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Errata:
A Quantum Computation Workbook

April 13, 2023

Contents

Notes

- The eBook was updated on April 3, 2023, incorporating most corrections.

- The typos marked as "corrected" in the margin have been corrected in the updated eBook.

- Those marked as "TO-DO" are still to be corrected.

Chapter 1

Postulates of Quantum Mechanics

Problem 1.10 (a), p. 30 The words "first" and "second" in the statements must be exchanged (p_2 should also be replaced with p_1 to be consistent with the rest subquestions). The correct statement should read as

(a) What is the probability p_0 to find the *second* qubit in $|0\rangle$ (regardless of the *first* qubit)? Similarly, what is the probability p_1 to find the *second* qubit in the state $|1\rangle$?

Chapter 2

Quantum Computation: Overview

TO-DO   **Section 2.1, p. 40, Eq. (2.17)**   The right-hand side of it should read as

$$\cdots = i \exp\left[-i\frac{\pi}{2\sqrt{2}}\left(\hat{X} + \hat{Z}\right)\right].$$

corrected   **Section 2.2, p. 47, Eq. (2.31)**   It should read as

$$\text{CNOT} = \frac{1}{2}\left(\hat{I} + \hat{S}_c^z + \hat{S}_t^x - \hat{S}_c^z\hat{S}_t^x\right)$$

corrected   **Section 2.2, p. 64, Eq. (2.65)**   It should read as

$$UT_1T_2 = \begin{bmatrix} U_{11} & U_{12}'' & 0 & 0 \\ U_{21} & U_{22}'' & U_{23}'' & U_{24}' \\ U_{31} & U_{32}'' & U_{33}'' & U_{34}' \\ U_{41} & U_{42}'' & U_{43}'' & U_{44}' \end{bmatrix}.$$

corrected   **Section 2.2, p. 64, Eq. (2.66)**   It should read as

$$T_3 = \begin{bmatrix} \tilde{U}_{11}^* & \tilde{U}_{12}'' & & \\ \tilde{U}_{12}''^* & -\tilde{U}_{11} & & \\ & & 1 & \\ & & & 1 \end{bmatrix}.$$

corrected   **Section 2.2, p. 64, Eq. (2.67)**   It should read as

$$UT_1T_2T_3 = \begin{bmatrix} U_{11}''' & 0 & 0 & 0 \\ 0 & U_{22}''' & U_{23}'' & U_{24}' \\ 0 & U_{32}''' & U_{33}'' & U_{34}' \\ 0 & U_{42}''' & U_{43}'' & U_{44}' \end{bmatrix}.$$

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corrected **Problem 2.3, p. 85, Eq. (2.93)** Equation (2.93) should read as

$$\cdots = \hat{S}^\nu \cos(\phi) - \sum_\lambda \hat{S}^\lambda \epsilon_{\lambda\mu\nu} \sin(\phi).$$

Chapter 3

Realizations of Quantum Computers

corrected **Section 3.2, p. 100, line 5 from the top** "It takes two Pauli X gates ..." $\rightarrow$ "It takes two Hadamard gates ...".

corrected **Section 3.3, p. 109, Eq. (3.51)** It should read as

$$\cdots = \sum_{i} \cdots .$$

corrected **Section 3.3, p. 110, Eq. (3.54)** It should read as

$$\cdots = \sum_{ij} \cdots .$$

TO-DO **Section 3.4, p. 116, opening sentence of Section 3.4.1** "Le us ..." $\rightarrow$ "Let us ...".

2023-04-13 **Section 3.4, p. 116, Eq. (3.73)** It should read as

$$\cdots = \cdots \hat{U}_z(\phi_1) |\psi\rangle .$$

TO-DO (partially corrected) **Section 3.4, p. 117, below Eq. (3.77)** "..., we set $\phi_2 = (-1)^m \beta$." $\rightarrow$ "..., we set $\phi_2 = (-1)^{x_1} \beta$."

In the updated eBook: "..., we set $\phi_2 = (-1)^{x1} \beta$." $\rightarrow$ "..., we set $\phi_2 = (-1)^{x_1} \beta$."

TO-DO **Section 3.4, p. 119, opening sentence of Section 3.4.2** "Le us ..." $\rightarrow$ "Let us ...".

Chapter 4

Quantum Algorithms

corrected   **Section 4.2, p. 145, the second line of the opening paramgraph**   "... the best known ..." $\rightarrow$ "... the known best ...".

corrected   **Section 4.2, p. 147, just above Eq. (4.35)**   "$(a_z \oplus s) \cdot y = (a_z \cdot y) \oplus (s \oplus y)$, it follows ..." $\rightarrow$ "$(a_z \oplus s) \cdot y = (a_z \cdot y) \oplus (s \cdot y)$, it follows ...".

corrected   **Section 4.4, p. 161, Eq. (4.63)**   Symbol n must be replaced by m as follows

$$\left(\hat{H}^{\otimes m} |0\rangle \right) \otimes |\phi\rangle = \frac{1}{2^{m/2}} \sum_{x=0}^{2^m - 1} |x\rangle \otimes |\phi\rangle \, .$$

corrected   **Section 4.4, p. 161, below Eq. (4.64)**   "... performing the transformation $\hat{U}$ repeatedly depending on the value y on the native register." $\rightarrow$ "... performing the transformation $\hat{U}$ repeatedly depending on value x on the native register."

corrected   **Fig. 4.4b, p. 179**   "... with respect to ω ..." $\rightarrow$ "... with respect to $|v\rangle$ ..."

corrected   **Fig. 4.5a, p. 182** $\theta, \theta/2 \rightarrow 2\theta, \theta$, respectively. Here is the correct figure for Fig. 4.5**a**:
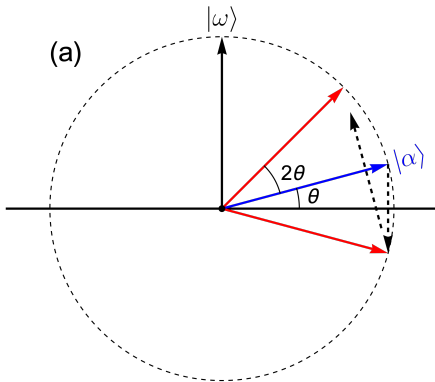
corrected   **Problem 4.1 (a), p. 188**   "Classically (...), ..." $\rightarrow$ "Show that classically (...), ...".

Chapter 5

Quantum Decoherence

TO-DO
(partially
corrected)
Section 5.1 In several places, "Zender" must be corrected to "Zehnder": p. 191 (main text, bottom), p. 191 (Fig. 5.2), p. 192 (main text), p. 192 (code piece), p. 193 (Fig. 5.3), p. 193 (main text), p. 194 (main text), p. 194 (code piece), p. 195 (code piece).

corrected
Section 5.1, p. 191, the last line "In the blue arm, photon passes through ..." $\rightarrow$ "In the red arm, photon passes through ...".

corrected
Section 5.1, p. 194, below Eq. (5.6) "Whence the photon detection probabilities ..." $\rightarrow$ "Hence the photon detection probabilities...".

corrected
Section 5.2, p. 206, Eq. (5.43) The second '$\otimes$' should be removed from Eq. (5.43). That is, Eq. (5.43) should read as

$$|\Phi\rangle\langle\Phi| = \sum_{kl} |v_k\rangle\langle v_l| \otimes |v_k\rangle\langle v_l|.$$

corrected
Fig. 5.4, p. 208, line 3 of the caption "... the success probability is $1/4$ ..." $\rightarrow$ "... the success probability is $1/d^2$ for $d = \dim \mathcal{V}$ ...".

corrected
Section 5.2, p. 209, line 1 "... a success probability of $1/4$ ..." $\rightarrow$ "... a success probability of $1/d^2$ for $d = \dim \mathcal{V}$ ...".

corrected
Section 5.2, p. 209, line 10 from top "... a success probability of $1/4$ ..." $\rightarrow$ "... a success probability of $1/d^2$ ...".

corrected
Section 5.2, p. 210, line 3 "... quantum operation: $\mathscr{F} : \mathcal{L}(\mathcal{V}) \rightarrow \mathcal{L}(\mathcal{W})$ ..." $\rightarrow$ "... quantum operation: $\mathscr{F} : \mathcal{L}(\mathcal{V}) \rightarrow \mathcal{L}(\mathcal{V})$ ...".

TO-DO
(partially
corrected)
Section 5.2, p. 210, above Eq. (5.58) "than $(\dim \mathcal{V})\times(\dim \mathcal{W})$. ... on $\mathcal{V}\otimes\mathcal{W}$ ..." $\rightarrow$ "than $(\dim \mathcal{V})^2$. ... on $\mathcal{V} \otimes \mathcal{E}$ ...".

corrected
Section 5.3, p. 216, line 8 "...probabilities $\mathscr{F}_m(\hat{\rho})$" must reads as "...probabilities $\mathrm{Tr}\,[\mathscr{F}_m(\hat{\rho})]$".

corrected **Section 5.4, Eq. (5.99)** It should read as

$$\hat{G} = \frac{1}{2} \sum_{\mu > 0} \hat{L}_\mu^\dagger \hat{L}_\mu \,.$$

corrected **Section 5.4, Eq. (5.147)** It should read as

$$\frac{d\hat{\rho}}{dt} = \cdots \,.$$

corrected **Section 5.5, the first sentence, p. 234** "..., who close (or different) ..." $\rightarrow$ "..., how close (or different) ...".

corrected **Section 5.5, p. 236, just below Eq. (5.164)** "... the canonical norm associate with ..." $\rightarrow$ "... the canonical norm associated with ...".

corrected **Section 5.5, p. 237, just below Eq. (5.177)** "... traceless Hermitian operators (a_0) ..." $\rightarrow$ "... traceless Hermitian operators $(a_0 = 0)$ ...".

corrected **Section 5.5, p. 244, the first line** "associate with a POVM ..." $\rightarrow$ "associated with a POVM ...".

corrected **Section 5.5, p. 247, below Eq. (5.209)** "... of two vectors normalized vectors ..." $\rightarrow$ "... of two normalized vectors ...".

corrected **Section 5.5, p. 248, below Eq. (5.215)** "... to note that $\hat{\rho}$ as two eigenvalues ..." $\rightarrow$ "... to note that $\hat{\rho}$ has two eigenvalues ...".

TO-DO **Sectoin 5.5, p. 249, Eq. (5.224)** It should reads

$$\cdots \geq \left| \left(\langle \Psi | \otimes \langle \epsilon_0 | \right) \hat{U} \hat{U}^\dagger \left(| \Phi \rangle \otimes | \epsilon_0 \rangle \right) \right| = \cdots \,.$$

corrected **Problem 5.4, p. 252, Eq.(5.234)** $\gamma_1 \rightarrow \gamma_\phi$

Chapter 6

Quantum Error-Correction Codes

corrected   **Section 6.1, p. 259, line 10 from the top**   "... the encoded state $|\psi\rangle$ ..." $\rightarrow$ "... the encoded state $|\bar{\psi}\rangle$ ..."

corrected   **Section 6.1, p. 259, the second from the bottom**   "... the original encoded state $|\psi\rangle$ ..." $\rightarrow$ "... the original encoded state $|\bar{\psi}\rangle$ ..."

corrected   **Section 6.1, p. 265, between Eqs. (6.8) and (6.9)**   "The phase-slip error ..." $\rightarrow$ "The phase-flip error ...".

corrected   **Section 6.3, p. 288, Eq. (6.75)**

$$\hat{U}(|0\rangle \otimes |\alpha\rangle) = |0\rangle \otimes |\alpha_0\rangle + |1\rangle \otimes \hat{A}|\alpha_1\rangle = \cdots$$

must be changed to

$$\hat{U}(|0\rangle \otimes |\alpha\rangle) = |0\rangle \otimes |\alpha_0\rangle + |1\rangle \otimes \hat{A}|\alpha_0\rangle = \cdots.$$

corrected   **Section 6.4, p. 298, above Eq. (6.101)**   "whence" $\rightarrow$ "hence".

corrected   **Section 6.4, p. 301**   In the last sentence of the second paragraph of Section 6.4.2: "... the error sydromes for bit-flip errors ..." $\rightarrow$ "... the error sydromes for phase-flip errors ...".

corrected   **Section 6.5, p. 309, line 5 from the bottom**   "These are difficult ..." $\rightarrow$ "The toric codes are difficult ...".

corrected   **Section 6.5, p. 314, the last line at the bottom**   "A vertex on a rough edge ... with such a vertex ..." $\rightarrow$ "A plaquette on a rough edge ... with such a plaquette ...".

corrected   **Section 6.5, p. 315, line 2 from the bottom**   "... logical operator $\bar{Z}$ ..." $\rightarrow$ "... logical operator $\bar{X}$ ...".

corrected   **Section 6.5, p. 318, just below Eq. (6.120)**   "Plaquette and vertex opera-tors ..." → "Measurement of plaquette and vertex operators ...".

corrected   **Figure 6.9, p. 319, caption (b)**   "... and vertex defects (red ..." → "... and plaquette defects (red ...".

corrected   **Section 6.5, p. 320, line 6 from the top**   "... upper example in Fig. 11b." → "... upper example in Fig. 11.".

Chapter 7

Quantum Information Theory

corrected **Section 7.1, p. 327, Eq. (7.14)** It should read as

$$\cdots \geq \frac{1-x}{\log_e 2}.$$

corrected **Section 7.3, p. 344, Eq. (7.77)** It should read as

$$|\Psi_m\rangle = \binom{n}{m}^{-1/2} \cdots.$$

corrected **Section 7.3, p. 344, above Eq. (7.80)** " ... diving ..." → " ... dividing ...".

Appendix A

Linear Algebra

Appendix A.1, p. 350, Definition A.3 "... there exists a solution ..." → "... there exists a non-trivial solution ..."

Appendix A.1, p. 351, above Eq. (A.5) "Whence u is orthogonal ..." → "Hence u is orthogonal ...".

Appendix A.4, p. 364, above Eq. (A.55) "Whence, $\hat{A} \geq 0$." → "Hence, $\hat{A} \geq 0$."

Appendix A.4, p. 364, below Eq. (A.59) "... eigenvalues ± 1" → "... eigenvalues $e^{\mp i\phi}$".

Appendix A.6, p. 369, below Eq. (A.79) $N := \mathcal{W} \to N := \dim \mathcal{W}$. Here, the word "dim" must be typeset in upright style (not italic style).

Appendix B

Superoperators

corrected **Appendix B.1, p. 377, Eq. (B.6)** $\hat{S}^x \to \hat{S}^\mu$.

corrected **Appendix B.2, p. 384, below Exercise B.4**

- "The following theorem confirms that any supermap ..." $\to$ "The following theorem confirms that any completely positive supermap ...".
- "... find a more compact ..." $\to$ "... find more compact ...".

TO-DO **Appendix B.2, p. 386, between Eqs. (B.30) and (B.31)**

- $\{v_j\} \to \{|v_j\rangle\}$
- $|w_k\rangle \to \{|w_k\rangle\}$

corrected **Appendix B.4, p. 391, just below Eq. (B.53)** "we have" $\to$ "We have".

corrected **Appendix B.4, p. 392, Eq. (B.56)** $|\Psi\rangle\langle\Psi| \to |\Phi\rangle\langle\Phi|$.

corrected **Appendix B.4, p. 393, the second-last line** "Whence, transposition ..." $\to$ "Hence, transposition ...".

Appendix C

Group Theory

corrected | **Appendix C.1, p. 396, Definition C.1 (c)** "... identity element $e \in \mathcal{G}$ ..." $\rightarrow$ "... identity element $E \in \mathcal{G}$ ...".

corrected | **Appendix C.2, p. 399, Theorem C.8 (b)** "... $\mathcal{G}$ an be ..." $\rightarrow$ "... $\mathcal{G}$ can be ...".

corrected | **Appendix C.4, pp. 402, Defintion C.17 (a)** "... $\mathcal{G} \otimes \mathcal{G}'$ ..." $\rightarrow$ "... $\mathcal{G} \times \mathcal{G}'$ ...".

corrected | **Appendix C.4, pp. 403, Eq. (C.22)** $\mathcal{G} \otimes \mathcal{G}' := \cdots$ $\rightarrow$ $\mathcal{G} \times \mathcal{G}' := \cdots$.

Appendix F

Solutions to Select Problems

corrected **Appendix F.3, p. 412** The heading "Quantum Computers" should be corrected to "Realizations of Quantum Comptuers" to match the original heading Chapter 3.

corrected **Appendix F.3, p. 412, Eq. (F.8)** $|D\rangle := \cdots \to |\Omega\rangle := \cdots$.

corrected **Appendix F.3, p. 412, Eq. (F.11)** It should read as

$$|D\rangle = \frac{|1\rangle \sin(\theta/2) e^{-i\phi/2} - \cdots}{\Omega}.$$

TO-DO **Appendix F.3, p. 412, below Eq. (F.12)** "... the Berry phase as $\gamma = -iA^\phi = \frac{1}{2}\cos\theta$" $\to$ "... the Berry phase as $\gamma := -i\int_0^{2\pi} d\phi\, A^\phi = -2\pi i A^\phi = \pi\cos\theta$".

corrected **Appendix F.3, p. 412, above Eq. (F.13)** "... the Abelina geometric ..." $\to$ "... the Abelian geometric ...".

TO-DO (partially corrected) **Appendix F.3, p. 412, Eq. (F.13)** It should read as

$$U(\mathcal{C}) = e^{-i\gamma} = e^{-i\pi\cos\theta}$$

corrected **Appendix F.3, p. 413, above Eq. (F.17)** "... a finite-finite dimensional ..." $\to$ "... a finite-dimensional ...".

corrected **Appendix F.5, p. 415** The heading "Decoherence" should be corrected to "Quantum Decoherence" to match the original heading of Chapter 5.

corrected **Problem 6.7, p. 422, the display equation between (F.58) and (F.59)** $\hat{W}$ must be replaced with $\hat{P}'''$, i.e.,

$$\cdots(\hat{Z}\otimes\hat{W})\cdots \quad \to \quad \cdots(\hat{Z}\otimes\hat{P}''')\cdots$$

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