# Multi-Control NOT Gate

In[0]:= Let[Qubit, S]

### **CNOT: Review**

```
In[*]:= cnot = QuantumCircuit[CNOT[S[1], S[2]]]
Out[*]:=
```



```
In[@ ]:= in = Basis[S@{1, 2}];
out = cnot ** in;
```

ProductForm[Thread[in → out], S@{1, 2}] // TableForm

Out[•]//TableForm=

$$|\hspace{.06cm} 0\hspace{.02cm}\rangle \otimes \hspace{.02cm} |\hspace{.06cm} 0\hspace{.02cm}\rangle \hspace{.1cm} \rightarrow \hspace{.02cm} |\hspace{.06cm} 0\hspace{.02cm}\rangle \otimes \hspace{.02cm} |\hspace{.06cm} 0\hspace{.02cm}\rangle$$

$$\left| \begin{array}{c} 0 \end{array} \right\rangle \otimes \left| \begin{array}{c} 1 \end{array} \right\rangle \rightarrow \left| \begin{array}{c} 0 \end{array} \right\rangle \otimes \left| \begin{array}{c} 1 \end{array} \right\rangle$$

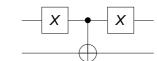
$$|1\rangle \otimes |0\rangle \rightarrow |1\rangle \otimes |1\rangle$$

$$\left| \hspace{.05cm} 1 \hspace{.05cm} \right\rangle \hspace{.05cm} \otimes \hspace{.05cm} \left| \hspace{.05cm} 1 \hspace{.05cm} \right\rangle \hspace{.05cm} \rightarrow \hspace{.05cm} \left| \hspace{.05cm} 1 \hspace{.05cm} \right\rangle \hspace{.05cm} \otimes \hspace{.05cm} \left| \hspace{.05cm} 0 \hspace{.05cm} \right\rangle$$

 $\begin{vmatrix} x_1 \rangle & & & | x_1 \rangle \\ | x_2 \rangle & & & | x_1 \oplus x_2 \rangle \end{vmatrix}$ 

## CNOT on the opposite condition

In[\*]:= xnot = QuantumCircuit[S[1, 1], CNOT[S[1], S[2]], S[1, 1]]
Out[\*] =



```
In[⊕]:= in = Basis[S@{1, 2}];
  out = xnot ** in;
  ProductForm[Thread[in → out], S@{1, 2}] // TableForm
```

Out[•]//TableForm=

$$| 0 \rangle \otimes | 0 \rangle \rightarrow | 0 \rangle \otimes | 1 \rangle$$

$$| 0 \rangle \otimes | 1 \rangle \rightarrow | 0 \rangle \otimes | 0 \rangle$$

$$\left| \begin{array}{c|c} 1 \end{array} \right\rangle \otimes \left| \begin{array}{c|c} 0 \end{array} \right\rangle \rightarrow \left| \begin{array}{c|c} 1 \end{array} \right\rangle \otimes \left| \begin{array}{c|c} 0 \end{array} \right\rangle$$

$$|1\rangle \otimes |1\rangle \rightarrow |1\rangle \otimes |1\rangle$$

In[@]:= TableForm[{MatrixForm /@Matrix /@ {cnot, xnot}}},

TableHeadings → {None, {CNOT, XNOT}}},

TableAlignments → Center]

Out[•]//TableForm=

CNOT					XNOT				
<i>(</i> 1			0		0		0		١
0		0			1	0	0	0	
0	0	0	1		0	0	1	0	
0	0	1	0 ,		0	0	0	1,	

In[.]:= CNOT[S[1], S[2]]

Out[0]=

$$CNOT\,[\;\{\,S_1\,\}\;\rightarrow\;\{\,1\,\}\;,\;\;\{\,S_2\,\}\;]$$

In[
$$\circ$$
]:= CNOT[S[1]  $\rightarrow$  0, S[2]]

Out[0]=

$$\text{CNOT} \left[ \; \left\{ \, \mathsf{S}_{1} \, \right\} \; \rightarrow \; \left\{ \, \mathsf{O} \, \right\} \; , \; \; \left\{ \, \mathsf{S}_{2} \, \right\} \; \right]$$

In[\*]:= new = QuantumCircuit[CNOT[S[1] → 0, S[2]]]

Out[•]=



In[\*]:= new - xnot // Elaborate

Out[•]=

0

In[\*]:= Let[Binary, x]

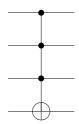
$$\label{eq:local_local_local_local} $$\inf_{\mathbf{0}} = \operatorname{QuantumCircuit}[\operatorname{in} = \operatorname{Ket}[S@\{1,\,2\} \to x@\{1,\,2\}], \ \operatorname{new}, $$ \operatorname{Ket}[S[1] \to x[1], S[2] \to \operatorname{Mod}[1+x[1]+x[2],\,2]], $$ "PortSize" \to \{0.7,\,2\}] $$$$

Out[•]=

$$\begin{vmatrix} x_1 \end{vmatrix} \longrightarrow \begin{vmatrix} x_1 \end{vmatrix}$$
  
 $\begin{vmatrix} x_2 \end{vmatrix} \longrightarrow \begin{vmatrix} 1 \oplus x_1 \oplus x_2 \end{vmatrix}$ 

### Multi-Control-NOT Gate

```
In[0]:= QuantumCircuit[CNOT[S[{1, 2, 3}, $], S[4]]]
Out[0]=
```



```
In[0]:= $n = 2;
     cc = S[Range[$n], $];
     tt = S[$n + 1, $];
     cnot = QuantumCircuit[CNOT[cc, tt]]
```

Out[0]=



In[.]:= Matrix[cnot] // MatrixForm

Out[•]//MatrixForm=

```
1 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0
0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0
0 0 0 1 0 0 0 0
0 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 1
00000010
```

```
In[0]:= in = Basis[cc, tt];
     out = cnot ** in;
```

#### In[:]:= ProductForm[Thread[in → out], {cc, tt}] // TableForm

Out[•]//TableForm=

$$igg|00
angle\otimesigg|0
angle
ightarrowigg|00
angle\otimesigg|0
angle\ |00
angle\otimesigg|1
angle
ightarrowigg|00
angle\otimesigg|1
angle$$

$$\left|\begin{array}{c} 01 \end{array}\right> \otimes \left|\begin{array}{c} 0 \end{array}\right> \rightarrow \left|\begin{array}{c} 01 \end{array}\right> \otimes \left|\begin{array}{c} 0 \end{array}\right>$$

$$|\hspace{.06cm} \texttt{01}\hspace{.02cm}\rangle \otimes \hspace{.05cm} |\hspace{.06cm} \texttt{1}\hspace{.02cm}\rangle \hspace{.1cm} \rightarrow \hspace{.05cm} |\hspace{.06cm} \texttt{01}\hspace{.02cm}\rangle \otimes \hspace{.05cm} |\hspace{.06cm} \texttt{1}\hspace{.02cm}\rangle$$

$$|10\rangle \otimes |0\rangle \rightarrow |10\rangle \otimes |0\rangle$$

$$|10\rangle \otimes |1\rangle \rightarrow |10\rangle \otimes |1\rangle$$

$$|11\rangle \otimes |0\rangle \rightarrow |11\rangle \otimes |1\rangle$$

$$|11\rangle \otimes |1\rangle \rightarrow |11\rangle \otimes |0\rangle$$

Out[•]=

CNOT[
$$\{S_1, S_2\} \rightarrow \{1, 1\}, \{S_3\}$$
]

$$In[ \circ ] := CNOT[S[\{1, 2\}, \$] \rightarrow \{1, 0\}, S[3]]$$

Out[0]=

$$\text{CNOT}\,[\,\,\{\,S_{1}\,,\,\,S_{2}\,\}\,\rightarrow\,\{\,1\,,\,\,0\,\}\,\,,\,\,\{\,S_{3}\,\}\,\,]$$

$$In[\ \circ\ ]:= qc = QuantumCircuit[CNOT[S@{1, 2} \rightarrow {1, 0}, S[3]]]$$

Out[0]=



#### $In[\circ]:=$ ProductForm[Thread[in $\rightarrow$ out], {S@{1, 2}, S[3]}] // TableForm

Out[•]//TableForm=

$$|\hspace{.06cm} 00\hspace{.02cm}\rangle \otimes \hspace{.02cm} |\hspace{.06cm} 0\hspace{.02cm}\rangle \hspace{.02cm} \rightarrow \hspace{.02cm} |\hspace{.06cm} 00\hspace{.02cm}\rangle \otimes \hspace{.02cm} |\hspace{.06cm} 0\hspace{.02cm}\rangle$$

$$|00\rangle \otimes |1\rangle \rightarrow |00\rangle \otimes |1\rangle$$

$$|\hspace{.06cm} \mathtt{01}\hspace{.02cm}\rangle \otimes |\hspace{.06cm} \mathtt{0}\hspace{.02cm}\rangle \rightarrow |\hspace{.06cm} \mathtt{01}\hspace{.02cm}\rangle \otimes |\hspace{.06cm} \mathtt{0}\hspace{.02cm}\rangle$$

$$|\hspace{.06cm} \texttt{01}\hspace{.02cm} \rangle \otimes \hspace{.02cm} |\hspace{.06cm} \texttt{1}\hspace{.02cm} \rangle \hspace{.02cm} \rightarrow \hspace{.02cm} |\hspace{.06cm} \texttt{01}\hspace{.02cm} \rangle \otimes \hspace{.02cm} |\hspace{.06cm} \texttt{1}\hspace{.02cm} \rangle$$

$$|10\rangle \otimes |0\rangle \rightarrow |10\rangle \otimes |1\rangle$$

$$|10\rangle \otimes |1\rangle \rightarrow |10\rangle \otimes |0\rangle$$

$$|11\rangle \otimes |0\rangle \rightarrow |11\rangle \otimes |0\rangle$$

$$|\hspace{.06cm} \texttt{11} \hspace{.02cm} \rangle \otimes \hspace{.02cm} |\hspace{.06cm} \texttt{1} \hspace{.02cm} \rangle \hspace{.02cm} \rightarrow \hspace{.02cm} |\hspace{.06cm} \texttt{11} \hspace{.02cm} \rangle \otimes \hspace{.02cm} |\hspace{.06cm} \texttt{1} \hspace{.02cm} \rangle$$

## Summary

### **Functions**

- CNOT
- Hadamard

#### **Related Links**

- Chapters 2 of the Quantum Workbook (2022, 2023).
- Tutorial: "Quantum Computation: Overview"