# Quantum Circuit

In[ ]:=
```
Quit[]
```

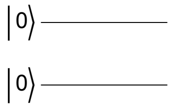
In[ ]:=
```
<< Q3`
```

In[ ]:=
```
Let[Qubit, S]
```

## Input

In[ ]:=
```
qc = QuantumCircuit[Ket[S@{1, 2}]]
```

Out[ ]=



In[ ]:=
```
Elaborate[qc]
```

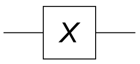
Out[ ]=

$\left|0_{S_1}0_{S_2}\right\rangle$

## Single-Qubit Operators

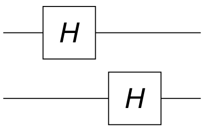
In[ ]:=
```
qc = QuantumCircuit[S[1, 1]]
```

Out[ ]=



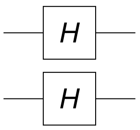

In[ ]:=
```
qc = QuantumCircuit[S[1, 6], S[2, 6]]
```

Out[ ]=



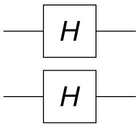

In[ ]:=
```
qc = QuantumCircuit[{S[1, 6], S[2, 6]}]
```

Out[ ]=



In[ ]:= `qc = QuantumCircuit[S[{1, 2}, 6]]`

Out[ ]=




In[ ]:= `op = Elaborate[qc]`

Out[ ]=

$$\frac{1}{2}\, S_1^x S_2^x + \frac{1}{2}\, S_1^x S_2^z + \frac{1}{2}\, S_1^z S_2^x + \frac{1}{2}\, S_1^z S_2^z$$

In[ ]:= `S[1, 6] ** S[2, 6]`

Out[ ]=

$$S_1^H S_2^H$$

In[ ]:=
```
mat = Matrix[qc];
MatrixForm[mat]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

---

In[ ]:= `out = qc ** Ket[]`

Out[ ]=

$$\frac{1}{2}\, \left|0_{S_1} 0_{S_2}\right\rangle + \frac{1}{2}\, \left|0_{S_1} 1_{S_2}\right\rangle + \frac{1}{2}\, \left|1_{S_1} 0_{S_2}\right\rangle + \frac{1}{2}\, \left|1_{S_1} 1_{S_2}\right\rangle$$

In[ ]:= `Basis[S@{1, 2}]`

Out[ ]=

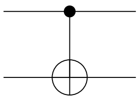
$$\left\{\left|0_{S_1} 0_{S_2}\right\rangle,\ \left|0_{S_1} 1_{S_2}\right\rangle,\ \left|1_{S_1} 0_{S_2}\right\rangle,\ \left|1_{S_1} 1_{S_2}\right\rangle\right\}$$

# Two-Qubit Operators

## CNOT (Controlled-NOT)

In[ ]:= `qc = QuantumCircuit[CNOT[S[1], S[2]]]`

Out[ ]=



In[ ]:= `in = Basis[S@{1, 2}]`

Out[ ]=

$$\left\{\left|0_{S_1} 0_{S_2}\right\rangle,\ \left|0_{S_1} 1_{S_2}\right\rangle,\ \left|1_{S_1} 0_{S_2}\right\rangle,\ \left|1_{S_1} 1_{S_2}\right\rangle\right\}$$

In[∘]:=
```
out = qc ** in
```
Out[∘]=

$\{|0_{S_1}0_{S_2}\rangle, |0_{S_1}1_{S_2}\rangle, |1_{S_1}1_{S_2}\rangle, |1_{S_1}0_{S_2}\rangle\}$

In[∘]:=
```
Thread[in → out] // TableForm
```
Out[∘]//TableForm=

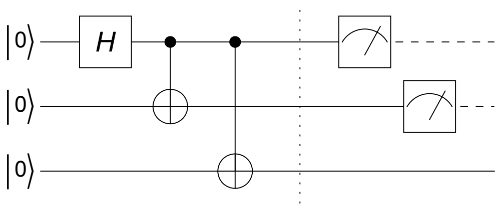
$$\begin{array}{l} |0_{S_1}0_{S_2}\rangle \to |0_{S_1}0_{S_2}\rangle \\ |0_{S_1}1_{S_2}\rangle \to |0_{S_1}1_{S_2}\rangle \\ |1_{S_1}0_{S_2}\rangle \to |1_{S_1}1_{S_2}\rangle \\ |1_{S_1}1_{S_2}\rangle \to |1_{S_1}0_{S_2}\rangle \end{array}$$

In[∘]:=
```
Matrix[qc] // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

# Measurements

In[∘]:=
```
qc = QuantumCircuit[Ket[S@{1, 2, 3}],
   S[1, 6], CNOT[S[1], S[2]], CNOT[S[1], S[3]], "Separator",
   Measurement[S[1, 3]], Measurement[S[2, 3]]]
```
Out[∘]=



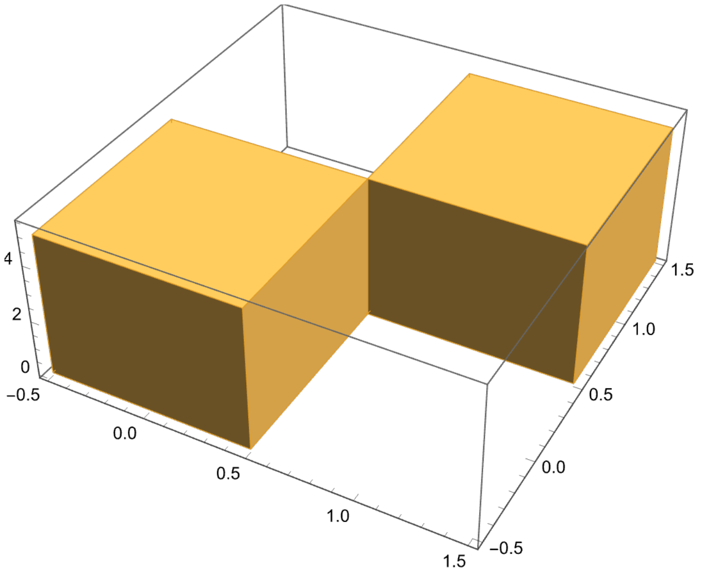

In[∘]:=
```
out = Elaborate[qc]
```
Out[∘]=

$|0_{S_1}0_{S_2}0_{S_3}\rangle$

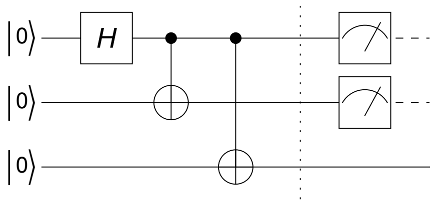
In[∘]:=
```
data = Table[Elaborate[qc]; Readout[S[{1, 2}, 3]], 10]
```
Out[∘]=

{{1, 1}, {0, 0}, {0, 0}, {0, 0}, {1, 1}, {1, 1}, {1, 1}, {0, 0}, {0, 0}, {1, 1}}

In[ ]:=
```
Histogram3D[data]
```
Out[ ]=

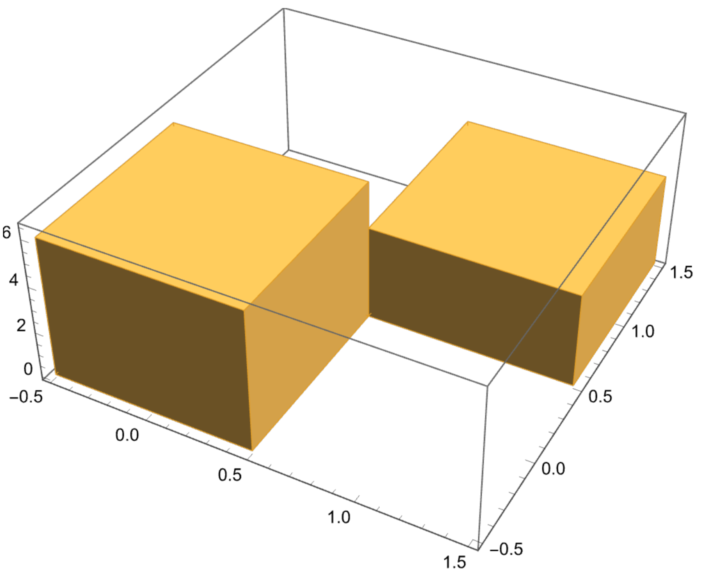



In[ ]:=
```
qc = QuantumCircuit[Ket[S@{1, 2, 3}],
  S[1, 6], CNOT[S[1], S[2]], CNOT[S[1], S[3]], "Separator",
  Measurement[S[{1, 2}, 3]]]
```
Out[ ]=



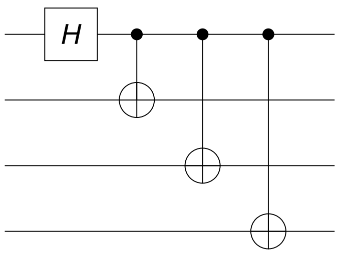

In[ ]:=
```
data = Table[Elaborate[qc]; Readout[S[{1, 2}, 3]], 10]
```
Out[ ]=
```
{{1, 1}, {0, 0}, {0, 0}, {1, 1}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {1, 1}, {1, 1}}
```

In[ ]:=
```
Histogram3D[data]
```
Out[ ]=



# List vs Sequence

We want to generate the following quantum circuit for different numbers of target qubits.




What should we do?

```
In[∘]:= Let[Qubit, S, T]
```

```
In[∘]:= $n = 3;
TT = T[Range[$n], $]
```

Out[∘]=

$\{T_1, T_2, T_3\}$

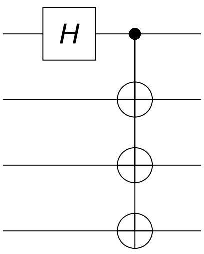
```
In[∘]:= cn = Table[CNOT[S[1], q], {q, TT}];
cn // TableForm
```

Out[∘]//TableForm=

CNOT[$\{S_1\} \to \{1\}$, $\{T_1\}$]
CNOT[$\{S_1\} \to \{1\}$, $\{T_2\}$]
CNOT[$\{S_1\} \to \{1\}$, $\{T_3\}$]

```
In[∘]:= qc = QuantumCircuit[S[1, 6], cn]
```

Out[∘]=

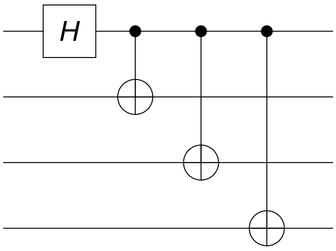



```
In[∘]:= cn
```

Out[∘]=

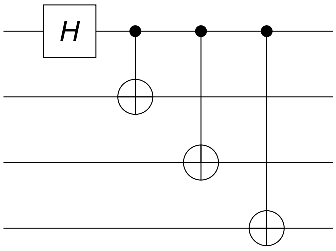
{CNOT[$\{S_1\} \to \{1\}$, $\{T_1\}$], CNOT[$\{S_1\} \to \{1\}$, $\{T_2\}$], CNOT[$\{S_1\} \to \{1\}$, $\{T_3\}$]}

In[ ]:=
```
QuantumCircuit[S[1, 6], CNOT[S[1], T[1]], CNOT[S[1], T[2]], CNOT[S[1], T[3]]]
```
Out[ ]=




Is there any simpler way?

In[ ]:=
```
qc = QuantumCircuit[S[1, 6], Sequence @@ cn]
```
Out[ ]=




`List`, `Sequence`, and `@@`.

# Summary

## Functions

- `QuantumCircuit`
- `Elaborate`, `ExpressionFor`
- `Matrix`, `MatrixForm`
- `Apply`

## Related Links

- Chapter 2 of the Quantum Workbook (2022, 2023).
- Tutorial: “Quantum Computation: Overview”