

# Quantum Measurement

In[\*]:= << Q3`

## Pauli Operators

In[\*]:= Let[Qubit, S]

In[\*]:= in = Ket[S[1] → 0] - I \* Ket[S[1] → 1]

Out[\*]=

$$|0_{S_1}\rangle - i |1_{S_1}\rangle$$

In[\*]:= out = Measurement[S[1, 3]][in]

Out[\*]=

$$-i |1_{S_1}\rangle$$

In[\*]:= out = Measurement[S[1, 3]] \*\* in

Out[\*]=

$$|0_{S_1}\rangle$$

In[\*]:= out = Table[Measurement[S[1, 3]] \*\* in, 10]

Out[\*]=

$$\{-i |1_{S_1}\rangle, |0_{S_1}\rangle, |0_{S_1}\rangle, |0_{S_1}\rangle, -i |1_{S_1}\rangle, -i |1_{S_1}\rangle, |0_{S_1}\rangle, -i |1_{S_1}\rangle, |0_{S_1}\rangle, |0_{S_1}\rangle\}$$

In[\*]:= out = Measurement[S[1, 3]] \*\* in

Out[\*]=

$$-i |1_{S_1}\rangle$$

In[\*]:= Readout[S[1, 3]]

Out[\*]=

1

In[\*]:= data = Table[  
    Measurement[S[1, 3]] \*\* in; Readout[S[1, 3]],  
    10]

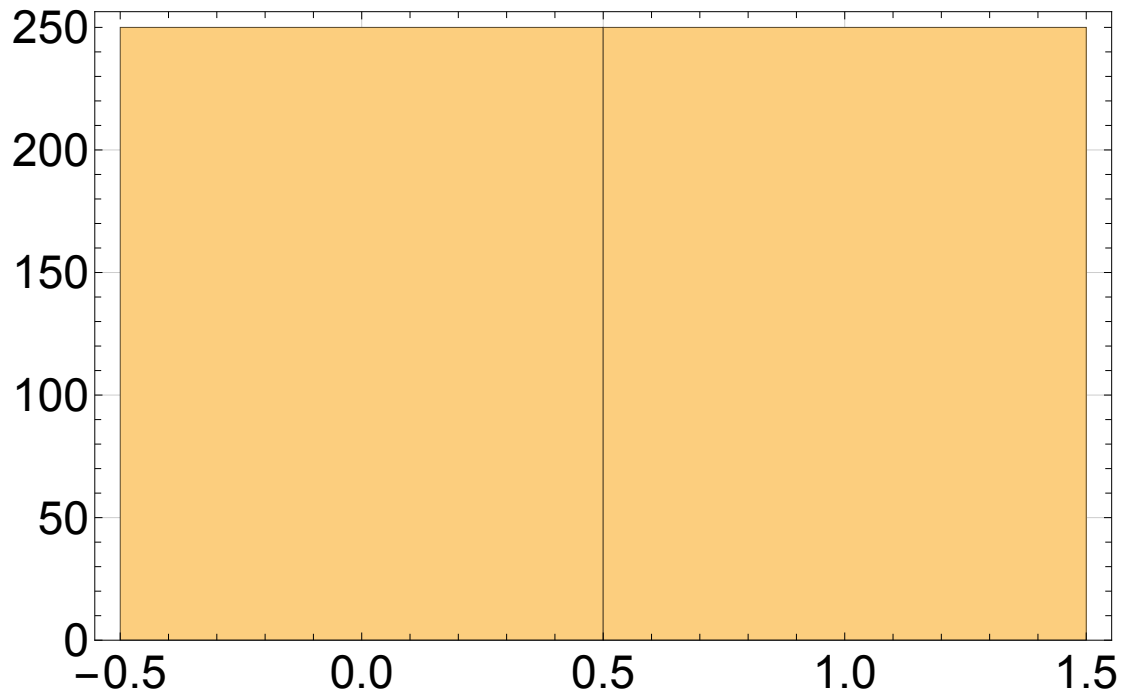
Out[\*]=

{1, 1, 0, 1, 1, 1, 0, 0, 0, 0}

In[\*]:= data = Table[  
    Measurement[S[1, 3]] \*\* in; Readout[S[1, 3]],  
    500];

```
In[*]:= Histogram[data]
```

```
Out[*]=
```



```
In[*]:= in = 2 * Ket[S[1] -> 0] - I * Ket[S[1] -> 1]
```

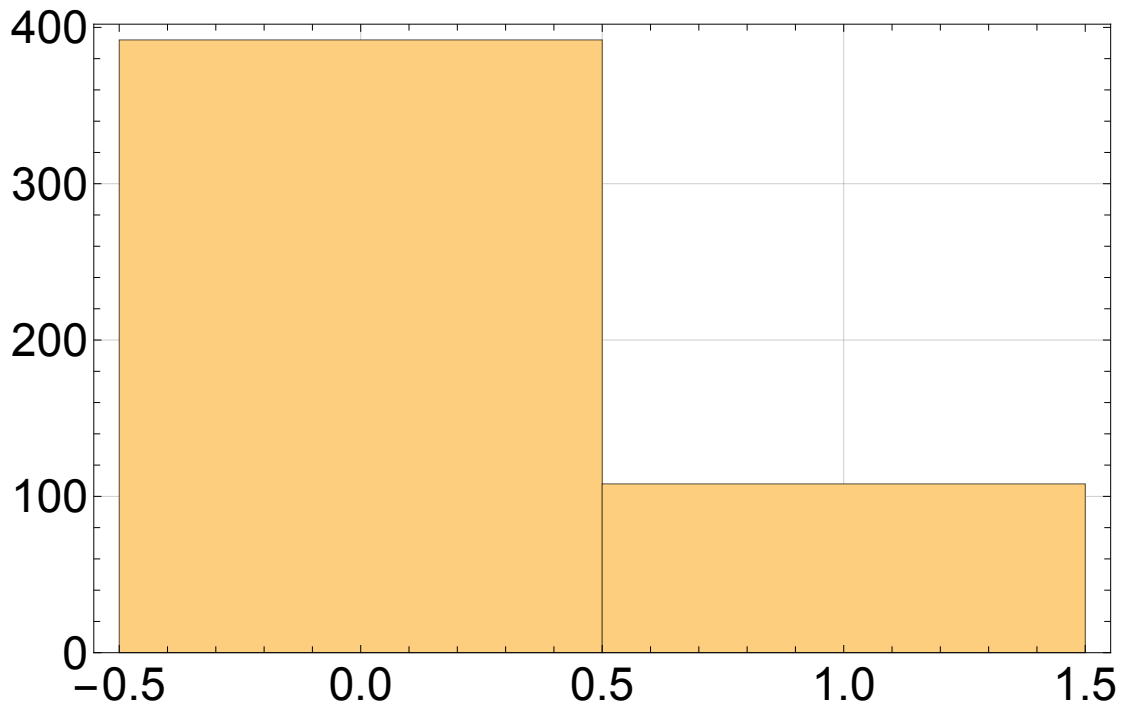
```
Out[*]=
```

$$2 \left| 0_{S_1} \right\rangle - i \left| 1_{S_1} \right\rangle$$

```
In[*]:= data = Table[
  Measurement[S[1, 3]] ** in; Readout[S[1, 3]],
  500];
```

```
In[*]:= Histogram[data]
```

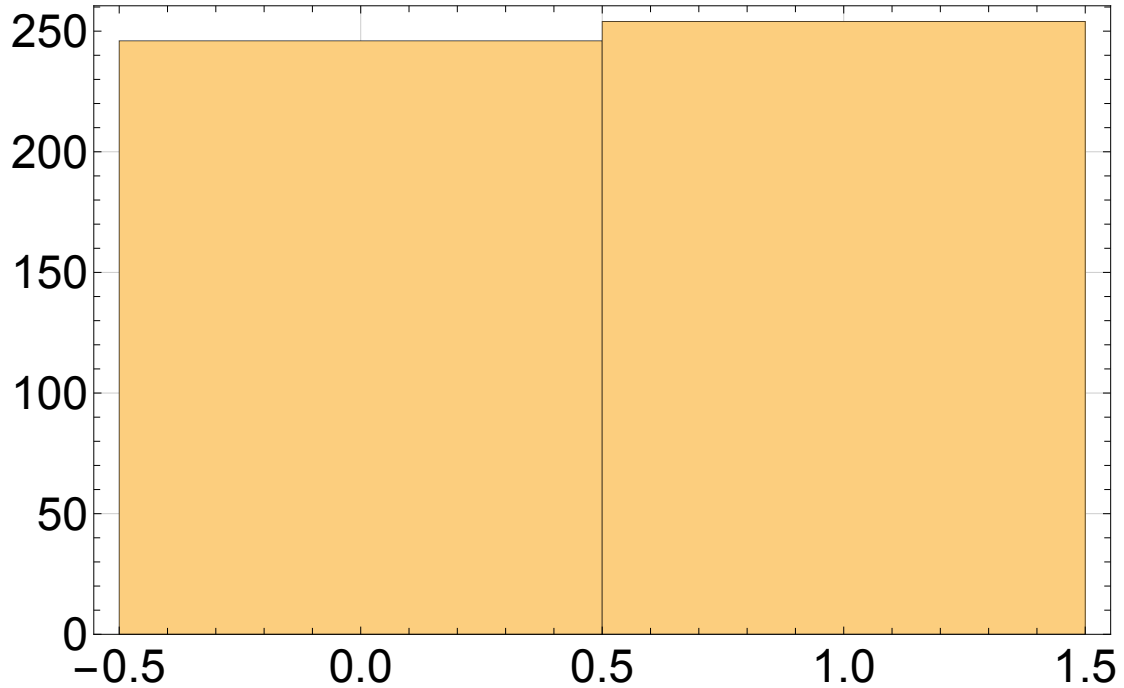
```
Out[*]=
```



```
In[*]:= data = Table[
  Measurement[S[1, 1]] ** in; Readout[S[1, 1]],
  500];
```

```
In[*]:= Histogram[data]
```

```
Out[*]=
```



## Analysis before Measurement

```
In[*]:= in = Ket[S[1] → 0] - I * Ket[S[1] → 1]
```

```
Out[*]=
```

$$\begin{vmatrix} 0_{S_1} \\ -i \\ 1_{S_1} \end{vmatrix}$$

```
In[*]:= Measurement[S[1, 3]] ** in
```

```
Out[*]=
```

$$\begin{vmatrix} -i \\ 1_{S_1} \end{vmatrix}$$

```
In[*]:= data = MeasurementOdds[S[1, 3]] [in]
```

```
Out[*]=
```

$$\left\langle \begin{vmatrix} 0 \rightarrow \left\{ \frac{1}{2}, \begin{vmatrix} 0_{S_1} \end{vmatrix} \end{vmatrix}, 1 \rightarrow \left\{ \frac{1}{2}, -i \begin{vmatrix} 1_{S_1} \end{vmatrix} \end{vmatrix} \right\} \right\rangle$$

```
In[*]:= data = MeasurementOdds[S[1, 2]] [in]
```

```
Out[*]=
```

$$\left\langle \begin{vmatrix} 0 \rightarrow \{0, 0\}, 1 \rightarrow \left\{ 1, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}} \right\} \right\rangle$$

```
In[*]:= out = Table[Measurement[S[1, 2]] ** in, 20]
```

```
Out[*]=
```

$$\left\{ \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \right. \\ \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \\ \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \\ \left. \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}}, \frac{\begin{vmatrix} 0_{S_1} \end{vmatrix}}{\sqrt{2}} - \frac{i \begin{vmatrix} 1_{S_1} \end{vmatrix}}{\sqrt{2}} \right\}$$

```
In[*]:= YBasisForm[out, S[1]]
```

```
Out[*]=
```

$$\left\{ \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \right. \\ \left. \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix}, \begin{vmatrix} R_{S_1} \end{vmatrix} \right\}$$

## Consecutive Measurements

```
In[*]:= in = Ket[] + Ket[S[1] → 1] + Ket[S[2] → 1] // KetRegulate
```

```
Out[*]=
```

$$\begin{vmatrix} 0_{S_1} 0_{S_2} \end{vmatrix} + \begin{vmatrix} 0_{S_1} 1_{S_2} \end{vmatrix} + \begin{vmatrix} 1_{S_1} 0_{S_2} \end{vmatrix}$$

```
In[*]:= out = Measurement[{S[1, 3], S[2, 3]]} ** in
```

```
Out[*]=
```

$$\begin{vmatrix} 1_{S_1} 0_{S_2} \end{vmatrix}$$

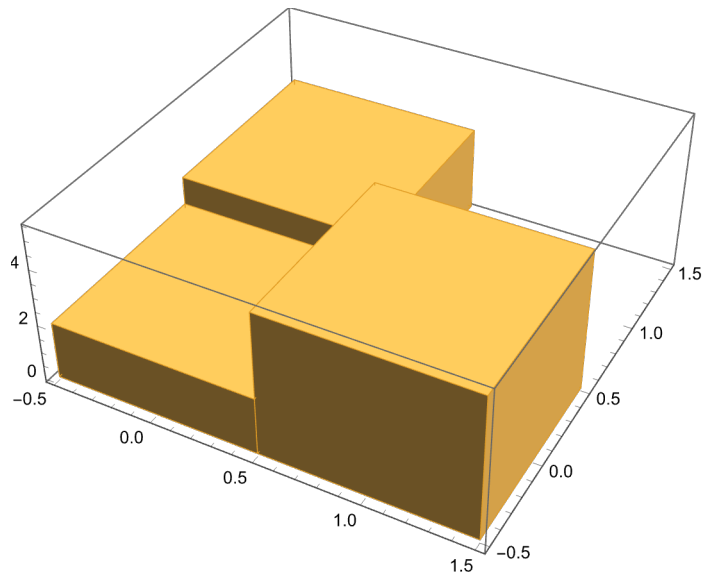
```

In[*]:= data = Table[
  Measurement[{S[1, 3], S[2, 3]}] ** in;
  Readout[{S[1, 3], S[2, 3]}],
  10]

Out[*]=
{{0, 0}, {1, 0}, {0, 1}, {0, 1}, {1, 0}, {1, 0}, {0, 0}, {1, 0}, {0, 1}, {1, 0}}

In[*]:= Histogram3D[data]
Out[*]=

```



## Collective Measurements

```

In[*]:= in = Ket[] + Ket[S[1] -> 1] + Ket[S[2] -> 1] // KetRegulate
Out[*]=
|0_{S_1}0_{S_2}> + |0_{S_1}1_{S_2}> + |1_{S_1}0_{S_2}>

In[*]:= out = Measurement[S[1, 3] ** S[2, 3]] ** in
Out[*]=
|0_{S_1}0_{S_2}>

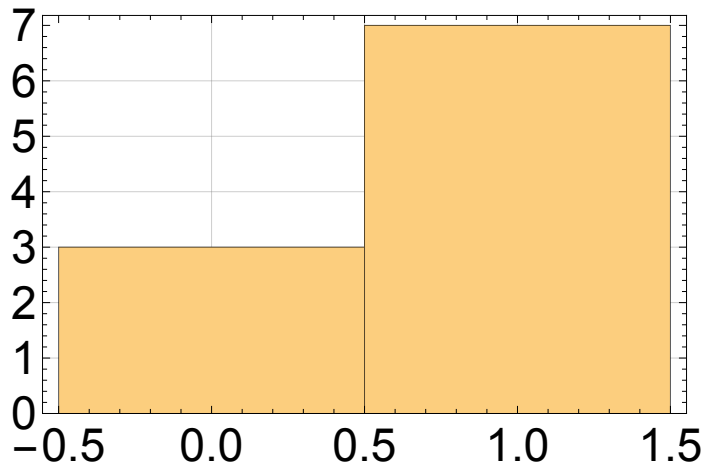
In[*]:= data = Table[
  Measurement[S[1, 3] ** S[2, 3]] ** in;
  Readout[S[1, 3] ** S[2, 3]],
  10]

Out[*]=
{1, 1, 1, 0, 1, 0, 1, 1, 1, 0}

```

```
In[*]:= Histogram[data, ImageSize → Medium]
```

```
Out[*]=
```



---

## Summary

### Functions

- `Measurement`
- `Readout`
- `MeasurementOdds`

### Related Links

- Chapter 1 of the Quantum Workbook (2022, 2023).
- Tutorial: “Measurements on Quantum States”