Two gas stations are located at a certain intersection. Each one has six gas pumps. Consider the experiment in which the number of pumps in use at a particular time of day is determined for each of the stations. An experimental outcome specifies how many pumps are in use at the first station and how many are in use at the second one.

One possible outcome is (2, 2), another is (4, 1), and yet another is (1, 4).

C

		Second Station						
		0	**1**	**2**	**3**	**4**	**5**	**6**
First Station	**0**	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 6)
	1	(1, 0)	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 0)	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 0)	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 0)	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 0)	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 0)	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

→ B

→ A

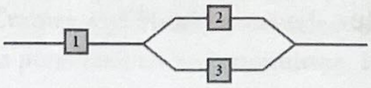
Compound events

A=the number of pumps in use is the same for both stations

B=the total number of pumps in use is four

C=at most one pump is in use at each station

Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2–3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions.



The experiment consists of determining the condition of each component [*S* (success) for a functioning component and *F* (failure) for a nonfunctioning component].

a. Which outcomes are contained in the event *A* that exactly two out of the three components function?

A = {(S,S,F),(S,F,S),(F,S,S)}

b. Which outcomes are contained in the event *B* that at least two of the components function?

B = {(S S F),(S F S),(F,S,S),(S,S,S)}

c. Which outcomes are contained in the event *C* that the system functions?

C = {(S,F,S)(S,S,F),(S,S,S)}

d. List outcomes in C', $A \cap C$, $A \cup C$, $C \cap B$, $C \cup B$

Beatles

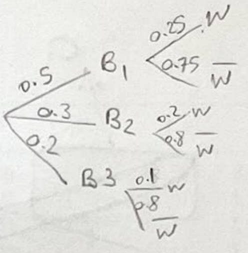
A particular iPod playlist contains 100 songs, 10 of which are by the Beatles. Suppose the shuffle feature is used to play the songs in random order.

What is the probability that the first Beatles song heard is the fifth song played? $\underset{90}{\_}\ \underset{89}{\_}\ \underset{88}{\_}\ \underset{87}{\_}\ \underset{10}{B}$ $\rightsquigarrow P(A) = \frac{\#A}{\#S} = \frac{90 \times 89 \times 88 \times 87 \times 10}{100 \times 99 \times 98 \times 97 \times 96}$

Example

? A chain of video stores sells three different brands of DVD players. Of its DVD player sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's DVD players require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

3



1. What is the probability that a randomly selected purchaser has bought a brand 1 DVD player that will need repair while under warranty?

2. What is the probability that a randomly selected purchaser has a DVD player that will need repair while under warranty?

3. If a customer returns to the store with a DVD player that needs warranty repair work, what is the probability that it is a brand 1 DVD player? A brand 2 DVD player? A brand 3 DVD player?

→ **dev8p77**

1) $P($تحتاج $W,$ اشترى $B_1) = 0.5 \times 0.25 = 0.125$

2) $P(W) = (0.5 \times 0.25) + (0.3 \times 0.2) + (0.2 \times 0.1)$
$= 0.205$

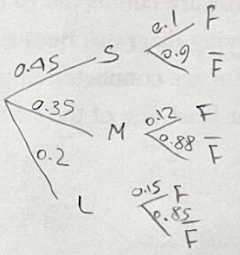
3) $P(B_1 | W) = \frac{P(W \cap B_1)}{P(B_1)} = \frac{0.125}{0.5}$

$P(b_2 | W) = \frac{0.2 \quad 0.3}{0.205} = -$

$P(b_3 | W) = \frac{0.2 \times 0.1}{0.205} = -$

Example

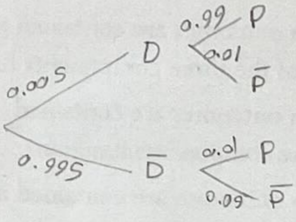
Customers who purchase a certain type of car can order an engine in any of three sizes. Of all the cars sold, 45% have the smallest engine, 35% have a medium-sized engine, and 20% have the largest. Of cars with smallest engines, 10% fail an emissions test within two years of purchase, while 12% of those with the medium size and 15% of those with the largest engine fail.



What is the probability that a randomly chosen car will fail an emissions test within two years?

$P(F) = (0.45 \times 0.1) + (0.35 \times 0.12) + (0.2 \times 0.12)$
$= 0.117$

The proportion of people in a given community who have a certain disease is 0.005. A test is available to diagnose the disease. If a person has the disease, the probability that the test will produce a positive signal is 0.99. If a person does not have the disease, the probability that the test will produce a positive signal is 0.01. If a person tests positive, what is the probability that the person actually has the disease?



$P(D | P) = \frac{P(D \cap P)}{P(P)} = \frac{(0.005 \times 0.99)}{(0.995 \times 0.01)(0.005 \times 0.99)}$

$= 0.332$

Ex. 23 pag. 64

The computers of six faculty members in a certain department are to be replaced. Two (2 L) of the faculty members have selected laptop machines and the other four (4 D) have chosen desktop machines. Suppose that only two of the setups can be done on a particular day, and the two computers to be set up are randomly selected from the six (implying 15 equally likely outcomes; if the computers are numbered 1, 2, . . . , 6, then one outcome consists of computers 1 and 2, another consists of computers 1 and 3, and so on).

a. What is the probability that both selected setups are for laptop computers?

b. What is the probability that both selected setups are desktop machines?

c. What is the probability that at least one selected setup is for a desktop computer?

d. What is the probability that at least one computer of each type is chosen for setup?

$$P(a) = \frac{\binom{2}{2}}{\binom{6}{2}}$$

$$P(b) = \frac{\binom{4}{2}}{\binom{6}{2}}$$

$$P(c) = \frac{\binom{4}{1}\binom{2}{1} + \binom{4}{2}}{\binom{6}{2}}$$

$$P(d) = \frac{\binom{4}{1}\binom{2}{1}}{\binom{6}{2}}$$

Ex. 27 pag. 64

An academic department with five (5) faculty members (Anderson, Box, Cox, Cramer, and Fisher) must select two (2) of its members to serve on a personnel review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representative will be selected by putting the names on identical pieces of paper and then randomly selecting two.

a. What is the probability that both Anderson and Box will be selected? [*Hint:* List the equally likely outcomes.]

$$P(a) = \frac{\binom{2}{2}}{\binom{5}{2}} = 0.1$$

b. What is the probability that at least one of the two members whose name begins with *C* is selected?

$$P(b) = \frac{\binom{2}{1}\binom{3}{1} + \binom{2}{2}}{\binom{5}{2}}$$

c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have a total of at least 15 years' teaching experience there?

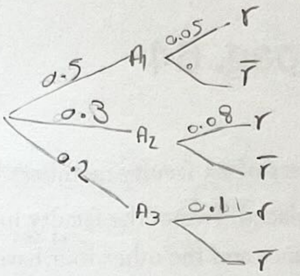
$$P(c) = \frac{6}{\binom{5}{2}} = 6/10$$

[illegible]

14, 3
14, 6
14, 7
14, 10
10, 7
10, 6

Ex. 104 pag. 90

A company uses three different assembly lines—A_1, A_2, and A_3—to manufacture a particular component. Of those manufactured by line A_1, 5% need rework to remedy a defect, whereas 8% of A_2's components need rework and 10% of A_3's need rework. Suppose that 50% of all components are produced by line A_1, 30% are produced by line A_2, and 20% come from line A_3. If a randomly selected component needs rework, what is the probability that it came from line A_1? From line A_2? From line A_3?



$$P(A_1|r) = \frac{P(A_1 \cap r)}{P r} = \frac{0.5 \times 0.05}{(0.2\times0.1)+(0.08\times0.3)+(0.05\times0.5)}$$

Ex. 109 pag. 91

Four engineers, A, B, C, and D, have been scheduled for job interviews at 10 A.M. on Friday, January 13, at Random Sampling. The personnel manager has scheduled the four for interview rooms 1, 2, 3, and 4, respectively.

However, the manager's secretary does not know this, so assigns them to the four rooms in a completely random fashion (what else!). What is the probability that

a. All four end up in the correct rooms? $\frac{1}{4!}$

b. None of the four ends up in the correct room? **→ umbrella** $b = \frac{9}{4!} = \frac{9}{24}$

چرا ۹؟ ← B چند حالت دارم که هر کدام جای درست نباشند؟ ۱ حالت.

۳ نفر ـ ـ ـ ؟ حالت جدیدی در نمی‌آید.

۲ نفر ؟ 2 حالت (دو نفر سر جای خودشون اند، ۲ تای دیگر جای غلط اول بوده، دومی هم جا غلط بوده یا برعکس ← ۲ حالت جدید کنن)

۱ نفر ؟ 3x2

که این سه حالت خودش هست، چون تعدادی ۳ انتخاب داره و بعد ۲ انتخاب
می‌شه 6 حالت جدید.

Exercise 13 p. 105

6

A mail-order computer business has six telephone lines. Let X denote the number of lines in use at a specified time. Suppose the pmf of X is as given in the accompanying table.

x	0	1	2	3	4	5	6
p(x)	.10	.15	.20	.25	.20	.06	.04

Calculate the probability of each of the following events.

a. {at most three lines are in use} ٠،١،٢،٣ → $P(a) = 0.25 + 0.2 + 0.15 + 0.1 = 0.7$

b. {fewer than three lines are in use} $P(b) = 0.25 + 0.2 + 0.15 = 0.45$

c. {at least three lines are in use} $P(c) = 0.25 + 0.2 + 0.06 + 0.04 = 0.55$

d. {between two and five lines, inclusive, are in use} $P(d) = 0.2 + 0.25 + 0.2 + 0.06 =$

؟ e. {between two and four lines, inclusive, are not in use} $\overline{P(e)} = 1 - (0.2 + 0.25 + 0.2)$

؟ f. {at least four lines are not in use}

١،٢،٣،٤

Ex. 18 p. 105

pmf [illegible] X

Two fair six-sided dice are tossed independently. Let M the maximum value of the two tosses.

a. What is the pmf of *M*? [*Hint:* First determine $p(1)$, then $p(2)$, and so on.]

b. Determine the cdf of *M* and graph it.

X	1	2	3	4	5	6
P(X)	$1/36$	$3/36$	$5/36$	$7/36$	$9/36$	$11/36$
F(x)	$1/36$	$4/36$	$9/36$	$16/36$	$25/36$	$36/36$

Ex. 17 pag. 105

A new battery's voltage may be acceptable (*A*) or unacceptable (*B*). A certain flashlight requires two batteries, so batteries will be independently selected and tested until two acceptable ones have been found. Suppose that 90% of all batteries have acceptable voltages. Let *Y* denote the number of batteries that must be tested.

a. What is $p(2)$, that is, $P(Y=2)$?

b. What is $p(3)$?

c. To have $Y=5$, what must be true of the fifth battery selected? List the four outcomes for which $Y=5$ and then determine $p(5)$.

d. Use the pattern in your answers for parts (a)–(c) to obtain a general formula for $p(y)$.

a) $P(Y=2) = P_S \times P_S = 0.9 \times 0.9$

b) $P(Y=3) = [(0.9)(0.1) \times 2] \times 0.9$

c)
F F F S S
F F S F S
F S F F S
S F F F

$\Rightarrow P(Y=5) = ((0.1)^3 (0.9) \times 4)(0.9)$

d) $P(y=Y) = (P_S)^{Y-1} (P_F)^{1-(Y-1)} \; (Y-1)$

Example

[illegible]

3

A computer store has purchased three computers of a certain type at \$500 apiece. It will sell them for \$1000 apiece. The manufacturer has agreed to repurchase any computers still unsold after a specified period at \$200 apiece.

Let X denote the number of computers sold, and suppose that $p(0)=0.1$, $p(1)=0.2$, $p(2)=0.3$, $p(3)=0.4$

With $h(X)$ denoting the profit associated with selling X units, the given information implies that

$h(X) = \text{revenue} - \text{cost} = 1000X + 200(3 - X) - 1500 = 800X - 900.$

Compute the expected profits.

$E(h(x)) = ?$

$E(h(x)) = \sum h(x).p(x) = -900 \times 0.1 - 100 \times 0.2 + 700 \times 0.3 + 1500 \times 0.4 = 700$

Exercise

A system is composed by four different components. The system works if at least two components work.

Let X be the number of properly functioning components. The pmf of X is:

$$p(x) = \begin{cases} cx & x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

- Find the value of c so that p(x) is a probability mass function. → $1c + 2c + 3c + 4c = 1 \rightsquigarrow c = 1/10$
- Find the probability that exactly $\overset{x=2}{2}$ components work. → $P(X=2) = \frac{1}{10}\, 2 = 0.2$
- Find the mean number of properly functioning components. → $E(x) = \sum^{4} x\, p(x) =$
- Find the variance and the standard deviation.

$= 1/10 + 4/10 + 9/10 + 16/10 =$

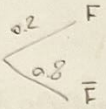
x	1	2	3	4
p(x)	1/10	2/10	3/10	4/10
$E(x^2) = x^2 p(x)$	1/10	8/10	27/10	$\overset{64}{\cancel{16\times 4}}$/10

$\boxed{E(x) = 30/10}$

$\boxed{E(x^2) = 10}$

$V(x) = E(x^2) - [E(x)]^2$

$V(x) = 10 - 9 = 1 \rightsquigarrow \boxed{\sigma = 1}$ ✓



Example

* به نحوه حل سوالات inclusive ها در [illegible] توزیع ها بیشتر دقت کن.

Suppose that 20% of all copies of a particular textbook fail a certain binding strength test. Let *X* denote the number among 15 randomly selected copies that fail the test.

Then *X* has a binomial distribution with n=15 and p=0.2

1. The probability that at most 8 fail the test is… $P(X \leq 8) = \binom{15}{8}(0.2)^8(0.8)^7$
2. The probability that exactly 8 fail is… $P(X=8) = F(8) - F(7) =$
3. The probability that at least 8 fail is… $P(X \geq 8) = 1 - F(7)$
4. The probability that between 4 and 7, inclusive, fail is…

$\hookrightarrow P(4 \leq X \leq 7) = F(7) - F(3)$

→ cdf table

Ex. 49 p. 120

A company that produces fine crystal knows from experience that 10% of its goblets have cosmetic flaws and must be classified as "seconds."

a. Among six randomly selected goblets, how likely is it that only one is a second?

$P(X=1) =$

a) $\binom{6}{1}(0.1)^1(0.9)^5 = 0.531$

b. Among six randomly selected goblets, what is the probability that at least two are seconds?

b) $P(X \geq 2) = 1 - F(1) =$

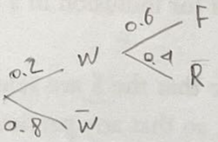
$P(X=0) + P(X=1)$

c. If goblets are examined one by one, what is the probability that at most five must be selected to find four that are not seconds?

[illegible]

c) $\binom{5}{1}(0.1)^1(0.9)^4$

Ex. 55 p. 121



%20

Twenty percent of all telephones of a certain type are submitted for service while under warranty. Of these, 60% can be repaired, whereas the other 40% must be replaced with new units. If a company purchases ten of these telephones, what is the probability that exactly two will end up being replaced under warranty?

n=10

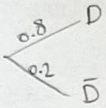
②

$P(\text{"replace} = 2\text{"}) = \binom{10}{2}(0.08)^2(0.92)^8$

$P(\text{replace}) = 0.4 \times 0.2 = 0.08$

α

Ex. 59, p. 121



An ordinance requiring that a smoke detector be installed in all previously constructed houses has been in effect in a particular city for 1 year. The fire department is concerned that many houses remain without detectors. Let p=the true proportion of such houses having detectors, and suppose that a random sample of 25 homes ($n=$) is inspected. If the sample strongly indicates that fewer than 80% of all houses have a detector, the fire department will campaign for a mandatory inspection program. Because of the costliness of the program, the department prefers not to call for such inspections unless sample evidence strongly argues for their necessity. Let *X* denote the number of homes with detectors among the 25 sampled. Consider rejecting the claim that p≥0,8 if *X*≤15.

a. What is the probability that the claim is rejected when the actual value of *p* is 0.8?

b. What is the probability of not rejecting the claim when *p* =0.7. When *p*=0.6?

c. How do the "error probabilities" of parts (a) and (b) change if the value 15 in the decision rule is replaced by 14?

Example 35

$$P(X=2)=\frac{\binom{12}{2}\binom{8}{3}}{\binom{20}{5}}$$

- During a particular period a university's information technology office received 20 service orders for problems with printers, of which 8 were laser printers and 12 were inkjet models. A sample of 5 of these service orders is to be selected for inclusion in a customer satisfaction survey. $n=5$

- Suppose that the 5 are selected in a completely random fashion, so that any particular subset of size 5 has the same chance of being selected as does any other subset. What then is the probability that exactly *x* (*x* = 0, 1, 2, 3, 4, or 5) of the selected service orders were for inkjet printers? $X=٢$ [illegible]

X Example 37

- Five individuals from an animal population thought to be near extinction in a certain region have been caught, tagged, and released to mix into the population. After they have had an opportunity to mix, a random sample of 10 of these animals is selected/ Let x = the number of tagged animals in the second sample.

$n=10$
$M=5$
$N=25$

$$p=\frac{M}{N}=\frac{5}{25}=0.2$$

$$E(x)=\overset{(x)}{10}(\overset{p(x)}{0.2})$$

$$V(x)=\frac{15}{24}(10)(0.2)(0.8)=1$$

- If there are actually 25 animals of this type in the region, what is the *E*(*X*) and *V*(*X*)?

$$V(x)=\frac{N-n}{N-1}$$

$$\frac{x}{n}\approx\frac{M}{N}$$

hypergeometric

$$E(x)=\sum x\,p(x)$$

$$V(x)=E(x^2)-[E[x]]^2$$

Example 38

- A pediatrician wishes to recruit 5 couples, each of whom is expecting their first child, to participate in a new natural childbirth regimen. Let $p = P$(a randomly selected couple agrees to participate).

- If $p = .2$, what is the probability that 15 couples must be asked before 5 are found who agree to participate? That is, with $S = \{$agrees to participate$\}$, what is the probability that 10 F's occur before the fifth S?

n (۱۳)

یادت باشه آخری چون ثابته، احتمالش حساب نکنیم.

$$\binom{14}{4}(0.2)^4(0.8)^{10}\cancel{(0.2)}$$

Example 39

- Let X denote the number of creatures of a particular type captured in a trap during a given time period.

- Suppose that X has a Poisson distribution with $\mu = 4.5$, so on average traps will contain 4.5 creatures.

- The probability that a trap contains exactly five creatures is

$$P(X = 5) = \frac{e^{-4.5}(4.5)^5}{5!} = .1708$$

چون $P(x,\mu) = \frac{e^{-\mu} \cdot \mu^x}{x!}$

Example 40

از کجا فهمید توزیعش پواسون هست؟

- If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is .005 and errors are independent from page to page, what is the probability that one of its $n =$ 400-page novels will contain exactly one page with errors? At most three pages with errors?

- With S denoting a page containing at least one error and F an error-free page, the number X of pages containing at least one error is a binomial rv with $n = 400$ and $p = .005$, so $np = 2$.

در صفحه

$P(X \geq 1) = 0.005$

پواسون

$$P(X=1) = b(\underset{x}{1}; \underset{n}{400}, \underset{p}{0.005}) \approx \hat{P}(\underset{x}{1}, \underset{\mu = np}{2})$$

$$= \frac{e^{-2}(2)^1}{1!} = 0.270671$$

$$P(X \leq 3) = \sum_{x=0}^{3} p(x,2) = \sum_{x=0}^{3} e^{-2} \times \frac{2^x}{x!} = 0.8571$$

posson process

Example 42

60 sec اتفاق ۶ پالس در
هست
30 sec ۳ پالس در
μ=3 میشه

- Suppose pulses arrive at a counter at an average rate of six per minute, so that $\alpha = 6$.
- To find the probability that in a 0.5-min interval at least one pulse is received, note that the number of pulses in such an interval has a Poisson distribution with parameter $\boxed{\alpha t = 6(.5) = 3}$ (.5 min is used because α is expressed as a rate per minute).
- Then with X = the number of pulses received in the 30-sec interval,

$$P(1 \le X) = 1 - P(X = 0) = 1 - \frac{e^{-3}(3)^0}{0!} = .950$$

30 sec

توزیع => $e^{-\mu} \cdot \mu^{x}/x! = e^{-3} \times 3^{x}/x!$

$P(X \geq 1) = 1 - P(X=0) = 1 - \left(e^{-3} \times \boxed{3^{0}/0!}\right) = 0.95$

• از کجا بفهمیم در این تایپ از سوالات، باید از پواسون استفاده کنیم؟
به واقع passon، تعداد رخدادها در یک بازهٔ زمانی
مشخص رو model می‌کنه. در این تیپ سوالات هم
با "بازهٔ زمانی ثابت"، "رخدادهای مستقل از هم در
این بازهٔ زمانی" سروکار داریم.
پواسون فقط یک پارامتر λ یا μ داره که میانگین تعداد رخدادها رو نشون میده.

Ex. 85 p. 135

Suppose small aircraft arrive at a certain airport according to a Poisson process with rate α=8 per hour, so that the number of arrivals during a time period of *t* hours is a Poisson rv with parameter μ=8 *t*.

a. What is the probability that exactly 6 small aircraft arrive during a 1-hour period? At least 6? At least 10? $\mu = 8$

b. What are the expected value and standard deviation of the number of small aircraft that arrive during a 90-min period? $\mu = 8 \times 1.5 = 12$

c. What is the probability that at least 20 small aircraft arrive during a 2.5-hour period? That at most 10 arrive during this period?

$\mu = 8 \times 2.5 = 20$

a) $P(X=6) = e^{-8}\, 8^{6}/6! \simeq 0.103$

$P(X \geq 6) = 1 - P(X \leq 5) = 1 - \sum_{x=0}^{5} e^{-8}\, 8^{x}/x! \simeq 0.377$

at least) $P(X \leq 10) = 1 - P(X \leq 9) = 1 - \sum_{x=0}^{9} e^{-8}\, 8^{x}/x! \simeq 0.022$

b) $E(X) = \mu = 12$

$V(X) = E(X) = 12 \rightsquigarrow \sigma = \sqrt{12}$

c) $P(X \geq 20) = 1 - P(X \leq 19)$

$= 1 - \sum_{x=0}^{19} e^{-20}\, 20^{x}/x!$

Ex. 97 p. 133

Of all customers purchasing automatic garage-door openers, 75% purchase a chain-driven model. Let X = the number among the next 15 purchasers who select the chain-driven model.

0.75 ch
$\overline{ch}$

a. What is the pmf of *X*? $\binom{15}{x}(0.75)^{x}(0.25)^{1-x}$

b. Compute P(X>10) $= 1 - P(X \leq 9) = 1 - \sum_{x=0}^{9} \binom{15}{x}(0.75)^{x}(0.25)^{1-x}$

c. Compute P(6≤X ≤ 10) $= P(X=10) - P(X=5) = \sum_{x=5}^{10} \binom{15}{x}(0.75)^{x}(0.25)^{1-x}$

d. Compute μ and σ².

e. If the store currently has in stock 10 chain-driven models and 8 shaft-driven models, what is the probability that the requests of these 15 customers can all be met from existing stock?

z_α Notation for z Critical Values

Table 4.1 lists the most useful z percentiles and z_α values.

Percentile	90	95	97.5	99	99.5	99.9	99.95
α (tail area)	.1	.05	.025	.01	.005	.001	.0005
$z_\alpha = 100(1-\alpha)$th percentile	1.28	1.645	1.96	2.33	2.58	3.08	3.27

Standard Normal Percentiles and Critical Values

If the population distribution of a variable is (approximately) normal, then

1. Roughly 68% of the values are within 1 SD of the mean.
2. Roughly 95% of the values are within 2 SDs of the mean.
3. Roughly 99.7% of the values are within 3 SDs of the mean.

Ex 12 p16 Ross9

- Five separate awards (best scholarship, best leadership qualities, and so on) are to be presented to selected students from a class of 30. How many different outcomes are possible if

(a) a student can receive any number of awards? →

(b) each student can receive at most 1 award?

a) 5^{30}

b) $\underline{30}\ \underline{29}\ \underline{28}\ \underline{27}\ \underline{26}$

Ⓞ or $\binom{30}{5}5! = \frac{30\times29\times28\times27\times26\times\cancel{25!}}{\cancel{25!}\ \cancel{5!}}\times\cancel{5!}$

Ex 19 p16 Ross9

- From a group of $\boxed{8 \text{ women}}$ and $\boxed{6 \text{ men}}$, a committee consisting of 3 men and 3 women is to be formed.
- How many different committees are possible if
  - (a) 2 of the men refuse to serve together?
  - (b) 2 of the women refuse to serve together?
  - (c) 1 man and 1 woman refuse to serve together?

[illegible]

a) $\binom{4}{3}\binom{8}{3}$

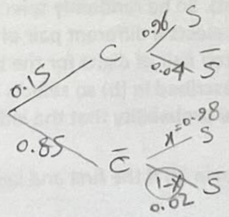
b) $\binom{6}{3}\binom{6}{3}$

c) $\binom{5}{3}\binom{7}{3}$

%98

Ninety-eight percent of all babies survive delivery. However, 15 percent of all births involve Cesarean (C) sections, and when a C section is performed, the baby survives 96 percent of the time.

If a randomly chosen pregnant woman does not have a C section, what is the probability that her baby survives? $P(S|\bar{C}) = x$



$P(S) = \cancel{0.96(0.15) + x(0.85) = 0.98} \rightarrow 0.836$

$x = 0.98$

$E_x\ 35 \Rightarrow P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) - P(A_1 \cap A_2 \cap A_3)$

$= (177100 + 593775 + 1376998 - 210 - 5005 - 38760 - 0) / 8145060$

EX 35 p.71 Dev8

A production facility employs 20 workers on the day shift, 15 workers on the swing shift, and 10 workers on the graveyard shift. A quality control consultant is to select 6 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 45).

20 dayshift
15 swingshift
10 graveyard

دقت کن که احتمال نخواسته
تعداد خواسته
این جا احتمال خواسته

A. How many selections result in all 6 workers coming from the day shift? What is the probability that all 6 selected workers will be from the day shift?

$\binom{20}{6}$, $P(a) = \dfrac{\binom{20}{6}}{\binom{45}{6}}$

B. What is the probability that all 6 selected workers will be from the same shift?

$P(b) = \dfrac{\binom{20}{6} + \binom{15}{6} + \binom{10}{6}}{\binom{45}{6}}$

حداقل ۲ شیفت یعنی یا ۲ یا ۳ شیفت

C. What is the probability that at least two different shifts will be represented among the selected workers?

$P(c) = 1 - P(b)$

D. What is the probability that at least one of the shifts will be unrepresented in the sample of workers?

متمم D) ← هر سه حضور داشته باشند، P("حداقل یک شیفت غایب") رو بسط بیاریم.

سه ترم: P("شیفت ۱ غایب")، P("شیفت ۲ غایب")، P("شیفت ۳ غایب") رو بسط بیاریم، از شمول اجتماع بنویسیم!

A_1, A_2, A_3

$N(A_1) = \binom{15+10}{6} = 177100$

$N(A_2) = \binom{20+10}{6} = 593775$ — فقط یک شیفت: $N(A_1 \cap A_2) = \binom{10}{6} = 210$

$N(A_3) = \binom{20+15}{6} = 1376998$ — فقط شیفت: $N(A_1 \cap A_3) = \binom{15}{6} = 5005$

$N(\text{total}) = \binom{45}{6} = 8145060$ — $N(A_2 \cap A_3) = \binom{20}{6} = 38760$

Ex 41 p.72 Dev8

An ATM personal identification number (PIN) consists of four digits, each a 0, 1, 2, . . . 8, or 9, in succession.

10

a. How many different possible PINs are there if there are no restrictions on the choice of digits? 10^4

$\underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 10^4$

b. According to a representative at the author's local branch of Chase Bank, there are in fact restrictions on the choice of digits. The following choices are prohibited: (i) all four digits identical (ii) sequences of consecutive ascending or descending digits, such as 6543 (iii) any sequence starting with 19 (birth years are too easy to guess). So if one of the PINs in (a) is randomly selected, what is the probability that it will be a legitimate PIN (that is, not be one of the prohibited sequences)?

صعودی - نزولی

b) i: 0000, … → 10 حالت

ii: Asc: (0123), (1234), (2,3,4,5), (3,4,5,6), (4,5,6,7), (5,6,7,8), (6,7,8,9)

Desc: (9,8,7,6), (8,7,6,5), (7654), (6543), (5432), (4321), (321,0)

14 حالت

iii: 1×1×10×10 — 100 حالت

c. Someone has stolen an ATM card and knows that the first and last digits of the PIN are 8 and 1, respectively. He has three tries before the card is retained by the ATM (but does not realize that). So he randomly selects the 2nd and 3rd digits for the first try, then randomly selects a different pair of digits for the second try, and yet another randomly selected pair of digits for the third try (the individual knows about the restrictions described in (b) so selects only from the legitimate possibilities). What is the probability that the individual gains access to the account?

d. Recalculate the probability in (c) if the first and last digits are 1 and 1, respectively.

$P(b) = \dfrac{10^4 - 124}{10^4}$

c) $\dfrac{1}{⑧}\ \dfrac{10}{}\ \dfrac{10}{}\ \dfrac{1}{①} \Rightarrow P(c) = 3 \times \dfrac{1}{100}$

تعداد بارهایی که شانس داره — میزان شانس هر بار

حالت
۱۱ حالت از ۱۰۰
حذف میشه
شانس هر بار از ۸۹
$1/89$ خودمون بیاد

d) $\dfrac{1}{}\ \dfrac{8}{1}\ \dfrac{2}{2}\ \dfrac{1}{}$ ① 1 1 1 → ۱ حالت

1 9 ≠ 1 → ۱۰ حالت

$P(d) = 3 \times \dfrac{1}{89} = 0.0337$

Ex 45 p.80 Dev8

The population of a particular country consists of three ethnic groups. Each individual belongs to one of the four major blood groups. The accompanying joint probability table gives the proportions of individuals in the various ethnic group–blood group combinations.

		Blood Group			
		O	A	B	AB
Ethnic Group	1	.082	.106	.008	.004
	2	.135	.141	.018	.006
	3	.215	.200	.065	.020

Ex 45 p.80 Dev8 cont'd

Suppose that an individual is randomly selected from the population, and define events by A="type A selected", B="type B selected" and C="ethnic group 3 selected".

$P(A) =$

a. Calculate *P(A), P(C) and P(A∩C)*. $= \emptyset$

b. Calculate both P(A|C) and P(C|A), and explain in context what each of these probabilities represents.

$$\rightarrow \begin{cases} P(A|C) = \frac{P(A\cap C)}{P(C)} = \frac{0.2}{0.5} = 0.4 \\ P(C|A) = \frac{P(A\cap C)}{P(A)} = \frac{0.2}{0.447} = - \end{cases}$$

c. If the selected individual does not have type B blood, what is the probability that he or she is from ethnic group 1?

$$\longrightarrow P(E_1|\bar{B}) = \frac{P(E_1\cap\bar{B})}{P(\bar{B})} = \frac{0.192}{0.909} = 0.2112$$

		Blood Group			
		O	A	B	AB
Ethnic Group	1	.082	.106	.008	.004
	2	.135	.141	.018	.006
	3	.215	.200	.065	.020

$\rightarrow P(B) = 0.081$

$\rightarrow P(C) = 0.5$

$P(A) = 0.447$

$P(A\cap C)$

If *A and B are independent events, show that A' and B are* also independent.

$A \perp\!\!\!\perp B \Rightarrow P(A\cap B) = P(A)P(B) \longrightarrow$ [illegible] A, B [illegible]

$P(A\cup B) = P(A\cup(\bar{A}\cap B)) = \boxed{P(A) + P(\bar{A}\cap B)}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B) = \boxed{P(A) + P(B) - P(A)P(B)}$

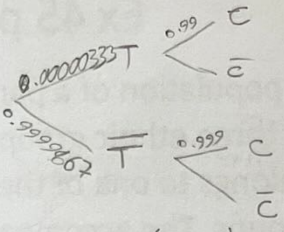
$\underline{P(\bar{A}\cap B)} = P(B)(1-P(A)) = \underline{\underline{P(B)P(\bar{A})}}$ *

$P(T) = \frac{1000}{300\,000\,000} = 0.00000333$

Ex 67 p.82 Dev8

There has been a great deal of controversy over the last several years regarding what types of surveillance are appropriate to prevent terrorism. Suppose a particular surveillance system has a 99% chance of correctly identifying a future terrorist and a 99.9% chance of correctly identifying someone who is not a future terrorist. If there are 1000 future terrorists in a population of 300 million, and one of these 300 million is randomly selected, scrutinized by the system, and identified as a future terrorist, what is the probability that he/she actually is a future terrorist? Does the value of this probability make you uneasy about using the surveillance system? Explain.

$P(T|C)$ ←



$P(T|C) = \frac{P(T\cap C)}{P(C)} = \frac{0.0000033}{(0.0000033)(0.99) + 1 -} \text{ —}$

$P(C) = 0.99896673 + \text{ —}$

- Let us consider the average number of washings which a washing machine performs before breaking down for the first time as an indicator of the quality of the washing machine. Let *X be the number of* washings before the first breaking. Let *T=X+1 the washing in which the washing machine breaks down* for the first time. Let *E(X)=μ=400 be the mean time between failures (MTBF).*
- Suppose that the washing machine is not affected by wear and that it is maintained in the same conditions of the first day thanks to a regular maintenance.
- Suppose also that the warranty period is 5 year and that 200 washings are done in one year for a total of 1000 washings during the warranty period.
- Determine the probability of calling (at least one time) the technician during the warranty period?

? Ex 36 p.72 Dev8

- An academic department with five faculty members narrowed its choice for department head to either candidate A or candidate *B. Each member then voted on a slip of paper* for one of the candidates. Suppose there are actually three votes for *A and two for B. If the slips are selected for tallying* in random order, what is the probability that *A remains* ahead of *B throughout the vote count (e.g., this event occurs* if the selected ordering is *AABAB, but not for ABBAA)?*

$3 \to A$
$2 \to B$

کل حالت‌ها : $\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 2^5$

حالت‌هایی که A جلو : _ _ _ _ _

A A A B B
A A B A B
A B A B B

$P = \frac{3}{2^5}$

4 40-W
5 60-W
6 75-W

Ex 38 p.72 Dev8

A box in a certain supply room contains four 40-W lightbulbs, five 60-W bulbs, and six 75-W bulbs. Suppose that three bulbs are randomly selected.

$$p(a) = \frac{\binom{6}{2}\binom{5}{1} + \binom{6}{2}\binom{4}{1}}{\binom{15}{3}}$$

a. What is the probability that exactly two of the selected bulbs are rated 75-W?

b. What is the probability that all three of the selected bulbs have the same rating?

$$P(b) = \frac{\binom{4}{3} + \binom{5}{3} + \binom{6}{3}}{\binom{15}{3}}$$

c. What is the probability that one bulb of each type is selected?

$$P(c) = \frac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}}$$

d. Suppose now that bulbs are to be selected one by one until a 75-W bulb is found. What is the probability that it is necessary to examine at least six bulbs?

$n=6$

احتمال قسمت d برابر با احتمال انتخاب ۵ تا غیر 75-W:
در 5 انتخاب اول هستن :

$$P(d) = \frac{\binom{9}{5}}{\binom{15}{5}}$$

Ex 55 p.81 Dev 8

✩ Deer ticks can be carriers of either Lyme disease or human granulocytic ehrlichiosis (HGE). Based on a recent study, suppose that 16% of all ticks in a certain location carry Lyme disease, 10% carry HGE, and 10% of the ticks that carry at least one of these diseases in fact carry both of them.

$P(A : Lyme) = 0.16$
$P(B : HGE) = 0.1$
$P(A \cap B \mid A \cup B) = 0.1$

→ we have this from the fact that A∩B is contained in A∪B

$$P(A \mid B) = \frac{P(A \cap B)^{*}}{P(B)} = \frac{0.023}{0.1}$$

If a randomly selected tick is found to have carried HGE, what is the probability that the selected tick is also a carrier of Lyme disease?

$$* P(A \cap B) \Rightarrow \frac{P(A \cap B \cap A \cup B) = P(A \cap B)}{P(A \cup B) = \underbrace{\underbrace{P(A)}_{0.1} + \underbrace{P(B)}_{0.16}}_{0.26} - P(A \cap B)} = 0.1 \Rightarrow 0.1 \times 0.26 - 0.1 P(A \cap B) = P(A \cap B)$$

$$P(A \cap B) = 0.623$$

Ex 65 p.122 Dev8

Customers at a gas station pay with a credit card (A), debit card (B), or cash (C). Assume that successive customers make independent choices, with P(A)=.5, P(B)=.2 and P(C)=.3.
credit debit cash

Binomial

چرا این توزیع، Binomial هست؟
n=number of trial ثابت باشه + هر trial
به ۲ حالت (F,S) دیتای ما که ۳ برچسب تعلق داره
تنها اگر debit card = S در غیر این صورت F.
+ trial ها مستقل از هم هستند +
هر [illegible] کنترل روی احتمال موفقیت ثابت خودش رو داره
constant P(A)=0.5 مثلا

a. Among the next 100 customers, what are the mean and variance of the number who pay with a debit card? Explain your reasoning.

n

b. Answer part (a) for the number among the 100 who don't pay with cash. →

a) $E(x) = np \rightsquigarrow 100 \times 0.2 = 20$
$V(x) = npq \rightsquigarrow 100 \times 0.2 \times 0.8 = 16 \rightsquigarrow \sigma = 4$

b) $np = 0.7 \times 100 = 70$
$npq = 100 \times 0.7 \times 0.3 = 21$

Similar to Ex 75 p.128 Dev8

(?)

3

- Three brothers and their wives decide to have children until each family has two female children.
  - What is the pmf of X= the total number of male children born to the brothers?
  - What is *E(X)*?

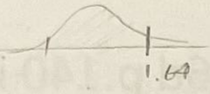
$P(\text{children} = F) = 0.5$

r = number of success

X = number of baby boys

$$E(X) = \frac{2(0.5)}{0.5} = 2$$

$$pmf_X = (1-0.5)^X (0.5)^2 \binom{X+2-1}{r}$$



Ex 43 p.163 Dev 8

$X \sim N(\mu, \sigma^2)$

- The distribution of resistance for resistors of a certain type is known to be normal, with 10% of all resistors having a resistance exceeding 10.256 ohms and 5% having a resistance smaller than 9.671 ohms.
- What are the mean value and standard deviation of the resistance distribution?

μ σ

$P(X \geq 10.256) = 0.1 \rightarrow P\left(Z \geq \frac{10.256-\mu}{\sigma}\right) = 0.1$

$P(X \leq 9.671) = 0.05 \rightarrow P\left(Z \leq \frac{9.671-\mu}{\sigma}\right) = 0.05$

$$\begin{cases} 10.256 - \mu = 1.28\sigma \\ 9.671 - \mu = -1.64\sigma \end{cases} \Rightarrow \begin{cases} \mu = 19.927 \\ \sigma = 0.228 \end{cases}$$ ✓

0.685

Ex 47 p164 Dev8

$N(12, (3.5)^2)$

- The weight distribution of parcels sent in a certain manner is normal with mean value 12 μ lb and standard deviation 3.5 lb. The parcel service wishes to establish a weight value c beyond which there will be a surcharge.

بار

اضافه شارژ!

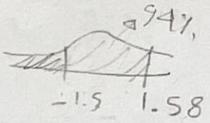
- What value of c is such that 99% of all parcels are at least 1 lb under the surcharge weight?

چون 99 درصد بار ها اضافه شارژ ندارند Z=2.33 است.

یعنی احتمال c-1 زیر میانگین 99٪ است.

$P(c-1 < 12) = 0.99$

$\frac{c-1-\mu}{3.5} = 2.33 \Longrightarrow c = 21.155$ ($\mu \to 12$)



Ex 55 p.164 Dev 8

- Suppose only 75% of all drivers in a certain state regularly wear a seat belt. A random sample of 500 drivers is selected.
- What is the probability that
  - a) – Between 360 and 400 (inclusive) of the drivers in the sample regularly wear a seat belt?
  - b) – Fewer than 400 of those in the sample regularly wear a seat belt?

a) $P(360 \leq X \leq 400) =$

$P\left(\frac{360-375}{9.68} \leq Z < \frac{400-375}{9.68}\right) =$

$= P(-1.6 \leq Z \leq 2.58) = \varphi(2.5) - \varphi(-1.58)$

$= 0.99 - 0.06 = 0.93$

b) $P(X < 400) = P(Z < 2.6) = 0.99$

for this one we should use the normal approximate to binomial!

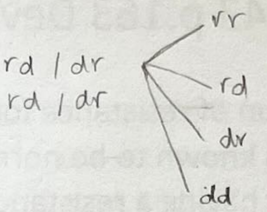
$bin(500, 75) \simeq N(375, 9.68)$

$p = 0.75$

$\mu = np = 500 \times 0.75 = 375$

$V(X) = npq = 500 \times 0.75 \times 0.25 = 93.75 \rightarrow \sigma = 9.68$

Ex 6d p.140 Ross9



- Suppose that a particular trait (such as eye color or left-handedness) of a person is classified on the basis of one pair of genes, and suppose also that d represents a dominant gene and r a recessive gene.
- Thus, a person with dd genes is purely dominant, one with rr is purely recessive, and one with rd is hybrid. The purely dominant and the hybrid individuals are alike in appearance. Children receive 1 gene from each parent.
- If, with respect to a particular trait, 2 hybrid parents have a total of 4 children, what is the probability that 3 of the 4 children have the outward appearance of the dominant gene?

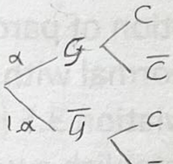
$P(\text{"apearnce of dominant"}) = 0.75$

$\text{bin}(4, 0.75)$

$P(x=3) = \binom{4}{3}(0.75)^3(0.25) = 27/64$

[illegible] α

Ex 6e p.141 Ross9



- Consider a jury trial in which it takes 8 of the 12 jurors to convict the defendant; that is, in order for the defendant to be convicted, at least 8 of the jurors must vote him guilty. If we assume that jurors act independently and that whether or not the defendant is guilty, each makes the right decision with probability θ, what is the probability that the jury renders a correct decision?

(α)

Ex 8e p.161 Ross9

- At all times, a pipe-smoking mathematician carries 2 matchboxes - 1 in his left-hand pocket and 1 in his right-hand pocket.
- Each time he needs a match, he is equally likely to take it from either pocket. Consider the moment when the mathematician first discovers that one of his matchboxes is empty. If it is assumed that both matchboxes initially contained N matches, what is the probability that there are exactly k matches, k = 0, 1, ... , N, in the other box?

Ex 11 p.294 Dev8

(* چون independent هستند joint رو میتونی به حاصل ضرب مرزی pmf ها بنویسی.

Two different professors have just submitted final exams for duplication. Let X denote the number of typographical errors on the first professor's exam and Y denote the number of such errors on the second exam. Suppose X has a Poisson distribution with parameter μ_1, Y has a Poisson distribution with parameter μ_2, and X and Y are independent.

a. What is the joint pmf of X and Y?

b. What is the probability that at most one error is made on both exams combined?

c. Obtain a general expression for the probability that the total number of errors in the two exams is m (where m is a nonnegative integer).

a) $pmf(x) = \dfrac{e^{-\mu_1}\mu_1^{x}}{x!}$

$pmf(y) = \dfrac{e^{-\mu_2}\mu_2^{y}}{y!}$

$\xrightarrow{joint} e^{-\mu_1-\mu_2}\dfrac{\mu_1^{x}\mu_2^{y}}{x!\,y!}$ *

b) $P(X+Y \leqslant 1) = P(0,0) + P(0,1) + P(1,0)$

$= e^{-\mu_1-\mu_2}[1+\mu_1+\mu_2]$

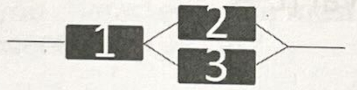
$P(X+Y=M) = P(0,M) + P(M,0) + P(M-1,1) + P(1,M-1)$

$= \sum p(X=k, Y=M-k) = e^{-\mu_1-\mu_2}\sum \dfrac{\mu_1^{k}\mu_2^{k-M}}{k!\,(k-M)!}$

Ex 15 p.205 Dev8

Consider a system consisting of three components as pictured. The system will continue to function as long as the first component functions and either component 2 or component 3 functions. Let X_1, X_2, and X_3 denote the lifetimes (exponensial) of components 1, 2, and 3, respectively. Suppose the X_i's are independent of one another and each X_i has an exponential distribution with parameter λ. → $F(y) = 1 - e^{-\lambda y}$

1. Let Y denote the system lifetime. Obtain the cumulative distribution function of Y and differentiate to obtain the pdf.
2. Compute the expected system lifetime.



اول باید احتمال اینکه سیستم در زمان y یا قبل از آن از کار بیفته رو پیدا کنیم:

$P(Y \leqslant y) = P((X_1 \leqslant y) \cup [(X_2 \leqslant y) \cap (X_3 \leqslant y)])$

$= P(X_1 \leqslant y) + P((X_2 \leqslant y) \cap (X_3 \leqslant y)) - P(X_1 \leqslant y \cap X_2 \leqslant y \cap X_3 \leqslant y)$

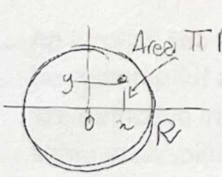
$= (1 - e^{-\lambda y}) + (1 - e^{-\lambda y})^2 - (1 - e^{-\lambda y})^3$

$pdf(y) = f(y) = F'(y) = \lambda e^{-\lambda y} + 2(1 - e^{-\lambda y})(\lambda e^{-\lambda y}) - 3(1 - e^{-\lambda y})^2(\lambda e^{-\lambda y})$

$= 4\lambda e^{-2\lambda y} - 3\lambda e^{-3\lambda y}$

α

تمرین



Ex17 p.205 Dev8

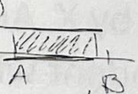
An ecologist wishes to select a point inside a circular sampling region according to a uniform distribution (in practice this could be done by first selecting a direction and then a distance from the center in that direction). Let X = the x coordinate of the point selected and Y = the y coordinate of the point selected. If the circle is centered at (0, 0) and has radius R, then the joint pdf of X and Y is

$\iint_{Circle(R)} k\,dx\,dy = 1$

$k(\pi R^2) = 1$

$(x,y) \in Circle(R)$

$$f(x,y) = \begin{cases} \dfrac{1}{\pi R^2} & x^2+y^2 \le R^2 \\ 0 & otherwise \end{cases}$$



$k = \dfrac{1}{B-A}$

a. What is the probability that the selected point is within R/2 of the center of the circular region

b. What is the probability that both X and Y differ from 0 by at most R/2?

c. Answer part (b) for R/√2 replacing R/2.

d. What is the marginal pdf of X? Of Y? Are X and Y independent?

G

Ex 20 p.206 Dev8

indep.

blue brown orange red yellow green

Let X1, X2, X3, X4, X5, and X6 denote the numbers of blue, brown, green, orange, red, and yellow M&M candies, respectively, in a sample of size n. Then these Xi's have a multinomial distribution. According to the M&M Web site, the color proportions are p1 =.24, p2= .13, p3=.16, p4=.20, p5 =.13, and p6 =.14.

a. If n =12, what is the probability that there are exactly two M&Ms of each color?

$$p(y) = \frac{12!}{2!\,2!\,2!\,2!\,2!\,2!}(0.24)^2(0.13)^2(.16)^2(.2)^2(.13)^2(.14)^2$$

b. For n =20, what is the probability that there are at most five orange candies?

$$P(X \le 5) = \sum_{i=0}^{i=5}\binom{20}{i}(0.2)^i(0.8)^{1-i} = 0.9873$$

c. In a sample of 20 M&Ms, what is the probability that the number of candies that are blue, green, or orange is at least 10?

$$P(X<10) = \underbrace{P(X_1+X_3+X_4<10)}_{} = \sum_{i=0}^{9} P(X_1+X_3+X_4=i)$$

$$\hookrightarrow = \frac{20!}{i!\,(20-i)!} \times (0.24)^i (0.10)^{20-i-i} (0.2)^{i+1}$$

2 x

Ex 25 p.211 Dev8

$A = XY$

A surveyor wishes to lay out a square region with each side having length L. However, because of a measurement error, he instead lays out a rectangle in which the north–south sides both have length X and the east–west sides both have length Y. ☐Y

Suppose that X and Y are independent and that each is uniformly distributed on the interval [L-A, L+A] (where 0<A<L).

What is the expected area of the resulting rectangle?

Ex 27 p.212 Dev8

α

- Annie and Alvie have agreed to meet for lunch between noon (0:00 P.M.) and 1:00 P.M. Denote Annie's arrival time by X, Alvie's by Y, and suppose X and Y are independent with pdf's

$$f_Y(y) = \begin{cases} 2y & 0 \le y \le 1 \\ 0 & otherwise \end{cases} \qquad f_X(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

- What is the expected amount of time that the one who arrives first must wait for the other person?

مدت انتظار $\rightarrow h(x,y) = |Y-X|$

joint pdf: $f_X(x) \cdot f_Y(y) = 3x^2\,2y = 6x^2y$

Expected waiting time : $E[h(x)] = \int_0^1\int_0^1 |x-y|\,f(x,y)\,dx.dy$

$$\int_0^1\int_0^1 |x-y|\,6x^2y\,dx\,dy =$$

$$= \int_0^1\int_0^x (x-y)\,6x^2y\,dx\,dy + \int_0^1\int_x^1 (x-y)\,6x^2\,dx\,dy =$$

$$= \frac{1}{6} + \frac{1}{12} = 1/4$$

① چرا انتگرال رو دو قسمت کردی؟

۲ چرا بجای انتگرال گرفتی؟

$$E(\text{erea}) = E(\pi R^2) = \int_{-\infty}^{+\infty} \pi R^2 f(r)\, dr$$

$$= \int_9^{11} \pi r^2 \, \tfrac{3}{4}(-r^2 - 20r - 99)\, .dr$$

$$= \pi \frac{r^3}{\cancel{3}} \times \frac{\cancel{3}}{4} \times \left(-\frac{r^3}{3} - \frac{20r^2}{2} - 99r\right)$$

$$= \tfrac{1}{4} \pi \left(-\frac{r^6}{3} - 10r^5 - 99r^4\right)\Big|_9^{11}$$

$$= \frac{501}{5} \pi = 314.79\ m^2$$

Ex 21 p.151 Dev8

An ecologist wishes to mark off a circular sampling region having radius 10 m. However, the radius of the resulting region is actually a random variable R with pdf

$100 - r^2 - 20r$

$$f(r) = \begin{cases} \frac{3}{4}[1 - (10 - r)^2] & 9 \le r \le 11 \\ 0 & \text{otherwise} \end{cases}$$

$1 - 100 - r^2 - 20r$

-99

What is the expected area of the resulting circular region?

Ex 37 p.163 Dev8

$\mu = 104 \quad \sigma = 5$

Suppose that blood chloride concentration (mmol/L) has a normal distribution with mean 104 and standard deviation 5 (information in the article “Mathematical Model of Chloride Concentration in Human Blood,” J. of Med. Engr. and Tech., 2006: 25–30, including a normal probability plot as described in Section 4.6, supports this assumption).

a. What is the probability that chloride concentration equals 105? Is less than 105? Is at most 105?

* b. What is the probability that chloride concentration differs from the mean by more than 1 standard deviation? Does this probability depend on the values of μ and σ ?

* c. How would you characterize the most extreme .1% of chloride concentration values?

a) $\boxed{P(X = 105) = 0}$ چون توزیع نرمال پیوسته است پس :

$P(X < 105) = P(Z < \frac{1}{5} = 0.2) = .5793$

$P(X \le 105) = .5793$

[illegible] توزیع پیوسته است.

b) $P(|X - \mu| < \sigma) = P(|Z| > 1) =$

$= P(Z < -1, Z > 1) = 2P(Z < -1) =$

$2\Phi(-1) = .3174$

c) $P(X > a) = 0.001 \rightarrow P(Z > \frac{a - 104}{5}) = 0.001$

$\Rightarrow \frac{a - 104}{5} = -3.09 \Rightarrow \boxed{a = 88.55}$

Ex 77 p.87 Dev8

An aircraft seam requires 25 rivets. The seam will have to be reworked if any of these rivets is defective. Suppose rivets are defective independently of one another, each with the same probability.

عیب

a. If 20% of all seams need reworking, what is the probability that a rivet is defective? p prob one rivet is defective

b. How small should the probability of a defective rivet be to ensure that only 10% of all seams need reworking?

a) $P(\text{"seam needs reworking"}) = 0.20$

$1 - (1-p)^{25}$

all 25 rivets ok

a) $1 - (1-p)^{25} = 0.20$

$(1-p)^{25} = 0.80 \rightarrow p \approx 0.00886$

b) $1 - (1-p)^{25} \le 0.10$

$\rightarrow p \le 0.004206$

a) $P(\text{"seam rework"}) = 1 - P(\text{"no rework needed"})$

$= 1 - P(\text{"}S_1 \text{ works} \cap S_2 \text{ works} \cap \dots \cap S_{25} \text{ works})$

if p = probebility that a rivet is defected

$\Rightarrow (1 - p)$ " " " " is working

$\Rightarrow 1 - (1-p) \dots (1-p) = 1 - (1-p)^{25}$

$1 - (1-p)^{25} = 0.8$

$1 - p = (0.8)^{1/25} \Rightarrow$

b) $P(\text{rework}) = 0.1$

$1 - p = (0.9)^{1/25}$

Ex 69 p.171 Dev8

A system consists of five identical components connected in series as shown:

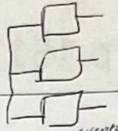
Suppose each component has a lifetime that is exponentially distributed with λ=0.01 and that components fail independently of one another. Define events A_i= {ith component lasts at least t hours}, i=1,...5, so that the A_is are independent events. Let X=the time at which the system fails—that is, the shortest (minimum) lifetime among the five components.

a) The event {X≥t} is equivalent to what event involving $A_1, A_2, \ldots A_5$?

b) Using the independence of the A_is, compute P({X≥t}). Then obtain F(t)=P(X≤t) and the pdf of X. What type of distribution does X have?

c) Suppose there are n components, each having exponential lifetime with parameter λ. What type of distribution does X have?

a) $\{X\geq t\} = \{$the life time of system is at least t$\}$
$= \{$all 5 life times are at least t$\}$



$= A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5$

b) $P(X\geq t) \overset{\text{events are}}{\underset{\text{independent}}{=}} P(A_1\cap A_2\cap A_3\cap A_4\cap A_5) =$

$= P(A_1)P(A_2)P(A_3)P(A_4)P(A_5)$

using exponencial ~~$X_i \sim \exp(0.01)$~~ CDF $\longrightarrow P(A_i) = P($"component liftime$\geq t$"$)$
$i=1,\ldots 5$
X_i are indep

$= 1 - F(t) = 1 - [1 - e^{-0.1t}] = e^{-0.1t}$

$P(X\geq t) = \underbrace{(e^{-0.1t})(e^{-0.1t})\ldots(e^{-0.1t})}_{5}$

$P(X\geq t) = e^{-0.5t}$

CDF: $F_X(t) = P(X\leq t) = 1 - e^{-0.5t}$

CDF $\xrightarrow{\text{derivative}}$ pdf : $f_X(t) = 0.5e^{-0.5t}$ → exponensial distribution with $\lambda = 0.5$

c) $P(X\leq t) = 1 - e^{-n\lambda t}$, X has an exponensial distribution with parameter $\underline{n\lambda}$.

چون event ها مستقل اند احتمال کلیشون، حاصلضرب احتمالاتشون هست یا اینکه
توزیع joint شون تعیین می‌کنه.

Ex 67 p.171 Dev8

Suppose that when a transistor of a certain type is subjected to an accelerated life test, the lifetime X (in weeks) has a gamma distribution with mean 24 weeks and standard deviation 12 weeks.

a) What is the probability that a transistor will last between 12 and 24 weeks?

b) What is the probability that a transistor will last at most 24 weeks? Is the median of the lifetime distribution less than 24? Why or why not?

c) What is the 99th percentile of the lifetime distribution?

d) Suppose the test will actually be terminated after t weeks. What value of t is such that only .5% of all transistors would still be operating at termination?

نیم درصد!

$\left.\begin{array}{l}\mu = 24\\ \sigma=12, V=144\end{array}\right\} \Rightarrow \begin{array}{l}\alpha\beta = 24\\ \alpha\beta^2 = 144\end{array} \Rightarrow \boxed{\begin{array}{l}\beta = 6\\ \alpha = 4\end{array}}$

a) $P(12\leq X\leq 24) = F(\frac{x}{\beta},\alpha) - F(\frac{x}{\beta},\alpha)$

$= F(4,4) - F(3,4) = .424$

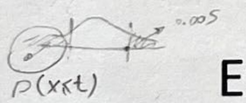
b) $P(X\leq 24) = F(4,4) = .567$

c) $F(\frac{x}{\beta},\alpha) = 0.99 \rightarrow F(\frac{x}{6},4) = 0.99$

$\xrightarrow[\text{table}]{\text{using}} F(10,4) = 0.99 \Rightarrow \frac{x}{6} = 10 \rightarrow \boxed{x = 60}$

d) $P(X\geq t) = 0.005 \longrightarrow P(X\leq t) = 0.005$

$F(\frac{x}{6},4) = 0.995$



$P(X\leq t)$

$\hookrightarrow F(11,4) = 0.995 \rightarrow \boxed{x = 66}$

این یعنی در 66 هفته فقط
نیم درصد ترانزیستورها
کار می‌کنند.

Ex 19 p.151 Dev8

Let X be a continuous rv with cdf

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \dfrac{x}{4}\left[1 + \ln\left(\dfrac{4}{x}\right)\right] & 0 < x \leq 4 \\ 1 & x > 4 \end{cases}$$

[This type of cdf is suggested in the article "Variability in Measured Bedload-Transport Rates" (*Water Resources Bull.*, 1985: 39–48) as a model for a certain hydrologic variable.] What is

a. $P(X \leq 1)$?

b. $P(1 \leq X \leq 3)$?

c. The pdf of X?

مشتق تابع توزیع تجمعی میشه!

a) $P(X\leq 1) = F(1) = \frac{1}{4}(1+\ln 4) = 0.597$

b) $P(1\leq X\leq 3) = F(3) - F(1) = \frac{3}{4}(1+\ln\frac{4}{3}) - \frac{1}{4}(1+\ln 4)$

$= .966 - .597 = .369$

c) derivation of CDF:

$(\ln f(x))' = \frac{1}{f(x)} \cdot f'(x)$

$\left(\frac{x}{4} - \frac{x}{4}\ln\frac{4}{x}\right)' = \frac{1}{4} - \left(\frac{1}{4}\ln\frac{4}{x} + \left(\frac{x}{4}\cdot\frac{-4}{x^2}\right)\right)$

$= \frac{1}{4} - \frac{1}{4}\ln\frac{4}{x} + \frac{1}{x}$

α

Ex 39 p.222 Dev8

- It is known that 80% of all brand A zip drives work in a satisfactory manner throughout the warranty period (are "successes").
- Suppose that n =10 drives are randomly selected.
- Let X the number of successes in the sample. The statistic X/n is the sample proportion (fraction) of successes.
- Obtain the sampling distribution of this statistic.

$n=10$, $p=0.8 \rightarrow bin(10, 0.8)$

x	0	1	2	3	4	5	6	7	8	9	10
$x/15$	0	$1/15$	0.133	0.2	.267	.333	.001	.003	.04	.043	.667
$p(x/15)$	0										

x	11	12	13	14	15
$x/15$					
$p(x/15)$					

چجوری [illegible] approximate [illegible]؟ [illegible]

[illegible]

Ex 41 p.222 Dev8

Let X be the number of packages being mailed by a randomly selected customer at a certain shipping facility. Suppose the distribution of X is as follows:

x	1	2	3	4
p(x)	.4	.3	.2	.1

$\bar{X}$

a. Consider a random sample of size n =2 (two customers), and let $\bar{X}$ be the sample mean number of packages shipped. Obtain the probability distribution of $\bar{X}$.

$P(\bar{X}) = ?$

b. Refer to part (a) and calculate $P(\bar{X} \le 2.5)$.

c. Again consider a random sample of size n =2, but now focus on the statistic R= the sample range (difference between the largest and smallest values in the sample). Obtain the distribution of R. [Hint: Calculate the value of R for each outcome and use the probabilities from part (a).]

d. If a random sample of size n=4 is selected, what is $P(\bar{X} \le 1.5)$? [Hint: You should not have to list all possible outcomes, only those for which $\bar{x} \le 1.5$.]

(x_1, x_2)	(1,1)	(1,2)	(1,3)	(1,4)	(2,1)	(2,2)	(2,3)	(2,4)	(3,1)	(3,2)	(3,3)	(3,4)	(4,1)	(4,2)	(4,3)	(4,4)
prob	.16	.12	.08	0.04	.12	.09	.06	0.03	.08	.06	.04	.02	.04	.03	.02	0.01
$\bar{x}$	1	1.5	2	2.5	1.5	2	2.5	3	2	2.5	3	3.5	2.5	3	3.5	4
r sample Range	0	1	2	3	1	0	1	2	2	1	0	1	3	2	1	0

a) $P(\bar{X}) = ?$

$\bar{x}$	1	1.5	2	2.5	3	3.5	4
$P(\bar{x})$	.16	.24	.25	.2	.1	.04	.01

احتمالات متناظر $\bar{x}$ را جمع می کنیم

b) $P(\bar{X} \le 2.5) = .16 + .24 + .25 + .2 = .85$

c)

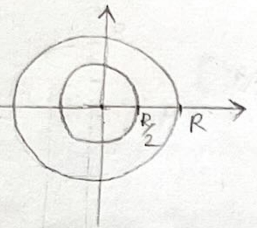
r	0	1	2	3
P(r)	.3	.4	.22	.08

ⓓ $\bar{x} \le 1.5 \longrightarrow \Sigma x \le 6$

$\Rightarrow P(1,1,1,1) + P(1,1,1,2) + \ldots$

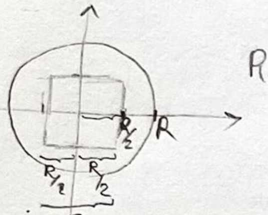
$(0.4)^4 + (0.4)^3(0.3) + \ldots$

17. a) what is the probability that selected point is within $R/2$ of the center region?



$$P(a) = \frac{S(R/2)}{S(R)} = \frac{\pi (R/2)^2}{\pi R^2} = \frac{R^2}{4R^2} = \frac{1}{4} = 0.25$$

b) what is the probability that X and Y differs from 0 by at most $R/2$?



$$P(b) = \frac{P_\square}{P_\circ} = \frac{R^2}{\pi R^2} = \frac{1}{\pi}$$

R [illegible] R

c) answer part b for $R/\sqrt{2}$!

$$P(c) = \frac{P_\square}{P_\circ} = \frac{\left(2R/\sqrt{2}\right)^2}{\pi R^2} = \frac{4R^2}{2\pi R^2} = \frac{2}{\pi}$$

d, what is ~~pdf~~ marginal pdf of X? of y? are X & y independent?

$$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{1}{\pi R^2}\,dy = \frac{2\sqrt{R^2-x^2}}{\pi R^2}, \quad -R \leq X \leq R$$

$$f_Y(y) \xrightarrow{\hspace{8em}} = \frac{2\sqrt{R^2-y^2}}{\pi R^2}, \quad -R \leq X \leq R$$

$$\frac{1}{\pi R^2} \neq \frac{2\sqrt{R^2-x^2}}{\pi R^2} \cdot \frac{2\sqrt{R^2-y^2}}{\pi R^2}$$: The product of marginal pdf

17. An ecologist wishes to select a point inside a circular sampling region according to a uniform distribution (in practice this could be done by first selecting a direction and then a distance from the center in that direction). Let X = the x coordinate of the point selected and Y = the y coordinate of the point selected. If the circle is centered at (0, 0) and has radius R, then the joint pdf of X and Y is

$$f(x, y) = \begin{cases} \dfrac{1}{\pi R^2} & x^2 + y^2 \le R^2 \\ 0 & \text{otherwise} \end{cases}$$

اینجا سوتی دادم ~~[illegible]~~ متن حل سوال که باید جواب داده شه رو نداشتم.
نتیجتاً سوالات + جواب هارو در ص 24.

Ex.39 pag.72

Fifteen telephones have just been received at an authorized service center. Five of these telephones are cellular, five are cordless, and the other five are corded phones. Suppose that these components are randomly allocated the numbers 1, 2, . . . , 15 to establish the order in which they will be serviced.

a. What is the probability that all the cordless phones are among the first ten to be serviced?

b. What is the probability that after servicing ten of these phones, phones of only two of the three types remain to be serviced?

c. What is the probability that two phones of each type are among the first six serviced?

5 cellu
5 cordless
5 corded

$P(a) = \dfrac{\binom{5}{5}\binom{10}{5}}{\binom{15}{5}}$

$P(b)$

$P(b) =$

Ex. 37 pag. 72

An experimenter is studying the effects of temperature, pressure, and type of catalyst on yield from a certain chemical reaction. Three different temperatures, four different pressures, and five different catalysts are under consideration.

a. If any particular experimental run involves the use of a single temperature, pressure, and catalyst, how many experimental runs are possible? $3 \times 4 \times 5 = 60$

b. How many experimental runs are there that involve use of the lowest temperature and two lowest pressures? $1 \times 2 \times 5 = 10$

c. Suppose that five different experimental runs are to be made on the first day of experimentation. If the five are randomly selected from among all the possibilities, so that any group of five has the same probability of selection, what is the probability that a different catalyst is used on each run?

c) از قسمت a می‌دانیم کلاً 60 آزمایش داریم
در قسمت c می‌خواهیم 5 تا آزمایش ازشون انتخاب کنیم
⇐ کل حالت ها $\binom{60}{5}$

حالت خاص مدنظر: هر catalyst فقط یکبار می‌تواند انتخاب شود
پس برای هر آزمایش [illegible] متمایز کاتالیست را انتخاب داریم.

$3 \times 4 \times 1 = 12$

در هر آزمایش 12 انتخاب برای T و P داریم

حالت در کل $12^5 = 12 \times 12 \times 12 \times 12 \times 12$

$\Rightarrow P(C) = \dfrac{(12)^5}{\binom{60}{5}}$