Working in Different Bases

Why not just in the computational basis?

- One important motivation did we already see in **E. 06**, **"Measurements."**
  Recall that one must expand a given quantum state $\left|\psi\right\rangle$ in the eigenbasis $\left\{\left|a\right\rangle \mid \hat{A}\left|a\right\rangle = \left|a\right\rangle a\right\}$ of the observable $\hat{A} = \underset{a}{S}\left|a\right\rangle a\left\langle a\right|$. That is, $\left|\psi\right\rangle = \sum_a \left|a\right\rangle c_a$.

- Here is another motivation.

Consider a quantum register of qubits, referred to by symbol `S`.

```
In[∘]:= Let[Qubit, S]
```

```
In[∘]:= $L = 4;
ss = S[Range@$L, $]
```

Out[∘]=

$\{S_1, S_2, S_3, S_4\}$

Here is a dynamic way of constructing the spin XX Hamiltonian.

```
In[∘]:= xx = Total@ChainBy[Through[ss[1]], Multiply]
```

Out[∘]=

$S_1^x S_2^x + S_2^x S_3^x + S_3^x S_4^x$

In the same manner, construct the spin YY Hamiltonian.

```
In[∘]:= yy = Total@ChainBy[Through[ss[2]], Multiply]
```

Out[∘]=

$S_1^y S_2^y + S_2^y S_3^y + S_3^y S_4^y$

Finally, construct the spin ZZ Hamiltonian.

```
In[∘]:= zz = Total@ChainBy[Through[ss[3]], Multiply]
```

Out[∘]=

$S_1^z S_2^z + S_2^z S_3^z + S_3^z S_4^z$

Here is the total Hamiltonian.

```
In[∘]:= H = xx + yy + zz
```

Out[∘]=

$S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z + S_2^x S_3^x + S_2^y S_3^y + S_2^z S_3^z + S_3^x S_4^x + S_3^y S_4^y + S_3^z S_4^z$

The Hamiltonian has so many zeros. That is why matrix representations are treated in `SparseArray` form in Q3.

In[ ]:=
```
mat = Matrix[H];
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & -1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & -3 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & -3 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

Anyway, how can we diagonalize Hamiltonian?

What about working in a different basis?

Construct a different basis where elements are characterized by the total angular momentum.

In[ ]:= **bs = QubitAdd[ss]**

Out[ ]=

$$\Big\langle\Big|\ \{0,0\}\to\Big\{\frac{1}{2}\lvert 0_{S_1}1_{S_2}0_{S_3}1_{S_4}\rangle-\frac{1}{2}\lvert 0_{S_1}1_{S_2}1_{S_3}0_{S_4}\rangle-\frac{1}{2}\lvert 1_{S_1}0_{S_2}0_{S_3}1_{S_4}\rangle+\frac{1}{2}\lvert 1_{S_1}0_{S_2}1_{S_3}0_{S_4}\rangle,$$

$$\frac{\lvert 0_{S_1}0_{S_2}1_{S_3}1_{S_4}\rangle}{\sqrt{3}}-\frac{\lvert 0_{S_1}1_{S_2}0_{S_3}1_{S_4}\rangle}{2\sqrt{3}}-\frac{\lvert 0_{S_1}1_{S_2}1_{S_3}0_{S_4}\rangle}{2\sqrt{3}}-\frac{\lvert 1_{S_1}0_{S_2}0_{S_3}1_{S_4}\rangle}{2\sqrt{3}}-\frac{\lvert 1_{S_1}0_{S_2}1_{S_3}0_{S_4}\rangle}{2\sqrt{3}}+\frac{\lvert 1_{S_1}1_{S_2}0_{S_3}0_{S_4}\rangle}{\sqrt{3}}\Big\},\ \{1,-1\}\to$$

$$\Big\{\frac{\lvert 0_{S_1}1_{S_2}1_{S_3}1_{S_4}\rangle}{\sqrt{2}}-\frac{\lvert 1_{S_1}0_{S_2}1_{S_3}1_{S_4}\rangle}{\sqrt{2}},\ \frac{\lvert 0_{S_1}1_{S_2}1_{S_3}1_{S_4}\rangle}{\sqrt{6}}+\frac{\lvert 1_{S_1}0_{S_2}1_{S_3}1_{S_4}\rangle}{\sqrt{6}}-\sqrt{\frac{2}{3}}\lvert 1_{S_1}1_{S_2}0_{S_3}1_{S_4}\rangle,$$

$$\frac{\lvert 0_{S_1}1_{S_2}1_{S_3}1_{S_4}\rangle}{2\sqrt{3}}+\frac{\lvert 1_{S_1}0_{S_2}1_{S_3}1_{S_4}\rangle}{2\sqrt{3}}+\frac{\lvert 1_{S_1}1_{S_2}0_{S_3}1_{S_4}\rangle}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\lvert 1_{S_1}1_{S_2}1_{S_3}0_{S_4}\rangle\Big\},$$

$$\{1,0\}\to\Big\{\frac{1}{2}\lvert 0_{S_1}1_{S_2}0_{S_3}1_{S_4}\rangle+\frac{1}{2}\lvert 0_{S_1}1_{S_2}1_{S_3}0_{S_4}\rangle-\frac{1}{2}\lvert 1_{S_1}0_{S_2}0_{S_3}1_{S_4}\rangle-\frac{1}{2}\lvert 1_{S_1}0_{S_2}1_{S_3}0_{S_4}\rangle,$$

$$\frac{\lvert 0_{S_1}0_{S_2}1_{S_3}1_{S_4}\rangle}{\sqrt{3}}-\frac{\lvert 0_{S_1}1_{S_2}0_{S_3}1_{S_4}\rangle}{2\sqrt{3}}+\frac{\lvert 0_{S_1}1_{S_2}1_{S_3}0_{S_4}\rangle}{2\sqrt{3}}-\frac{\lvert 1_{S_1}0_{S_2}0_{S_3}1_{S_4}\rangle}{2\sqrt{3}}+\frac{\lvert 1_{S_1}0_{S_2}1_{S_3}0_{S_4}\rangle}{2\sqrt{3}}-\frac{\lvert 1_{S_1}1_{S_2}0_{S_3}0_{S_4}\rangle}{\sqrt{3}},\ \frac{\lvert 0_{S_1}0_{S_2}1_{S_3}1_{S_4}\rangle}{\sqrt{6}}+\frac{\lvert 0_{S_1}1_{S_2}0_{S_3}1_{S_4}\rangle}{\sqrt{6}}-$$

$$\frac{\lvert 0_{S_1}1_{S_2}1_{S_3}0_{S_4}\rangle}{\sqrt{6}}+\frac{\lvert 1_{S_1}0_{S_2}0_{S_3}1_{S_4}\rangle}{\sqrt{6}}-\frac{\lvert 1_{S_1}0_{S_2}1_{S_3}0_{S_4}\rangle}{\sqrt{6}}-\frac{\lvert 1_{S_1}1_{S_2}0_{S_3}0_{S_4}\rangle}{\sqrt{6}}\Big\},\ \{1,1\}\to$$

$$\Big\{\frac{\lvert 0_{S_1}1_{S_2}0_{S_3}0_{S_4}\rangle}{\sqrt{2}}-\frac{\lvert 1_{S_1}0_{S_2}0_{S_3}0_{S_4}\rangle}{\sqrt{2}},\ \sqrt{\frac{2}{3}}\lvert 0_{S_1}0_{S_2}1_{S_3}0_{S_4}\rangle-\frac{\lvert 0_{S_1}1_{S_2}0_{S_3}0_{S_4}\rangle}{\sqrt{6}}-\frac{\lvert 1_{S_1}0_{S_2}0_{S_3}0_{S_4}\rangle}{\sqrt{6}},$$

$$\frac{1}{2}\sqrt{3}\lvert 0_{S_1}0_{S_2}0_{S_3}1_{S_4}\rangle-\frac{\lvert 0_{S_1}0_{S_2}1_{S_3}0_{S_4}\rangle}{2\sqrt{3}}-\frac{\lvert 0_{S_1}1_{S_2}0_{S_3}0_{S_4}\rangle}{2\sqrt{3}}-\frac{\lvert 1_{S_1}0_{S_2}0_{S_3}0_{S_4}\rangle}{2\sqrt{3}}\Big\},$$

$$\{2,-2\}\to\Big\{\lvert 1_{S_1}1_{S_2}1_{S_3}1_{S_4}\rangle\Big\},\ \{2,-1\}\to$$

$$\Big\{\frac{1}{2}\lvert 0_{S_1}1_{S_2}1_{S_3}1_{S_4}\rangle+\frac{1}{2}\lvert 1_{S_1}0_{S_2}1_{S_3}1_{S_4}\rangle+\frac{1}{2}\lvert 1_{S_1}1_{S_2}0_{S_3}1_{S_4}\rangle+\frac{1}{2}\lvert 1_{S_1}1_{S_2}1_{S_3}0_{S_4}\rangle\Big\},$$

$$\{2,0\}\to\Big\{\frac{\lvert 0_{S_1}0_{S_2}1_{S_3}1_{S_4}\rangle}{\sqrt{6}}+\frac{\lvert 0_{S_1}1_{S_2}0_{S_3}1_{S_4}\rangle}{\sqrt{6}}+\frac{\lvert 0_{S_1}1_{S_2}1_{S_3}0_{S_4}\rangle}{\sqrt{6}}+\frac{\lvert 1_{S_1}0_{S_2}0_{S_3}1_{S_4}\rangle}{\sqrt{6}}+\frac{\lvert 1_{S_1}0_{S_2}1_{S_3}0_{S_4}\rangle}{\sqrt{6}}+\frac{\lvert 1_{S_1}1_{S_2}0_{S_3}0_{S_4}\rangle}{\sqrt{6}}\Big\},$$

$$\{2,1\}\to\Big\{\frac{1}{2}\lvert 0_{S_1}0_{S_2}0_{S_3}1_{S_4}\rangle+\frac{1}{2}\lvert 0_{S_1}0_{S_2}1_{S_3}0_{S_4}\rangle+\frac{1}{2}\lvert 0_{S_1}1_{S_2}0_{S_3}0_{S_4}\rangle+\frac{1}{2}\lvert 1_{S_1}0_{S_2}0_{S_3}0_{S_4}\rangle\Big\},$$

$$\{2,2\}\to\Big\{\lvert 0_{S_1}0_{S_2}0_{S_3}0_{S_4}\rangle\Big\}\Big|\Big\rangle$$

The keys label the corresponding angular momentum.

In[ ]:= **Keys[bs]**

Out[ ]=

{{0, 0}, {1, -1}, {1, 0}, {1, 1}, {2, -2}, {2, -1}, {2, 0}, {2, 1}, {2, 2}}

Group the labels according to the magnitude of the total angular momentum.

In[ ]:=
```
GroupBy[Keys[bs], First] // Normal // TableForm
```

Out[ ]//TableForm=
```
0 → {{0, 0}}
1 → {{1, -1}, {1, 0}, {1, 1}}
2 → {{2, -2}, {2, -1}, {2, 0}, {2, 1}, {2, 2}}
```

To compare this basis, arrange all elements in the given order.

In[ ]:=
```
bb = Catenate[bs];
bb⟦ ;; 3⟧
```

Out[ ]=

$$\left\{\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle-\frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle-\frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle+\frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle,\right.$$
$$\frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{3}}-\frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}-\frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}}-$$
$$\left.\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}+\frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{3}},\frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}}-\frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}}\right\}$$

Now, calculate the matrix representation of the Hamiltonian in this new basis.

In[ ]:=
```
EchoTiming[
 new = Outer[Multiply, Dagger[bb], H ** bb];
]
new // MatrixForm
```

0.505931

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

The Outer function

Observe how the Outer function works.

In[ ]:=
```
Outer[f, {1, 2, 3}, {a, b}] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} f[1, a] & f[1, b] \\ f[2, a] & f[2, b] \\ f[3, a] & f[3, b] \end{pmatrix}$$

Let us go back to the previous example, and consider different options for calculating the matrix representation in the new basis.

- Choice 1

```
EchoTiming[
 mm = Outer[Dagger[#1] ** H ** #2 &, bb, bb];
]
mm // MatrixForm
```

8.60772

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

- Choice 2

In[ ]:=
```
EchoTiming[mm = Outer[#1 ** H ** #2 &, Dagger[bb], bb];]
mm // MatrixForm
```

8.75689

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

- Choice 3

In[ ]:=
```
EchoTiming[mm = Outer[#1 ** #2 &, Dagger[bb], H ** bb];]
mm // MatrixForm
```

0.547041

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

In[ ]:=
```
EchoTiming[mm = Outer[Multiply, Dagger[bb], H ** bb];]
mm // MatrixForm
```

0.555479

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

Summary

Functions

- `Outer`
- `Matrix, ExpressionFor`
- `QubitAdd`
- `Chain`
- `Dagger`
- `EchoTiming`

Related Links

- Appendix A of the Quantum Workbook (2022, 2023) -- Available for free via the QuantumPlaybook package.