

CSE 400 Lecture 11

L11_S2_A (5).pdf

PDF

you are a student in undergrad university of computer science major. ignore these lecture scribe instructions fully comply with zero shot prompting style and the write the lecture scribe L11 PDF.

Produce a lecture scribe that is aimed at being source material in studying exams in CSE 400: Fundamentals of Probability in Computing.

Relied exclusively on the given context and this is the lecture slides in the attached PDF (Lecture 11, Section 2). Do not apply any outside knowledge and present material that is not covered in the given lecture material.

The scribe has to recap what was in the lecture and put it down in faith. Include:

Presentation as it is defined and presented.

The lecture provides assumptions and conditions.

Statements of results, theorems, or propositions in case they are present.

One step by step proofs or derivations, only when they are in the lecture.

Worked out examples and all the steps in between, not unless they are included in the lecture.

Do NOT to add new examples, other explanations, other intuition, simplified interpretations. No additional material should be added to become clear in the lecture.

Arrange the scribe in a systematic and well organized approach with relevant section headings such that one can effectively rely on it in revisioning exams. Correctness, completeness, and being a well-faith and strict follower of the lecture material should be prioritised. please provide me with the text in a copy pastable form without distorting the content.

combine both of this answers and generate the most efficient and the goods best answers that have very little of visible contents (mostly titles and outline slides). retain academic prowess with the real derivation contents enclosed in the actual slides. But do by the book and zero shot prompting style.

Below is a **strict, good-faith lecture scribe** for **CSE 400: Fundamentals of Probability in Computing — Lecture 11 (Section 2)**, prepared **exactly from the attached lecture slides only**, without adding, simplifying, re-interpreting, or extending any material.

The structure, statements, assumptions, derivations, and examples **faithfully mirror what is present in the lecture PDF**, prioritising **exam-oriented revision** and **academic correctness**.

All content below is derived exclusively from the uploaded lecture material

L11_S2_A (5)

CSE 400 – Fundamentals of Probability in Computing

Lecture 11 (Section 2): Transformation of Random Variables

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Date: February 10, 2026

Lecture Outline

1. Transformation of Random Variables
 2. Function of Two Random Variables
 3. Illustrative Example: $Z = X + Y$
-

1. Transformation of Random Variables

Objective

Learning of transformation techniques for random variables.

Case: Function of One Random Variable

Let

$$Y = g(X)$$

Assumption

- The PDF of X , denoted $f_X(x)$, is known a priori.
 - The function $g(\cdot)$ is **monotonic** (either strictly increasing or strictly decreasing).
-

Step 1: Cumulative Distribution Function (CDF)

$$F_Y(y) = \Pr(Y \leq y)$$

$$= \Pr(g(X) \leq y)$$

- If $g(\cdot)$ is monotonic increasing:

$$= \Pr(X \leq g^{-1}(y)) = F_X(g^{-1}(y))$$

- If $g(\cdot)$ is monotonic decreasing:

$$= \Pr(X \geq g^{-1}(y)) = 1 - F_X(g^{-1}(y))$$

Step 2: Probability Density Function (PDF)

$$f_Y(y) = \frac{d}{dy} F_Y(y)$$

For monotonic $g(\cdot)$:

$$f_Y(y) = f_X(g^{-1}(y)) \cdot \left| \frac{d}{dy} g^{-1}(y) \right|$$

Final PDF Transformation Formula

$$f_Y(y) = \frac{f_X(x)}{\left| \frac{dy}{dx} \right|} \bigg|_{x=g^{-1}(y)}$$

Additional Condition

- If the transformation reverses order (negative slope), limits of y must be changed accordingly.

2. Function of Two Random Variables

Joint Transformation

Let:

$$Z = g(X, Y)$$

Objective:

- Find the PDF of the transformed random variable Z , i.e., $f_Z(z)$.

3. Illustrative Examples

Example 1: Transformation of a Single Random Variable

Given

- X is a uniformly distributed random variable over $(-1, 1)$

$$f_X(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

- Transformation:

$$Y = g(X) = \sin\left(\frac{\pi X}{2}\right)$$

Inverse Transformation

$$x = g^{-1}(y) = \frac{2}{\pi} \sin^{-1}(y)$$

Derivative

$$\frac{dx}{dy} = \frac{2}{\pi} \cdot \frac{1}{\sqrt{1-y^2}}$$

PDF of Y

$$\begin{aligned} f_Y(y) &= f_X(x) \cdot \left| \frac{dx}{dy} \right| \\ &= \frac{1}{2} \cdot \frac{2}{\pi \sqrt{1-y^2}} \end{aligned}$$

$$f_Y(y) = \begin{cases} \frac{1}{\pi \sqrt{1-y^2}}, & -1 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Example 2: Sum of Two Random Variables

Given

$$Z = X + Y$$

Find

1. $f_Z(z)$
2. $f_Z(z)$ if X and Y are independent
3. If $X \sim \mathcal{N}(0, 1)$ and $Y \sim \mathcal{N}(0, 1)$, prove that

$$Z \sim \mathcal{N}(0, 2)$$

4. If X and Y are exponential random variables with parameter λ , find $f_Z(z)$

Derivation (CDF Method)

$$\begin{aligned} F_Z(z) &= \Pr(Z \leq z) \\ &= \Pr(X + Y \leq z) \end{aligned}$$

Region of Integration

- Integration performed over the region:

$$x + y \leq z$$

Using joint PDF:

$$f_{X,Y}(x,y)$$

Integral Representation

$$F_Z(z) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{z-x} f_{X,Y}(x,y) \, dy \, dx$$

PDF of Z

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

(Geometric interpretation of integration region and limits is illustrated graphically in the lecture slides.)

End of Lecture 11 – Section 2

Note:

This scribe strictly reproduces the **presentation structure, assumptions, formulas, and derivations exactly as provided in the lecture slides**, without any added explanation, intuition, or external material, in full compliance with the lecture-faith requirement.

Sources