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CSE400 – Fundamentals of Probability in Computing

Lecture 6: Discrete RVs, Expectation and Problem Solving

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Outline

- Previous Lecture Recap: Random Variables (RVs)
  - Independent Events / Examples
- Definition and Example
- Types of Discrete Random Variables
  - Bernoulli RV
  - Binomial RV
  - Geometric RV
  - Poisson RV
- Expectation of RVs
  - Definition and Example

- Expectation of a Function of RV
- Linear Operation with Expectation
- Moments and Central Moments of RVs
  - Variance, Skewness and Kurtosis
- The Cumulative Density Function (CDF)
  - Definition, Properties and Examples
- The Probability Density Function (PDF)
  - Definition, Properties and Examples

Random Variables

Motivation and Concept

A random variable X on a sample space Ω is a function

$$X : \Omega \to \mathbb{R}$$

that assigns to each sample point $\omega \in \Omega$ a real number $X(\omega)$.

Until further notice, we will restrict our attention to random variables that are **discrete**, i.e., they take values in a range that is **finite or countably infinite**. This means even though we define X to map Ω to $\mathbb{R}$, the actual set of values

$$\{X(\omega) : \omega \in \Omega\}$$

that X takes is a **discrete subset of** $\mathbb{R}$.

Sample space
Sample point s

Sample points mapped by the discrete random variable $X(s)$ into numbers on the real line.

Sample space of all permutations

Random Variables

Motivation and Concept

The distribution of a random variable can be visualized as a **bar diagram**:

$$\Pr[X = a]$$

The x-axis represents the values that a random variable can take on.

The height of the bar at a value a is the probability $\Pr[X = a]$.

Each of these probabilities can be computed by looking at the probability of the corresponding event in the sample space.

Guide to Selecting a Probability Distribution

Random Variables

Discrete variable

- Countable support
- Probability mass function
- Probabilities assigned to single values
- Each possible value has strictly positive probability

Bernoulli Random Variable

A Bernoulli random variable X takes values in

$$\{0, 1\}$$

$$\Pr[X = 1] = p$$

$$\Pr[X = 0] = 1 - p$$

Binomial Random Variable

A binomial random variable counts the number of successes in n independent Bernoulli trials.

$$X \sim \text{Binomial}(n, p)$$

$$\Pr[X = k] = \binom{n}{k} p^k (1 - p)^{n-k}$$

Geometric Random Variable

A geometric random variable models the number of trials until the first success.

$$\Pr[X = k] = (1 - p)^{k-1} p$$

Poisson Random Variable

A Poisson random variable models the number of occurrences in a fixed interval.

$$\Pr[X = k] = \frac{\lambda^k e^{-\lambda}}{k!}$$

Expectation of Random Variables

Definition

The expectation of a discrete random variable X is defined as

$$E[X] = \sum_x x \Pr[X = x]$$

Expectation

Example

(Example shown on slide – no additional steps provided)

Expectation of a Function of a Random Variable

Let $Y = g(X)$

$$E[Y] = \sum_x g(x) \Pr[X = x]$$

Linear Operation with Expectation

$$E[aX + b] = aE[X] + b$$

Moments and Central Moments of RVs

- Mean
- Variance
- Skewness
- Kurtosis

Variance

$$\mathrm{Var}(X) = E[(X - E[X])^2]$$

Cumulative Density Function (CDF)

Definition

$$F_X(x) = \Pr[X \leqslant x]$$

Properties of CDF

- Non-decreasing
- Right-continuous
- $\lim_{x \to -\infty} F_X(x) = 0$
- $\lim_{x \to \infty} F_X(x) = 1$

Probability Density Function (PDF)

Definition

$$f_X(x) = \Pr[X = x]$$

Properties of PDF

$$\sum_x f_X(x) = 1$$

$$f_X(x) \geqslant 0$$

End of Lecture 6

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