z-y$ or $y=z-x$) and writes the double-integral form using the joint pdf $f_{XY}(x,y)$, giving two equivalent strip integrations:

```
\subsubsection{(A) Horizontal strip (H-strip) form}
\[
F_Z(z)
=
\int_{y=-\infty}^{\infty}
\int_{x=-\infty}^{z-y}
f_{XY}(x,y) \, dx \, dy.
\]
```

```
\subsubsection{(B) Vertical strip form}
\[
F_Z(z)
=
\int_{x=-\infty}^{\infty}
\int_{y=-\infty}^{z-x}
f_{XY}(x,y) \, dy \, dx.
\]
```

```
\vspace{0.3cm}
\hrule
\vspace{0.2cm}
\noindent\textrit{End of Lecture 11 scribe.}

\end{document}
```

there are errors coming in this

Yep — a few things in your pasted version can trigger LaTeX errors, mainly because:

- your notes have **Markdown** [] blocks (those are *not* LaTeX),
- there are a few places where the integrals/limits got typed like `,-\infty` or `^{},z-`
`y}` (the comma breaks it),
- and `=====` lines / stray punctuation inside math can break compilation.

Here's a **clean, compile-ready** LaTeX version of your "Lecture 11 Scribe" with the same content, but fixed syntax.

latex

```
\documentclass[11pt]{article}

% ====== PACKAGES ======
\usepackage[a4paper,margin=1in]{geometry}
\usepackage{amsmath,amssymb}
\usepackage{enumitem}
\usepackage{fancyhdr}
\usepackage{xcolor}
\usepackage[hidelinks]{hyperref}

% ====== HEADER & FOOTER ======
\pagestyle{fancy}
\fancyhf{}
\lhead{CSE400: Fundamentals of Probability in Computing}
\rhead{Lecture 11 Scribe}
\cfoot{\thepage}

% ====== TITLE INFO ======
\title{\textbf{Lecture 11 Scribe: Transformation of Random Variables}}\v
```

```

\begin{document}
\maketitle
\vspace{0.15cm}\large February 10, 2026}
\author{\textbf{Niharika Ashar}\\\ AU ID: \textbf{AU2440175}\\\vspace{0.15cm}\small Instructor: Dhaval Patel, PhD (Associate Professor)\\}
SEAS -- Ahmedabad University, Ahmedabad, Gujarat, India}
\date{}


\begin{document}
\maketitle
\vspace{-0.3cm}
\hrule
\vspace{0.4cm}

% ===== 1. OUTLINE =====
\section{Outline and Learning Objective}

\subsection{Topics in this lecture}
\begin{enumerate}[leftmargin=*]
    \item \textbf{Transformation of Random Variables:} learning transformation techniques for random variables.
    \item \textbf{Function of Two Random Variables:} joint transformations and derived distributions.
    \item \textbf{Illustrative Example:} detailed derivation for the case  $Z=X+Y$ .
\end{enumerate}

\subsection{Assumption / setup stated in the outline}
\textbf{Assumption:} the PDF of the original RV (e.g.,  $f_X(x)$ ) is known a priori; the goal is how to find the PDF of the new transformed RV (e.g.,  $f_Y(y)$ ).

\subsection{Noted example transformations (as written on the outline slide)}
\begin{bmatrix}
Z_1=X+Y, \quad \\
Z_2=X-Y, \quad \\
Z_3=\frac{X}{Y}, \quad \\
Z_4=\sqrt{X^2+Y^2}.
\end{bmatrix}

% ===== 2. ONE RV TRANSFORMATION =====
\section{Transformation of One Random Variable:  $Y=g(X)$ }

```

```
\subsection{Definitions and notation (as used on the slides)}
```

A transformation is defined by

```
\[
```

```
Y=g(X).
```

```
\]
```

CDF notation: $F_Y(y)$ and $F_X(x)$. \\

PDF notation: $f_Y(y)$ and $f_X(x)$. \\

The slides proceed via \textbf{Step S1 (CDF)} then \textbf{Step S2 (differentiate w.r.t. y)}.

```
\subsection{Assumption / condition: monotonicity}
```

The slide distinguishes monotonic cases:

```
\begin{itemize}[leftmargin=*
```

```
    \item \textbf{Monotonically increasing} case
```

```
    \item \textbf{Monotonically decreasing} case
```

```
\end{itemize}
```

```
\subsection{Case A:  $g(\cdot)$  is monotonically increasing}
```

```
\subsubsection*{Step S1 (CDF method)}
```

```
\[
```

```
F_Y(y)
```

```
= \Pr(Y \leq y)
```

```
= \Pr(g(X) \leq y)
```

```
= \Pr!\big(X \leq g^{-1}(y)\big)
```

```
= F_X!\big(g^{-1}(y)\big).
```

```
\]
```

```
\subsubsection*{Step S2 (Differentiate w.r.t.  $y$ )}
```

```
\[
```

```
f_Y(y)
```

```
= \frac{d}{dy} \Big[ F_X!(g^{-1}(y)) \Big]
```

```
= f_X!(g^{-1}(y)) \cdot \frac{d}{dy} \big(g^{-1}(y)\big).
```

```
\]
```

Let $x = g^{-1}(y)$:

```
\[
```

```
f_Y(y) = f_X(x) \cdot \frac{dx}{dy} \Big|_{x=g^{-1}(y)}.
```

```
\]
```

```
\subsection{Case B:  $g(\cdot)$  is monotonically decreasing}
```

```

\subsubsection*{Step S1 (CDF method)}
\[
F_Y(y) \\
= \Pr(Y \leq y) \\
= \Pr!\big(X \geq g^{-1}(y)\big) \\
= 1 - F_X!\big(g^{-1}(y)\big).
\]

\subsubsection*{Step S2 (Differentiate w.r.t. $y$)}
\[
f_Y(y) \\
=\frac{f_X(x)}{\left|\frac{dy}{dx}\right|} \Bigg|_{x=g^{-1}(y)}.
\]

\textbf{S3 note:} change the limits for $y$ (determine valid $y$-range from the mapping).

% ===== 3. WORKED EXAMPLE =====
\section{Worked Example: $X \sim \mathrm{Uniform}(-1,1)$, \\
$Y = \sin!\left(\frac{x}{2}\right)$}

\subsection{Given (from the example slide)}
\begin{itemize}[leftmargin=*]
\item $X$ is uniform on $(-1,1)$.
\item Transformation:
\[
Y = \sin!\left(\frac{x}{2}\right).
\]
\item PDF of $X$:
\[
f_X(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1, \\ 0, & \text{otherwise}. \end{cases}
\]
\item Objective: find $f_Y(y)$.
\end{itemize}
\end{itemize}

\subsection{Step-by-step solution (as shown)}

\subsubsection*{Step 1: Invert the transformation}
\[

```

```

y=\sin!\left(\frac{\pi x}{2}\right)
\quad\Rightarrow\quad
x=\frac{2}{\pi}\sin^{-1}(y).
\]

\subsubsection*{Step 2: Differentiate  $x$  w.r.t.  $y$ }
\[
\frac{dx}{dy}=\frac{2}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}.
\]

\subsubsection*{Step 3: Apply the transformation formula}
\[
f_Y(y)= f_X(x)\cdot|\frac{dx}{dy}|.
\]

Substitute  $f_X(x)=\frac{1}{2}$ :
\[
f_Y(y)
=\frac{1}{2}\cdot\frac{2}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}
=\frac{1}{\pi\sqrt{1-y^2}}.
\]

\subsubsection*{Step 4: Determine the valid range of  $y$  from endpoints}
\begin{itemize}[leftmargin=*]
\item At  $x=-1$ :  $y=\sin\left(-\frac{\pi}{2}\right)=-1$ 
\item At  $x=1$ :  $y=\sin\left(\frac{\pi}{2}\right)=1$ 
\end{itemize}
So the support is  $-1 \leq y \leq 1$  (otherwise  $0$ ).

\subsection*{Final answer (as presented)}
\[
f_Y(y)=
\begin{cases}
\frac{1}{\pi\sqrt{1-y^2}}, & -1 < y < 1, \\[6pt]
0, & \text{otherwise}.
\end{cases}
\]

% ===== 4. TWO RV FUNCTION =====
\section*{Function of Two Random Variables: Example Setup ( $Z=X+Y$ )}

\subsection*{Problem statements listed on the slide}
For  $Z=X+Y$ , the slide lists:

```

```
\begin{enumerate}[leftmargin=*]
    \item Find the PDF of  $Z$ :  $f_Z(z)$ .
    \item Find  $f_Z(z)$  if  $X$  and  $Y$  are independent.
    \item Let  $X \sim N(0,1)$  and  $Y \sim N(0,1)$ ; prove that  $Z \sim N(0,2)$ .
    \item If  $X$  and  $Y$  are exponential RVs with parameter  $\lambda$ , find  $f_Z(z)$ .
\end{enumerate}
```

% ====== 5. CDF DERIVATION SETUP ======

```
\section{Detailed Derivation: CDF setup for  $Z=X+Y$  via Region Integration}
```

```
\subsection{Definition of  $Z$  and CDF start}
```

```
\[
Z=X+Y,
\qquad
F_Z(z)=\Pr(Z \leq z)=\Pr(X+Y \leq z).
\]
```

```
\subsection{Region description and the “strip” integrals}
```

Using the joint pdf $f_{XY}(x,y)$ and the boundary line $x+y=z$:

```
\subsubsection{(A) Horizontal strip (H-strip) form}
```

```
\[
F_Z(z)
=
\int_{-\infty}^{\infty}
\int_{-\infty}^{z-y}
f_{XY}(x,y) \, dx \, dy.
\]
```

```
\subsubsection{(B) Vertical strip form}
```

```
\[
F_Z(z)
=
\int_{-\infty}^{\infty}
\int_{-\infty}^{z-x}
f_{XY}(x,y) \, dy \, dx.
\]
```

```
\vspace{0.3cm}
```

```
\hrule
\vspace{0.2cm}
\noindent\textit{End of Lecture 11 scribe.}

\end{document}
```

If you paste the **exact error message** (the red lines from Overleaf like “Undefined control sequence” + line number), I can point to the *exact* line causing it.