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Errata:
A Quantum Computation Workbook

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Contents

Chapter 1

Postulates of Quantum Mechanics

Problem 1.10 (a), p. 30 The words "first" and "second" in the statements must be exchanged (p_2 should also be replaced with p_1 to be consistent with the rest subquestions). The correct statement should read as

> What is the probability p_0 to find the *second* qubit in $|0\rangle$ (regardless of the *first* qubit)? Similarly, what is the probability p_1 to find the *second* qubit in the state $|1\rangle$?

Chapter 2

Quantum Computation: Overview

Section 2.2, p. 47, Eq. (2.31) It should read as

$$\text{CNOT} = \frac{1}{2} \left(\hat{I} + \hat{S}_c^z + \hat{S}_t^x - \hat{S}_c^z \hat{S}_t^x \right)$$

Section 2.2, p. 64, Eq. (2.65) It should read as

$$UT_1T_2 = \begin{bmatrix} U_{11} & U_{12}'' & 0 & 0 \\ U_{21} & U_{22}'' & U_{23}'' & U_{24}' \\ U_{31} & U_{32}'' & U_{33}'' & U_{34}' \\ U_{41} & U_{42}'' & U_{43}'' & U_{44}' \end{bmatrix}.$$

Section 2.2, p. 64, Eq. (2.66) It should read as

$$T_3 = \begin{bmatrix} \tilde{U}_{11}^* & \tilde{U}_{12}'' & & \\ \tilde{U}_{12}''^* & -\tilde{U}_{11} & & \\ & & 1 & \\ & & & 1 \end{bmatrix}.$$

Section 2.2, p. 64, Eq. (2.67) It should read as

$$UT_1T_2T_3 = \begin{bmatrix} U_{11}''' & 0 & 0 & 0 \\ 0 & U_{22}''' & U_{23}'' & U_{24}' \\ 0 & U_{32}''' & U_{33}'' & U_{34}' \\ 0 & U_{42}''' & U_{43}'' & U_{44}' \end{bmatrix}.$$

Problem 2.3, p. 85, Eq. (2.93) Equation (2.93) should read as

$$\cdots = \hat{S}^\nu \cos(\phi) - \sum_\lambda \hat{S}^\lambda \epsilon_{\lambda\mu\nu} \sin(\phi).$$

Chapter 3

Realizations of Quantum Computers

Section 3.2, p. 100, line 5 from the top "It takes two Pauli X gates ..." $\rightarrow$ "It takes two Hadamard gates ...".

Section 3.3, p. 109, Eq. (3.51) It should read as

$$\cdots = \sum_i \cdots .$$

Section 3.3, p. 110, Eq. (3.54) It should read as

$$\cdots = \sum_{ij} \cdots .$$

Section 3.4, p. 116 The first sentence of Section 3.4.1 shoudl start with "Let us ...".

Section 3.4, p. 117, below Eq. (3.77) "..., we set $\phi_2 = (-1)^m \beta$." $\rightarrow$ "..., we set $\phi_2 = (-1)^{x_1} \beta$."
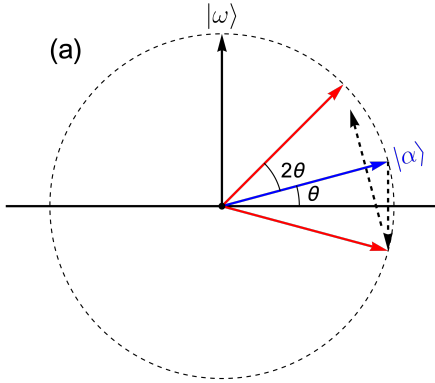
Section 3.4, p. 119 The first sentence of Section 3.4.2 shoudl start with "Let us ...".

Chapter 4

Quantum Algorithms

Fig. 4.4b, p. 179 "... with respect to ω ..." $\rightarrow$ "... with respect to $|v\rangle$ ..."

Fig. 4.5a, p. 182 $\theta, \theta/2 \rightarrow 2\theta, \theta$, respectively. Here is the correct figure for Fig. 4.5**a**:

Section 4.2, p. 145, the second line of the opening paramgraph "... the best known ..." $\rightarrow$ "... the known best ...".

Section 4.2, p. 147, just above Eq. (4.35) "$(a_z \oplus s) \cdot y = (a_z \cdot y) \oplus (s \oplus y)$, it follows ..." $\rightarrow$ "$(a_z \oplus s) \cdot y = (a_z \cdot y) \oplus (s \cdot y)$, it follows ...".

Section 4.4, p. 161, Eq. (4.63) Symbol n must be replaced by m as follows

$$\left(\hat{H}^{\otimes m} |0\rangle \right) \otimes |\phi\rangle = \frac{1}{2^{m/2}} \sum_{x=0}^{2^m-1} |x\rangle \otimes |\phi\rangle .$$

Section 4.4, p. 161, below Eq. (4.64) "... performing the transformation $\hat{U}$ repeatedly depending on the value y on the native register." $\rightarrow$ "... performing the transformation $\hat{U}$ repeatedly depending on value x on the native register."

Problem 4.1 (a) "Classically (...), ..." $->$ "Show that classically (...), ...".

Chapter 5

Quantum Decoherence

Section 5.1 In several places, "Zender" must be corrected to "Zehnder".

Section 5.1, p. 191, the last line "In the blue arm, photon passes through ..." → "In the red arm, photon passes through ...".

Section 5.1, p. 194, below Eq. (5.6) "Whence the photon detection probabilities ..." → "Hence the photon detection probabilities...".

Section 5.2, p. 206, Eq. (5.43) The second '$\otimes$' should be removed from Eq. (5.43). That is, Eq. (5.43) should read as

$$|\Phi\rangle \langle\Phi| = \sum_{kl} |v_k\rangle \langle v_l| \otimes |v_k\rangle \langle v_l| \,.$$

Fig. 5.4, p. 208, line 3 of the caption "... the success probability is $1/4$ ..." → "... the success probability is $1/d^2$ for $d = \dim \mathcal{V}$ ...".

Section 5.2, p. 209, line 1 "... a success probability of $1/4$ ..." → "... a success probability of $1/d^2$ for $d = \dim \mathcal{V}$ ...".

Section 5.2, p. 209, line 10 from top "... a success probability of $1/4$ ..." → "... a success probability of $1/d^2$ ...".

Section 5.2, p. 210, line 3 "... quantum operation: $\mathscr{F} : \mathcal{L}(\mathcal{V}) \to \mathcal{L}(\mathcal{W})$ ..." → "... quantum operation: $\mathscr{F} : \mathcal{L}(\mathcal{V}) \to \mathcal{L}(\mathcal{V})$ ...".

Section 5.2, p. 210, above Eq. (5.58) "than $(\dim \mathcal{V}) \times (\dim \mathcal{W})$. ... on $\mathcal{V} \otimes \mathcal{W}$ ..." → "than $(\dim \mathcal{V})^2$. ... on $\mathcal{V} \otimes \mathcal{E}$ ...".

Section 5.3, p. 216, line 8 "...probabilities $\mathscr{F}_m(\hat{\rho})$" must reads as "...probabilities $\mathrm{Tr}\,[\mathscr{F}_m(\hat{\rho})]$".

Section 5.4, Eq. (5.99) It should read as

$$\hat{G} = \frac{1}{2} \sum_{\mu>0} \hat{L}_\mu^\dagger \hat{L}_\mu \,.$$

Section 5.4, Eq. (5.147) It should read as

$$\frac{d\hat{\rho}}{dt} = \cdots .$$

Section 5.5, the first sentence, p. 234 "..., who close (or different) ..." → "..., how close (or different) ...".

Section 5.5, p. 236, just below Eq. (5.164) "... the canonical norm associate with ..." → "... the canonical norm associated with ...".

Section 5.5, p. 237, just below Eq. (5.177) "... traceless Hermitian operators (a_0) ..." → "... traceless Hermitian operators $(a_0 = 0)$ ...".

Section 5.5, p. 244, the first line "associate with a POVM ..." → "associated with a POVM ...".

Section 5.5, p. 247, below Eq. (5.209) "... of two vectors normalized vectors ..." → "... of two normalized vectors ...".

Section 5.5, p. 248, below Eq. (5.215) "... to note that $\hat{\rho}$ as two eigenvalues ..." → "... to note that $\hat{\rho}$ has two eigenvalues ...".

Sectoin 5.5, p. 249, Eq. (5.224) It should reads

$$\cdots \geq \left| \left(\langle \Psi | \otimes \langle \epsilon_0 | \right) \hat{U} \hat{U}^\dagger \left(|\Phi\rangle \otimes |\epsilon_0\rangle \right) \right| = \cdots .$$

Problem 5.4, p. 252, Eq.(5.234) $\gamma_1 \to \gamma_\phi$

Chapter 6

Quantum Error-Correction Codes

Section 6.1, p. 259, line 10 from the top "... the encoded state $|\psi\rangle$ ..." $\rightarrow$ "... the encoded state $|\bar{\psi}\rangle$ ..."

Section 6.1, p. 259, the second from the bottom "... the original encoded state $|\psi\rangle$ ..." $\rightarrow$ "... the original encoded state $|\bar{\psi}\rangle$ ..."

Section 6.1, p. 265, between Eqs. (6.8) and (6.9) "The phase-slip error ..." $\rightarrow$ "The phase-flip error ...".

Section 6.3, p. 288, Eq. (6.75)

$$\hat{U}(|0\rangle \otimes |\alpha\rangle) = |0\rangle \otimes |\alpha_0\rangle + |1\rangle \otimes \hat{A}|\alpha_1\rangle = \cdots$$

must be changed to

$$\hat{U}(|0\rangle \otimes |\alpha\rangle) = |0\rangle \otimes |\alpha_0\rangle + |1\rangle \otimes \hat{A}|\alpha_0\rangle = \cdots .$$

Section 6.4, p. 298, above Eq. (6.101) "whence" $\rightarrow$ "hence".

Section 6.4, p. 301 In the last sentence of the second paragraph of Section 6.4.2: "... the error sydromes for bit-flip errors ..." $\rightarrow$ "... the error sydromes for phse-flip errors ...".

Section 6.5, p. 309, line 5 from the bottom "These are difficult ..." $\rightarrow$ "The toric codes are difficult ...".

Section 6.5, p. 314, the last line at the bottom "A vertex on a rough edge ... with such a vertex ..." $\rightarrow$ "A plaquette on a rough edge ... with such a plaquette ...".

Section 6.5, p. 315, line 2 from the bottom "... logical operator $\bar{Z}$ ..." $\rightarrow$ "... logical operator $\bar{X}$ ...".

Section 6.5, p. 318, just below Eq. (6.120) "Plaquette and vertex operators
..." → "Measurement of plaquette and vertex operators ...".

Figure 6.9, p. 319, caption (b) "... and vertex defects (red ..." → "... and
plaquette defects (red ...".

Section 6.5, p. 320, line 6 from the top "... upper example in Fig. 11b." →
"... upper example in Fig. 11.".

Chapter 7

Quantum Information Theory

Section 7.1, p. 327, Eq. (7.14) It should read as

$$\cdots \geq \frac{1-x}{\log_e 2} \, .$$

Section 7.3, p. 344, Eq. (7.77) It should read as

$$|\Psi_m\rangle = \binom{n}{m}^{-1/2} \cdots .$$

Section 7.3, p. 344, above Eq. (7.80) " ... diving ..." $\rightarrow$ " ... dividing ...".

Appendix A

Linear Algebra

Appendix A.1, p. 350, Definition A.3 "... there exists a solution ..." → "... there exists a non-trivial solution ..."

Appendix A.1, p. 351, above Eq. (A.5) "Whence u is orthogonal ..." → "Hence u is orthogonal ...".

Appendix A.4, p. 364, above Eq. (A.55) "Whence, $\hat{A} \geq 0$." → "Hence, $\hat{A} \geq 0$."

Appendix A.4, p. 363, below Eq. (A.59) "... eigenvalues ± 1" → "... eigenvalues $e^{\mp i\phi}$".

Appendix A.6, p. 369, below Eq. (A.79) $N := \mathcal{W} \to N := \dim \mathcal{W}$.

Appendix B

Superoperators

Appendix B.1, p. 377, Eq. (B.6) $\hat{S}^x \to \hat{S}^\mu$.

Appendix B.2, below Exercise B.4

- "The following theorem confirms that any supermap ..." *to* "The following theorem confirms that any completely positive supermap ...".
- "... find a more compact ..." $\to$ "... find more compact ...".

Appendix B.2, between Eqs. (B.30) and (B.31)

- $\{v_j\} \to \{|v_j\rangle\}$
- $|w_k\rangle \to \{|w_k\rangle\}$

Appendix B.4, p. 391, just below Eq. (B.53) "we have" $\to$ "We have".

Appendix B.4, p. 392, Eq. (B.56) $|\Psi\rangle\langle\Psi|$ should be replaced by $|\Phi\rangle\langle\Phi|$.

Appendix B.4, p. 393, the second last line "Whence, transposition ..." $\to$ "Hence, transposition ...".

Appendix C

Group Theory

Appendix C.1, p. 396, Definition C.1 (c) "... identity element $e \in \mathcal{G}$ ..." $\rightarrow$ "... identity element $E \in \mathcal{G}$ ...".

Appendix C.2, p. 399, Theorem C.8 (b) "... $\mathcal{G}$ an be ..." $\rightarrow$ "... $\mathcal{G}$ can be ...".

Appendix C.4, pp. 402, Defintion C.17 (a) "... $\mathcal{G} \otimes \mathcal{G}'$ ..." $\rightarrow$ "... $\mathcal{G} \times \mathcal{G}'$ ...".

Appendix C.4, pp. 403, Eq. (C.22) $\mathcal{G} \otimes \mathcal{G}' := \cdots \rightarrow \mathcal{G} \times \mathcal{G}' := \cdots$.

Appendix F

Solutions to Select Problems

Appendix F.3, p. 412 The heading "Quantum Computers" should be corrected to "Realizations of Quantum Comptuers" to match the original heading Chapter 3.

Appendix F.3, p. 412, Eq. (F.8) $|D\rangle := \cdots \rightarrow |\Omega\rangle := \cdots$.

Appendix F.3, p. 412, Eq. (F.11) It should read as

$$|D\rangle = \frac{|1\rangle \sin(\theta/2)e^{-i\phi/2} - \cdots}{\Omega}.$$

Appendix F.3, p. 412, below Eq. (F.12) "... the Berry phase as $\gamma = -iA^\phi = \frac{1}{2}\cos\theta$" $\rightarrow$ "... the Berry phase as $\gamma := -i\int_0^{2\pi} d\phi\, A^\phi = -2\pi i A^\phi = \pi\cos\theta$".

Appendix F.3, p. 412, above Eq. (F.13) "... the Abelina geometric ..." $\rightarrow$ "... the Abelian geometric ...".

Appendix F.3, p. 412, Eq. (F.13) It should read as

$$U(\mathcal{C}) = e^{-i\gamma} = e^{-i\pi\cos\theta}$$

Appendix F.3, p. 413, above Eq. (F.17) "... a finite-finite dimensional ..." $\rightarrow$ "... a finite-dimensional ...".

Appendix F.5, p. 415 The heading "Decoherence" should be corrected to "Quantum Decoherence" to match the original heading Chapter 5.

Problem 6.7, p. 422, the display equation between (F.58) and (F.59) $\hat{W}$ must be replaced with $\hat{P}'''$, i.e.,

$$\cdots(\hat{Z} \otimes \hat{W})\cdots \quad \rightarrow \quad \cdots(\hat{Z} \otimes \hat{P}''')\cdots$$