

Working in Different Bases: MatrixIn & ExpressionIn

Example: A Chain of Qubits

Consider a quantum register of qubits, referred to by symbol S .

```
In[*]:= Let[Qubit, S]

In[*]:= $L = 4;
ss = S[Range@$L, $]

Out[*]=
{S1, S2, S3, S4}
```

```
In[*]:= xx = Total@ChainBy[Through[ss[1]], Multiply];
yy = Total@ChainBy[Through[ss[2]], Multiply];
zz = Total@ChainBy[Through[ss[3]], Multiply];
H = xx + yy + zz

Out[*]=
S1x S2x + S1y S2y + S1z S2z + S2x S3x + S2y S3y + S2z S3z + S3x S4x + S3y S4y + S3z S4z
```

The Hamiltonian has so many zeros. That is why matrix representations are treated in `SparseArray` form in Q3.

```
In[*]:= old = Matrix[H];
old // MatrixForm

Out[*] // MatrixForm =
```

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & -3 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & -3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

MatrixIn & ExpressionIn

In[*]:= **bs = QubitAdd[ss]**

Out[*]=

$$\begin{aligned}
 & \langle \{0, 0\} \rightarrow \left\{ \frac{1}{2} |\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle - \frac{1}{2} |\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle - \frac{1}{2} |\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle + \frac{1}{2} |\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle, \right. \\
 & \quad \frac{|\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{3}} - \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle}{2\sqrt{3}} - \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle}{2\sqrt{3}} - \\
 & \quad \left. \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle}{2\sqrt{3}} - \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle}{2\sqrt{3}} + \frac{|\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{3}} \right\}, \{1, -1\} \rightarrow \\
 & \quad \left\{ \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{2}} - \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{2}}, \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{6}} + \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{6}} - \sqrt{\frac{2}{3}} |\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle, \right. \\
 & \quad \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{2\sqrt{3}} + \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{2\sqrt{3}} + \frac{|\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle}{2\sqrt{3}} - \frac{1}{2} \sqrt{3} |\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle \Big\}, \\
 & \{1, 0\} \rightarrow \left\{ \frac{1}{2} |\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle + \frac{1}{2} |\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle - \frac{1}{2} |\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle - \frac{1}{2} |\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle, \right. \\
 & \quad \frac{|\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{3}} - \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle}{2\sqrt{3}} + \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle}{2\sqrt{3}} - \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle}{2\sqrt{3}} + \\
 & \quad \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle}{2\sqrt{3}} - \frac{|\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{3}}, \frac{|\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{6}} + \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{6}} - \\
 & \quad \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{6}} + \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{6}} - \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{6}} - \frac{|\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{6}} \Big\}, \{1, 1\} \rightarrow \\
 & \quad \left\{ \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{2}} - \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{2}}, \sqrt{\frac{2}{3}} |\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle - \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{6}} - \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{6}}, \right. \\
 & \quad \left. \frac{1}{2} \sqrt{3} |\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle - \frac{|\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle}{2\sqrt{3}} - \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{2\sqrt{3}} - \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{2\sqrt{3}} \right\}, \\
 & \{2, -2\} \rightarrow \{ |\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle \}, \{2, -1\} \rightarrow \\
 & \quad \left\{ \frac{1}{2} |\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle + \frac{1}{2} |\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle + \frac{1}{2} |\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle + \frac{1}{2} |\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle \right\}, \\
 & \{2, 0\} \rightarrow \left\{ \frac{|\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{6}} + \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{6}} + \frac{|\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{6}} + \right. \\
 & \quad \left. \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle}{\sqrt{6}} + \frac{|\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{6}} + \frac{|\mathbf{1}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle}{\sqrt{6}} \right\}, \\
 & \{2, 1\} \rightarrow \left\{ \frac{1}{2} |\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{1}_{S_4}\rangle + \frac{1}{2} |\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \mathbf{0}_{S_4}\rangle + \frac{1}{2} |\mathbf{0}_{S_1} \mathbf{1}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle + \frac{1}{2} |\mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle \right\}, \\
 & \{2, 2\} \rightarrow \{ |\mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \mathbf{0}_{S_4}\rangle \} \Big|
 \end{aligned}$$

```
In[*]:= Keys[bs]
```

```
Out[*]=
```

```
{ {0, 0}, {1, -1}, {1, 0}, {1, 1}, {2, -2}, {2, -1}, {2, 0}, {2, 1}, {2, 2} }
```

```
In[*]:= GroupBy[Keys[bs], First] // Normal // TableForm
```

```
Out[*] // TableForm =
```


```
0 → { {0, 0} }
1 → { {1, -1}, {1, 0}, {1, 1} }
2 → { {2, -2}, {2, -1}, {2, 0}, {2, 1}, {2, 2} }
```

```
In[*]:= bb = Catenate[bs]
```

```
Out[*]=
```

$$\begin{aligned}
& \left\{ \frac{1}{2} \left| 0_{S_1} 1_{S_2} 0_{S_3} 1_{S_4} \right\rangle - \frac{1}{2} \left| 0_{S_1} 1_{S_2} 1_{S_3} 0_{S_4} \right\rangle - \frac{1}{2} \left| 1_{S_1} 0_{S_2} 0_{S_3} 1_{S_4} \right\rangle + \frac{1}{2} \left| 1_{S_1} 0_{S_2} 1_{S_3} 0_{S_4} \right\rangle, \frac{\left| 0_{S_1} 0_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{\sqrt{3}} - \right. \\
& \quad \frac{\left| 0_{S_1} 1_{S_2} 0_{S_3} 1_{S_4} \right\rangle}{2\sqrt{3}} - \frac{\left| 0_{S_1} 1_{S_2} 1_{S_3} 0_{S_4} \right\rangle}{2\sqrt{3}} - \frac{\left| 1_{S_1} 0_{S_2} 0_{S_3} 1_{S_4} \right\rangle}{2\sqrt{3}} - \frac{\left| 1_{S_1} 0_{S_2} 1_{S_3} 0_{S_4} \right\rangle}{2\sqrt{3}} + \frac{\left| 1_{S_1} 1_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{\sqrt{3}}, \\
& \quad \frac{\left| 0_{S_1} 1_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{\sqrt{2}} - \frac{\left| 1_{S_1} 0_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{\sqrt{2}}, \frac{\left| 0_{S_1} 1_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{\sqrt{6}} + \frac{\left| 1_{S_1} 0_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{\sqrt{6}} - \sqrt{\frac{2}{3}} \left| 1_{S_1} 1_{S_2} 0_{S_3} 1_{S_4} \right\rangle, \\
& \quad \frac{\left| 0_{S_1} 1_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{2\sqrt{3}} + \frac{\left| 1_{S_1} 0_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{2\sqrt{3}} + \frac{\left| 1_{S_1} 1_{S_2} 0_{S_3} 1_{S_4} \right\rangle}{2\sqrt{3}} - \frac{1}{2} \sqrt{3} \left| 1_{S_1} 1_{S_2} 1_{S_3} 0_{S_4} \right\rangle, \\
& \quad \frac{1}{2} \left| 0_{S_1} 1_{S_2} 0_{S_3} 1_{S_4} \right\rangle + \frac{1}{2} \left| 0_{S_1} 1_{S_2} 1_{S_3} 0_{S_4} \right\rangle - \frac{1}{2} \left| 1_{S_1} 0_{S_2} 0_{S_3} 1_{S_4} \right\rangle - \frac{1}{2} \left| 1_{S_1} 0_{S_2} 1_{S_3} 0_{S_4} \right\rangle, \\
& \quad \frac{\left| 0_{S_1} 0_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{\sqrt{3}} - \frac{\left| 0_{S_1} 1_{S_2} 0_{S_3} 1_{S_4} \right\rangle}{2\sqrt{3}} + \frac{\left| 0_{S_1} 1_{S_2} 1_{S_3} 0_{S_4} \right\rangle}{2\sqrt{3}} - \frac{\left| 1_{S_1} 0_{S_2} 0_{S_3} 1_{S_4} \right\rangle}{2\sqrt{3}} + \\
& \quad \frac{\left| 1_{S_1} 0_{S_2} 1_{S_3} 0_{S_4} \right\rangle}{2\sqrt{3}} - \frac{\left| 1_{S_1} 1_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{\sqrt{3}}, \frac{\left| 0_{S_1} 0_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{\sqrt{6}} + \frac{\left| 0_{S_1} 1_{S_2} 0_{S_3} 1_{S_4} \right\rangle}{\sqrt{6}} - \\
& \quad \frac{\left| 0_{S_1} 1_{S_2} 1_{S_3} 0_{S_4} \right\rangle}{\sqrt{6}} + \frac{\left| 1_{S_1} 0_{S_2} 0_{S_3} 1_{S_4} \right\rangle}{\sqrt{6}} - \frac{\left| 1_{S_1} 0_{S_2} 1_{S_3} 0_{S_4} \right\rangle}{\sqrt{6}} - \frac{\left| 1_{S_1} 1_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{\sqrt{6}}, \\
& \quad \frac{\left| 0_{S_1} 1_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{\sqrt{2}} - \frac{\left| 1_{S_1} 0_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{\sqrt{2}}, \sqrt{\frac{2}{3}} \left| 0_{S_1} 0_{S_2} 1_{S_3} 0_{S_4} \right\rangle - \frac{\left| 0_{S_1} 1_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{\sqrt{6}} - \frac{\left| 1_{S_1} 0_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{\sqrt{6}}, \\
& \quad \frac{1}{2} \sqrt{3} \left| 0_{S_1} 0_{S_2} 0_{S_3} 1_{S_4} \right\rangle - \frac{\left| 0_{S_1} 0_{S_2} 1_{S_3} 0_{S_4} \right\rangle}{2\sqrt{3}} - \frac{\left| 0_{S_1} 1_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{2\sqrt{3}} - \frac{\left| 1_{S_1} 0_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{2\sqrt{3}}, \left| 1_{S_1} 1_{S_2} 1_{S_3} 1_{S_4} \right\rangle, \\
& \quad \frac{1}{2} \left| 0_{S_1} 1_{S_2} 1_{S_3} 1_{S_4} \right\rangle + \frac{1}{2} \left| 1_{S_1} 0_{S_2} 1_{S_3} 1_{S_4} \right\rangle + \frac{1}{2} \left| 1_{S_1} 1_{S_2} 0_{S_3} 1_{S_4} \right\rangle + \frac{1}{2} \left| 1_{S_1} 1_{S_2} 1_{S_3} 0_{S_4} \right\rangle, \frac{\left| 0_{S_1} 0_{S_2} 1_{S_3} 1_{S_4} \right\rangle}{\sqrt{6}} + \\
& \quad \frac{\left| 0_{S_1} 1_{S_2} 0_{S_3} 1_{S_4} \right\rangle}{\sqrt{6}} + \frac{\left| 0_{S_1} 1_{S_2} 1_{S_3} 0_{S_4} \right\rangle}{\sqrt{6}} + \frac{\left| 1_{S_1} 0_{S_2} 0_{S_3} 1_{S_4} \right\rangle}{\sqrt{6}} + \frac{\left| 1_{S_1} 0_{S_2} 1_{S_3} 0_{S_4} \right\rangle}{\sqrt{6}} + \frac{\left| 1_{S_1} 1_{S_2} 0_{S_3} 0_{S_4} \right\rangle}{\sqrt{6}}, \\
& \quad \left. \frac{1}{2} \left| 0_{S_1} 0_{S_2} 0_{S_3} 1_{S_4} \right\rangle + \frac{1}{2} \left| 0_{S_1} 0_{S_2} 1_{S_3} 0_{S_4} \right\rangle + \frac{1}{2} \left| 0_{S_1} 1_{S_2} 0_{S_3} 0_{S_4} \right\rangle + \frac{1}{2} \left| 1_{S_1} 0_{S_2} 0_{S_3} 0_{S_4} \right\rangle, \left| 0_{S_1} 0_{S_2} 0_{S_3} 0_{S_4} \right\rangle \right\}
\end{aligned}$$

```
In[ ]:= EchoTiming[
  new = Outer[Multiply, Dagger[bb], H**bb];
]
new // MatrixForm
```


 0.568049

Out[]//MatrixForm=



$$\begin{pmatrix} -6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

The same job can be done by simply using `MatrixIn`. One important difference is that the result is in `SparseArray` now.

```
In[ ]:= EchoTiming[
  more = MatrixIn[H, bb]
]
```

 0.564406

Out[]:=

SparseArray[  Specified elements: 29
Dimensions: {16, 16}]

In[*]:= more // MatrixForm

Out[*]//MatrixForm=

$$\begin{pmatrix} -6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

```
In[*]:= mm = MatrixIn[H, bs]
```

```
Out[*]=
```

$$\begin{aligned}
 & \langle \left| \{0, 0\} \rightarrow \text{SparseArray} \left[\begin{array}{c} \text{Specified elements: } 3 \\ \text{Dimensions: } \{2, 2\} \end{array} \right] \right. , \\
 & \{1, -1\} \rightarrow \text{SparseArray} \left[\begin{array}{c} \text{Specified elements: } 7 \\ \text{Dimensions: } \{3, 3\} \end{array} \right] , \\
 & \{1, 0\} \rightarrow \text{SparseArray} \left[\begin{array}{c} \text{Specified elements: } 7 \\ \text{Dimensions: } \{3, 3\} \end{array} \right] , \\
 & \{1, 1\} \rightarrow \text{SparseArray} \left[\begin{array}{c} \text{Specified elements: } 7 \\ \text{Dimensions: } \{3, 3\} \end{array} \right] , \\
 & \{2, -2\} \rightarrow \text{SparseArray} \left[\begin{array}{c} \text{Specified elements: } 1 \\ \text{Dimensions: } \{1, 1\} \end{array} \right] , \\
 & \{2, -1\} \rightarrow \text{SparseArray} \left[\begin{array}{c} \text{Specified elements: } 1 \\ \text{Dimensions: } \{1, 1\} \end{array} \right] , \\
 & \{2, 0\} \rightarrow \text{SparseArray} \left[\begin{array}{c} \text{Specified elements: } 1 \\ \text{Dimensions: } \{1, 1\} \end{array} \right] , \\
 & \{2, 1\} \rightarrow \text{SparseArray} \left[\begin{array}{c} \text{Specified elements: } 1 \\ \text{Dimensions: } \{1, 1\} \end{array} \right] , \\
 & \{2, 2\} \rightarrow \text{SparseArray} \left[\begin{array}{c} \text{Specified elements: } 1 \\ \text{Dimensions: } \{1, 1\} \end{array} \right] \rangle
 \end{aligned}$$

```
In[*]:= MatrixForm /@ mm
```

```
Out[*]=
```

$$\begin{aligned}
 & \langle \left| \{0, 0\} \rightarrow \begin{pmatrix} -6 & \sqrt{3} \\ \sqrt{3} & 0 \end{pmatrix}, \{1, -1\} \rightarrow \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}, \right. \\
 & \{1, 0\} \rightarrow \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}, \{1, 1\} \rightarrow \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}, \{2, -2\} \rightarrow (3), \\
 & \{2, -1\} \rightarrow (3), \{2, 0\} \rightarrow (3), \{2, 1\} \rightarrow (3), \{2, 2\} \rightarrow (3) \left. \right| \rangle
 \end{aligned}$$

```
In[*]:= op = ExpressionIn[mm, bs] // Elaborate
```

```
Out[*]=
```

$$S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z + S_2^x S_3^x + S_2^y S_3^y + S_2^z S_3^z + S_3^x S_4^x + S_3^y S_4^y + S_3^z S_4^z$$

```
In[*]:= op - H
Out[*]=
0
```

Using a basis-change matrix?

- **Question:** How about just constructing a unitary matrix corresponding to the basis change?
- **Answer:** It may be inefficient in many cases. Sometimes, it is practically impossible.

Let us consider an example to see why.

```
In[*]:= old // MatrixForm
Out[*]//MatrixForm=
```

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & -3 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & -3 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

```
In[*]:= MatrixForm /@ mm
Out[*]=
```

$$\begin{aligned} & \langle \{0, 0\} \rightarrow \begin{pmatrix} -6 & \sqrt{3} \\ \sqrt{3} & 0 \end{pmatrix}, \{1, -1\} \rightarrow \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}, \\ & \{1, 0\} \rightarrow \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}, \{1, 1\} \rightarrow \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}, \{2, -2\} \rightarrow (3), \\ & \{2, -1\} \rightarrow (3), \{2, 0\} \rightarrow (3), \{2, 1\} \rightarrow (3), \{2, 2\} \rightarrow (3) \rangle \end{aligned}$$

```
In[*]:= Keys[bs]
Out[*]=
{{0, 0}, {1, -1}, {1, 0}, {1, 1}, {2, -2}, {2, -1}, {2, 0}, {2, 1}, {2, 2}}
```

```

In[ ]:= U = Conjugate@Matrix[bs[{0, 0}]];
        U // MatrixForm

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & 0 & 0 & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 \end{pmatrix}$$


In[ ]:= mm00 = U.old.Topple[U];
        mm00 // MatrixForm

Out[ ]//MatrixForm=

$$\begin{pmatrix} -6 & \sqrt{3} \\ \sqrt{3} & 0 \end{pmatrix}$$


```

Summary

Functions

- MatrixIn, ExpressionIn
- Matrix, ExpressionFor
- Outer

Related Links

- Appendix A of the Quantum Workbook (2022, 2023) -- Available for free via the QuantumPlaybook package.