Quantum Oracle: Definition

Episode 26. Classical Oracle

Episode 27. Quantum Oracle: Definition

Episode 28. Quantum Oracle: Properties

Classical Oracle: Review

Classical Oracle

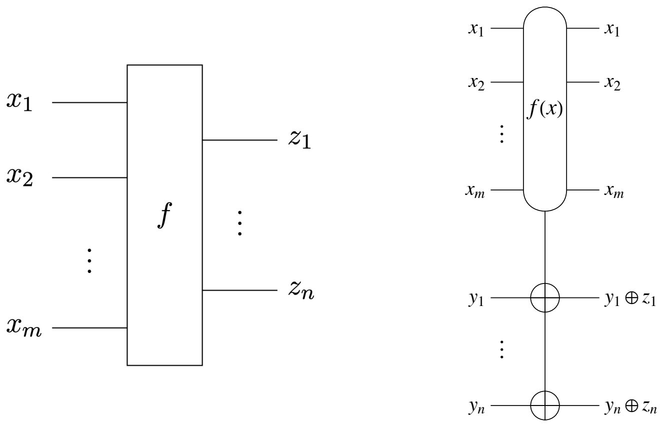


Figure 1. **(Left)** A circuit diagram of classical oracle $f : \{0, 1\}^m \to \{0, 1\}^n$. x is an m-bit string, $x \in \{0, 1\}^m$, and z denotes the image of f at x, $z = f(x) \in \{0, 1\}^n$. **(Right)** A reversible version of classical oracle $f : \{0, 1\}^m \to \{0, 1\}^n$. $x \in \{0, 1\}^m$ and $y \in \{0, 1\}^n$ are m-bit and n-bit strings, respectively, and $z = f(x) \in \{0, 1\}^n$ denotes the image of f at x.

```
In[ ]:= $m = 3;
        $n = 2;
```

```
In[ ]:= f[1] = f[2] = 3;
        f[7] = 2;
        f[_Integer] = 0;
```

```
In[ ]:= ff = Oracle[f, $m, $n]
Out[ ]= Oracle[f, 3, 2]
```

```
In[ ]:= xx = Tuples[{0, 1}, $m]
Out[ ]= {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}, {0, 1, 1}, {1, 0, 0}, {1, 0, 1}, {1, 1, 0}, {1, 1, 1}}
```

```
In[ ]:= zz = ff /@ xx
Out[ ]=
{{0, 0}, {1, 1}, {1, 1}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {1, 0}}
```

```
In[ ]:= Thread[xx → zz] // TableForm
Out[ ]//TableForm=
{0, 0, 0} → {0, 0}
{0, 0, 1} → {1, 1}
{0, 1, 0} → {1, 1}
{0, 1, 1} → {0, 0}
{1, 0, 0} → {0, 0}
{1, 0, 1} → {0, 0}
{1, 1, 0} → {0, 0}
{1, 1, 1} → {1, 0}
```

Quantum Oracle

Definition

The *quantum oracle* corresponding to the classical oracle f is simply an implementation of the above extended mapping for reversible computation on quantum registers. It is a quantum gate operation defined by the association

$$U_f : |x\rangle \otimes |y\rangle \mapsto |x\rangle \otimes |f(x) \oplus y\rangle,$$

where $|x\rangle$ and $|y\rangle$ are the computational basis states belonging to the native and auxiliary register of m and n qubits, respectively.

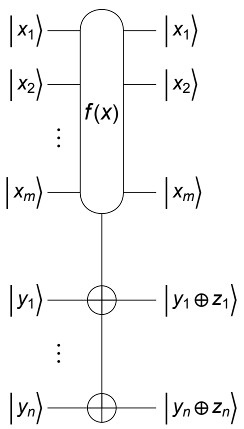


Figure 3. A quantum circuit for the quantum oracle corresponding to classical oracle $f : \{0, 1\}^m \to \{0, 1\}^n$., which is a direct analogue of the classical reversible oracle in Figure 2. $x \in \{0, 1\}^m$ and $y \in \{0, 1\}^n$ is m-bit and n-bit strings, respectively, and $z = f(x) \in \{0, 1\}^n$ denotes the image of f at x.

- Since the extended mapping is one-to-one and the computational basis states are orthonormal, the operator U_f is unitary.
- Recall that U_f is a linear operator and can act on any arbitrary superposition states.

Example

In[∘]:=
```
Let[Qubit, S, T]
```

In[∘]:=
```
SS = S[Range[$m], $];
TT = T[Range[$n], $];
```

In[∘]:=
```
op = Oracle[f, SS, TT]
```

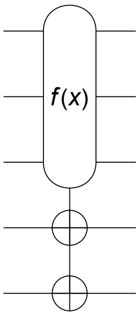
Out[∘]=

$\text{Oracle}[f, \{S_1, S_2, S_3\}, \{T_1, T_2\}]$

Note: $\text{Oracle}[f, \{c_1, c_2, \ldots\}, \{t_1, t_2, \ldots\}]$ represents quantum oracle while $\text{Oracle}[f, m, n]$ refers to the classical oracle.

In[∘]:=
```
qc = QuantumCircuit[op]
```

Out[∘]=



In[∘]:=
```
in = Basis[SS, TT];
ProductForm[in, {SS, TT}]
```

Out[∘]=

$\{|000\rangle \otimes |00\rangle, |000\rangle \otimes |01\rangle, |000\rangle \otimes |10\rangle, |000\rangle \otimes |11\rangle, |001\rangle \otimes |00\rangle, |001\rangle \otimes |01\rangle,$
$|001\rangle \otimes |10\rangle, |001\rangle \otimes |11\rangle, |010\rangle \otimes |00\rangle, |010\rangle \otimes |01\rangle, |010\rangle \otimes |10\rangle, |010\rangle \otimes |11\rangle,$
$|011\rangle \otimes |00\rangle, |011\rangle \otimes |01\rangle, |011\rangle \otimes |10\rangle, |011\rangle \otimes |11\rangle, |100\rangle \otimes |00\rangle,$
$|100\rangle \otimes |01\rangle, |100\rangle \otimes |10\rangle, |100\rangle \otimes |11\rangle, |101\rangle \otimes |00\rangle, |101\rangle \otimes |01\rangle,$
$|101\rangle \otimes |10\rangle, |101\rangle \otimes |11\rangle, |110\rangle \otimes |00\rangle, |110\rangle \otimes |01\rangle, |110\rangle \otimes |10\rangle,$
$|110\rangle \otimes |11\rangle, |111\rangle \otimes |00\rangle, |111\rangle \otimes |01\rangle, |111\rangle \otimes |10\rangle, |111\rangle \otimes |11\rangle\}$

In[∘]:=
```
out = op ** in;
ProductForm[out, {SS, TT}]
```

Out[∘]=

$\{|000\rangle \otimes |00\rangle, |000\rangle \otimes |01\rangle, |000\rangle \otimes |10\rangle, |000\rangle \otimes |11\rangle, |001\rangle \otimes |11\rangle, |001\rangle \otimes |10\rangle,$
$|001\rangle \otimes |01\rangle, |001\rangle \otimes |00\rangle, |010\rangle \otimes |11\rangle, |010\rangle \otimes |10\rangle, |010\rangle \otimes |01\rangle, |010\rangle \otimes |00\rangle,$
$|011\rangle \otimes |00\rangle, |011\rangle \otimes |01\rangle, |011\rangle \otimes |10\rangle, |011\rangle \otimes |11\rangle, |100\rangle \otimes |00\rangle,$
$|100\rangle \otimes |01\rangle, |100\rangle \otimes |10\rangle, |100\rangle \otimes |11\rangle, |101\rangle \otimes |00\rangle, |101\rangle \otimes |01\rangle,$
$|101\rangle \otimes |10\rangle, |101\rangle \otimes |11\rangle, |110\rangle \otimes |00\rangle, |110\rangle \otimes |01\rangle, |110\rangle \otimes |10\rangle,$
$|110\rangle \otimes |11\rangle, |111\rangle \otimes |10\rangle, |111\rangle \otimes |11\rangle, |111\rangle \otimes |00\rangle, |111\rangle \otimes |01\rangle\}$

In[∘]:=
```
ProductForm[Thread[in → out], {SS, TT}] // TableForm
```

Out[∘]//TableForm=

$|000\rangle \otimes |00\rangle \to |000\rangle \otimes |00\rangle$
$|000\rangle \otimes |01\rangle \to |000\rangle \otimes |01\rangle$
$|000\rangle \otimes |10\rangle \to |000\rangle \otimes |10\rangle$
$|000\rangle \otimes |11\rangle \to |000\rangle \otimes |11\rangle$
$|001\rangle \otimes |00\rangle \to |001\rangle \otimes |11\rangle$
$|001\rangle \otimes |01\rangle \to |001\rangle \otimes |10\rangle$
$|001\rangle \otimes |10\rangle \to |001\rangle \otimes |01\rangle$
$|001\rangle \otimes |11\rangle \to |001\rangle \otimes |00\rangle$
$|010\rangle \otimes |00\rangle \to |010\rangle \otimes |11\rangle$
$|010\rangle \otimes |01\rangle \to |010\rangle \otimes |10\rangle$
$|010\rangle \otimes |10\rangle \to |010\rangle \otimes |01\rangle$
$|010\rangle \otimes |11\rangle \to |010\rangle \otimes |00\rangle$
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$|011\rangle \otimes |10\rangle \to |011\rangle \otimes |10\rangle$
$|011\rangle \otimes |11\rangle \to |011\rangle \otimes |11\rangle$
$|100\rangle \otimes |00\rangle \to |100\rangle \otimes |00\rangle$
$|100\rangle \otimes |01\rangle \to |100\rangle \otimes |01\rangle$
$|100\rangle \otimes |10\rangle \to |100\rangle \otimes |10\rangle$
$|100\rangle \otimes |11\rangle \to |100\rangle \otimes |11\rangle$
$|101\rangle \otimes |00\rangle \to |101\rangle \otimes |00\rangle$
$|101\rangle \otimes |01\rangle \to |101\rangle \otimes |01\rangle$
$|101\rangle \otimes |10\rangle \to |101\rangle \otimes |10\rangle$
$|101\rangle \otimes |11\rangle \to |101\rangle \otimes |11\rangle$
$|110\rangle \otimes |00\rangle \to |110\rangle \otimes |00\rangle$
$|110\rangle \otimes |01\rangle \to |110\rangle \otimes |01\rangle$
$|110\rangle \otimes |10\rangle \to |110\rangle \otimes |10\rangle$
$|110\rangle \otimes |11\rangle \to |110\rangle \otimes |11\rangle$
$|111\rangle \otimes |00\rangle \to |111\rangle \otimes |10\rangle$
$|111\rangle \otimes |01\rangle \to |111\rangle \otimes |11\rangle$
$|111\rangle \otimes |10\rangle \to |111\rangle \otimes |00\rangle$
$|111\rangle \otimes |11\rangle \to |111\rangle \otimes |01\rangle$

```
In[]:= Thread[xx → zz] // TableForm
Out[]//TableForm=
{0, 0, 0} → {0, 0}
{0, 0, 1} → {1, 1}
{0, 1, 0} → {1, 1}
{0, 1, 1} → {0, 0}
{1, 0, 0} → {0, 0}
{1, 0, 1} → {0, 0}
{1, 1, 0} → {0, 0}
{1, 1, 1} → {1, 0}
```

Summary

Keywords

- Oracle
- Quantum oracle
- Quantum decision making

Functions

- `Oracle`
- `ControlledExp`

Related Links

- Section 4.2 of the Quantum Workbook (2022, 2023).
- Tutorial: Quantum Oracle
- Tutorial: Quantum Decision Algorithms