Quantum Measurement

```
In[ ]:= << Q3`
```

Pauli Operators

```
In[ ]:= Let[Qubit, S]
```

```
In[ ]:= in = Ket[S[1] → 0] - I * Ket[S[1] → 1]
```

```
Out[ ]=
```

$$|0_{S_1}\rangle - i\ |1_{S_1}\rangle$$

```
In[ ]:= out = Measurement[S[1, 3]][in]
```

```
Out[ ]=
```

$$-i\ |1_{S_1}\rangle$$

```
In[ ]:= out = Measurement[S[1, 3]] ** in
```

```
Out[ ]=
```

$$|0_{S_1}\rangle$$

```
In[ ]:= out = Table[Measurement[S[1, 3]] ** in, 10]
```

```
Out[ ]=
```

$$\{-i\ |1_{S_1}\rangle,\ |0_{S_1}\rangle,\ |0_{S_1}\rangle,\ |0_{S_1}\rangle,\ -i\ |1_{S_1}\rangle,\ -i\ |1_{S_1}\rangle,\ |0_{S_1}\rangle,\ -i\ |1_{S_1}\rangle,\ |0_{S_1}\rangle,\ |0_{S_1}\rangle\}$$

```
In[ ]:= out = Measurement[S[1, 3]] ** in
```

```
Out[ ]=
```

$$-i\ |1_{S_1}\rangle$$

```
In[ ]:= Readout[S[1, 3]]
```

```
Out[ ]=
1
```

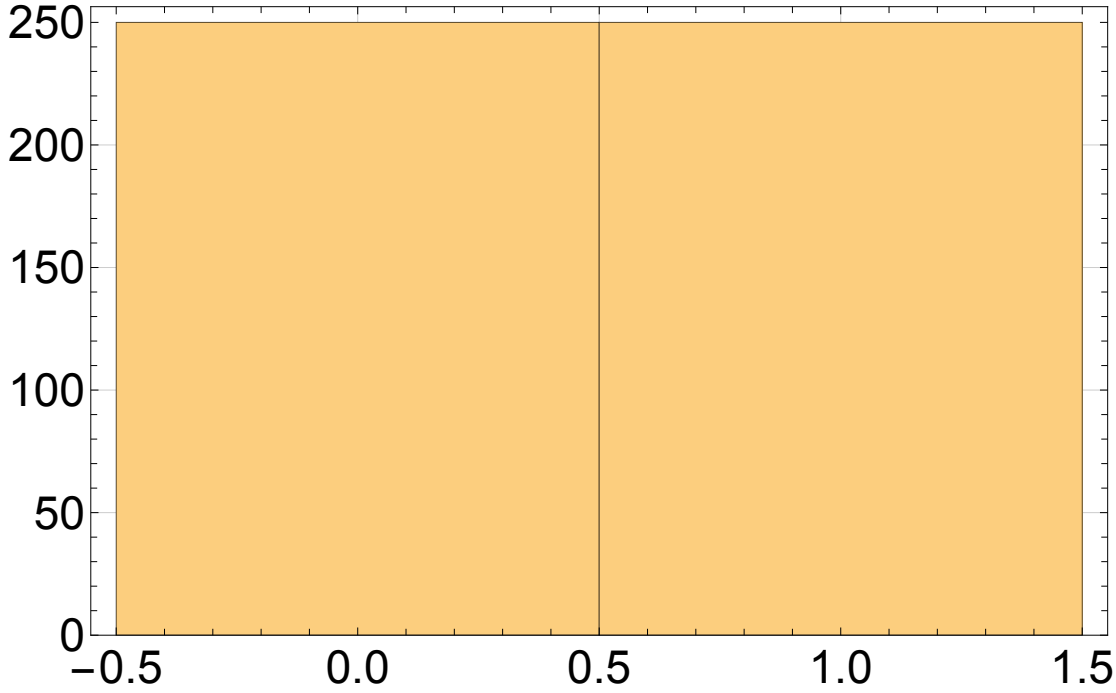
```
In[ ]:= data = Table[
          Measurement[S[1, 3]] ** in; Readout[S[1, 3]],
          10]
```

```
Out[ ]=
{1, 1, 0, 1, 1, 1, 0, 0, 0, 0}
```

```
In[ ]:= data = Table[
           Measurement[S[1, 3]] ** in; Readout[S[1, 3]],
           500];
```

In[ ]:=
```
Histogram[data]
```

Out[ ]=



In[ ]:=
```
in = 2 * Ket[S[1] → 0] - I * Ket[S[1] → 1]
```

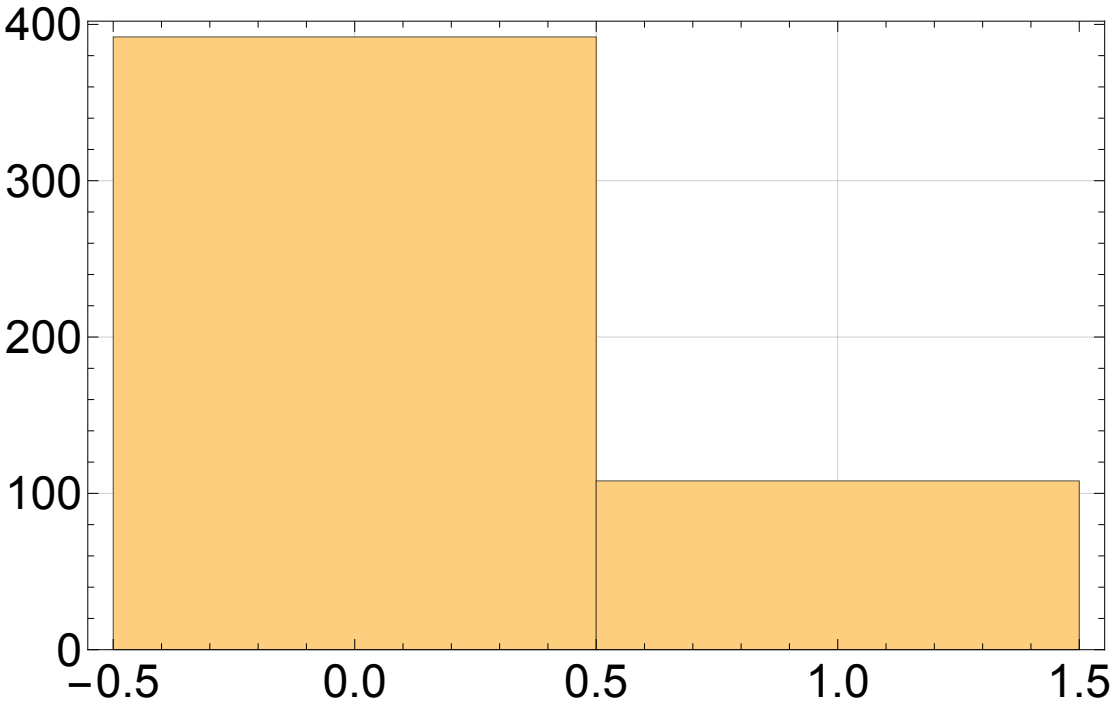
Out[ ]=

$$2\ \left|0_{S_1}\right\rangle - i\ \left|1_{S_1}\right\rangle$$

In[ ]:=
```
data = Table[
    Measurement[S[1, 3]] ** in; Readout[S[1, 3]],
    500];
```

In[ ]:=
```
Histogram[data]
```

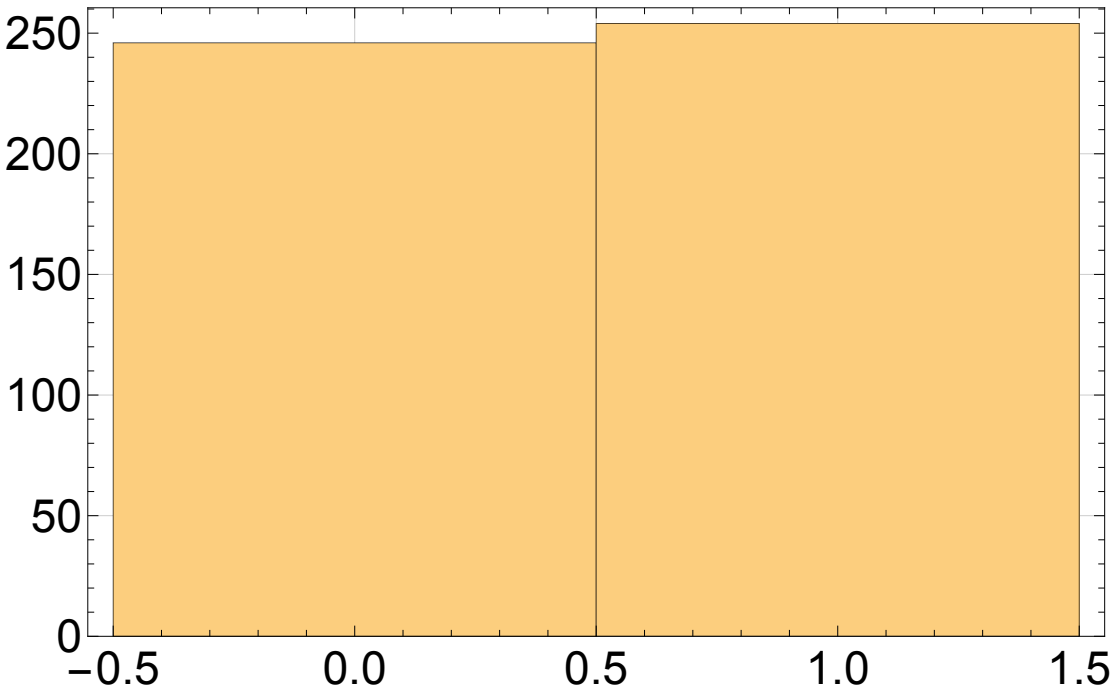
Out[ ]=



In[ ]:=
```
data = Table[
    Measurement[S[1, 1]] ** in; Readout[S[1, 1]],
    500];
```

In[ ]:=
```
Histogram[data]
```

Out[ ]=



Analysis before Measurement

In[•]:=
```
in = Ket[S[1] → 0] - I * Ket[S[1] → 1]
```
Out[•]=
$$\left|0_{S_1}\right\rangle - i \left|1_{S_1}\right\rangle$$

In[•]:=
```
Measurement[S[1, 3]] ** in
```
Out[•]=
$$-i \left|1_{S_1}\right\rangle$$

In[•]:=
```
data = MeasurementOdds[S[1, 3]][in]
```
Out[•]=
$$\left\langle\left| 0 \to \left\{\frac{1}{2}, \left|0_{S_1}\right\rangle\right\}, 1 \to \left\{\frac{1}{2}, -i \left|1_{S_1}\right\rangle\right\} \right|\right\rangle$$

In[•]:=
```
data = MeasurementOdds[S[1, 2]][in]
```
Out[•]=
$$\left\langle\left| 0 \to \{0, 0\}, 1 \to \left\{1, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}\right\} \right|\right\rangle$$

In[•]:=
```
out = Table[Measurement[S[1, 2]] ** in, 20]
```
Out[•]=
$$\left\{\frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}},\right.$$
$$\frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}},$$
$$\frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}},$$
$$\left.\frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}, \frac{\left|0_{S_1}\right\rangle}{\sqrt{2}} - \frac{i \left|1_{S_1}\right\rangle}{\sqrt{2}}\right\}$$

In[•]:=
```
YBasisForm[out, S[1]]
```
Out[•]=
$$\left\{\left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle,\right.$$
$$\left.\left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle, \left|R_{S_1}\right\rangle\right\}$$

Consecutive Measurements

In[•]:=
```
in = Ket[] + Ket[S[1] → 1] + Ket[S[2] → 1] // KetRegulate
```
Out[•]=
$$\left|0_{S_1} 0_{S_2}\right\rangle + \left|0_{S_1} 1_{S_2}\right\rangle + \left|1_{S_1} 0_{S_2}\right\rangle$$

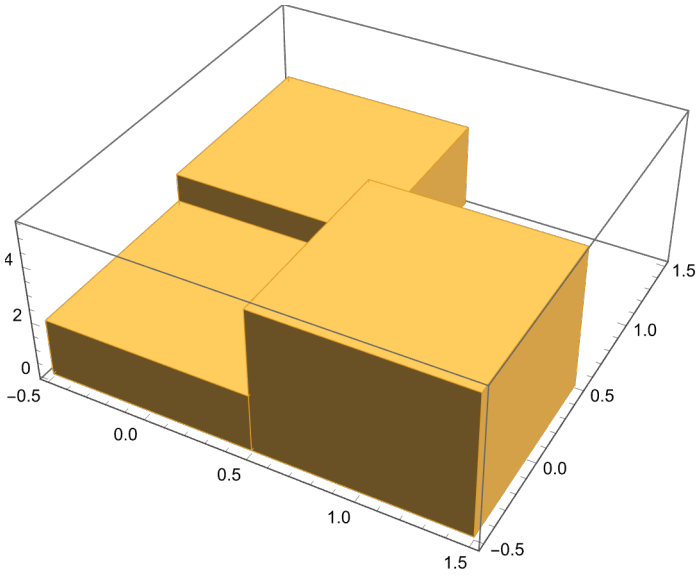
In[•]:=
```
out = Measurement[{S[1, 3], S[2, 3]}] ** in
```
Out[•]=
$$\left|1_{S_1} 0_{S_2}\right\rangle$$

```
In[ ]:= data = Table[
          Measurement[{S[1, 3], S[2, 3]}] ** in;
          Readout[{S[1, 3], S[2, 3]}],
          10]
```

```
Out[ ]=
{{0, 0}, {1, 0}, {0, 1}, {0, 1}, {1, 0}, {1, 0}, {0, 0}, {1, 0}, {0, 1}, {1, 0}}
```

```
In[ ]:= Histogram3D[data]
```

```
Out[ ]=
```



Collective Measurements

```
In[ ]:= in = Ket[] + Ket[S[1] → 1] + Ket[S[2] → 1] // KetRegulate
```

```
Out[ ]=
|0_{S_1} 0_{S_2}⟩ + |0_{S_1} 1_{S_2}⟩ + |1_{S_1} 0_{S_2}⟩
```

```
In[ ]:= out = Measurement[S[1, 3] ** S[2, 3]] ** in
```

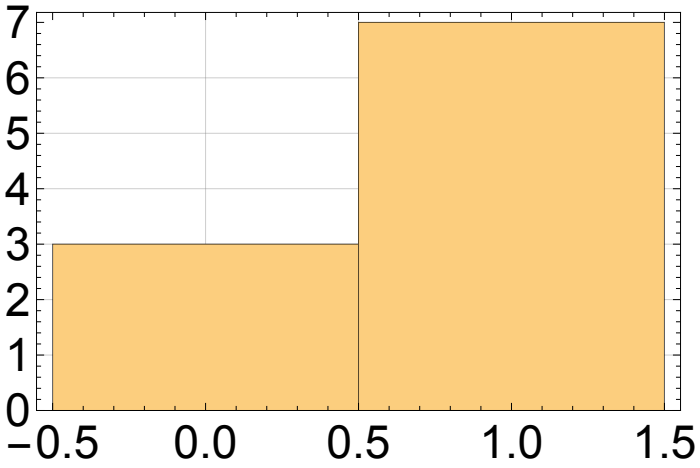
```
Out[ ]=
|0_{S_1} 0_{S_2}⟩
```

```
In[ ]:= data = Table[
          Measurement[S[1, 3] ** S[2, 3]] ** in;
          Readout[S[1, 3] ** S[2, 3]],
          10]
```

```
Out[ ]=
{1, 1, 1, 0, 1, 0, 1, 1, 1, 0}
```

In[ ]:=
```
Histogram[data, ImageSize → Medium]
```

Out[ ]=



Summary

Functions

- **Measurement**
- **Readout**
- **MeasurementOdds**

Related Links

- Chapter 1 of the Quantum Workbook (2022, 2023).
- Tutorial: “Measurements on Quantum States”