Last meek we introduced Θ , O, S, o, ω and discussed the relations among them. Then we gave a divide and conquer algorithm for sorting, called there sort and whate expressed its running time vin a recoverance equation. We solved this equation via a technique called recursion true (although we did not use this term). We then stated the Master Than which helps solve recurrence equations of certain form.

This week:

More divide and congrer algorithms.

- Quick sort
- Karatook Binary search
- Karatsuba's integer multiplication alg.
- Strassen's fast thatis well-iplication alg.

Remark: please need part VIII of CLRS for the required andthe background!

Quick-sort

Quick sort is author sorting algorithm. Its running time in the worst owe is 2(12) so why introduce another show corting alg. when we already have Merge-sort? The point is that Q.s. teless on average O(u/ya) steps.

OK, but we still have Merge-Sort so what he iden?

Q.S. is a simple to implement in-place sorting alg. The people are use gave for the bout in place It can be made in-place with some effort but this complicates the code. Outsided there was

Note: Last week a suggestion was made to make Ms. in-place 6. + Hotilan had a buy in it that I dichit notice when answering the question. It anyone is interested they can contact me and ask.

Anyway, Q.S. is simple in-place up and in practice it vons pretty well and is usually implemental instead of Ms.

So what's He we:

The alg picks an element that is called a pivot and mores all elements smaller them the pivot to its left and all elements larger than it to its right and then recursively sorts the left and right part

There are 3 parts for the alg.

- 1. Selecting the pirot
- 2 Partitioning the elements to the smaller than it and those larger than it
- 3. Recorsier

3.2		to the second control of the second control
n og til kall krived blevere de skrivet for skrivet for skrivet krivet krivet krivet krivet krivet krivet kriv	Lets start with the simplest selection	- Hat of the last elevent
Assumed what is a second of the second of th	The pseudocade for quick-sorting a so	Land Control of the C
\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\		*
	Quick-sort (A, P, r)	
gggggggggggggggggggggggggggggggggggggg	L. If Par	
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payagayya kayagay ya fahamaya ayadda kadayaa qadanaya qayay qayayyy	3. Quide-sort (A, P, 9-1)	? is the correct location of the
Photographic and the state of t	4. Quide-sut (A, 9-1, r)	
unamannen etaan etaan kaan etaan etaan etaan etaan eta		
เกมเลงเพลงเมมเลงเมลงขนายของเหลดเมลเพน้น	To sort A we run Quick-sort (A, 1,	•
	Next we describe the partition why	•
	Partition (Age)	
	1. X = A6:3	x is the pivot
	2. i = q-'	i will be the last element
***************************************	3 For 1 = 2 to r-1	smaller than x
1889 PARRA MARINA BARNA BA	and the contraction of the contr	
eeneermaanseneenmenen vareneersienmen.		
	6 Swap AEIJ with AEJJ	
emperatura de de productiva de la compania de la c	7. Swap Aciti W:th Acts	pubs the first in place
menteraranan darah d	8. Return itl	
	I dea: at each step the picture is	
aan gaaray ay ahaa ahaa ahaa ahaa ahaa ahaa a	The loop invaliant is that at the beginning	oteach steration, for any inter k
	1. it PEKEI Hen ACKJ =x	<i>' †</i>
	2. if it1 = K = J-1 then AIR] > iX	
rust 1998 til stadt til stadt skrift skullet skrift skullet skrift skullet skrift skrift skrift skrift skrift	3. it K=r then A[x]=x	and the second

This is include what the picture fells us and it can be verified quite easily

Back to Q.S. What can we say about its running time? (correctness is simple to verify given that partition is correct).

worst case: Imagine that the array is already sorted! Thun, each step executions of partition will return an array succeller by a and an empty array. As Portition to kee linear Now, are is to be have $c \stackrel{\sim}{\Sigma}_i = \mathfrak{G}(n^2)$

What happens in the best case? In this case Partition returns two suburrays of raryuly equal sites and we get the recurrence $T(n) \le 2T(\frac{n}{2}) + \Theta(n)$

which gives T(n) = O(non) as we saw.

what is the typical case? It AINS is carplain than we expect to get a belanced partition. Notice that even an ubbalanced partition to eight and only gives a recurrence

T(n) = T(eight T(quin) + T(rio) + O(n).

Solving this still gives T(n) = O(nyn)

(Here are & loging in levels of recursion, and contact or each step is O(11)

What about the average case:

For a random input we expect the partition to be roughly belanced Notice that even it half the steps give an unbalanced partition (which is actually unlikely) that he running hime is still O(u)u) as the vecursion depth grows by a tacker of 2

The alg. we should have the drawback that for some imputs it performs poorly leg. sorted arrays.

A way to overcome this is by selecting a random intex to be the pivot.

Motice that the probability that the chosen pivot is either smaller than shoot 9/10 of the elements or larger than 9/10 is at must = 20 x. Thus with good probability each step is at least (9/10) belowced. It can also be proved that with high probability at least 1/2 of the steps are -3000.

	in the state of th
Ammunia ya masa (ya masa na ya masa na	More examples of divide and conquer.
ndressessessessessessessessessessessessess	
	The simplest example is that at binary search. It is also something called
ett kall standard til kall til standard til kall standard standard standard standard standard standard standard	Thou to cotch a lim in the depart Suhara desert
ett til til film fra framtid fra framtid et allefans som framskall framskall framskall framskall framskall fra	Au Line
PAPAPAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA	The de divide the desert to two parts. The consider the half in which the
	lion is and catch the lien in that halt of the allert
e decembra y company y company a superior y company a superior y company a superior y company a superior y com	
1417800-1-1/1/100-1/11/100-1/11/100-1/1/100-1/1/100-1/1/100-1/1/100-1/1/100-1/1/100-1/1/100-1/1/100-1/1/100-1/	What is the binary search problem:
*	Import: A sorted Array A[1 n] and an element e.
	boal: find i s.f. Asiz=e
PP-COST VIEW COV I SERVEY I SERVEY I SECOND CONTRACT CONT	
TV-0-01111000A-111110A-11110A-A-A-A-A-A-A-	Idea: Compare e to ALTI. It e is smaller Hen search ALI-E) a.v.
71-77-77-4-7-8-7-8-7-8-8-8-8-8-8-8-8-8-8-8	search Alite, Ag).
197-00/A/ 000000000000000000000000000000000	Alga
// A (TAM / A (TAM A	Binary-Jearch (A, <, n)
PROPERTY AND	1. If now return Not Found
778-88AA 78-88-11 ABB 18-88-11 AB	2. mol 34
	2. Ele m: 1%1
	3. If e = A[m]
	4. neturn m.
,	5. else If exAlmj
a dalaman kangan mangan pangan pa	6. return Binouy-search (A[1m-1], e, m-1)
NAMES THE RESIDENCE OF THE STREET OF THE STR	7. else return Binary-Seurch (A[mti,,n], e, n-m)
	Claim: Binary-Search (Acein) Solves binary search
10 TO THE TOTAL	94: By induction on h

3.1	sightoticated	
oostiinin oo ka	More examples of divide and conquer	
denkunnkar Handel el lelekat 190-aur en hader en delen let er en		
uuunkuudussa kaakajaksisjaksiskiksiksiksi (1970) (1970) (1970) (1970) (1970) (1970)	Karatorba's alg for integer multiplication.	
rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr	Input: two n-digit binary numbers a = a,an b > b,bn	
mysses, yanyan ayayayayayayay jeft saasan fishif dhiba dha dha dha dha dha dha dha dha dha dh	Goul: compute the birary rep. of a-b	
ուսույ _{ն գո} րու _{սո} յցցցցցցցցցում բազանաներ գործական գործանական առաջինանական առաջինան համանականում առաջինան համա	School alg. was in the O(a2) why?	
Kavalzubas	I dea! Notice a = (anti-an) + 2 . (a, an) = alt z an	
	b = (by-1 by) + 2"/2 (b1 by) = b2 - 2"/2 bh	
UN AND UNITED BOOK OF THE STATE	New when a and be to are in-digit numbers.	
kun kairinkaya kirinda ka kakalah PRAMA	Now, $a \cdot b = (a_{e} + i^{n/2} a_{h}) \cdot (b_{e} + i^{n/2} b_{h}) = a_{e} \cdot b_{e} + i^{n/2} (a_{e} b_{h} + a_{h} b_{e}) + i^{n} a_{h} b_{h}$	
	If we naively compute ask, ask, ask, ask, ash thereofer the me	
nonnyana nyaya sankara yangi nayakahahah hambah nyanadah hykanaha hambah nyadany	get T(n) = 4T(x) + O(n)	
ուսուգուսագույ ու գույն ընչ է իրջանույթում ընչանական հանագահանում է բունական համանական համանական համանական համան	alling whige wanter	
majama, gangangan papilamgan panghapagangan pamagan gangan gangan pamahan pamahan pamahan pamahan pamahan pamah	Solving (e.g. Master than) we get T(n1 = O(n2), like school als	
	It we could save even one multiplication we would get a much fester ulg.	
	Notice, paying on O(4) cost for each step is ok as it doesn't change	
enstatutus kuningas katanga tahung tahung tahung tahung kang kang kang kang kang kang kang ka	hat as compete to betting products was the assymptotic of the running	
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and the control of th	Let us comple	
TEPSTER APPENDING AND	$(\alpha_{\ell} + \alpha_{h})(b_{\ell} + b_{h}) = \alpha_{\ell} b_{\ell} + \alpha_{h} b_{h} + (\alpha_{\ell} b_{h} + \alpha_{h} b_{\ell}) \qquad I$	
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r enverenden Northwell Arthum (Arthum (Arthum Anna Anna Anna Anna Anna Anna Anna Ann		
	Nou, able is given, abbatante is obtained via & (I-II)	
	and a, by via $\frac{1}{2}(I+II)-II$	

- Comment

=> T(n) = O(n(302) = O(n2.31...)

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t vertical and antimomental to the transfer of	
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n ole	4. Cy = A21. (By-Bi)
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