# Homework 13



A problem is NP-hard if a polynomial time algorithm for it implies that NP is in P. A problem is NP complete if it is NP-hard and in addition it is in NP.

1. Consider the following problem IS10: given a graph $G$, determine whether $G$ has an independent set of size 10 (i.e. with 10 vertices). What is the complexity of IS10? Is it in P, NP, neither? Is it NP-hard?

2. Consider the problem 4SAT: given a boolean formula on $n$ variables of the form $C_1 \wedge \cdots \wedge C_m$ where each $C_i$ is a clause containing 4 literals (e.g. $x \vee \overline{z} \vee y \vee \overline{w}$), determine whether there is a satisfying assignment. Show that 4SAT is NP-complete.

3. Given a graph $G = (V, E)$, the $k$-colouring probem ($k$COL) asks if there exists an assignment of colours to vertices $c\colon V \to \{1, 2, \ldots, k\}$ such that for any edge $(u, v) \in E$, $c(u) \neq c(v)$. For example, a complete graph (a graph where every pair of vertices has an edge between them) on 3 vertices is 3-colourable, whereas a complete graph on 4 vertices is not.

   (a) Is 2COL in P, NP, or neither?

   (b) Prove that 3COL is in NP.