Transformation of RVs

L11_S2_A.pdf
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CSE400 - Fundamentals of Probability in Computing

Lecture - 11: Tranformation of Random Variables

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Outline

1. **Transformation of Random Variables**
   Learning of transformation techniques for random variables.
2. **Function of Two Random Variables**
   Joint transformations and derived distributions.
3. **Illustrative Example**
   Detailed derivation for the case: $Z = X + Y$

Transformation of RVs + Function of Two RVs + Example

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1. Transformation of Random Variables

Let

$$Y = g(X)$$

Two cases:

1. +ve (Monotonaly)

2. -ve

S1:

$$F_Y(y) = P(Y \leqslant y)$$

$$= P(g(X) \leqslant y)$$

$$= P\left(X \leqslant g^{-1}(y)\right)$$

$$= F_X\left(g^{-1}(y)\right)$$

S2: Diff. w.r.t. y

$$f_Y(y) = \frac{d}{dy}\left[F_X\left(g^{-1}(y)\right)\right]$$

$$= f_X\left(g^{-1}(y)\right) \cdot \frac{d}{dy} g^{-1}(y)$$

$$= f_X(x) \frac{dx}{dy}\Big|_{x=g^{-1}(y)}$$

For -ve case:

S1:

$$F_Y(y) = P(Y \leqslant y)$$

$$= P\left(X \geqslant g^{-1}(y)\right)$$

$$= 1 - F_X\left(g^{-1}(y)\right)$$

S2:

$$f_Y(y) = f_X(x) \left|\frac{dx}{dy}\right| \Big|_{x=g^{-1}(y)}$$

S3: Change the limits for Y

$$f_Y(y) = \frac{f_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{x=g^{-1}(y)}$$

(+ve / -ve indicated)

Example

X ~ Uniformly over (-1,1)

$$Y = g(X) = \sin\left(\frac{\pi X}{2}\right)$$

$$f_X(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1 \\ 0, & otherwise \end{cases}$$

Sol:

$$y = \sin\left(\frac{\pi x}{2}\right)$$

$$x = \frac{2}{\pi}\sin^{-1}(y)$$

$$\frac{dx}{dy} = \frac{2}{\pi}\frac{1}{\sqrt{1-y^2}}$$

$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$$

$$= \frac{1}{2}\cdot\frac{2}{\pi}\frac{1}{\sqrt{1-y^2}}$$

$$= \frac{1}{\pi\sqrt{1-y^2}}, \quad -1 < y < 1$$

0, otherwise

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2. Function of Two Random Variables

Illustrative Example

$$Z = X + Y$$

Find:

(i) PDF of Z, $f_Z(z)$

(ii) $f_Z(z)$, if X & Y are independent

(iii) Let $X \sim N(0, 1)$ & $Y \sim N(0, 1)$. Prove that $Z \sim N(0, 2)$

(iv) If X & Y are exponential distribi RVs with parameter λ, find $f_Z(z)$

Detailed derivation for the case: $Z = X + Y$

$$F_Z(z) = P(Z \leqslant z)$$

$$= P(X + Y \leqslant z)$$

$$= \int \int f_{X,Y}(x, y)\, dx\, dy$$

Horizontal Strip:

$$= \int_{y=-\infty}^{\infty} \int_{x=-\infty}^{z-y} f_{X,Y}(x, y)\, dx\, dy$$

Vertical Strip:

$$= \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{z-x} f_{X,Y}(x, y)\, dy\, dx$$

$$Z = X + Y$$

$$x : 0,\ z = y$$

$$y : 0,\ z = x$$

$$z = -\infty$$

$$y = 0$$

$$z = x + y$$

$$x - y$$

$$\sqrt{x^2 + y^2}$$

(Region and strip representation as shown in diagram)

End of Lecture 11

Do you like this personality?