Working in Different Bases

Why not just in the computational basis?

- One important motivation did we already see in **E. 06**, **“Measurements.”**
  Recall that one must expand a given quantum state $|\psi\rangle$ in the eigenbasis $\{|a\rangle \mid \hat{A}|a\rangle = |a\rangle a\}$ of the observable $\hat{A} = \underset{a}{S}|a\rangle a\langle a|$. That is, $|\psi\rangle = \sum_a |a\rangle c_a$.
- Here is another motivation.

Consider a quantum register of qubits, referred to by symbol **S**.

In[ ]:=
```
Let[Qubit, S]
```

In[ ]:=
```
$L = 4;
ss = S[Range@$L, $]
```

Out[ ]=

$\{S_1, S_2, S_3, S_4\}$

Here is a dynamic way of constructing the spin XX Hamiltonian.

In[ ]:=
```
xx = Total@ChainBy[Through[ss[1]], Multiply]
```

Out[ ]=

$S_1^x S_2^x + S_2^x S_3^x + S_3^x S_4^x$

In the same manner, construct the spin YY Hamiltonian.

In[ ]:=
```
yy = Total@ChainBy[Through[ss[2]], Multiply]
```

Out[ ]=

$S_1^y S_2^y + S_2^y S_3^y + S_3^y S_4^y$

Finally, construct the spin ZZ Hamiltonian.

In[ ]:=
```
zz = Total@ChainBy[Through[ss[3]], Multiply]
```

Out[ ]=

$S_1^z S_2^z + S_2^z S_3^z + S_3^z S_4^z$

Here is the total Hamiltonian.

In[ ]:=
```
H = xx + yy + zz
```

Out[ ]=

$S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z + S_2^x S_3^x + S_2^y S_3^y + S_2^z S_3^z + S_3^x S_4^x + S_3^y S_4^y + S_3^z S_4^z$

The Hamiltonian has so many zeros. That is why matrix representations are treated in `SparseArray` form in Q3.

```
In[ ]:= mat = Matrix[H];
        mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & -1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & -3 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & -3 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

Anyway, how can we diagonalize Hamiltonian?

What about working in a different basis?

Construct a different basis where elements are characterized by the total angular momentum.

In[∘]:= `bs = QubitAdd[ss]`

Out[∘]=

$$\left\langle\left| \{0,0\} \to \left\{\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle - \frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle - \frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle + \frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle, \frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{3}} - \frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}} - \frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}} - \frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}} - \frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}} + \frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{3}}\right\}, \{1,-1\} \to \left\{\frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}} - \frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}} + \frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}} - \sqrt{\frac{2}{3}}\left|1_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle, \frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}} + \frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}} + \frac{\left|1_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\left|1_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle\right\},$$

$$\{1,0\} \to \left\{\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle + \frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle - \frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle - \frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle, \frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{3}} - \frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}} + \frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}} - \frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}} + \frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}} - \frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{3}}, \frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}} + \frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}} - \frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}} + \frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}} - \frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}} - \frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}\right\}, \{1,1\} \to \left\{\frac{\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{2}} - \frac{\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{2}}, \sqrt{\frac{2}{3}}\left|0_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle - \frac{\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}} - \frac{\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}, \frac{1}{2}\sqrt{3}\left|0_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle - \frac{\left|0_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}} - \frac{\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}} - \frac{\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}}\right\},$$

$$\{2,-2\} \to \left\{\left|1_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle\right\}, \{2,-1\} \to \left\{\frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle + \frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle + \frac{1}{2}\left|1_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle + \frac{1}{2}\left|1_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle\right\},$$

$$\{2,0\} \to \left\{\frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{6}} + \frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}} + \frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}} + \frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{\sqrt{6}} + \frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{\sqrt{6}} + \frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{6}}\right\},$$

$$\{2,1\} \to \left\{\frac{1}{2}\left|0_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle + \frac{1}{2}\left|0_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle + \frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle + \frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle\right\},$$

$$\{2,2\} \to \left\{\left|0_{S_1}0_{S_2}0_{S_3}0_{S_4}\right\rangle\right\}\right|\right\rangle$$

The keys label the corresponding angular momentum.

In[∘]:= `Keys[bs]`

Out[∘]=

`{{0, 0}, {1, -1}, {1, 0}, {1, 1}, {2, -2}, {2, -1}, {2, 0}, {2, 1}, {2, 2}}`

Group the labels according to the magnitude of the total angular momentum.

In[∘]:=
```
GroupBy[Keys[bs], First] // Normal // TableForm
```

Out[∘]//TableForm=

$$
\begin{array}{l}
0 \to \{\{0, 0\}\} \\
1 \to \{\{1, -1\}, \{1, 0\}, \{1, 1\}\} \\
2 \to \{\{2, -2\}, \{2, -1\}, \{2, 0\}, \{2, 1\}, \{2, 2\}\}
\end{array}
$$

To compare this basis, arrange all elements in the given order.

In[∘]:=
```
bb = Catenate[bs];
bb[[ ;; 3]]
```

Out[∘]=

$$
\Big\{\frac{1}{2}\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle - \frac{1}{2}\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle - \frac{1}{2}\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle + \frac{1}{2}\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle,
$$
$$
\frac{\left|0_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{3}} - \frac{\left|0_{S_1}1_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}} - \frac{\left|0_{S_1}1_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}} - \frac{\left|1_{S_1}0_{S_2}0_{S_3}1_{S_4}\right\rangle}{2\sqrt{3}} -
$$
$$
\frac{\left|1_{S_1}0_{S_2}1_{S_3}0_{S_4}\right\rangle}{2\sqrt{3}} + \frac{\left|1_{S_1}1_{S_2}0_{S_3}0_{S_4}\right\rangle}{\sqrt{3}}, \frac{\left|0_{S_1}1_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}} - \frac{\left|1_{S_1}0_{S_2}1_{S_3}1_{S_4}\right\rangle}{\sqrt{2}}\Big\}
$$

Now, calculate the matrix representation of the Hamiltonian in this new basis.

In[∘]:=
```
EchoTiming[
 new = Outer[Multiply, Dagger[bb], H ** bb];
]
new // MatrixForm
```

0.505931

Out[∘]//MatrixForm=

$$
\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}
$$

The Outer function

Observe how the Outer function works.

In[ ]:=
```
Outer[f, {1, 2, 3}, {a, b}] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} f[1, a] & f[1, b] \\ f[2, a] & f[2, b] \\ f[3, a] & f[3, b] \end{pmatrix}$$

Let us go back to the previous example, and consider different options for calculating the matrix representation in the new basis.

- Choice 1

```
EchoTiming[
 mm = Outer[Dagger[#1] ** H ** #2 &, bb, bb];
]
mm // MatrixForm
```

8.60772

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

- Choice 2

```
In[ ]:= EchoTiming[mm = Outer[#1 ** H ** #2 &, Dagger[bb], bb];]
        mm // MatrixForm
```

8.75689

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

- Choice 3

```
In[ ]:= EchoTiming[mm = Outer[#1 ** #2 &, Dagger[bb], H ** bb];]
        mm // MatrixForm
```

0.547041

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}$$

```
In[ ]:= EchoTiming[mm = Outer[Multiply, Dagger[bb], H ** bb];]
        mm // MatrixForm
```

```
0.555479
```

Out[ ]//MatrixForm=

$$
\begin{pmatrix}
-6 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & -\frac{4}{3} & \frac{4\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}
$$

Summary

Functions

- `Outer`
- `Matrix, ExpressionFor`
- `QubitAdd`
- `Chain`
- `Dagger`
- `EchoTiming`

Related Links

- Appendix A of the Quantum Workbook (2022, 2023) -- Available for free via the QuantumPlaybook package.