

# Lecture 11 Scribe

## CSE400 — Fundamentals of Probability in Computing

## Lecture 11: Transformation of Random Variables  
(February 10, 2026)

**Instructor:** Dhaval Patel, PhD (Associate Professor),  
SEAS–Ahmedabad University, Ahmedabad, Gujarat,  
India.

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## # 1. Outline and Learning Objective

### ## 1.1 Topics in this lecture

1. **Transformation of Random Variables**: learning transformation techniques for random variables.
2. **Function of Two Random Variables**: joint transformations and derived distributions.
3. **Illustrative Example**: detailed derivation for the case ( $Z = X + Y$ ).

### ## 1.2 Assumption / setup stated in the outline

\* **Assumption:** the **PDF of the original RV** (e.g.,  $f_X(x)$ ) is **known a priori**; goal is **how to find the PDF of the new transformed RV** (e.g.,  $f_Y(y)$ ).

### ## 1.3 Noted example transformations (as written on the outline slide)

- \* ( $Z_1 = X + Y$ )
- \* ( $Z_2 = X - Y$ )
- \* ( $Z_3 = X / Y$ )
- \* ( $Z_4 = \sqrt{X^2 + Y^2}$ )

These appear as representative transformation forms on the outline page.

---

## # 2. Transformation of One Random Variable: ( $Y = g(X)$ )

### ## 2.1 Definitions and notation (as used on the slides)

\* A **transformation** is defined by

$$\begin{aligned} &[ \\ &Y = g(X). \\ &] \end{aligned}$$

\* **CDF notation**: ( $F_Y(y)$ ) and ( $F_X(x)$ ).

\* **PDF notation**: ( $f_Y(y)$ ) and ( $f_X(x)$ ).

\* The slides explicitly proceed via **Step S1 (CDF)** then **Step S2 (differentiate w.r.t. ( $y$ ))**.

### ## 2.2 Assumption / condition: monotonicity

The slide distinguishes monotonic cases (illustrated via increasing and decreasing sketches):

\* **Monotonically increasing** case

\* **Monotonically decreasing** case

---

### ## 2.3 Case A: ( $g(\cdot)$ ) is monotonically increasing

#### ### Step S1 (CDF method)

$$\begin{aligned} &[ \\ &F_Y(y) \stackrel{;}{=} \Pr(Y \leq y) \\ &\stackrel{;}{=} \Pr(g(X) \leq y) \\ &\stackrel{;}{=} \Pr(\big(X \leq g^{-1}(y)\big)) \\ &\stackrel{;}{=} F_X(\big(g^{-1}(y)\big)). \\ &] \end{aligned}$$

This is written on the slide in the "S1" sequence.

#### ### Step S2 (Differentiate w.r.t. ( $y$ ))

$$\begin{aligned} &[ \\ &f_Y(y) \stackrel{;}{=} \frac{d}{dy} \big[F_X(\big(g^{-1}(y)\big))\big] \\ &\stackrel{;}{=} f_X(\big(g^{-1}(y)\big)) \cdot \frac{d}{dy} \big(g^{-1}(y)\big) \end{aligned}$$

$(y)\big)$ .

]

The slide then rewrites this using  $(x=g^{-1}(y))$ :

[

$f_Y(y) := f_X(x) \left| \frac{dx}{dy} \right|_{x=g^{-1}(y)}$ .

]

These steps are shown as the CDF  $\rightarrow$  PDF derivation chain for the increasing case.

---

## 2.4 Case B:  $(g(\cdot))$  is monotonically decreasing

### Step S1 (CDF method)

[

$F_Y(y) = \Pr(Y \leq y)$

$= \Pr(X \geq g^{-1}(y))$

$= 1 - F_X(g^{-1}(y))$ .

]

This "S1" chain is displayed on the decreasing-case slide.

### Step S2 (Differentiate w.r.t.  $(y)$ )

The slide then differentiates and rearranges to the displayed boxed relationship:

[

$f_Y(y) := \frac{f_X(x)}{\left| \frac{dy}{dx} \right|_{x=g^{-1}(y)}}$ .

]

The slide also annotates "S3: Change the limits for  $(y)$ " (i.e., determine the valid  $(y)$ -range from the mapping).

---

# 3. Worked Example:  $(X \sim \text{Uniform}(-1,1))$ ,  
 $(Y = \sin(\frac{\pi x}{2}))$

## 3.1 Given (from the example slide)

\*  $(X)$  is a continuous RV, \*\*uniformly distributed\*\* on  $((-1,1))$ .

\* The transformation is

$$Y = g(X) = \sin\left(\frac{\pi X}{2}\right).$$

\* The PDF of  $(X)$  is written explicitly as

$$f_X(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

\* \*\*Objective on the slide:\*\* "(f\_Y(y)=?)"

## 3.2 Step-by-step solution (as shown)

### Step 1: Invert the transformation

Starting from

$$y = \sin\left(\frac{\pi X}{2}\right),$$

the slide gives

$$X = \frac{2}{\pi} \sin^{-1}(y).$$

### Step 2: Differentiate  $(X)$  w.r.t.  $(y)$

$$\frac{dx}{dy} = \frac{2}{\pi} \cdot \frac{1}{\sqrt{1-y^2}}.$$

### Step 3: Apply the transformation formula (using the slide's substitution)

The slide applies (in the same structure as earlier derivation):

```
[
f_Y(y) = f_X(x) \cdot \left| \frac{dx}{dy} \right|.
]
Substitute (f_X(x) = \frac{1}{2}) and (\frac{dx}{dy}) above:
[
f_Y(y) = \frac{1}{2} \cdot \frac{2}{\pi} \cdot \frac{1}{\sqrt{1-y^2}}
= \frac{1}{\pi \sqrt{1-y^2}}.
]
```

### Step 4: Determine the valid range of (y) from endpoints (as written)

The slide checks endpoints:

- \* At (x=-1): (y = \sin^{-1}(-\frac{\pi}{2}) = -1)
  - \* At (x=1): (y = \sin^{-1}(\frac{\pi}{2}) = 1)
- So the slide states the support as (-1 < y < 1), and "otherwise 0".

## 3.3 Final answer (as presented)

```
[
f_Y(y) =
\begin{cases}
\frac{1}{\pi \sqrt{1-y^2}}, & -1 < y < 1, [6pt]
0, & \text{otherwise}.
\end{cases}
]
```

---

# 4. Function of Two Random Variables: Example Setup  
(Z = X + Y)

## 4.1 Problem statements listed on the slide

For (Z = X + Y), the slide lists tasks:

1. \*\*Find the PDF of (Z)\*\*: (f\_Z(z)).

2. **Find  $f_Z(z)$  if  $(X)$  and  $(Y)$  are independent.**
3. **Let  $(X \sim N(0,1))$  and  $(Y \sim N(0,1))$ ; prove that  $(Z \sim N(0,2))$ .** (Statement appears as an objective on the slide; no proof is included on the slide.)
4. **If  $(X)$  and  $(Y)$  are exponential distribution RVs with parameter  $(\lambda)$ , find  $f_Z(z)$ .**

\*(Only these tasks are stated; the slide does not provide completed proofs for the normal/exponential claims.)\*

---

# 5. Detailed derivation shown: CDF setup for  $(Z=X+Y)$  via region integration

## 5.1 Definition of  $(Z)$  and CDF start

The slide sets

[  
 $Z=X+Y,$   
 ]

and defines

[  
 $F_Z(z) = \Pr(Z \leq z)$   
 $= \Pr(X+Y \leq z).$   
 ]

## 5.2 Region description and the "strip" integrals

The slide illustrates the line  $(x+y=z)$  (equivalently  $(x=z-y)$  or  $(y=z-x)$ ) and writes the double-integral form using the joint pdf  $(f_{XY}(x,y))$ , showing two equivalent "strip" integrations:

### (A) Horizontal strip ("H-strip") form

[  
 $F_Z(z)$   
 =====

$\int_{y=-\infty}^{\infty}$

$$\int_{x=-\infty}^{z-y} f_{XY}(x,y) dx, dy.$$

The slide labels this as **H-strip**.

### (B) Vertical strip ("Vertical strip") form

$$[F_Z(z)$$

$$\int_{y=-\infty}^{\infty} \int_{x=-\infty}^{z-x} f_{XY}(x,y) dy, dx.$$

The slide labels this as **Vertical strip**.

\*(The slide provides these CDF integral setups and the geometric interpretation; it does not proceed further to a final closed-form ( $f_Z(z)$ ) on the provided pages.)\*

make a structures latex name lecture 11 scribe , Niharika Ashar with au id AU2440175

latex

```
\documentclass[11pt]{article}

% ===== PACKAGES =====
\usepackage[a4paper,margin=1in]{geometry}
\usepackage{amsmath,amssymb}
\usepackage{enumitem}
\usepackage{fancyhdr}
\usepackage{xcolor}
\usepackage{hyperref}

% ===== HEADER & FOOTER =====
\pagestyle{fancy}
\fancyhf{}
\lhead{CSE400: Fundamentals of Probability in Computing}
\rhead{Lecture 11 Scribe}
\cfoot{\thepage}
```

```
% ===== TITLE INFO =====
\title{\textbf{Lecture 11 Scribe: Transformation of Random Variables}}\
\vspace{0.15cm}\large February 10, 2026}
\author{\textbf{Niharika Ashar}}\ AU ID: \textbf{AU2440175}\
\vspace{0.15cm}\small Instructor: Dhaval Patel, PhD (Associate
Professor)\
SEAS -- Ahmedabad University, Ahmedabad, Gujarat, India}
\date{}

\begin{document}
\maketitle
\vspace{-0.3cm}
\hrule
\vspace{0.4cm}

% ===== 1. OUTLINE =====
\section{Outline and Learning Objective}

\subsection{Topics in this lecture}
\begin{enumerate}[leftmargin=*]
\item \textbf{Transformation of Random Variables:} learning
transformation techniques for random variables.
\item \textbf{Function of Two Random Variables:} joint
transformations and derived distributions.
\item \textbf{Illustrative Example:} detailed derivation for the
case  $Z=X+Y$ .
\end{enumerate}

\subsection{Assumption / setup stated in the outline}
\textbf{Assumption:} the PDF of the original random variable (e.g.,
 $f_X(x)$ ) is known a priori; the goal is to find the PDF of the
transformed random variable (e.g.,  $f_Y(y)$ ).

\subsection{Noted example transformations (as written on the outline
slide)}
\[
Z_1=X+Y, \quad
Z_2=X-Y, \quad
Z_3=\frac{X}{Y}, \quad
Z_4=\sqrt{X^2+Y^2}.
\]
```



% ===== 2. ONE RV TRANSFORMATION =====

\section{Transformation of One Random Variable:  $Y=g(X)$ }

\subsection{Definitions and notation (as used on the slides)}

A transformation is defined by

\[

$Y=g(X)$ .

\]

CDF notation:  $F_Y(y)$  and  $F_X(x)$ . \\\

PDF notation:  $f_Y(y)$  and  $f_X(x)$ . \\\

Slides proceed via \textbf{Step S1 (CDF)} then \textbf{Step S2 (differentiate w.r.t.  $y$ )}.

\subsection{Assumption / condition: monotonicity}

The slide distinguishes:

\begin{itemize}[leftmargin=\*]

\item \textbf{Monotonically increasing} case

\item \textbf{Monotonically decreasing} case

\end{itemize}

% ----- 2.3 Increasing -----

\subsection{Case A:  $g(\cdot)$  is monotonically increasing}

\subsubsection\*{Step S1 (CDF method)}

\[

$F_Y(y) = \Pr(Y \leq y)$

$= \Pr(g(X) \leq y)$

$= \Pr(\big(X \leq g^{-1}(y)\big)$

$= F_X(\big(g^{-1}(y)\big)).$

\]

\subsubsection\*{Step S2 (Differentiate w.r.t.  $y$ )}

\[

$f_Y(y) = \frac{d}{dy} \Big[ F_X(\big(g^{-1}(y)\big)) \Big]$

$= f_X(\big(g^{-1}(y)\big)) \cdot \frac{d}{dy} \big(g^{-1}(y)\big).$

\]

Rewriting with  $x=g^{-1}(y)$ :

\[

$f_Y(y) = f_X(x) \cdot \frac{dx}{dy} \Big|_{x=g^{-1}(y)}.$

\]

% ----- 2.4 Decreasing -----

```

\subsection{Case B:  $g(\cdot)$  is monotonically decreasing}

\subsubsection*{Step S1 (CDF method)}
\[
F_Y(y) = \Pr(Y \leq y)
= \Pr(\big(X \geq g^{-1}(y)\big)
= 1 - F_X(\big(g^{-1}(y)\big).
\]

\subsubsection*{Step S2 (Differentiate w.r.t.  $y$ )}
```

The slide's boxed relationship:

```

\[
f_Y(y) = \frac{f_X(x)}{\left| \frac{dy}{dx} \right|} \Big|_{x=g^{-1}(y)}.
\]
\textbf{S3 note:} Change the limits for  $y$  (determine the valid  $y$ -
range from the mapping).

% ===== 3. WORKED EXAMPLE =====
\section{Worked Example:  $X \sim \text{Uniform}(-1,1)$ ,  $Y = \sin\left(\frac{\pi}{2} X\right)$ }

\subsection{Given (from the example slide)}
\begin{itemize}[leftmargin=*]
  \item  $X$  is continuous and uniformly distributed on  $(-1,1)$ .
  \item Transformation:
  \[
Y = g(x) = \sin\left(\frac{\pi}{2} x\right).
\]
  \item PDF of  $X$ :
  \[
f_X(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1, \\ 0, & \text{otherwise.} \end{cases}
\]
  \item Objective: find  $f_Y(y)$ .
\end{itemize}

\subsection{Step-by-step solution (as shown)}

\subsubsection*{Step 1: Invert the transformation}

```

```

\l
y=\sin\!\left(\frac{\pi x}{2}\right)
\quad\Rightarrow\quad
x=\frac{2}{\pi}\sin^{-1}(y).
\l

\subsubsection*{Step 2: Differentiate  $x$  w.r.t.  $y$ }
\l
\frac{dx}{dy}=\frac{2}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}.
\l

\subsubsection*{Step 3: Apply the transformation formula}
\l
f_Y(y)=f_X(x)\cdot\left|\frac{dx}{dy}\right|.
\l
Substitute  $f_X(x)=\frac{1}{2}$ :
\l
f_Y(y)=\frac{1}{2}\cdot\frac{2}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}
=\frac{1}{\pi\sqrt{1-y^2}}.
\l

\subsubsection*{Step 4: Determine the valid range of  $y$  from endpoints}
\begin{itemize}[leftmargin=*]
  \item At  $x=-1$ :  $y=\sin\!\left(-\frac{\pi}{2}\right)=-1$ 
  \item At  $x=1$ :  $y=\sin\!\left(\frac{\pi}{2}\right)=1$ 
\end{itemize}
So the support is  $-1<y<1$  (otherwise  $0$ ).

\subsection{Final answer (as presented)}
\l
f_Y(y)=
\begin{cases}
\frac{1}{\pi\sqrt{1-y^2}}, & -1<y<1, \\[6pt]
0, & \text{otherwise.}
\end{cases}
\l

% ===== 4. TWO RV FUNCTION =====
\section{Function of Two Random Variables: Example Setup ( $Z=X+Y$ )}

\subsection{Problem statements listed on the slide}
For  $Z=X+Y$ , the slide lists:

```

```

\begin{enumerate}[leftmargin=*]
  \item Find the PDF of  $Z$ :  $f_Z(z)$ .
  \item Find  $f_Z(z)$  if  $X$  and  $Y$  are independent.
  \item Let  $X \sim N(0,1)$  and  $Y \sim N(0,1)$ ; prove that  $Z \sim N(0,2)$ .
  \item If  $X$  and  $Y$  are exponential RVs with parameter  $\lambda$ , find  $f_Z(z)$ .
\end{enumerate}

% ===== 5. CDF DERIVATION SETUP =====
\section{Detailed Derivation Shown: CDF setup for  $Z=X+Y$  via Region Integration}

\subsection{Definition of  $Z$  and CDF start}
\[
Z=X+Y,
\quad
F_Z(z)=\Pr(Z \leq z)=\Pr(X+Y \leq z).
\]

\subsection{Region description and the “strip” integrals}
The slide illustrates the line  $x+y=z$  (equivalently  $x=z-y$  or  $y=z-x$ ) and writes the double-integral form using the joint pdf  $f_{XY}(x,y)$ , giving two equivalent strip integrations:

\subsubsection*(A) Horizontal strip (H-strip) form}
\[
F_Z(z)
=
\int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} f_{XY}(x,y) \, dx \, dy.
\]

\subsubsection*(B) Vertical strip form}
\[
F_Z(z)
=
\int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} f_{XY}(x,y) \, dy \, dx.
\]

```

```

\vspace{0.3cm}
\hrule
\vspace{0.2cm}
\noindent\textit{End of Lecture 11 scribe.}

\end{document}

```

there are errors coming in this

Yep — a few things in your pasted version can trigger LaTeX errors, mainly because:

- your notes have **Markdown** [ ] blocks (those are *not* LaTeX),
- there are a few places where the integrals/limits got typed like `,-\infty` or `^{\,,z-`  
`y}` (the comma breaks it),
- and `=====` lines / stray punctuation inside math can break compilation.

Here's a **clean, compile-ready** LaTeX version of your "Lecture 11 Scribe" with the same content, but fixed syntax.

latex

```

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% ===== PACKAGES =====
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Z_1=X+Y, \quad
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% ===== 2. ONE RV TRANSFORMATION =====
\section{Transformation of One Random Variable:  $Y=g(X)$ }

```

`\subsection{Definitions and notation (as used on the slides)}`

A transformation is defined by

`\[`

$Y=g(X).$

`\]`

CDF notation:  $F_Y(y)$  and  $F_X(x)$ . `\\`

PDF notation:  $f_Y(y)$  and  $f_X(x)$ . `\\`

The slides proceed via `\textbf{Step S1 (CDF)}` then `\textbf{Step S2 (differentiate w.r.t.  $y$ )}`.

`\subsection{Assumption / condition: monotonicity}`

The slide distinguishes monotonic cases:

`\begin{itemize}[leftmargin=*`

`\item \textbf{Monotonically increasing} case`

`\item \textbf{Monotonically decreasing} case`

`\end{itemize}`

`\subsection{Case A:  $g(\cdot)$  is monotonically increasing}`

`\subsubsection*{Step S1 (CDF method)}`

`\[`

$F_Y(y)$

$= \Pr(Y \leq y)$

$= \Pr(g(X) \leq y)$

$= \Pr(\big(X \leq g^{-1}(y)\big)$

$= F_X(\big(g^{-1}(y)\big)).$

`\]`

`\subsubsection*{Step S2 (Differentiate w.r.t.  $y$ )}`

`\[`

$f_Y(y)$

$= \frac{d}{dy} \big[F_X(\big(g^{-1}(y)\big)\big]$

$= f_X(\big(g^{-1}(y)\big)) \cdot \frac{d}{dy} \big(g^{-1}(y)\big).$

`\]`

Let  $x=g^{-1}(y)$ :

`\[`

$f_Y(y) = f_X(x) \cdot \frac{dx}{dy} \big|_{x=g^{-1}(y)}.$

`\]`

`\subsection{Case B:  $g(\cdot)$  is monotonically decreasing}`

```

\subsubsection*{Step S1 (CDF method)}
\[
F_Y(y)
= \Pr(Y \leq y)
= \Pr(\big(X \geq g^{-1}(y)\big))
= 1 - F_X(\big(g^{-1}(y)\big)).
\]

\subsubsection*{Step S2 (Differentiate w.r.t. $y$)}
\[
f_Y(y)
= \frac{f_X(x)}{\left| \frac{dy}{dx} \right|} \Big|_{x=g^{-1}(y)}.
\]
\textbf{S3 note:} change the limits for $y$ (determine valid $y$-range
from the mapping).

% ===== 3. WORKED EXAMPLE =====
\section{Worked Example: $X \sim \mathrm{Uniform}(-1,1)$, \;}
$Y = \sin\left(\frac{\pi x}{2}\right)$

\subsection{Given (from the example slide)}
\begin{itemize}[leftmargin=*]
  \item $X$ is uniform on $(-1,1)$.
  \item Transformation:
  \[
  Y = g(x) = \sin\left(\frac{\pi x}{2}\right).
  \]
  \item PDF of $X$:
  \[
  f_X(x) =
  \begin{cases}
    \frac{1}{2}, & -1 < x < 1, \\
    0, & \text{otherwise}.
  \end{cases}
  \]
  \item Objective: find $f_Y(y)$.
\end{itemize}

\subsection{Step-by-step solution (as shown)}

\subsubsection*{Step 1: Invert the transformation}
\]
```



```
y=\sin\!\left(\frac{\pi x}{2}\right)
\quad\rightarrow\quad
x=\frac{2}{\pi}\sin^{-1}(y).
\]
```

```
\subsubsection*{Step 2: Differentiate  $x$  w.r.t.  $y$ }
```

```
\[
\frac{dx}{dy}=\frac{2}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}.
\]
```

```
\subsubsection*{Step 3: Apply the transformation formula}
```

```
\[
f_Y(y)=f_X(x)\cdot\left|\frac{dx}{dy}\right|.
\]
```

```
Substitute  $f_X(x)=\frac{1}{2}$ :
```

```
\[
f_Y(y)
=\frac{1}{2}\cdot\frac{2}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}
=\frac{1}{\pi\sqrt{1-y^2}}.
\]
```

```
\subsubsection*{Step 4: Determine the valid range of  $y$  from endpoints}
```

```
\begin{itemize}[leftmargin=*
```

```
\item At  $x=-1$ :  $y=\sin\!\left(-\frac{\pi}{2}\right)=-1$ 
```

```
\item At  $x=1$ :  $y=\sin\!\left(\frac{\pi}{2}\right)=1$ 
```

```
\end{itemize}
```

```
So the support is  $-1<y<1$  (otherwise  $0$ ).
```

```
\subsection{Final answer (as presented)}
```

```
\[
f_Y(y)=
\begin{cases}
\frac{1}{\pi\sqrt{1-y^2}}, & -1<y<1, \\[6pt]
0, & \text{otherwise.}
\end{cases}
\]
```

```
% ===== 4. TWO RV FUNCTION =====
```

```
\section{Function of Two Random Variables: Example Setup ( $Z=X+Y$ )}
```

```
\subsection{Problem statements listed on the slide}
```

```
For  $Z=X+Y$ , the slide lists:
```

```

\begin{enumerate}[leftmargin=*]
  \item Find the PDF of  $Z$ :  $f_Z(z)$ .
  \item Find  $f_Z(z)$  if  $X$  and  $Y$  are independent.
  \item Let  $X \sim N(0,1)$  and  $Y \sim N(0,1)$ ; prove that  $Z \sim N(0,2)$ .
  \item If  $X$  and  $Y$  are exponential RVs with parameter  $\lambda$ , find  $f_Z(z)$ .
\end{enumerate}

% ===== 5. CDF DERIVATION SETUP =====
\section{Detailed Derivation: CDF setup for  $Z=X+Y$  via Region Integration}

\subsection{Definition of  $Z$  and CDF start}
\[
Z=X+Y,
\quad
F_Z(z)=\Pr(Z \leq z)=\Pr(X+Y \leq z).
\]

\subsection{Region description and the “strip” integrals}
Using the joint pdf  $f_{XY}(x,y)$  and the boundary line  $x+y=z$ :

\subsubsection*(A) Horizontal strip (H-strip) form}
\[
F_Z(z)
=
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x,y) \, dx \, dy.
\]

\subsubsection*(B) Vertical strip form}
\[
F_Z(z)
=
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x,y) \, dy \, dx.
\]

\vspace{0.3cm}

```

```
\hrule  
\vspace{0.2cm}  
\noindent\textit{End of Lecture 11 scribe.}  
  
\end{document}
```

If you paste the **exact error message** (the red lines from Overleaf like “Undefined control sequence” + line number), I can point to the *exact* line causing it.