Mahn-Soo Choi (Korea University)

A Quantum Computation Workbook

February 4, 2022

Springer

For students

Notice

N.B. This book will be published with Springer on February 1, 2022. This draft is provided for educational purposes with the permission of Springer. Redistribution of this draft is strictly prohibited.

N.B. Two digital materials accompany the book. Both are freely available through the respective GitHub repositories at the links below.

- Q3, a Mathematica application (Appendix D):
  `https://github.com/quantum-mob/Q3App`
- QuantumWorkbook, a compilation of the demonstrations in this book written in the Wolfram Language (Appendix E):
  `https://github.com/quantum-mob/QuantumWorkbook`

Preface

This book is intended as an introductory text for a university course on quantum computation and as a self-learning guide. It is an attempt to collect some fundamental principles and elementary methods in the field of quantum computation and quantum information and then reorganize them in a compact and integrated form. A proper introduction to (and hence learning of) quantum computation and quantum information turns out to be demanding, especially to undergraduate students, because it needs to cover various subjects from different fields of science including the physics of quantum mechanics, the mathematics of advanced linear algebra, and the computer science of information theory, to list the least. Each field of science usually favors its specific language, so another layer of difficulties is to organize the subjects consistently in a unified language.

Two digital materials accompany the book. Both are freely available through the respective GitHub repositories at the links below.

- Q3, a Mathematica application (Appendix D):
  `https://github.com/quantum-mob/Q3App`
- QuantumWorkbook, a compilation of the demonstrations in this book written in the Wolfram Language (Appendix E):
  `https://github.com/quantum-mob/QuantumWorkbook`

Q3 consists of tools and utilities that perform symbolic calculations and numerical simulations useful in the study of quantum information processing, quantum many-body systems, and quantum spin systems. With Q3, one can avoid many of the tedious calculations involved in various principles and theorems of quantum theory. Furthermore, numerous visualization and simulation tools can help deepen the understanding of core concepts. *QuantumWorkbook* is a compilation of Mathematica Notebook files that contain the code used to generate the demonstrations in the book. Readers themselves can run and modify the code to build their own examples from the demonstrations and to experiment with fresh ideas. Both materials can be helpful companions, particularly in a course of self-learning, of the various subjects of quantum physics present in the book and for the general study of the overall subject as well.

This book is a result of several years of experience in teaching quantum computation and quantum information. It is a helpful resource to select and present particular topics of the subject at the undergraduate level. Although the book aims to be self-contained, it nonetheless assumes some basic knowledge of quantum mechanics. Chapter 1 summarizes the fundamental postulates of quantum mechanics and effectively provides a brief review of basic concepts and fundamental principles of quantum mechanics. Chapter 2 presents and describes the properties of elementary quantum gates for universal quantum computation. These are the building blocks of quantum algorithms and quantum communication protocols. Chapter 3 discusses physical methods and principles to implement elementary quantum gates and also introduces different quantum computation schemes. Chapter 4 introduces some widely known quantum algorithms to help grasp the idea of the so-called 'quantum supremacy' of quantum algorithms over their classical counterparts. Chapter 5 is dedicated to decoherence effects and introduces mathematical methods including the Kraus representation and the Lindblad equation to describe the phenomena. Chapter 6 is devoted to quantum error-correction codes through a discussion of the basic principles, procedures, and examples. Chapter 7 introduces quantum information theory. It discusses distance measures for quantum information, measures for quantum entanglement degree, and entropies of information content. To maintain a coherent structure and to focus on the primary topics, we collect the corresponding mathematical theories in the appendices and refer to them from the main text as necessary.

The author is indebted to students in his classes for pointing out numerous mistakes and typographical errors in the manuscript. Many people have contributed to the development of Q3 by testing and actively using it. The author gives particular thanks to Ha-Eum Kim, Myeongwon Lee, and Su-Ho Choi for their energetic discussions and constructive feedback in the early stages of the development of Q3. He also appreciates bug reports and valuable comments by Boris Laurent, Mi Jung So, Yeong-ho Je, and Dongni Chen.

Seoul, Korea
August 2021 *Mahn-Soo Choi*

Contents