Working in Different Bases: MatrixIn & ExpressionIn

Example: A Chain of Qubits

Consider a quantum register of qubits, referred to by symbol S.

In[∘]:=
```
Let[Qubit, S]
```

In[∘]:=
```
$L = 4;
ss = S[Range@$L, $]
```

Out[∘]=

$$\{S_1, S_2, S_3, S_4\}$$

In[∘]:=
```
xx = Total@ChainBy[Through[ss[1]], Multiply];
yy = Total@ChainBy[Through[ss[2]], Multiply];
zz = Total@ChainBy[Through[ss[3]], Multiply];
H = xx + yy + zz
```

Out[∘]=

$$S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z + S_2^x S_3^x + S_2^y S_3^y + S_2^z S_3^z + S_3^x S_4^x + S_3^y S_4^y + S_3^z S_4^z$$

The Hamiltonian has so many zeros. That is why matrix representations are treated in `SparseArray` form in Q3.

In[∘]:=
```
old = Matrix[H];
old // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & -1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & -3 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & -3 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

MatrixIn & ExpressionIn

In[ ]:=
```
bs = QubitAdd[ss]
```

Out[ ]=

$$\Big\langle\Big|\ \{0,0\}\to\Big\{\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle-\frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle-\frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle,$$

$$\frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{3}}-\frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{3}}\Big\},\ \{1,-1\}\to$$

$$\Big\{\frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}}-\frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}},\ \frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}-\sqrt{\frac{2}{3}}\left|1_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle,$$

$$\frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|1_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\left|1_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle\Big\},$$

$$\{1,0\}\to\Big\{\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle-\frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle-\frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle,$$

$$\frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{3}}-\frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{3}},$$

$$\frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}-\frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}-\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}-\frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}\Big\},\ \{1,1\}\to$$

$$\Big\{\frac{\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{2}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{2}},\ \sqrt{\frac{2}{3}}\left|0_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle-\frac{\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}},$$

$$\frac{1}{2}\sqrt{3}\left|0_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle-\frac{\left|0_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}\Big\},$$

$$\{2,-2\}\to\Big\{\left|1_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle\Big\},\ \{2,-1\}\to$$

$$\Big\{\frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle\Big\},$$

$$\{2,0\}\to\Big\{\frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}\Big\},$$

$$\{2,1\}\to\Big\{\frac{1}{2}\left|0_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|0_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle+\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle\Big\},$$

$$\{2,2\}\to\Big\{\left|0_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle\Big\}\Big|\Big\rangle$$

In[•]:=
```
Keys[bs]
```
Out[•]=
```
{{0, 0}, {1, -1}, {1, 0}, {1, 1}, {2, -2}, {2, -1}, {2, 0}, {2, 1}, {2, 2}}
```

In[•]:=
```
GroupBy[Keys[bs], First] // Normal // TableForm
```
Out[•]//TableForm=
```
0 → {{0, 0}}
1 → {{1, -1}, {1, 0}, {1, 1}}
2 → {{2, -2}, {2, -1}, {2, 0}, {2, 1}, {2, 2}}
```

In[•]:=
```
bb = Catenate[bs]
```
Out[•]=

$$\Big\{\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle-\frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle-\frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle,\ \frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{3}}-$$

$$\frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{3}},$$

$$\frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}}-\frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}},\ \frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}-\sqrt{\frac{2}{3}}\left|1_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle,$$

$$\frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|1_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\left|1_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle,$$

$$\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle-\frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle-\frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle,$$

$$\frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{3}}-\frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}+$$

$$\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{3}},\ \frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}-$$

$$\frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}-\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}-\frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}},$$

$$\frac{\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{2}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{2}},\ \sqrt{\frac{2}{3}}\left|0_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle-\frac{\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}},$$

$$\frac{1}{2}\sqrt{3}\left|0_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle-\frac{\left|0_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}},\ \left|1_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle,$$

$$\frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle,\ \frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+$$

$$\frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}+\frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}},$$

$$\frac{1}{2}\left|0_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|0_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle+\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle,\ \left|0_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle\Big\}$$

In[•]:=
```
EchoTiming[
  new = Outer[Multiply, Dagger[bb], H ** bb];
]
new // MatrixForm
```

0.568049

Out[•]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

The same job can be done by simply using `MatrixIn`. One important difference is that the result is in `SparseArray` now.

In[•]:=
```
EchoTiming[
  more = MatrixIn[H, bb]
]
```

0.564406

Out[•]=

```
SparseArray[ Specified elements: 29  Dimensions: {16, 16} ]
```

In[ ]:=
```
more // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

In[∘]:=
```
mm = MatrixIn[H, bs]
```

Out[∘]=

⟨| {0, 0} → SparseArray[Specified elements: 3 Dimensions: {2, 2}],
{1, -1} → SparseArray[Specified elements: 7 Dimensions: {3, 3}],
{1, 0} → SparseArray[Specified elements: 7 Dimensions: {3, 3}],
{1, 1} → SparseArray[Specified elements: 7 Dimensions: {3, 3}],
{2, -2} → SparseArray[Specified elements: 1 Dimensions: {1, 1}],
{2, -1} → SparseArray[Specified elements: 1 Dimensions: {1, 1}],
{2, 0} → SparseArray[Specified elements: 1 Dimensions: {1, 1}],
{2, 1} → SparseArray[Specified elements: 1 Dimensions: {1, 1}],
{2, 2} → SparseArray[Specified elements: 1 Dimensions: {1, 1}] |⟩

In[∘]:=
```
MatrixForm /@ mm
```

Out[∘]=

$$\langle| \{0,0\} \to \begin{pmatrix} -6 & \sqrt{3} \\ \sqrt{3} & 0 \end{pmatrix}, \{1,-1\} \to \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix},$$

$$\{1,0\} \to \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}, \{1,1\} \to \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}, \{2,-2\} \to (3),$$

$$\{2,-1\} \to (3), \{2,0\} \to (3), \{2,1\} \to (3), \{2,2\} \to (3) |\rangle$$

In[∘]:=
```
op = ExpressionIn[mm, bs] // Elaborate
```

Out[∘]=

$$S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z + S_2^x S_3^x + S_2^y S_3^y + S_2^z S_3^z + S_3^x S_4^x + S_3^y S_4^y + S_3^z S_4^z$$

In[ ]:=
```
op - H
```
Out[ ]=
```
0
```

Using a basis-change matrix?

- **Question**: How about just constructing a unitary matrix corresponding to the basis change?
- **Answer**: It may be inefficient in many cases. Sometimes, it is practically impossible.

Let us consider an example to see why.

In[ ]:=
```
old // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix}
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & -1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & -3 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & -3 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

In[ ]:=
```
MatrixForm /@ mm
```
Out[ ]=

$$\left\langle\left| \{0,0\} \to \begin{pmatrix} -6 & \sqrt{3} \\ \sqrt{3} & 0 \end{pmatrix},\ \{1,-1\} \to \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix},\right.\right.$$

$$\{1,0\} \to \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix},\ \{1,1\} \to \begin{pmatrix} -2 & \sqrt{3} & 0 \\ \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix},\ \{2,-2\} \to (3),$$

$$\left.\left.\{2,-1\} \to (3),\ \{2,0\} \to (3),\ \{2,1\} \to (3),\ \{2,2\} \to (3) \right|\right\rangle$$

In[ ]:=
```
Keys[bs]
```
Out[ ]=
```
{{0, 0}, {1, -1}, {1, 0}, {1, 1}, {2, -2}, {2, -1}, {2, 0}, {2, 1}, {2, 2}}
```

In[ ]:=
```
U = Conjugate@Matrix[bs[{0, 0}]];
U // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & 0 & 0 & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
mm00 = U.old.Topple[U];
mm00 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -6 & \sqrt{3} \\ \sqrt{3} & 0 \end{pmatrix}$$

Summary

Functions

- `MatrixIn, ExpressionIn`
- `Matrix, ExpressionFor`
- `Outer`

Related Links

- Appendix A of the Quantum Workbook (2022, 2023) -- Available for free via the QuantumPlaybook package.