Quantum Oracle: Properties

Episode 26. Classical Oracle

Episode 27. Quantum Oracle: Definition

Episode 28. Quantum Oracle: Properties

Preliminary

Here, let us set up classical and quantum oracles to be used in the examples below.

```
In[ • ]:= $m = 3;
          n = 2;
  In[ \circ ] := f[1] = f[2] = 3;
          f[7] = 2;
          f[_Integer] = 0;
  In[0]:= ff = Oracle[f, $m, $n];
          xx = Tuples[{0, 1}, $m];
          yy = ff/@xx;
  In[\cdot]:= Thread[xx \rightarrow yy] // TableForm
Out[ • ]//TableForm=
          \{0, 0, 0\} \rightarrow \{0, 0\}
          \{0, 0, 1\} \rightarrow \{1, 1\}
          \{0, 1, 0\} \rightarrow \{1, 1\}
          \{0, 1, 1\} \rightarrow \{0, 0\}
          \{1, 0, 0\} \rightarrow \{0, 0\}
          \{1, 0, 1\} \rightarrow \{0, 0\}
          \{1, 1, 0\} \rightarrow \{0, 0\}
          \{1, 1, 1\} \rightarrow \{1, 0\}
  In[*]:= Let[Qubit, S, T]
```

```
In[*]:= Ecc[qustc, 5, .]

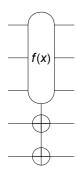
In[*]:= SS = S[Range[$m], $];

TT = T[Range[$n], $];

In[*]:= op = Oracle[f, SS, TT]

Out[*] =

Oracle[f, {S1, S2, S3}, {T1, T2}]
```



Making Copies

The CNOT gate makes a copy of the computational basis state of the control register to the target register when the latter has been prepared initially in the state $|0\rangle$,

$$|x\rangle \otimes |0\rangle \mapsto |x\rangle \otimes |x\rangle.$$

The quantum oracle has a similar property, but it makes a copy of the image |f(x)| rather than the input state $|x\rangle$ itself of the native register to the ancillary register,

$$|x\rangle \otimes |0\rangle \mapsto |x\rangle \otimes |f(x)\rangle.$$

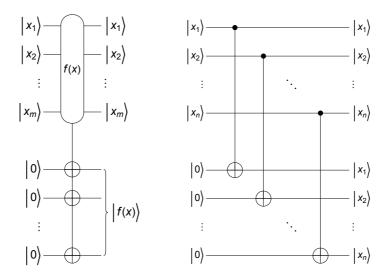
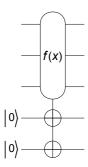


Figure 4. (left) A quantum circuit demonstrating the feature $|x\rangle \otimes |0\rangle \mapsto |x\rangle \otimes |f(x)\rangle$ of the quantum oracle corresponding to classical oracle $f:\{0, 1\}^m \to \{0, 1\}^n$. (right) A quantum circuit demonstrating the feature $|x\rangle \otimes |0\rangle \mapsto |x\rangle \otimes |x\rangle$ of the CNOT gates.

Example

```
In[*]:= QuantumCircuit[Ket[TT], op]
Out[0]=
```



In[*]:= in = Basis[SS]; ProductForm[in, {SS, TT}]

Out[•]=

$$\begin{array}{c|c} \{ \hspace{.1cm} \big|\hspace{.1cm} 000 \hspace{.1cm} \big\rangle \otimes \hspace{.1cm} \big|\hspace{.1cm} 001 \hspace{.1cm} \big\rangle \otimes \hspace{.1cm} \big|\hspace{.1cm} 000 \hspace{.1cm} \big\rangle, \hspace{.1cm} \big|\hspace{.1cm} 010 \hspace{.1cm} \big\rangle \otimes \hspace{.1cm} \big|\hspace{.1cm} 001 \hspace{.1cm} \big\rangle \otimes \hspace{.1cm} \big|\hspace{.1cm} 000 \hspace{.1cm} \big\rangle, \hspace{.1cm} \big|\hspace{.1cm} 111 \hspace{.1cm} \big\rangle \otimes \hspace{.1cm} \big|\hspace{.1cm} 00 \hspace{.1cm} \big\rangle, \end{array}$$

$$\begin{array}{c|c} \textit{Out[=]=} \\ & \left\{ \left| 0000 \right\rangle \otimes \left| 000 \right\rangle, \; \left| 0010 \right\rangle \otimes \left| 11 \right\rangle, \; \left| 010 \right\rangle \otimes \left| 11 \right\rangle, \; \left| 011 \right\rangle \otimes \left| 00 \right\rangle, \\ & \left| 1000 \right\rangle \otimes \left| 000 \right\rangle, \; \left| 101 \right\rangle \otimes \left| 000 \right\rangle, \; \left| 110 \right\rangle \otimes \left| 000 \right\rangle, \; \left| 111 \right\rangle \otimes \left| 100 \right\rangle \end{array}$$

In[⊕]:= ProductForm[Thread[in → out], {SS, TT}] // TableForm

Out[•]//TableForm=

$$\begin{array}{c|c} \left| \begin{array}{c|c} 000 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \rightarrow \left| \begin{array}{c|c} 000 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \\ \left| \begin{array}{c|c} 001 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \rightarrow \left| \begin{array}{c|c} 001 \right\rangle \otimes \left| \begin{array}{c|c} 11 \end{array} \right\rangle \\ \left| \begin{array}{c|c} 010 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \rightarrow \left| \begin{array}{c|c} 010 \right\rangle \otimes \left| \begin{array}{c|c} 11 \end{array} \right\rangle \\ \left| \begin{array}{c|c} 011 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \rightarrow \left| \begin{array}{c|c} 011 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \\ \left| \begin{array}{c|c} 100 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \rightarrow \left| \begin{array}{c|c} 100 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \\ \left| \begin{array}{c|c} 101 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \rightarrow \left| \begin{array}{c|c} 110 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \\ \left| \begin{array}{c|c} 110 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \rightarrow \left| \begin{array}{c|c} 110 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \\ \left| \begin{array}{c|c} 111 \right\rangle \otimes \left| \begin{array}{c|c} 00 \end{array} \right\rangle \rightarrow \left| \begin{array}{c|c} 111 \end{array} \right\rangle \otimes \left| \begin{array}{c|c} 10 \end{array} \right\rangle \end{array}$$

In[•]:= Thread[xx → yy] // TableForm

Out[•]//TableForm=

$$\begin{cases} 0,\,0,\,0 \rbrace \to \{0,\,0 \rbrace \\ \{0,\,0,\,1 \rbrace \to \{1,\,1 \rbrace \\ \{0,\,1,\,0 \rbrace \to \{1,\,1 \rbrace \\ \{0,\,1,\,1 \rbrace \to \{0,\,0 \rbrace \\ \{1,\,0,\,0 \rbrace \to \{0,\,0 \rbrace \\ \{1,\,0,\,1 \rbrace \to \{0,\,0 \rbrace \\ \{1,\,1,\,0 \rbrace \to \{0,\,0 \rbrace \\ \{1,\,1,\,1 \rbrace \to \{1,\,0 \rbrace } \end{cases}$$

Superposition State

Furthermore, suppose that a native quantum register is in the superposition

$$\frac{1}{2^{m/2}}\sum_{x=0}^{2^m-1} \left| x \right\rangle$$

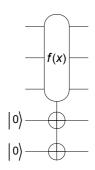
and the ancillary quantum register in in the state $|0\rangle \equiv |0\rangle^{\otimes n}$. The quantum oracle transforms the state as

$$\frac{1}{2^{m/2}}\sum_{x=0}^{2^m-1}\left|x\right\rangle\otimes\left|0\right\rangle\mapsto\frac{1}{2^{m/2}}\sum_{x=0}^{2^m-1}\left|x\right\rangle\otimes\left|f(x)\right\rangle$$

Just like the CNOT gate, the above state from the quantum oracle is also entangled in general unless f is a constant or very special function. In this case, the entanglement is controlled by the (classical) oracle f.

In[@]:= QuantumCircuit[Ket[TT], op]

Out[•]=



In[*]:= in = Total@Basis[SS];

ProductForm[in, {SS, TT}]

$$\begin{array}{c|c} \textit{Out[=]=} \\ & \left| .000 \right\rangle \otimes \left| .00 \right\rangle + \left| .001 \right\rangle \otimes \left| .00 \right\rangle + \left| .010 \right\rangle \otimes \left| .00 \right\rangle + \left| .011 \right\rangle \otimes \left| .00 \right\rangle + \left| .101 \right\rangle \otimes \left| .00 \right\rangle + \left| .111 \right\rangle \otimes \left| .00 \right\rangle \otimes \left| .00 \right\rangle + \left| .111 \right\rangle \otimes \left| .00 \right\rangle \otimes \left| .00 \right\rangle + \left| .111 \right\rangle \otimes \left| .00 \right\rangle \otimes \left| .00 \right\rangle \otimes \left| .00 \right\rangle \otimes \left| .00 \right\rangle + \left| .00 \right\rangle \otimes \left| .00$$

In[0]:= out = op ** in;

ProductForm[out, {SS, TT}]

Out[*] =
$$\begin{vmatrix} 0000 \rangle \otimes |000\rangle + |001\rangle \otimes |11\rangle + |010\rangle \otimes |11\rangle + |011\rangle \otimes |00\rangle + |100\rangle \otimes |00\rangle + |101\rangle \otimes |00\rangle + |111\rangle \otimes |10\rangle$$

In[•]:= Thread[xx → yy] // TableForm

Out[•]//TableForm=

 $\{0, 0, 0\} \rightarrow \{0, 0\}$

 $\{0, 0, 1\} \rightarrow \{1, 1\}$

 $\{0, 1, 0\} \rightarrow \{1, 1\}$

 $\{0, 1, 1\} \rightarrow \{0, 0\}$ $\{1, 0, 0\} \rightarrow \{0, 0\}$

 $\{1, 0, 1\} \rightarrow \{0, 0\}$

 $\{1, 1, 0\} \rightarrow \{0, 0\}$

 $\{1, 1, 1\} \rightarrow \{1, 0\}$

Marking

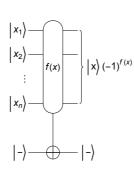
The controlled-unitary gate induces a phase shift on the control register (rather than on the target register) when the target register is in an eigenstate of the unitary operator.

A similar method can be used to induce a phase shift conditionally on every term that satisfies a certain condition:

$$|x\rangle \mapsto |x\rangle (-1)^{f(x)}$$

More generally,

$$\sum_{x=0}^{2^{n}-1} \left| x \right\rangle \mapsto \sum_{x=0}^{2^{n}-1} \left| x \right\rangle \left(-1\right)^{f(x)}$$



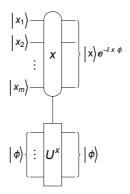
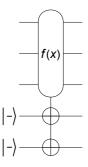


Figure 5. (left) A quantum circuit to induces conditional phase shifts depending on function value f(x) at input value $x := (x_1, x_2, ..., x_n)$. (right) For comparison, the controlled exponentiation of a unitary gate induces an x-dependent phase shift, where $x := (x_1 x_2 ... x_m)_2$.

Example

 $In[\cdot]:=$ QuantumCircuit[ProductState[T[{1, 2}] \rightarrow {1, -1}, "Label" \rightarrow Ket[{"-"}]], op] Out[0]=



In[0]:= in = Basis[SS] ** ProductState[T[1] \rightarrow {1, 1}, T[2] \rightarrow {1, -1}]; KetFactor[in]

Out[•]= $\{ \left| \mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \right\rangle \otimes \left(\left| \mathbf{0}_{T_1} \right\rangle + \left| \mathbf{1}_{T_1} \right\rangle \right) \otimes \left(\left| \mathbf{0}_{T_2} \right\rangle - \left| \mathbf{1}_{T_2} \right\rangle \right), \left| \mathbf{0}_{S_1} \mathbf{0}_{S_2} \mathbf{1}_{S_3} \right\rangle \otimes \left(\left| \mathbf{0}_{T_1} \right\rangle + \left| \mathbf{1}_{T_1} \right\rangle \right) \otimes \left(\left| \mathbf{0}_{T_2} \right\rangle - \left| \mathbf{1}_{T_2} \right\rangle \right),$ $\left|\left.0_{\mathsf{S}_{1}}\mathbf{1}_{\mathsf{S}_{2}}0_{\mathsf{S}_{3}}\right.
ight>\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right>+\left.\left|\left.1_{\mathsf{T}_{1}}\right.\right>\right.\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right>-\left.\left|\left.1_{\mathsf{T}_{2}}\right.\right>\right.\right)$, $\left|\left.0_{\mathsf{S}_{1}}\mathbf{1}_{\mathsf{S}_{2}}\mathbf{1}_{\mathsf{S}_{3}}\right.\right> \otimes \left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right> + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right>\right.\right) \otimes \left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right> - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right>\right.\right)$, $\left| \mathbf{1}_{\mathsf{S}_1} \mathbf{0}_{\mathsf{S}_2} \mathbf{0}_{\mathsf{S}_3} \right> \otimes \left(\left| \mathbf{0}_{\mathsf{T}_1} \right> + \left| \mathbf{1}_{\mathsf{T}_1} \right> \right) \otimes \left(\left| \mathbf{0}_{\mathsf{T}_2} \right> - \left| \mathbf{1}_{\mathsf{T}_2} \right> \right)$, $\left| \mathbf{1}_{\mathsf{S}_1} \mathbf{0}_{\mathsf{S}_2} \mathbf{1}_{\mathsf{S}_3} \right> \otimes \left(\left| \mathbf{0}_{\mathsf{T}_1} \right> + \left| \mathbf{1}_{\mathsf{T}_1} \right> \right) \otimes \left(\left| \mathbf{0}_{\mathsf{T}_2} \right> - \left| \mathbf{1}_{\mathsf{T}_2} \right> \right)$, $\left| \mathbf{1}_{\mathsf{S}_1} \mathbf{1}_{\mathsf{S}_2} \mathbf{0}_{\mathsf{S}_3} \right> \otimes \left(\left| \mathbf{0}_{\mathsf{T}_1} \right> + \left| \mathbf{1}_{\mathsf{T}_1} \right> \right) \otimes \left(\left| \mathbf{0}_{\mathsf{T}_2} \right> - \left| \mathbf{1}_{\mathsf{T}_2} \right> \right)$, $\left| 1_{S_1} 1_{S_2} 1_{S_3} \right\rangle \otimes \left(\left| 0_{T_1} \right\rangle + \left| 1_{T_1} \right\rangle \right) \otimes \left(\left| 0_{T_2} \right\rangle - \left| 1_{T_2} \right\rangle \right) \right\}$

In[0]:= out = op ** in; KetFactor[out]

Out[0]= $\{ | \Theta_{S_1} \Theta_{S_2} \Theta_{S_3} \rangle \otimes (| \Theta_{T_1} \rangle + | \mathbf{1}_{T_1} \rangle) \otimes (| \Theta_{T_2} \rangle - | \mathbf{1}_{T_2} \rangle)$, $-\left(\left|\left.0_{S_{1}}0_{S_{2}}1_{S_{3}}\right\rangle \otimes\left(\left|\left.0_{T_{1}}\right\rangle \right.+\left|\left.1_{T_{1}}\right\rangle \right.\right)\otimes\left(\left|\left.0_{T_{2}}\right\rangle \right.-\left|\left.1_{T_{2}}\right\rangle \right.\right)\right)\text{,}$ $-\left(\left|\left.0_{S_{1}}\mathbf{1}_{S_{2}}0_{S_{3}}\right\rangle \otimes\left(\left|\left.0_{T_{1}}\right\rangle +\left|\left.1_{T_{1}}\right\rangle \right.\right) \otimes\left(\left|\left.0_{T_{2}}\right\rangle -\left|\left.1_{T_{2}}\right\rangle \right.\right)\right)\text{,}$ $| O_{S_1} 1_{S_2} 1_{S_3} \rangle \otimes (| O_{T_1} \rangle + | 1_{T_1} \rangle) \otimes (| O_{T_2} \rangle - | 1_{T_2} \rangle)$, $\left| \mathbf{1}_{S_1} \mathbf{0}_{S_2} \mathbf{0}_{S_3} \right\rangle \otimes \left(\left| \mathbf{0}_{T_1} \right\rangle + \left| \mathbf{1}_{T_1} \right\rangle \right) \otimes \left(\left| \mathbf{0}_{T_2} \right\rangle - \left| \mathbf{1}_{T_2} \right\rangle \right)$, $\left| \mathbf{1}_{\mathsf{S}_1} \mathbf{0}_{\mathsf{S}_2} \mathbf{1}_{\mathsf{S}_3} \right\rangle \otimes \left(\left| \mathbf{0}_{\mathsf{T}_1} \right\rangle + \left| \mathbf{1}_{\mathsf{T}_1} \right\rangle \right) \otimes \left(\left| \mathbf{0}_{\mathsf{T}_2} \right\rangle - \left| \mathbf{1}_{\mathsf{T}_2} \right\rangle \right)$, $\left| \mathbf{1}_{\mathsf{S}_1} \mathbf{1}_{\mathsf{S}_2} \mathbf{0}_{\mathsf{S}_3} \right\rangle \otimes \left(\left| \mathbf{0}_{\mathsf{T}_1} \right\rangle + \left| \mathbf{1}_{\mathsf{T}_1} \right\rangle \right) \otimes \left(\left| \mathbf{0}_{\mathsf{T}_2} \right\rangle - \left| \mathbf{1}_{\mathsf{T}_2} \right\rangle \right)$, $\left| \mathbf{1}_{\mathsf{S}_{1}} \mathbf{1}_{\mathsf{S}_{2}} \mathbf{1}_{\mathsf{S}_{3}} \right\rangle \otimes \left(\left| \mathbf{0}_{\mathsf{T}_{1}} \right\rangle + \left| \mathbf{1}_{\mathsf{T}_{1}} \right\rangle \right) \otimes \left(\left| \mathbf{0}_{\mathsf{T}_{2}} \right\rangle - \left| \mathbf{1}_{\mathsf{T}_{2}} \right\rangle \right) \right\}$

In[*]:= Thread[KetFactor[in] → KetFactor[out]] // TableForm

Out[•]//TableForm=

```
\left|\left.0_{\mathsf{S}_{1}}0_{\mathsf{S}_{2}}0_{\mathsf{S}_{3}}\right\rangle \otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right) \otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right. \\ \rightarrow \left.\left|\left.0_{\mathsf{S}_{1}}0_{\mathsf{S}_{2}}0_{\mathsf{S}_{3}}\right.\right\rangle \otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right) \otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right. \\ \left.\left.\left|\left.0_{\mathsf{S}_{1}}0_{\mathsf{S}_{2}}0_{\mathsf{S}_{3}}\right.\right\rangle \otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\right. \\ \left.\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle \otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle \otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle\right.\right] \\ \left.\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle \otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle \otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle \otimes\left.\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle \right.\right\rangle \right] \right. \\ \left.\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle \otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle \otimes\left.\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle \otimes\left.\left(\left.\left|\left.0
         \left|\left.0_{\mathsf{S}_1}0_{\mathsf{S}_2}1_{\mathsf{S}_2}\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_1}\right.\right\rangle+\left.\left|\left.1_{\mathsf{T}_1}\right.\right\rangle\right.\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_2}\right.\right\rangle-\left.\left|\left.1_{\mathsf{T}_2}\right.\right\rangle\right.\right)\right.\\ \left.\left.\left.\left.\left|\left.0_{\mathsf{S}_1}0_{\mathsf{S}_2}1_{\mathsf{S}_2}\right.\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_1}\right.\right\rangle+\left.\left|\left.1_{\mathsf{T}_1}\right.\right\rangle\right.\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_2}\right.\right\rangle-\left.\left|\left.1_{\mathsf{T}_2}\right.\right\rangle\right.\right)\right.
         \left|\left.0_{\mathsf{S}_{1}}\right.1_{\mathsf{S}_{2}}\right.0_{\mathsf{S}_{3}}\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right.\\ \rightarrow -\left(\left.\left|\left.0_{\mathsf{S}_{1}}\right.1_{\mathsf{S}_{2}}\right.0_{\mathsf{S}_{3}}\right.\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{2}}\right.0_{\mathsf{S}_{3}}\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{2}}\right.0_{\mathsf{S}_{3}}\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{2}}\right.0_{\mathsf{S}_{3}}\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{1}}\right.0_{\mathsf{S}_{3}}\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{1}}\right.0_{\mathsf{S}_{3}}\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{1}}\right.0_{\mathsf{S}_{3}}\right.\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{1}}\right.0_{\mathsf{S}_{1}}\right.\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{1}}\right.0_{\mathsf{S}_{1}}\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{1}}\right.0_{\mathsf{S}_{1}}\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{1}}\right.0_{\mathsf{S}_{1}}\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{1}}\right.\right.\\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\right.\right|_{\mathsf{S}_{1}}\right.\right.
    \left|\left.0_{\mathsf{S}_{1}}\mathbf{1}_{\mathsf{S}_{2}}\mathbf{1}_{\mathsf{S}_{3}}\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right. \\ \rightarrow \left.\left|\left.0_{\mathsf{S}_{1}}\mathbf{1}_{\mathsf{S}_{2}}\mathbf{1}_{\mathsf{S}_{3}}\right.\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right. \\ + \left.\left|\left.0_{\mathsf{S}_{1}}\mathbf{1}_{\mathsf{S}_{2}}\mathbf{1}_{\mathsf{S}_{3}}\right.\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right. \\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\mathbf{1}_{\mathsf{S}_{2}}\mathbf{1}_{\mathsf{S}_{3}}\right.\right\rangle\otimes\left(\left.\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right.\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right)\right. \\ + \left.\left.\left|\left.0_{\mathsf{S}_{1}}\mathbf{1}_{\mathsf{S}_{2}}\mathbf{1}_{\mathsf{S}_{3}}\right.\right\rangle\otimes\left(\left.\left|\left.0_{\mathsf{T}_{1}}\right.\right\rangle + \left.\left|\left.1_{\mathsf{T}_{1}}\right.\right\rangle\right.\right)\otimes\left(\left.\left|\left.0_{\mathsf{T}_{2}}\right.\right\rangle - \left.\left|\left.1_{\mathsf{T}_{2}}\right.\right\rangle\right.\right)
    \left| \mathbf{1}_{\mathsf{S_1}} \mathbf{0}_{\mathsf{S_2}} \mathbf{0}_{\mathsf{S_3}} \right\rangle \otimes \left( \left| \mathbf{0}_{\mathsf{T_1}} \right\rangle + \left| \mathbf{1}_{\mathsf{T_1}} \right\rangle \right) \otimes \left( \left| \mathbf{0}_{\mathsf{T_2}} \right\rangle - \left| \mathbf{1}_{\mathsf{T_2}} \right\rangle \right) \\ \rightarrow \left| \mathbf{1}_{\mathsf{S_1}} \mathbf{0}_{\mathsf{S_2}} \mathbf{0}_{\mathsf{S_3}} \right\rangle \otimes \left( \left| \mathbf{0}_{\mathsf{T_1}} \right\rangle + \left| \mathbf{1}_{\mathsf{T_1}} \right\rangle \right) \otimes \left( \left| \mathbf{0}_{\mathsf{T_2}} \right\rangle - \left| \mathbf{1}_{\mathsf{T_2}} \right\rangle \right)
\left| \mathbf{1}_{\mathsf{S}_1} \mathbf{0}_{\mathsf{S}_2} \mathbf{1}_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \mathbf{0}_{\mathsf{T}_1} \right\rangle + \left| \mathbf{1}_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| \mathbf{0}_{\mathsf{T}_2} \right\rangle - \left| \mathbf{1}_{\mathsf{T}_2} \right\rangle \right) \rightarrow \left| \mathbf{1}_{\mathsf{S}_1} \mathbf{0}_{\mathsf{S}_2} \mathbf{1}_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \mathbf{0}_{\mathsf{T}_1} \right\rangle + \left| \mathbf{1}_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| \mathbf{0}_{\mathsf{T}_2} \right\rangle - \left| \mathbf{1}_{\mathsf{T}_2} \right\rangle \right)
\left| \left| \left| 1_{\mathsf{S}_1} 1_{\mathsf{S}_2} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \right. \\ \left. \left. \left| \left| 1_{\mathsf{S}_1} 1_{\mathsf{S}_2} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \right. \\ \left. \left| \left| 1_{\mathsf{S}_1} 1_{\mathsf{S}_2} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \right. \\ \left. \left| \left| 1_{\mathsf{S}_1} 1_{\mathsf{S}_2} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \right. \\ \left. \left| \left| 1_{\mathsf{S}_1} 1_{\mathsf{S}_2} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \right. \\ \left. \left| \left| 1_{\mathsf{S}_1} 1_{\mathsf{S}_2} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \right. \\ \left. \left| \left| 1_{\mathsf{S}_1} 1_{\mathsf{S}_2} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right. \\ \left. \left| \left| 1_{\mathsf{S}_2} 1_{\mathsf{S}_3} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right. \\ \left. \left| \left| 1_{\mathsf{S}_2} 1_{\mathsf{S}_3} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right. \\ \left. \left| \left| 1_{\mathsf{S}_2} 1_{\mathsf{S}_3} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right. \\ \left. \left| \left| 1_{\mathsf{T}_2} 1_{\mathsf{S}_3} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle + \left| 1_{\mathsf{T}_2} \right\rangle \right. \\ \left. \left| \left| 1_{\mathsf{T}_2} 1_{\mathsf{S}_3} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle + \left| 1_{\mathsf{T}_2} 0_{\mathsf{S}_3} \right\rangle \right. \\ \left. \left| \left| \left| 0_{\mathsf{T}_2} \right\rangle + \left| \left| 1_{\mathsf{T}_2} 0_{\mathsf{S}_3} 0_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle + \left| 1_{\mathsf{T}_2} 0_{\mathsf{S}_3} \right\rangle \right. \\ \left. \left| \left| 0_{\mathsf{T}_2} 0_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle + \left| \left| 0_{\mathsf{T}_2} 0_{\mathsf{T}_2} \right\rangle \right. \\ \left. \left| \left| 0_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle + \left| \left| 0_{\mathsf{T}_2} 0_{\mathsf{T}_2} \right\rangle \right. \right] \right. \\ \left. \left| \left| 0_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle + \left| \left| 0_{\mathsf{T}_2} \right\rangle \right| \right. \\ \left. \left| \left| 0_{\mathsf{T}_2} \right\rangle \otimes \left( \left
\left| \left| \left| 1_{\mathsf{S}_1} 1_{\mathsf{S}_2} 1_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \right. \\ \rightarrow \left| \left| 1_{\mathsf{S}_1} 1_{\mathsf{S}_2} 1_{\mathsf{S}_3} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_1} \right\rangle + \left| 1_{\mathsf{T}_1} \right\rangle \right) \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \\ + \left| \left| 1_{\mathsf{T}_1} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \\ + \left| \left| 1_{\mathsf{T}_1} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \\ + \left| \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right) \\ + \left| \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| \left| 0_{\mathsf{T}_2} \right\rangle \otimes \left( \left| 0_{\mathsf{T}_2} \right\rangle - \left| 1_{\mathsf{T}_2} \right\rangle \otimes \left( \left| 0_{\mathsf{T}_2} \right\rangle \otimes \left(
```

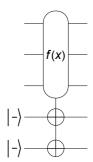
In[⊕]:= Thread[xx → yy] // TableForm

Out[•]//TableForm=

- $\{0, 0, 0\} \rightarrow \{0, 0\}$
- $\{0, 0, 1\} \rightarrow \{1, 1\}$
- $\{0, 1, 0\} \rightarrow \{1, 1\}$
- $\{0, 1, 1\} \rightarrow \{0, 0\}$
- $\{1, 0, 0\} \rightarrow \{0, 0\}$
- $\{1, 0, 1\} \rightarrow \{0, 0\}$
- $\{1, 1, 0\} \rightarrow \{0, 0\}$
- $\{1, 1, 1\} \rightarrow \{1, 0\}$

Superposition

 $In[\cdot]:=$ QuantumCircuit[ProductState[T[{1, 2}] \rightarrow {1, -1}, "Label" \rightarrow Ket[{"-"}]], op] Out[0]=



In[a]:= in = Total@Basis[SS] ** ProductState[T[1] \rightarrow {1, 1}, T[2] \rightarrow {1, -1}]; KetFactor[in]

Out[0]=

```
In[.]:= out = op ** in;
                                                                                                                                                                                                                          ProductForm[out, {SS, TT}]
Out[0]=
                                                                                                                                                                                                                              |\hspace{.06cm} 000\rangle \otimes |\hspace{.06cm} 00\rangle - |\hspace{.06cm} 000\rangle \otimes |\hspace{.06cm} 01\rangle + |\hspace{.06cm} 000\rangle \otimes |\hspace{.06cm} 10\rangle - |\hspace{.06cm} 000\rangle \otimes |\hspace{.06cm} 11\rangle - |\hspace{.06cm} 001\rangle \otimes |\hspace{.06cm} 00\rangle + |\hspace{.06cm} 001\rangle \otimes |\hspace{.06cm} 01\rangle - |\hspace{.06cm} 000\rangle \otimes |\hspace{.06cm} 01\rangle + |\hspace{.06cm} 001\rangle \otimes |\hspace{.06cm} 01\rangle + |\hspace{.06cm} 01\rangle \otimes |\hspace{.06cm} 01\rangle \otimes |\hspace{.06cm} 01\rangle + |\hspace{.06cm} 01\rangle \otimes |\hspace{.06cm} 0
                                                                                                                                                                                                                                                                            |\hspace{.06cm}001\rangle \otimes |\hspace{.06cm}10\rangle + |\hspace{.06cm}001\rangle \otimes |\hspace{.06cm}11\rangle - |\hspace{.06cm}010\rangle \otimes |\hspace{.06cm}00\rangle + |\hspace{.06cm}010\rangle \otimes |\hspace{.06cm}01\rangle - |\hspace{.06cm}010\rangle \otimes |\hspace{.06cm}10\rangle + |\hspace{.06cm}010\rangle \otimes |\hspace{.06cm}01\rangle - |\hspace{.06cm}010\rangle \otimes |\hspace{.06cm}10\rangle + |\hspace{.06cm}010\rangle \otimes |\hspace{.06cm}01\rangle + |\hspace{.06cm}01\rangle \otimes |\hspace{.06cm}01\rangle + |\hspace{.06cm}01\rangle \otimes |\hspace{.06cm}01\rangle \otimes |\hspace{.06cm}01\rangle + |\hspace{.06cm}01\rangle \otimes |\hspace{.06cm}01\rangle \otimes |\hspace{.06cm}01\rangle + |\hspace{.06cm}01\rangle \otimes |\hspace{.
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Summary

Keywords

- Oracle
- Quantum oracle

 $\{1, 0, 1\} \rightarrow \{0, 0\}$ $\{1, 1, 0\} \rightarrow \{0, 0\}$ $\{1, 1, 1\} \rightarrow \{1, 0\}$

Quantum decision making

Functions

- Oracle
- ControlledExp

Related Links

- Section 4.2 of the Quantum Workbook (2022, 2023).
- Tutorial: Quantum Oracle
- Tutorial: Quantum Decision Algorithms