# **Quantum Oracle: Definition**

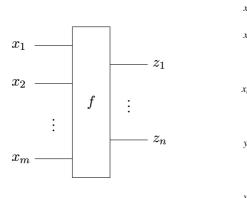
Episode 26. Classical Oracle

**Episode 27. Quantum Oracle: Definition** 

Episode 28. Quantum Oracle: Properties

# Classical Oracle: Review

#### Classical Oracle



**Figure 1. (Left)** A circuit diagram of classical oracle  $f:\{0,1\}^m \to \{0,1\}^n$ . x is an m-bit string,  $x \in \{0,1\}^m$ , and z denotes the image of f at x,  $z = f(x) \in \{0,1\}^n$ . (**Right**) A reversible version of classical oracle  $f:\{0,1\}^m \to \{0,1\}^n$ .  $x \in \{0,1\}^m$  and  $y \in \{0,1\}^n$  are m-bit and n-bit strings, respectively, and  $z = f(x) \in \{0,1\}^n$  denotes the image of f at x.

```
In[0]:= $m = 3;
    $n = 2;
In[0]:= f[1] = f[2] = 3;
    f[7] = 2;
    f[_Integer] = 0;
```

 $-y_n \oplus z_n$ 

```
In[.]:= ZZ = ff /@ XX
Out[0]=
          \{\{0,0\},\{1,1\},\{1,1\},\{0,0\},\{0,0\},\{0,0\},\{0,0\},\{1,0\}\}\}
  In[•]:= Thread[xx → zz] // TableForm
Out[•]//TableForm=
          \{0, 0, 0\} \rightarrow \{0, 0\}
          \{0, 0, 1\} \rightarrow \{1, 1\}
          \{0, 1, 0\} \rightarrow \{1, 1\}
          \{0, 1, 1\} \rightarrow \{0, 0\}
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          \{1, 0, 1\} \rightarrow \{0, 0\}
          \{1, 1, 0\} \rightarrow \{0, 0\}
          \{1, 1, 1\} \rightarrow \{1, 0\}
```

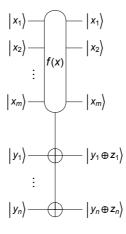
# **Quantum Oracle**

#### Definition

The quantum oracle corresponding to the classical oracle f is simply an implementation of the above extended mapping for reversible computation on quantum registers. It is a quantum gate operation defined by the association

$$U_f: |x\rangle \otimes |y\rangle \mapsto |x\rangle \otimes |f(x) \oplus y\rangle,$$

where  $|x\rangle$  and  $|y\rangle$  are the computational basis states belonging to the native and auxiliary register of *m* and *n* qubits, respectively.



**Figure 3.** A quantum circuit for the quantum oracle corresponding to classical oracle  $f:\{0,1\}^m \to \{0,1\}^n$ , which is a direct analogue of the classical reversible oracle in Figure 2.  $x \in \{0, 1\}^m$  and  $y \in \{0, 1\}^n$  is m-bit and n-bit strings, respectively, and  $z = f(x) \in \{0, 1\}^n$ denotes the image of f at x.

- Since the extended mapping is one-to-one and the computational basis states are orthonormal, the operator  $U_f$  is unitary.
- $\blacksquare$  Recall that  $U_f$  is a linear operator and can act on any arbitrary superposition states.

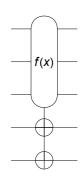
### Example

```
In[*]:= Let[Qubit, S, T]
 In[0]:= SS = S[Range[$m], $];
       TT = T[Range[$n], $];
 in[*]:= op = Oracle[f, SS, TT]
Out[0]=
       Oracle[f, \{S_1, S_2, S_3\}, \{T_1, T_2\}]
```

**Note**: Oracle  $[f, \{c_1, c_2, ...\}, \{t_1, t_2, ...\}]$  represents quantum oracle while Oracle [f, m, n] refers to the classical oracle.

```
In[o]:= qc = QuantumCircuit[op]
```

Out[0]=



```
In[*]:= in = Basis[SS, TT];
     ProductForm[in, {SS, TT}]
```

```
Out[0]=
                                     \{ \mid 000 \rangle \otimes \mid 00 \rangle, \mid 000 \rangle \otimes \mid 01 \rangle, \mid 000 \rangle \otimes \mid 10 \rangle, \mid 000 \rangle \otimes \mid 11 \rangle, \mid 001 \rangle \otimes \mid 00 \rangle, \mid 001 \rangle \otimes \mid 01 \rangle,
                                                |001\rangle\otimes|10\rangle, |001\rangle\otimes|11\rangle, |010\rangle\otimes|00\rangle, |010\rangle\otimes|01\rangle, |010\rangle\otimes|10\rangle, |010\rangle\otimes|11\rangle,
                                               |\hspace{.06cm}011\hspace{.02cm}\rangle \otimes |\hspace{.06cm}00\hspace{.02cm}\rangle , |\hspace{.06cm}011\hspace{.02cm}\rangle \otimes |\hspace{.06cm}01\hspace{.02cm}\rangle , |\hspace{.06cm}011\hspace{.02cm}\rangle \otimes |\hspace{.06cm}10\hspace{.02cm}\rangle , |\hspace{.06cm}011\hspace{.02cm}\rangle \otimes |\hspace{.06cm}11\hspace{.02cm}\rangle , |\hspace{.06cm}100\hspace{.02cm}\rangle \otimes |\hspace{.06cm}00\hspace{.02cm}\rangle ,
                                                |100\rangle \otimes |01\rangle, |100\rangle \otimes |10\rangle, |100\rangle \otimes |11\rangle, |101\rangle \otimes |00\rangle, |101\rangle \otimes |01\rangle,
                                               |\hspace{.06cm} 101 \rangle \otimes |\hspace{.06cm} 10 \rangle , |\hspace{.06cm} 101 \rangle \otimes |\hspace{.06cm} 11 \rangle , |\hspace{.06cm} 110 \rangle \otimes |\hspace{.06cm} 00 \rangle , |\hspace{.06cm} 110 \rangle \otimes |\hspace{.06cm} 01 \rangle , |\hspace{.06cm} 110 \rangle \otimes |\hspace{.06cm} 10 \rangle ,
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```
\label{eq:out} \begin{array}{l} \textit{In} [\circ] := & \textit{out} = \textit{op} * \star \textit{in}; \\ \textit{ProductForm} [\textit{out}, \{SS, TT\}] \\ \\ \textit{Out} [\circ] := & \\ & \left\{ \left| \ 0000 \right> \otimes \left| \ 000 \right>, \ \left| \ 0000 \right> \otimes \left| \ 010 \right>, \ \left| \ 0000 \right> \otimes \left| \ 110 \right>, \ \left| \ 0010 \right> \otimes \left| \ 110 \right>, \ \left| \ 0010 \right> \otimes \left| \ 110 \right>, \ \left| \ 0010 \right> \otimes \left| \ 110 \right>, \ \left| \ 0010 \right> \otimes \left| \ 110 \right>, \ \left| \ 0100 \right> \otimes \left| \ 010 \right>, \ \left| \ 0100 \right> \otimes \left| \ 000 \right>, \ \left| \ 0110 \right> \otimes \left| \ 000 \right>, \ \left| \ 0110 \right> \otimes \left| \ 010 \right> \otimes \left| \ 010 \right>, \ \left| \ 0110 \right> \otimes \left| \ 000 \right>, \ \left| \ 0110 \right> \otimes \left| \ 010 \right>, \ \left| \ 0110 \right> \otimes \left| \ 010 \right>, \ \left| \ 0110 \right> \otimes \left| \ 010 \right>, \ \left| \ 0110 \right> \otimes \left| \ 010 \right>, \ \left| \ 0110 \right> \otimes \left| \ 010 \right>, \ \left| \ 0110 \right> \otimes \left| \ 010 \right>, \ \left| \ 1110 \right> \otimes \left| \ 010 \right>, \ \left| \ 1110 \right> \otimes \left| \ 110 \right>, \ \left| \ 1110 \right> \otimes \left| \ 010 \right>, \ \left| \ 1111 \right> \otimes \left| \ 011 \right>, \ \left| \ 1111 \right> \otimes \left| \ 011 \right>, \ \left| \ 1111 \right> \otimes \left| \ 011 \right>, \ \left| \ 1111 \right> \otimes \left| \ 011 \right>, \ \left| \ 1111 \right> \otimes \left| \ 011 \right>, \ \left| \ 1111 \right> \otimes \left| \ 011 \right>, \ \left| \ 1111 \right> \otimes \left| \ 011 \right>, \ \left| \ 1111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 011 \right>, \ \left| \ 0111 \right> \otimes \left| \ 0111 \right>, \ \left| \ 0111 \right> \otimes \left| \ 0111 \right>, \ \left| \ 0111 \right> \otimes \left| \ 0111 \right>, \ \left| \ 0111 \right> \otimes \left| \ 0111 \right>, \ \left| \ 0111 \right> \otimes \left| \ 0111 \right>, \ \left| \ 0111 \right> \otimes \left| \ 0111 \right>, \ \left| \ 01111 \right> \otimes \left| \ 0111 \right> \otimes \left| \ 0111 \right>, \ \left| \ 01111 \right> \otimes \left| \ 0111 \right>, \ \left| \ 01111 \right> \otimes \left| \
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#### In[⊕]:= ProductForm[Thread[in → out], {SS, TT}] // TableForm

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#### $In[ \circ ] := Thread[xx \rightarrow zz] // TableForm$

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Out[•]//TableForm=
            \{0, 0, 0\} \rightarrow \{0, 0\}
            \{0, 0, 1\} \rightarrow \{1, 1\}
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            \{1, 1, 0\} \rightarrow \{0, 0\}
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# **Summary**

## Keywords

- Oracle
- Quantum oracle
- Quantum decision making

### **Functions**

- Oracle
- ControlledExp

### **Related Links**

- Section 4.2 of the Quantum Workbook (2022, 2023).
- Tutorial: Quantum Oracle
- Tutorial: Quantum Decision Algorithms