

Mahn-Soo Choi (Korea University)

A Quantum Computation Workbook

April 21, 2021 (v1.3 since October 3, 2020)

Preface

The book will be published later this year, and obviously, this draft edition is very rough and far from complete for the moment.

N.B. This draft is provided for educational purposes with the permission of Springer. Redistribution of this draft is strictly prohibited.

Seoul, Korea
April 2021

Mahn-Soo Choi

Contents

1	The Postulates of Quantum Mechanics	9
1.1	Quantum States	10
1.1.1	Pure States	10
1.1.2	Mixed States	15
1.2	Time Evolution of Quantum States	22
1.2.1	Unitary Dynamics	23
1.2.2	Quantum Noisy Dynamics	26
1.3	Measurements on Quantum States	27
1.3.1	Projection Measurements	28
1.3.2	Generalized Measurements	31
2	Quantum Computation: Overview	37
2.1	Single-Qubit Gates	38
2.1.1	Pauli Gates	38
2.1.2	Hadamard Gate	42
2.1.3	Rotations	45
2.2	Two-Qubit Gates	48
2.2.1	CNOT, CZ, and SWAP	48
2.2.2	Controlled- U Gate	57
2.2.3	General Unitary Gate	65
2.3	Multi-Qubit Controlled Gates	72
2.3.1	Gray Code	73
2.3.2	Multi-Qubit Controlled-NOT	75
2.4	Universal Quantum Computation	80
2.5	Measurements	83
3	Virtual Realizations of Quantum Computers	89
3.1	Quantum Bits	90
3.2	Dynamical Scheme	92
3.2.1	Implementation of Single-Qubit Gates	93
3.2.2	Implementation of CNOT	99
3.3	Geometric/Topological Scheme	102
3.4	Measurement-Based Scheme	106

3.5	Spin-Boson Model*	109
4	Quantum Algorithms: Introduction	113
4.1	Quantum Teleportation	113
4.1.1	Nonlocality in Entanglement	114
4.1.2	Implementation of Quantum Teleportation	116
4.2	Deutsch-Jozsa Algorithm & Variants	117
4.2.1	Quantum Oracle	118
4.2.2	Deutsch-Jozsa Algorithm	122
4.2.3	Bernstein-Vazirani Algorithm	123
4.2.4	Simon's Algorithm	123
4.3	Quantum Fourier Transform (QFT)	124
4.3.1	Definition and Physical Meaning	125
4.3.2	Quantum Implementation	126
4.3.3	Semiclassical Implementation	127
4.4	Quantum Phase Estimation (QPE)	128
4.4.1	Definition	128
4.4.2	Quantum Implementation	129
4.4.3	Simulation of von Neumann Measurement	132
5	Decoherence	135
5.1	Quantum Operations	135
5.1.1	The Kraus Representation	136
5.1.2	Unitary Representation	141
5.1.3	Examples	142
5.2	Generalized Measurements as Quantum Operations	144
5.3	Quantum Master Equation	144
5.3.1	Derivation	146
5.3.2	Examples	147
5.3.3	Solution Methods	149
5.4	Fidelity and Trace Distance	151
5.5	Entanglement, Entropy, Mutual Information	151
6	Quantum Error Correction Codes: Introduction	153
6.1	Discretization of errors	153
6.2	9-Qubit Code (Shor's Code)	153
6.3	Fault-Tolerant Quantum Computation	153
6.4	CSS Code (Optional)	153
6.5	Stabilizer Code (Optional)	153
6.6	Surface Code (Optional)	153

A	Linear Algebra	155
A.1	Vectors	155
A.1.1	Vector Space	155
A.1.2	Hermitian Product	156
A.1.3	Basis	157
A.1.4	Representations	158
A.2	Linear Operators	160
A.2.1	Linear Maps	160
A.2.2	Representations	161
A.2.3	Hermitian Conjugate of Operators	161
A.3	Dirac's Bra-Ket Notation	164
A.4	Spectral Theorems	167
A.4.1	Spectral Decomposition	167
A.4.2	Functions of Operators	168
A.5	Tensor-Product Spaces	170
A.5.1	Vectors in a Product Space	170
A.5.2	Operators on a Product Space	172
B	Superoperators	173
B.1	Operators as Vectors	173
B.2	Superoperators	179
B.2.1	Matrix Representation	179
B.2.2	Operator-Sum Representation	180
B.2.3	Choi Isomorphism	183
B.3	Partial Trace	185
B.4	Partial Transposition	186
C	Mathematica Application Q3	189
C.1	Installation	189
C.2	Quick Start	190
	Bibliography	191
	Index	195

