

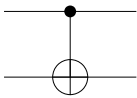
Multi-Control NOT Gate

```
In[*]:= Let[Qubit, S]
```

CNOT: Review

```
In[*]:= cnot = QuantumCircuit[CNOT[S[1], S[2]]]
```

Out[*]=



```
In[*]:= in = Basis[S@{1, 2}];  
out = cnot ** in;  
ProductForm[Thread[in → out], S@{1, 2}] // TableForm
```

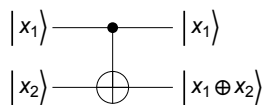
Out[*]//TableForm=

$ 0\rangle \otimes 0\rangle$	\rightarrow	$ 0\rangle \otimes 0\rangle$
$ 0\rangle \otimes 1\rangle$	\rightarrow	$ 0\rangle \otimes 1\rangle$
$ 1\rangle \otimes 0\rangle$	\rightarrow	$ 1\rangle \otimes 1\rangle$
$ 1\rangle \otimes 1\rangle$	\rightarrow	$ 1\rangle \otimes 0\rangle$

```
In[*]:= Let[Binary, x]
```

```
In[*]:= qc = QuantumCircuit[in = Ket[S@{1, 2} → x@{1, 2}], cnot,  
Ket[S[1] → x[1], S[2] → Mod[x[1] + x[2], 2]],  
"PortSize" → {0.7, 1.5}]
```

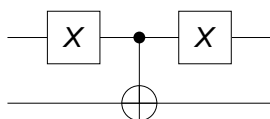
Out[*]=



CNOT on the opposite condition

```
In[*]:= xnnot = QuantumCircuit[S[1, 1], CNOT[S[1], S[2]], S[1, 1]]
```

Out[*]=



```
In[*]:= in = Basis[S@{1, 2}];
out = xnot ** in;
ProductForm[Thread[in → out], S@{1, 2}] // TableForm
```

```
Out[*]//TableForm=

$$\begin{array}{l} |0\rangle \otimes |0\rangle \rightarrow |0\rangle \otimes |1\rangle \\ |0\rangle \otimes |1\rangle \rightarrow |0\rangle \otimes |0\rangle \\ |1\rangle \otimes |0\rangle \rightarrow |1\rangle \otimes |0\rangle \\ |1\rangle \otimes |1\rangle \rightarrow |1\rangle \otimes |1\rangle \end{array}$$

```

```
In[*]:= TableForm[{MatrixForm /@ Matrix /@ {cnot, xnot}},
TableHeadings → {None, {CNOT, XNOT}},
TableAlignments → Center]
```

```
Out[*]//TableForm=


| CNOT                                                                                             | XNOT                                                                                             |
|--------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|
| $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ |


```

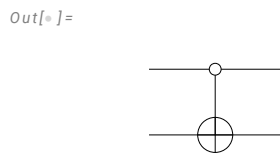
```
In[*]:= CNOT[S[1], S[2]]
```

```
Out[*]=
CNOT[{S1} → {1}, {S2}]
```

```
In[*]:= CNOT[S[1] → 0, S[2]]
```

```
Out[*]=
CNOT[{S1} → {0}, {S2}]
```

```
In[*]:= new = QuantumCircuit[CNOT[S[1] → 0, S[2]]]
```

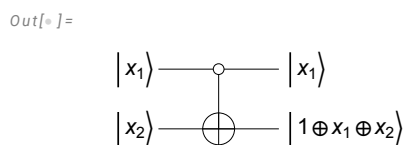


```
In[*]:= new - xnot // Elaborate
```

```
Out[*]=
0
```

```
In[*]:= Let[Binary, x]
```

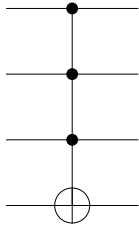
```
In[*]:= qc = QuantumCircuit[in = Ket[S@{1, 2} → x@{1, 2}], new,
Ket[S[1] → x[1], S[2] → Mod[1 + x[1] + x[2], 2]],
"PortSize" → {0.7, 2}]
```



Multi-Control-NOT Gate

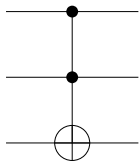
```
In[*]:= QuantumCircuit[CNOT[S[{1, 2, 3}], $], S[4]]]
```

Out[*]=



```
In[*]:= $n = 2;
cc = S[Range[$n], $];
tt = S[$n + 1, $];
cnot = QuantumCircuit[CNOT[cc, tt]]
```

Out[*]=



```
In[*]:= Matrix[cnot] // MatrixForm
```

Out[*]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

```
In[*]:= in = Basis[cc, tt];
out = cnot ** in;
```

```
In[*]:= ProductForm[Thread[in → out], {cc, tt}] // TableForm
```

```
Out[*]//TableForm=
```

$$\begin{array}{l} |00\rangle \otimes |0\rangle \rightarrow |00\rangle \otimes |0\rangle \\ |00\rangle \otimes |1\rangle \rightarrow |00\rangle \otimes |1\rangle \\ |01\rangle \otimes |0\rangle \rightarrow |01\rangle \otimes |0\rangle \\ |01\rangle \otimes |1\rangle \rightarrow |01\rangle \otimes |1\rangle \\ |10\rangle \otimes |0\rangle \rightarrow |10\rangle \otimes |0\rangle \\ |10\rangle \otimes |1\rangle \rightarrow |10\rangle \otimes |1\rangle \\ |11\rangle \otimes |0\rangle \rightarrow |11\rangle \otimes |1\rangle \\ |11\rangle \otimes |1\rangle \rightarrow |11\rangle \otimes |0\rangle \end{array}$$

```
In[*]:= CNOT[S[{1, 2}, $], S[3]]
```

```
Out[*]=
```

$$\text{CNOT}[\{S_1, S_2\} \rightarrow \{1, 1\}, \{S_3\}]$$

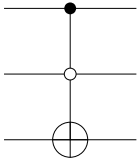
```
In[*]:= CNOT[S[{1, 2}, $] → {1, 0}, S[3]]
```

```
Out[*]=
```

$$\text{CNOT}[\{S_1, S_2\} \rightarrow \{1, 0\}, \{S_3\}]$$

```
In[*]:= qc = QuantumCircuit[CNOT[S@{1, 2} → {1, 0}, S[3]]]
```

```
Out[*]=
```



```
In[*]:= in = Basis[S@{1, 2, 3}];
```

```
out = qc ** in;
```

```
In[*]:= ProductForm[Thread[in → out], {S@{1, 2}, S[3]}] // TableForm
```

```
Out[*]//TableForm=
```

$$\begin{array}{l} |00\rangle \otimes |0\rangle \rightarrow |00\rangle \otimes |0\rangle \\ |00\rangle \otimes |1\rangle \rightarrow |00\rangle \otimes |1\rangle \\ |01\rangle \otimes |0\rangle \rightarrow |01\rangle \otimes |0\rangle \\ |01\rangle \otimes |1\rangle \rightarrow |01\rangle \otimes |1\rangle \\ |10\rangle \otimes |0\rangle \rightarrow |10\rangle \otimes |1\rangle \\ |10\rangle \otimes |1\rangle \rightarrow |10\rangle \otimes |0\rangle \\ |11\rangle \otimes |0\rangle \rightarrow |11\rangle \otimes |0\rangle \\ |11\rangle \otimes |1\rangle \rightarrow |11\rangle \otimes |1\rangle \end{array}$$

Summary

Functions

- CNOT
- Hadamard

Related Links

- Chapters 2 of the Quantum Workbook (2022, 2023).
- Tutorial: “Quantum Computation: Overview”