Last week we proved lower bounds on comparison based sorting, then gave counting sort that when all valves are in a soft too large interval works in them time. Then we gave them time alg. for thirting the modium and in general tor returning the i'th element.

Reminders Mid term on Oct 25. Will contain all material covered till Hall form.

Gracing: Hhr grading has been very very gentle. It was important for we to encourage you to do your HIV on your own, but also to give you good teedback on your awar solutions.

Historm, exam will not be so gently graded!

Evaluation form: Please fill thee.

Today: Biney Seach Tome

Binary search Trace is a data structure for storing sets that evalue ever time via insertions/deletions, similar to happe. Thus, BET can be used for priority queue as well.

But is elsigned so that searching the tree is similar to binary rearch quel hence its name.

We will see the definition of BST and how to portern the built operations on it similar to hoops, most operations take the O(h), but white hoops, BST are not olways balanced.

We shall speak more about heaps is. BST at the end.

## so whit is a BST!

Fach element has a key according to which we boil the BIT and satellite cluta.

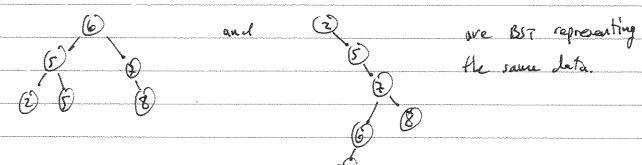
The elements are ordered in a binary true, which saube stored on using pointers from each nale to its parent and children.

They each note x contain the attributes xhey xilett, xiright, x.y.

If a child or a perent is missing (e.g. leet or root) than that attribute contribute will NIL.

The main proporty of BST is that NH for every nada x in a BIT we now that if y is in the left subfirm of x then y, key & x.key and it y is in the right subfirme at x then y, key & x.key.

E.G.



The nice thing about the BST property is that it allows up to print out all the keys in a sorted order in linear time by a simple recurrence algorithm.

The algorithm is called inordertree walk:

Inorder-tree-walk (x)

x is a nale

L It X & ML

- 2. Inorder-transalk (x.left)
- 3. Prat kakey
- 4. Inorder- tree-walk (x-right)

funning time is  $\Theta(n)$  and correctness can be easily proved by induction. Indeed it is has knowless on the left and n-k-c on the right then

T(n) & T(k) + T(n-k-1) + O(1).

The solution is Traison.

We can also obtine preorder-trae mulk which points the root before the value in either sistence and postorolar trae mulk which prohits it after the value in its software For example

Preorder-Gree-walk (x)

1. If X+ NIL

- z Print. X. Key
- 3. Preorder-tree-walk (x.lett)
- 4. Preorder-tran-walk (xiright)

,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	$(1) \\ (2) \\ (3) \\ (3) \\ (4) \\ (3) \\ (4) $
gamenta a a a a a a a a a a a a a a a a a a	A back property of BST (well what weaker them so useful) is that searching the
ensystem o ytherhors bengamen a magaine o program y program o y	tree and finding the max and min can be done efficiently. Moreover,
ayana ( ( ana ( ) aha a ankan a panganapan pangan pangan pangan pangan pangan pangan pangan pangan pangan pang	tinding soccour and predecessor is also done efficiently depending only
	on h.
na Syman ann ainm an d-gaillean agus an ann agus an guaga a gu ga	
ensymmetric manners of the property of the pro	Tree-search (x, k) x is the reat k is the key me
ennythina diamanan'i an diamany any ao	1. If X= MIL or X-key ok search
	2 return X
etilijaan kantalaan saanna	3. It k< x.key
apar (jahan sa sanga), da sadana, an a pamana pagangaya pamana, pagangaya panga	4. Tree-search (x.leff.k)
olandok (kanaman Anas) maja njajimja ya aya ya y	F. Else Tree-sewish (xiright, le)
Anthriji himodra kosso Asijossa sa songa si ini sa sa ja ja ja ja	avaning than: Notice that it xikey + k then we go down the free.
	This, running time is O(L).
APP & 1.1.1 F & 1.1.1 A &	Another nice way to be search is:
Felicia // Annico a particul A a semina i A successo de A frence (A s. semina i A semina	I terative tra-search (xxx)
7576	1. While XFNIC and X-key 41c
	2. It kikey elc.
	$x \in x.left$
	4. Ble x x x x x x x x x x x x x x x x x x x
	s, Return x

V · ,	
	Min and Max are also easy, we postraight left or straight right
viinen in	
e (1882) Sich der Sich (1884) Sich der Sich (1882) Sahah Sich aus der Anderson (1884) Sich aus der Anderson (1	True-Mi-Wi-Wi-Wi-
mazi Ulumin Kiril Ukun kebebahin ing kebili kebihin kebihin keb	1. While xleH + HIL
1/223/millimmilli (Same) (Same	2 X = xleft
n szere filmén pajásásasán felyészítések felyészítések köndési köndési köndési köndési köndési köndési köndési	
of 1755 H zmille 1845 (1944 feet flysk k tyter flysk k tyter flysk feet flysk feet flysk feet flysk feet flysk	
25	Similar for Tree-max.
00H/HH1/HH21T5BK/SSENYHHHHH44EW/HHAVAAAA	Successor & more interesting. When can we find X's successor?
aapäinessillistattysysysysystyttiitiilliiliiliiniiniilliiniiliiniiliiniiliiniiliiniiliiniiliiniiliiniiliiniili	It is has a right child then the min value in that subtree is layer
kikajainin kalimataja ja kalimataja kalimata kalimata kalimata kalimata kalimata kalimata kalimata kalimata ka	thm x. Are there other candidates ! what about x's parant?
	well, it x was a right child (x.p) right =x flow x.f is smaller
ry ministry striministrated may be about from the property of the Port of the Section 1 and 1 and 1 and 1 and 1	Plan X: it x was a left child flow x.p. key is larger flow x's
kilossia jiikilisiisiisii kajajiin ja 1400 ja 1500 ja 1	and all of x's children. Thus, the minimum of the subtree of
h 1994 to the first of the Section o	* A i A HA Dish pis A propriet suggested
rinstandar pygyyyy y syfylys syfyr ambalanamini bantatanda hetatanan harara.	BIH HAT WE FY gut ANGE 2
ochmunisynninilahete tetyläisi (k. f. p. klaisi inkuurpuuntus eri esistä keet seuteimint eri 1910).	HILL IE JUNE AN ABLES
returnituus vinatuluus keesessää (s.k.l.) täidettee kkonmook kentriis (t. 2-e.t.) (s.m.) suud suud	Well, not quite. What about xip.p: etc.
ett til stand i konstrålighet statistisk konstruktivet kriftet de konstruktivet promiter at som til statistisk Til til stand i konstruktivet skrivet skrivet kriftet de konstruktivet skrivet promiter skrivet skrivet skrive	Notice that as long as x is a right child of x.p and x.p. is
r industrial a little little little little little little litte little little little little little little little	a right child of (xp). p etc the value can only decrease. It at some
1.00 Terr) siste permit Medicing 1914 (Sing 192) estimator e est estrare de missoarre en dis	point the parent is such that we were in its left outtree than this
t energishishis and the history fight for the following to september on minimum terms to come	parent is larger than all values in its rebtre induling the view of xwight
numikale (5) job 2/63/5/63/5/5/5/5/5/5/5/5/5/5/5/5/5/5/5/5	16 tree
ht addition (AS) jirgan kunin dahan dahadi ain daha di dahan di Judi sahadi dahan dahadi dahan dah dahadi daha	OK, so if xiright = NIL we come how to think X's successor. But
memmunan dalahak sastikan keri mentendi sebada keri tendi 1967 (1977) (1979).	what it x is epht = N/L?

Вy	H	. W	gvh	ent ut	۱۰۰۰ <u>۷</u>	e shale	ه ا	w. (-, ·	the the	st	ances	1	ot-	x that
•			•					nutice						
								y, let		stanbaumstar samutth			allendrating and and and an area and a second	

Tree-juccessor (x)

- Limited XXX Sq X+ + XIL
- 2. Return Free-mis (x.right)
- 3. y ≠ X.q
- 4. While y + NIL and X = Yiright
- r. x+y
- G. Y440
- 7. Return y

Note that it x is the largest (and thus has no successor) we will reach the root and then yo to were its parent which is NIL.

(melyion:

Thomas

we can implement the dynamic-set operations search, Min, May, successor, predecessor in time O(4) on BST at height he

<b>(. 6</b>	
n og græne fredering skinner skille skil	We shall now see how to insert elevents and delete elements for an a BST
ne oktypini moogulaisekkii koogulaisekkii koogulaiseksii koogulaiseksii koogulaiseksii koogulaiseksii koogulai	Inserting is relatively any but deleting is more complicated.
ettermitteid 1144 Statismittiinin valtaja 1149 tää 11 ministyy ja	The injertion alg. takes a tree T and it should injert the value or to T
	So we assume use have a nale & with Zikey = V . Zileft, Zight Zigs ML
nainidit hidisid Juliikid 1944 (lilliliki lillililiki 1954 teksik lihak etililililik 1964 kida anat 17 millili	aul inverts z to T.
	The ide is to use search to that where to went z.
rical grade i manus de armani de la constanti d	Tre-iner(I,Z)
ilikan 39. ah kara 44. 200 200 da falliar filli la balla da kalilin kalila karilin kalila karilin kalila karil	
9998995099445554555555555555555555555555	2. X = 7. v = 1.
oliminadastalistisky po zysiini jedy pristy pyknosisky intervi jesi siste zo zazy jednosia o zwiani osionetji	3. While x + ML
mmis n John kart koppet ei State ei St	4
orinaritaineisiataisiitiitiinii ja	
ani a salah sa	L. X + X Lett
erinte de la la companya de la comp	t. Eve x 6 xxxx <sup>t</sup>
and the state of t	8. 7.9=x y ww points to x and x is
	9. It y= ML Where & should be unless &
allumlaresish 1111 ja siga 2111 kallumasisa 3396 ka kalabasish 350 da masi Aryan na na na sabirila.	10. T. reaf = 2 softh though tree is early
adalarko kalid fizi pojminio k (Cafgaffaf fizio def più quin se inministra poet più quant se inministra de la	11. Ele It z.key < y.key
rust aur 4 statut 45 jahait suutus 5 jahaitaksis jähnijän joi joi joi jahamatusis ja jahaitaksis jahaitaksis j	12. Y 1e4+ = 7
must der del mille insche del siste (zeine de sind zijn de zein zijn de zein zijn de zein zijn de de siste zij	13. Ebe Yright = 2
ommondada aregistojiliinikuses salistavuse Viilitas salvatalluuda assatusta salasti suur nee sussa	Running time is again O(h).
ert grater tall til gjelsen killede på på jale jame i fart fligt for flyt og frem som en semme jamen sed en sk I	

6.8		
	Notice that transplant does not tooch the painter	to v from v.p. This will have
30.44.2550000 m <sup>2</sup> 0.20000000000000000000000000000000000	to be dealt with by the alg. running transplant	
\$\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\	Ru-time is del	
52-1-3-55454568893355555553333445533	We can now give the deletion aly.	
1944) sakak fiffinisi kidi fifunda 1855 fire ett ett ett ett ett ett ett ett ett e		
e ja	Tre-pelate (T. 3)	
555   BANTON   BANTON	L If E. left = NIL	) we first houselle the case
001 13 G-5000000000000000000000000000000000000	2. Transplant (T, Z, Z. right)	ot at most one child.
	3. Ele Pless 2. right = NIL	
	4. Transplant (T, Z, Z.leff)	
	T. Ebe y= Tree-min (2. vight)	manual function the successor
and of the state o	ki tita	1 s not Exciple
	7. Transplant (Tex x right)	e we fix y's robtrae
99.44-4444411111111111111111111111111111	2. Y. right = 2. right	+ make y point to zvight
	9. y.right.y x	4 and vice sers a
alanda selle of phosphological of the Polythological School Schoo	lo Transfort (T, Zix)	+ Now we get y intended ?
induction of the desired and a state of the desired proportion of the desired as	4. 7. Left & 2. left	E we make 2's left ma y's
	12. 7.left-p = y	E and make it point by
erszentősiii i Anflum Walisanna kalakakarem nyazkisesiye kiri		- /
ende growe growing to this is transmissed as filliage in the Section of the Secti	Corretness tollows from the previous discussion	
	12cm dime is O(h) again (O(a) Transplant	
	mest one T	
		,