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A Quantum Computation Workbook

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Preface

As is clear from the title, the book is intended for use as an introductory text and self-learning guide. It provides little original work. Rather, it is an attempt to collect and reorganize in a compact and unified form some of the fundamental principles and elementary methods necessary for an understanding of current research topics.

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Two accompanying digital materials are freely available to the public. Both can be downloaded from the GitHub repository by clicking the links below.

- Mathematica application “Q3”: <https://github.com/quantum-mob/Q3App>
- Wolfram Language codes used to generate the demonstrations in the text: <https://github.com/quantum-mob/QuantumWorkbook>

N.B. The book will be published later this year, and obviously, this draft edition is very rough and far from complete for the moment.

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