Single-Qubit Rotations

```
In[ ]:= Let[Qubit, S]
        Let[Real, ϕ]
```

Rotation Around the X Axis

```
In[ ]:= op = Rotation[ϕ, S[1]]
```

Out[ ]= $\text{Rotation}[\phi, S^x]$

```
In[ ]:= in = Ket[{S}]
```

Out[ ]= $\left|0_S\right\rangle$

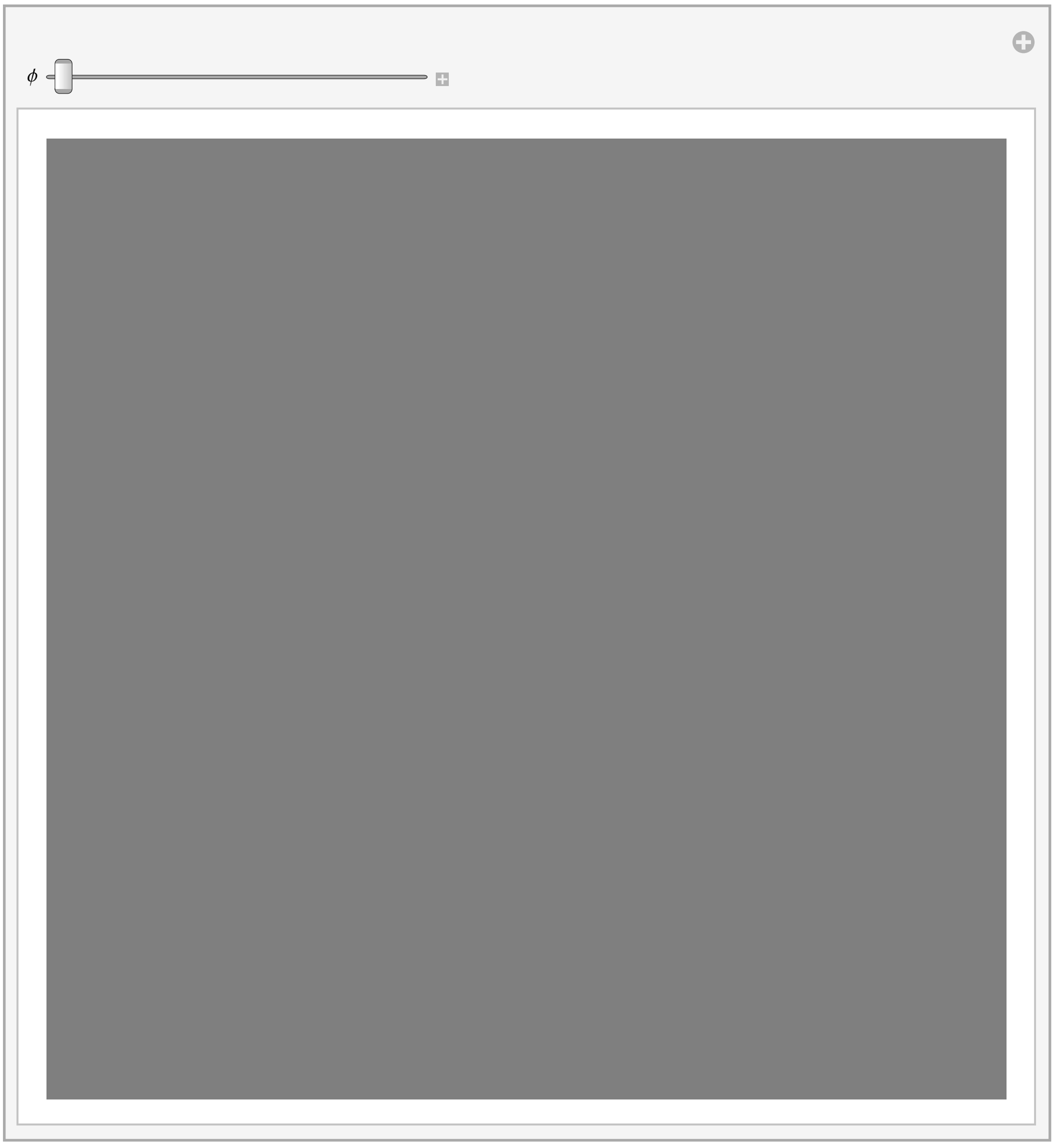
```
In[ ]:= out = op ** in
```

Out[ ]= $\text{Cos}\left[\frac{\phi}{2}\right] \left|0_S\right\rangle - i \left|1_S\right\rangle \text{Sin}\left[\frac{\phi}{2}\right]$

```
In[ ]:= bv[ϕ_] = BlochVector[out] // ExpToTrig // FullSimplify
```

Out[ ]= $\{0, -\text{Sin}[\phi], \text{Cos}[\phi]\}$

In[ ]:=
```
Manipulate[BlochSphere[{Red, Bead@bv@ϕ}, ImageSize → Medium], {ϕ, 0, 2 Pi}]
```
Out[ ]=



Rotation Around the Y Axis

In[ ]:=
```
op = Rotation[ϕ, S[2]]
```
Out[ ]=

$\text{Rotation}[\phi, S^y]$

In[ ]:=
```
in = Ket[{S}]
```
Out[ ]=

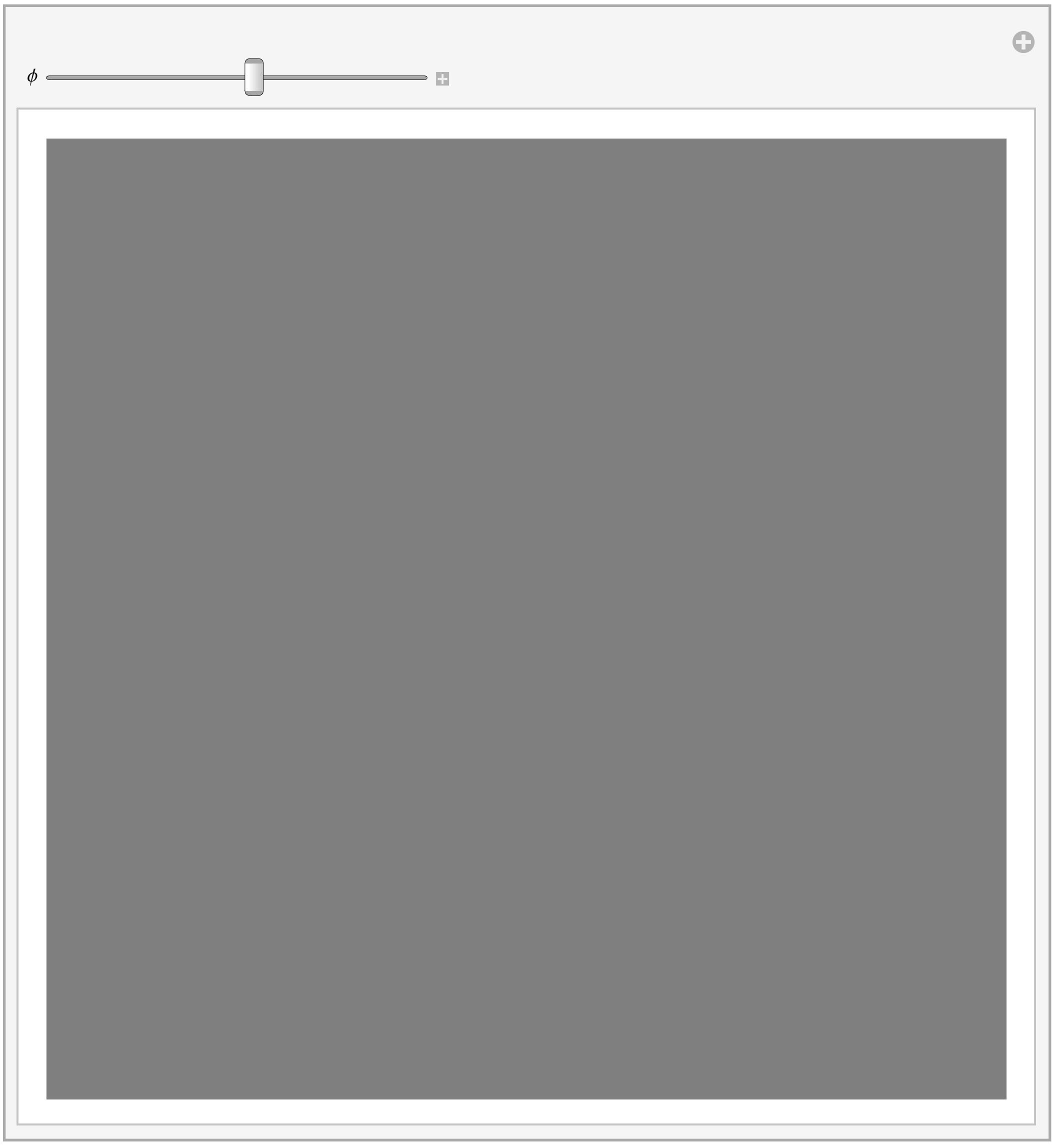
$\left|0_S\right\rangle$

In[ ]:=
```
out = op ** in
```
Out[ ]=

$\text{Cos}\left[\frac{\phi}{2}\right] \left|0_S\right\rangle + \left|1_S\right\rangle \text{Sin}\left[\frac{\phi}{2}\right]$

In[ ]:=
```
bv[ϕ_] = BlochVector[out] // ExpToTrig // FullSimplify
```
Out[ ]=
```
{Sin[ϕ], 0, Cos[ϕ]}
```

In[ ]:=
```
Manipulate[BlochSphere[{Red, Bead@bv@ϕ}, ImageSize → Medium], {ϕ, 0, 2 Pi}]
```
Out[ ]=



Rotation Around the Z Axis

In[ ]:=
```
op = Rotation[ϕ, S[3]]
```
Out[ ]=

$\text{Rotation}[\phi, S^z]$

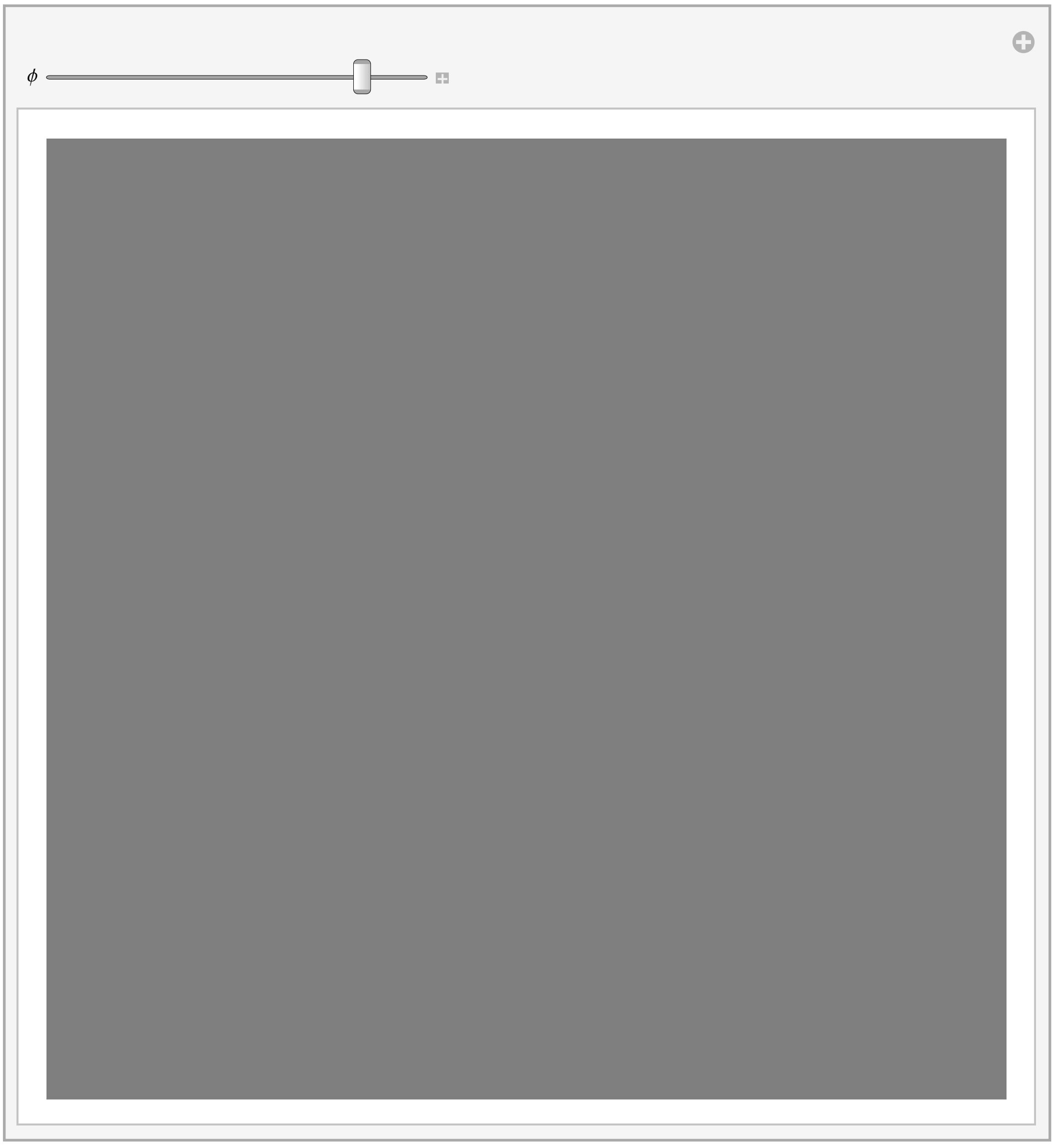
In[ ]:=
```
in = S[6] ** Ket[{S}]
```
Out[ ]=

$$\frac{|\text{␣}\rangle}{\sqrt{2}} + \frac{|1_S\rangle}{\sqrt{2}}$$

In[ ]:=
```
out = op ** in
```
Out[ ]=
$$\frac{|\text{␣}\rangle \left(\mathrm{Cos}\left[\frac{\phi}{2}\right] - i\,\mathrm{Sin}\left[\frac{\phi}{2}\right]\right)}{\sqrt{2}} + \frac{|1_S\rangle \left(\mathrm{Cos}\left[\frac{\phi}{2}\right] + i\,\mathrm{Sin}\left[\frac{\phi}{2}\right]\right)}{\sqrt{2}}$$

In[ ]:=
```
bv[ϕ_] = BlochVector[out] // ExpToTrig // FullSimplify
```
Out[ ]=
```
{Cos[ϕ], Sin[ϕ], 0}
```

In[ ]:=
```
Manipulate[BlochSphere[{Red, Bead@bv@ϕ}, ImageSize → Medium], {ϕ, 0, 2 Pi}]
```
Out[ ]=



Operator Algebra

In[ ]:=
```
op = Rotation[ϕ, S[3]]
```
Out[ ]=
$$\mathrm{Rotation}[\phi, S^z]$$

In[ ]:=
```
Elaborate[op]
```
Out[ ]=

$$\mathrm{Cos}\left[\frac{\phi}{2}\right] - i\ S^z\ \mathrm{Sin}\left[\frac{\phi}{2}\right]$$

In[ ]:=
```
SS = S[All]
```
Out[ ]=

$\{S^x, S^y, S^z\}$

In[ ]:=
```
TT = op ** SS ** Dagger[op]
```
Out[ ]=

$\{\mathrm{Cos}[\phi]\ S^x + S^y\,\mathrm{Sin}[\phi],\ \mathrm{Cos}[\phi]\ S^y - S^x\,\mathrm{Sin}[\phi],\ S^z\}$

In[ ]:=
```
mat = RotationMatrix[ϕ, {0, 0, 1}]
```
Out[ ]=

$\{\{\mathrm{Cos}[\phi], -\mathrm{Sin}[\phi], 0\}, \{\mathrm{Sin}[\phi], \mathrm{Cos}[\phi], 0\}, \{0, 0, 1\}\}$

In[ ]:=
```
SS.mat - TT
```
Out[ ]=

{0, 0, 0}

In[ ]:=
```
op = Rotation[ϕ, S[1]]
```
Out[ ]=

$\mathrm{Rotation}[\phi, S^x]$

In[ ]:=
```
Elaborate[op]
```
Out[ ]=

$$\mathrm{Cos}\left[\frac{\phi}{2}\right] - i\ S^x\ \mathrm{Sin}\left[\frac{\phi}{2}\right]$$

In[ ]:=
```
SS = S[All]
```
Out[ ]=

$\{S^x, S^y, S^z\}$

In[ ]:=
```
TT = op ** SS ** Dagger[op]
```
Out[ ]=

$\{S^x,\ \mathrm{Cos}[\phi]\ S^y + S^z\,\mathrm{Sin}[\phi],\ \mathrm{Cos}[\phi]\ S^z - S^y\,\mathrm{Sin}[\phi]\}$

In[ ]:=
```
mat = RotationMatrix[ϕ, {1, 0, 0}]
```
Out[ ]=

$\{\{1, 0, 0\}, \{0, \mathrm{Cos}[\phi], -\mathrm{Sin}[\phi]\}, \{0, \mathrm{Sin}[\phi], \mathrm{Cos}[\phi]\}\}$

In[ ]:=
```
SS.mat - TT
```
Out[ ]=

{0, 0, 0}

In[ ]:=
```
op = Rotation[ϕ, S[2]]
```
Out[ ]=

$\mathrm{Rotation}[\phi, S^y]$

In[ ]:=
```
Elaborate[op]
```
Out[ ]=
$$\mathrm{Cos}\left[\frac{\phi}{2}\right] - i\, S^{y}\, \mathrm{Sin}\left[\frac{\phi}{2}\right]$$

In[ ]:=
```
SS = S[All]
```
Out[ ]=
$\{S^{x}, S^{y}, S^{z}\}$

In[ ]:=
```
TT = op ** SS ** Dagger[op]
```
Out[ ]=
$\{\mathrm{Cos}[\phi]\ S^{x} - S^{z}\,\mathrm{Sin}[\phi],\ S^{y},\ \mathrm{Cos}[\phi]\ S^{z} + S^{x}\,\mathrm{Sin}[\phi]\}$

In[ ]:=
```
mat = RotationMatrix[ϕ, {0, 1, 0}]
```
Out[ ]=
$\{\{\mathrm{Cos}[\phi], 0, \mathrm{Sin}[\phi]\}, \{0, 1, 0\}, \{-\mathrm{Sin}[\phi], 0, \mathrm{Cos}[\phi]\}\}$

In[ ]:=
```
SS.mat - TT
```
Out[ ]=
$\{0, 0, 0\}$

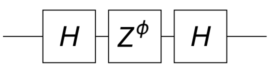
Application: Phase and Hadamard

In[ ]:=
```
op = Rotation[ϕ, S[3]]
```
Out[ ]=
$\mathrm{Rotation}[\phi, S^{z}]$

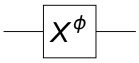
In[ ]:=
```
mat = Matrix[op];
MatrixForm[mat]
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} e^{-\frac{i\phi}{2}} & 0 \\ 0 & e^{\frac{i\phi}{2}} \end{pmatrix}$$

In[ ]:=
```
Phase[ϕ, S[3]] // Matrix // MatrixForm
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

In[ ]:=
```
Exp[I * ϕ / 2] * mat // MatrixForm
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

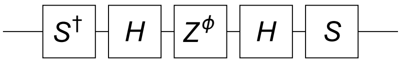
In[ ]:=
```
qc = QuantumCircuit[S[6], Phase[ϕ, S[3]], S[6]]
```
Out[ ]=


In[ ]:=
```
new = QuantumCircuit[Phase[ϕ, S[1]]]
```
Out[ ]=

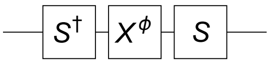


In[ ]:=
```
qc - new // Elaborate // Simplify
```
Out[ ]=
```
0
```

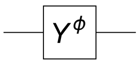
In[ ]:=
```
qc = QuantumCircuit[S[-7], S[6], Phase[ϕ, S[3]], S[6], S[7]]
```
Out[ ]=



In[ ]:=
```
more = QuantumCircuit[S[-7], Phase[ϕ, S[1]], S[7]]
```
Out[ ]=



In[ ]:=
```
new = QuantumCircuit[Phase[ϕ, S[2]]]
```
Out[ ]=



In[ ]:=
```
qc - new // Elaborate // Simplify
more - new // Elaborate // Simplify
```
Out[ ]=
```
0
```
Out[ ]=
```
0
```

Summary

Functions

- `Rotation`
- `BlochVector, BlochSphere, Bead`
- `Phase`

Related Links

- Chapters 1 and 2 of the Quantum Workbook (2022, 2023).
- Tutorial: “Quantum States”
- Tutorial: “Quantum Computation: Overview”