More Mathematica Cool Tips

In[*]:= Let[Qubit, S]

Thread

Suppose you have two lists of inputs and outputs.

$$\begin{split} &\inf \text{ in = Basis[S@{1,2}]} \\ &\text{ out = CNOT[S[1], S[2]] **S[1,6] ** in } \\ &\text{ out[\circ] = } \\ & \left\{ \left| \Theta_{S_1} \Theta_{S_2} \right\rangle, \, \left| \Theta_{S_1} 1_{S_2} \right\rangle, \, \left| 1_{S_1} \Theta_{S_2} \right\rangle, \, \left| 1_{S_1} 1_{S_2} \right\rangle \right\} \\ &\text{ out[\circ] = } \\ & \left\{ \frac{\left| \Theta_{S_1} \Theta_{S_2} \right\rangle}{\sqrt{2}} + \frac{\left| 1_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}}, \, \frac{\left| \Theta_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}} + \frac{\left| 1_{S_1} \Theta_{S_2} \right\rangle}{\sqrt{2}}, \, \frac{\left| \Theta_{S_1} \Theta_{S_2} \right\rangle}{\sqrt{2}} - \frac{\left| 1_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}}, \, \frac{\left| \Theta_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}} - \frac{\left| 1_{S_1} \Theta_{S_2} \right\rangle}{\sqrt{2}} \right\} \end{split}$$

You want to compare the corresponding elements in the two list side by side. Unfortunately, this would not work for obvious reasons.

$$\begin{aligned} &\inf \circ \]:= \ \ \textbf{in} \rightarrow \textbf{out} \\ & \text{Out} [\circ \]:= \\ & \left\{ \left| \ \theta_{S_1} \theta_{S_2} \right\rangle, \ \left| \ \theta_{S_1} 1_{S_2} \right\rangle, \ \left| \ 1_{S_1} \theta_{S_2} \right\rangle, \ \left| \ 1_{S_1} 1_{S_2} \right\rangle \right\} \rightarrow \\ & \left\{ \left| \ \frac{\left| \ \theta_{S_1} \theta_{S_2} \right\rangle}{\sqrt{2}} + \frac{\left| \ 1_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}}, \ \frac{\left| \ \theta_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}} + \frac{\left| \ 1_{S_1} \theta_{S_2} \right\rangle}{\sqrt{2}}, \ \frac{\left| \ \theta_{S_1} \theta_{S_2} \right\rangle}{\sqrt{2}} - \frac{\left| \ 1_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}}, \ \frac{\left| \ \theta_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}} - \frac{\left| \ 1_{S_1} \theta_{S_2} \right\rangle}{\sqrt{2}} \right\} \end{aligned}$$

Thread is useful in such cases.

$$\begin{array}{ll} \text{In[\bullet]$:= Thread[in \to out] // TableForm} \\ \text{Out[\bullet] // TableForm=} \\ & \left| \left. 0_{S_1} 0_{S_2} \right\rangle \right. \rightarrow \frac{\left| 0_{S_1} 0_{S_2} \right\rangle}{\sqrt{2}} + \frac{\left| 1_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}} \\ & \left| \left. 0_{S_1} 1_{S_2} \right\rangle \right. \rightarrow \frac{\left| 0_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}} + \frac{\left| 1_{S_1} 0_{S_2} \right\rangle}{\sqrt{2}} \\ & \left| \left. 1_{S_1} 0_{S_2} \right\rangle \right. \rightarrow \frac{\left| 0_{S_1} 0_{S_2} \right\rangle}{\sqrt{2}} - \frac{\left| 1_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}} \\ & \left| \left. 1_{S_1} 1_{S_2} \right\rangle \right. \rightarrow \frac{\left| 0_{S_1} 1_{S_2} \right\rangle}{\sqrt{2}} - \frac{\left| 1_{S_1} 0_{S_2} \right\rangle}{\sqrt{2}} \end{array}$$

These are typical examples.

MapThread

Suppose you have two lists.

```
In[0]:= Let[Complex, a, b]
        aa = Array[a, 5]
        bb = Array[b, 5]
Out[0]=
        \{a_1, a_2, a_3, a_4, a_5\}
Out[0]=
        \{b_1, b_2, b_3, b_4, b_5\}
        You want to construct a new list such that
        \{F[a_1, b_1], F[a_2, b_2], F[a_3, b_3], F[a_4, b_4], F[a_5, b_5]\}.
 In[@]:= MapThread[F, {aa, bb}]
Out[0]=
        {F[a_1, b_1], F[a_2, b_2], F[a_3, b_3], F[a_4, b_4], F[a_5, b_5]}
```

In fact, for the particular example above, Thread is already enough.

```
In[•]:= F[aa, bb]
Out[0]=
        F[{a_1, a_2, a_3, a_4, a_5}, {b_1, b_2, b_3, b_4, b_5}]
 In[0]:= Thread[F[aa, bb]]
Out[0]=
        \{F[a_1, b_1], F[a_2, b_2], F[a_3, b_3], F[a_4, b_4], F[a_5, b_5]\}
```

However, some times, F itself may have some particular meaning for List inputs.

```
In[0]:= F[x_List, y_] := x * y
```

In such case, Thread does not have a chance to play its role.

```
In[0]:= Thread[F[aa, bb]]
Out[0]=
        \{a_1 \ b_1, a_2 \ b_2, a_3 \ b_3, a_4 \ b_4, a_5 \ b_5\}
        However, MapThread works correctly.
 In[@]:= MapThread[F, {aa, bb}]
Out[0]=
        {F[a_1, b_1], F[a_2, b_2], F[a_3, b_3], F[a_4, b_4], F[a_5, b_5]}
```

Through

Suppose that we have a list of qubits.

```
ln[ \circ ] := SS = S[\{1, 2, 3, 4\}, \$]
Out[0]=
          \{S_1, S_2, S_3, S_4\}
```

We want to convert this list to a new list of Pauli X operators on all qubits in the list.

```
ln[ \circ ] := SX = S[\{1, 2, 3, 4\}, 1]
Out[0]=
           \{S_1^x, S_2^x, S_3^x, S_4^x\}
```

Note that once S is declared as a qubit,

```
In[0]:= S[1, $]
Out[0]=
        S_1
 In[0]:= S[1, $][1]
Out[0]=
        S_1^x
 In[*]:= % // InputForm
Out[•]//InputForm=
        S[1, 1]
```

But, unfortunately, **SS**[1] would not work.

```
In[0]:= SS[1]
Out[0]=
        {S_1, S_2, S_3, S_4}[1]
 In[*]:= Through[SS[1]]
Out[0]=
        \{S_1^x, S_2^x, S_3^x, S_4^x\}
        This is a typical example.
 In[@]:= Through[{a, b, c, d}[x]]
Out[0]=
        {a[x], b[x], c[x], d[x]}
```

Summary

Functions

- Thread
- MapThread

- Through
- Table
- Apply
- Map, MapThread
- @, @@, @@@
- f@x, f[x], x//f

Related Links

- S. Wolfram (2017), "An Elementary Introduction to Wolfram Language," 2nd edition (2017).
- The Wolfram Language: Fast Introduction for Math Students
- The Wolfram Language: Fast Introduction for Programmers