Schmidt Decomposition

양자얽힘 판별법

최만수 (고려대 물리학과)

Singular Value Decomposition

of any rectangular matrix

$$n\left\{\overbrace{\boxed{\quad n\times m \quad}}^{m}\right. = U_{n\times n} \overbrace{\begin{bmatrix} s_1 & & & & 0 & \cdots & 0 \\ & s_2 & & & 0 & \cdots & 0 \\ & & \ddots & & 0 & \cdots & 0 \\ & & & s_n & 0 & \cdots & 0 \end{bmatrix}}^{m} V^\dagger_{m\times m}$$

Bi-partite System

$$\{|a_1\rangle, \cdots, |b_n\rangle\} \otimes \{|b_1\rangle, \cdots, |b_n\rangle\}$$

$$\{|a_1\rangle \otimes |b_1\rangle, \cdots, |a_i\rangle \otimes |b_j\rangle, \cdots, |a_m\rangle \otimes |b_n\rangle\}$$

$$\{\cdots, |a_i b_j\rangle, \cdots\}$$

$$|\Psi\rangle = \sum_{i,j} |a_i\rangle \otimes |b_j\rangle \; C_{ij}$$

$$|\Psi\rangle = \sum_{i,j} |a_i\rangle \otimes |b_j\rangle \; C_{ij}$$

$$C_{ij} = \left(U\Sigma V^\dagger\right)_{ij} = \sum_k U_{ik} s_k V^*_{jk}$$

$$|\Psi\rangle = \sum_k \left(\sum_i |a_i\rangle \, U_{ik}\right) \otimes \left(\sum_j |b_j\rangle \, V^*_{jk}\right) \; s_k$$

$$|\Psi\rangle = \sum_k |\alpha_k\rangle \otimes |\beta_k\rangle \; s_k$$

ENTANGLEMENT

- *Entangled* if there are more than one Schmidt numbers.
- *Separable* if there is only one Schmidt number.

감사합니다!