회전

단일 큐비트 연산자의 기하학적 해석

최만수 (고려대 물리학과)

Theorem I

$$
\begin{aligned}
\hat{U} &= \exp\left[-\frac{i}{2}\left(c_0\hat{I} + c_x\hat{X} + c_y\hat{Y} + c_z\hat{Z}\right)\right] \\
&= e^{ic_0/2}\exp\left[-\frac{i}{2}\left(c_x\hat{X} + c_y\hat{Y} + c_z\hat{Z}\right)\right]
\end{aligned}
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파울리 공식

$$
\exp\left[-\frac{i}{2}\left(c_x\hat{X} + c_y\hat{Y} + c_z\hat{Z}\right)\right] = \cos(\phi/2)\hat{I} - i\sin(\phi/2)\left(n_x\hat{X} + n_y\hat{Y} + n_z\hat{Z}\right)
$$

$$
\begin{aligned}
\phi &:= \sqrt{c_x^2 + c_y^2 + c_z^2} \\
n_k &:= c_k/\phi
\end{aligned}
$$

Single-Qubit Rotation

$$\hat{U}_{\boldsymbol{n}}(\phi) := \exp\left[-i\frac{\phi}{2}\left(n_x\hat{X} + n_y\hat{Y} + n_z\hat{Z}\right)\right]$$

$$\boldsymbol{n} := (n_x, n_y, n_z)\,, \quad \|\boldsymbol{n}\| = 1$$

Theorem 2

$$\hat{U}_{\boldsymbol{n}}(\phi)\hat{S}^{\nu}\hat{U}_{\boldsymbol{n}}^{\dagger}(\phi) = \sum_{\mu}\hat{S}^{\mu}\left[R_{\boldsymbol{n}}(\phi)\right]_{\mu\nu}$$

감사합니다!