Assignment 2

Table of Contents

Part1 Question a

With constant resistance across the material and boundary conditions on left and right, the voltage drop should be a straight line (dV/dx is negative constant).

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a2p1q1_1D
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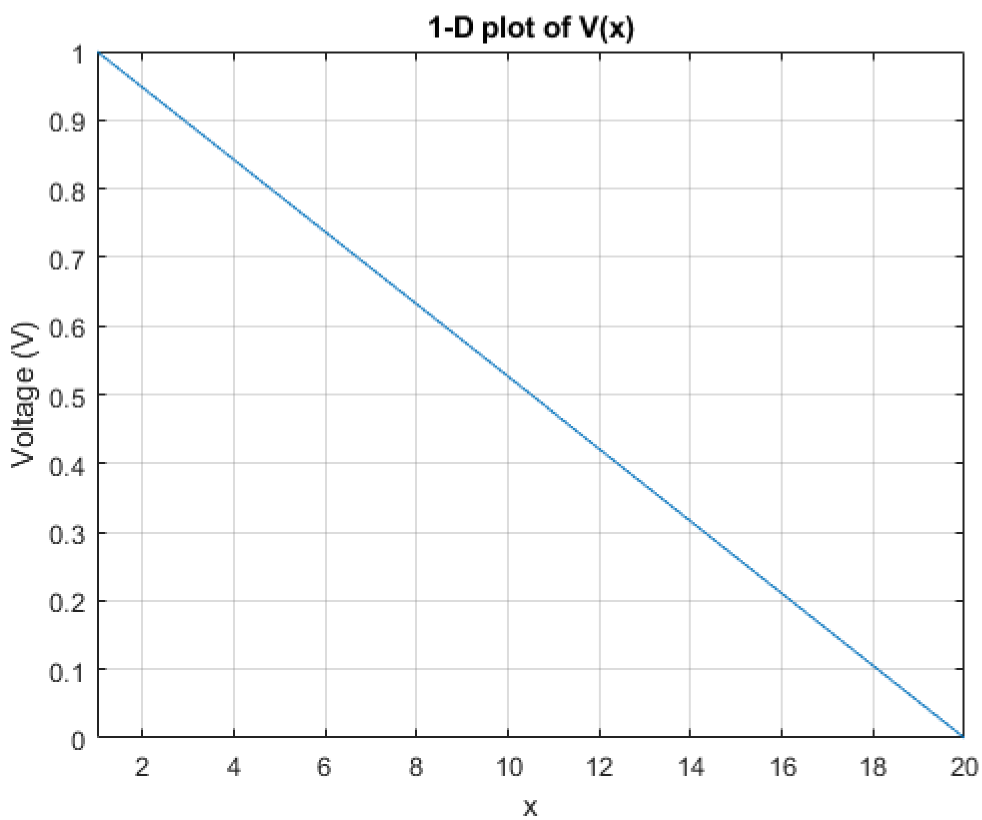

Part1 Question b analytical

The boundary conditions are two x boundaries fixed to V0(1V in this case), two other sides fixed to the ground. Due to constant conductance(sigma), the result is a smooth surface.

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a2p1q1_analytical
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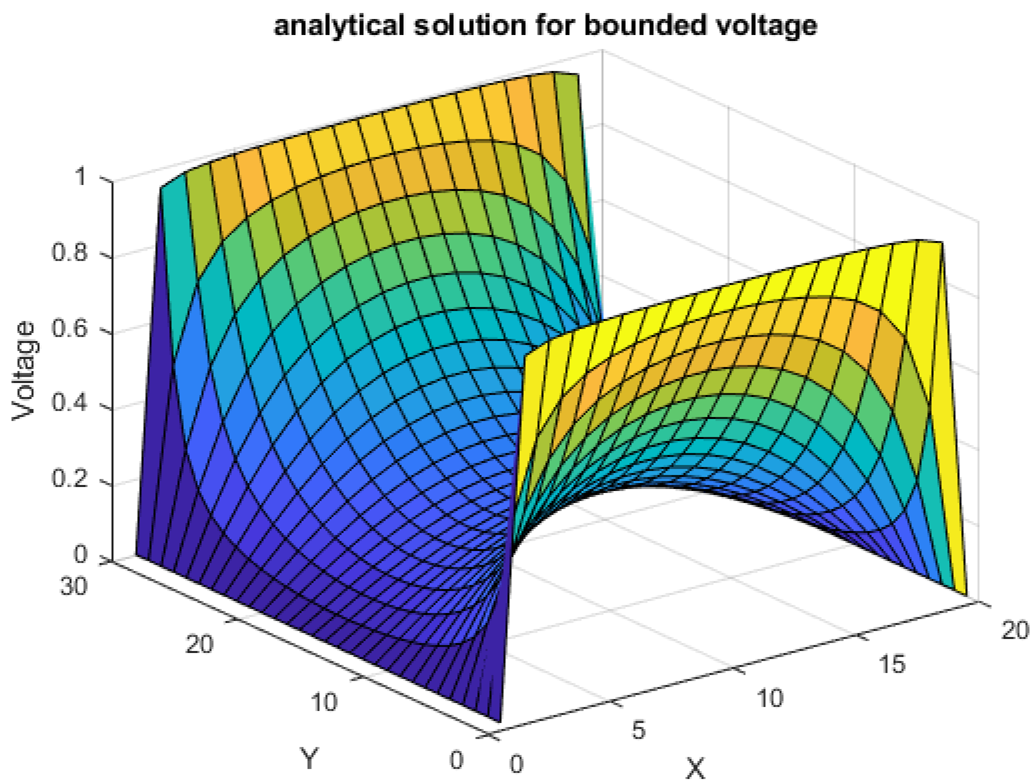


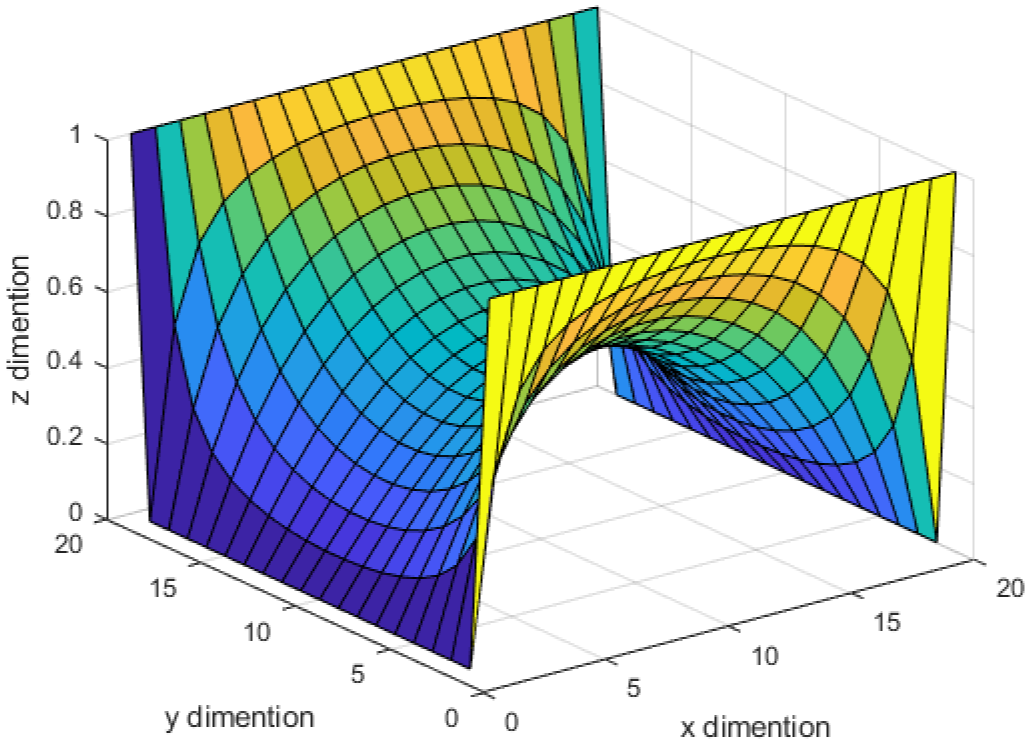
Part1 Question b numerical

The boundary conditions are two sides fixed to V0 (1V) and two sides fixed to the ground. The current flow from 1V to 0V, with a 90degree change of direction. The curve is smooth because of constant conductance.

The accuracy of the numerical method depends on the mesh size. On the other hand, the accuracy of the analytical solution depends on the number of the series (N=1,3,5...) included. The more series included, the more accurate result can be expected, but due to the infinite number of series available, the result is still an (close enough) approximation.

Notice for the numeric case the four corners are different from analytical. This is because in analytical V(0,0)=0, due to Y boundary condition; in numerical V(0,0)=1, due to the X boundary condition. All other points are similar if not the same.

```
a2p1q1_numeric
```



Part2 Question a

This part changes the material so the sigma is no longer constant. The main idea is the current flow across three parallel resistors! One resistor has a sigma of 1 (bottleneck), and the other two box resistors have a sigma of 0.01.

Sigma plot shows a lower conductivity in the box region, as expected. Therefore more current will flow across the higher conductivity region since the resistors on the sides are shorted.

The voltage map shows voltage mainly drops across the middle region, via highly conductive region. The box regions have almost zero voltage drop, since almost no current flow across it (shorted).

The current density and the electrical field plots are almost identical. This is due to ohms law:

$$I = V/R$$

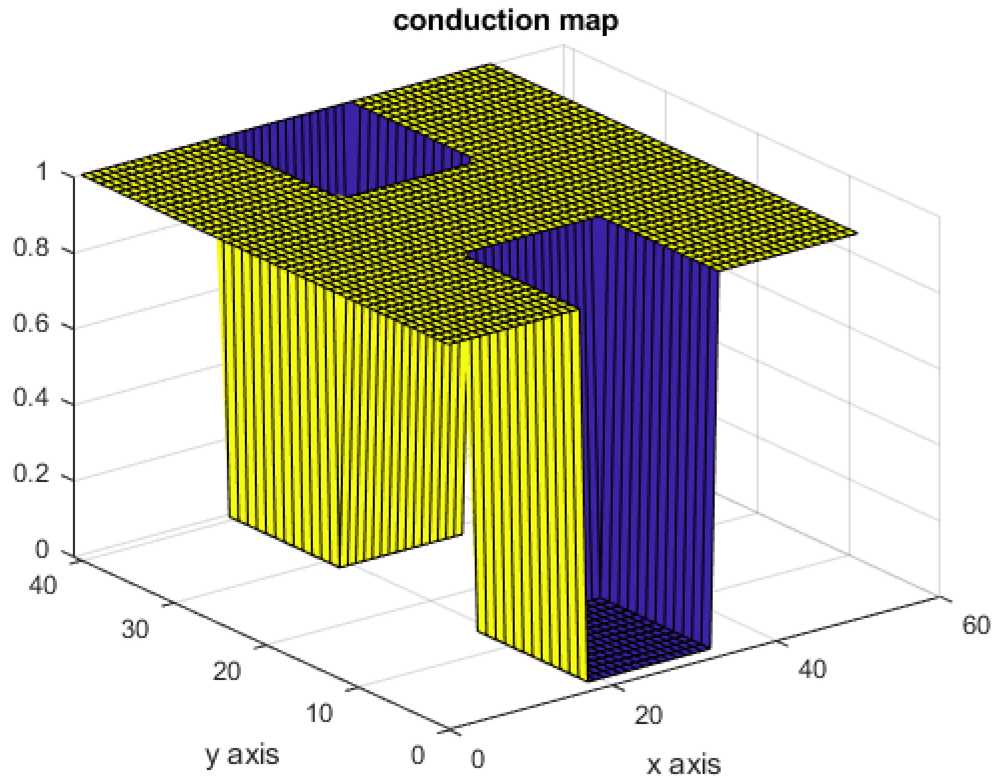
$$dI/dx = (dV/dx)/(dR/dx)$$

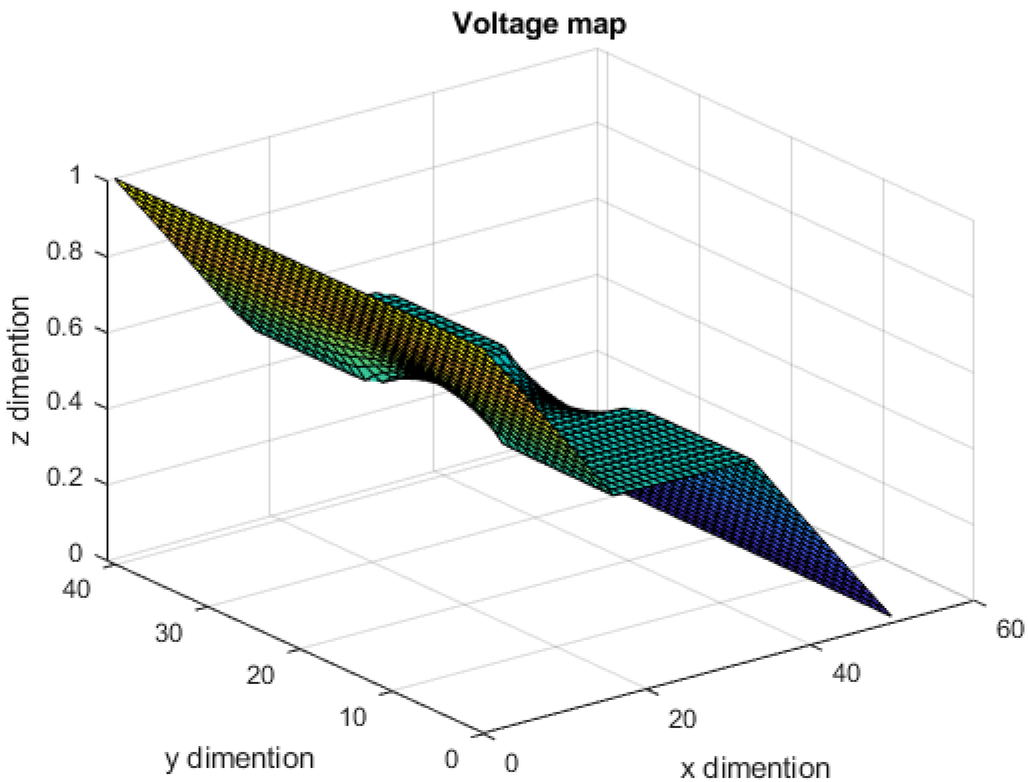
where R is a constant inside box and outside the box, dR/dx is zero; dV/dx is the electrical field, dI/dx is the current density

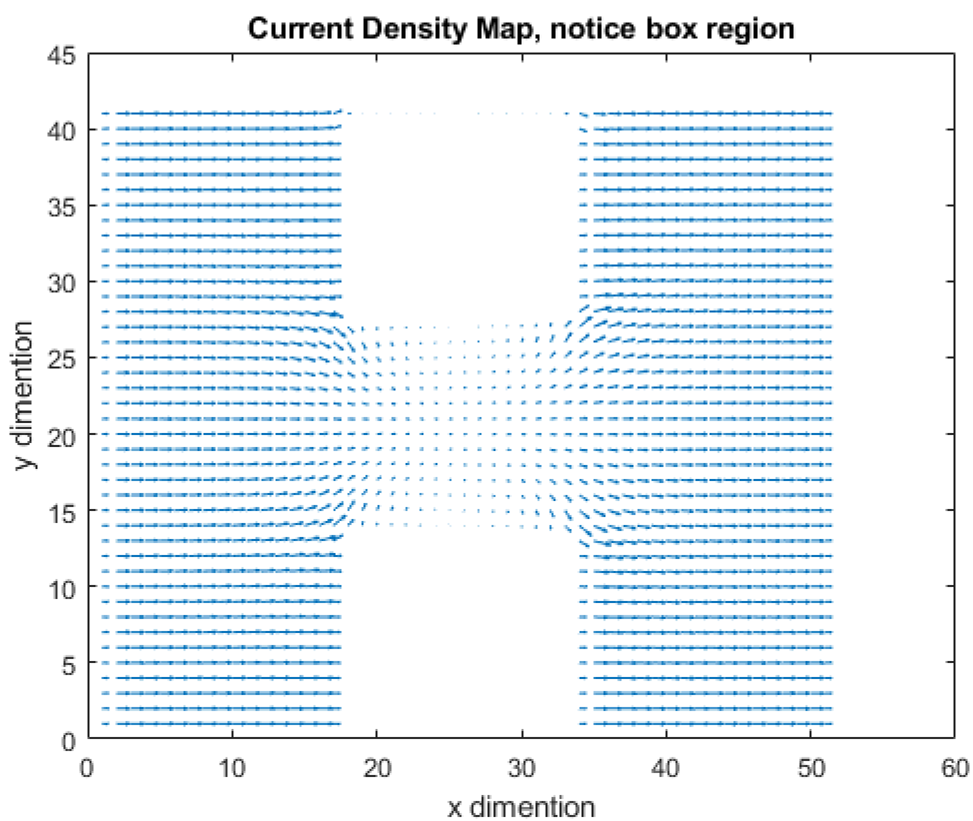
$$J = E/R$$

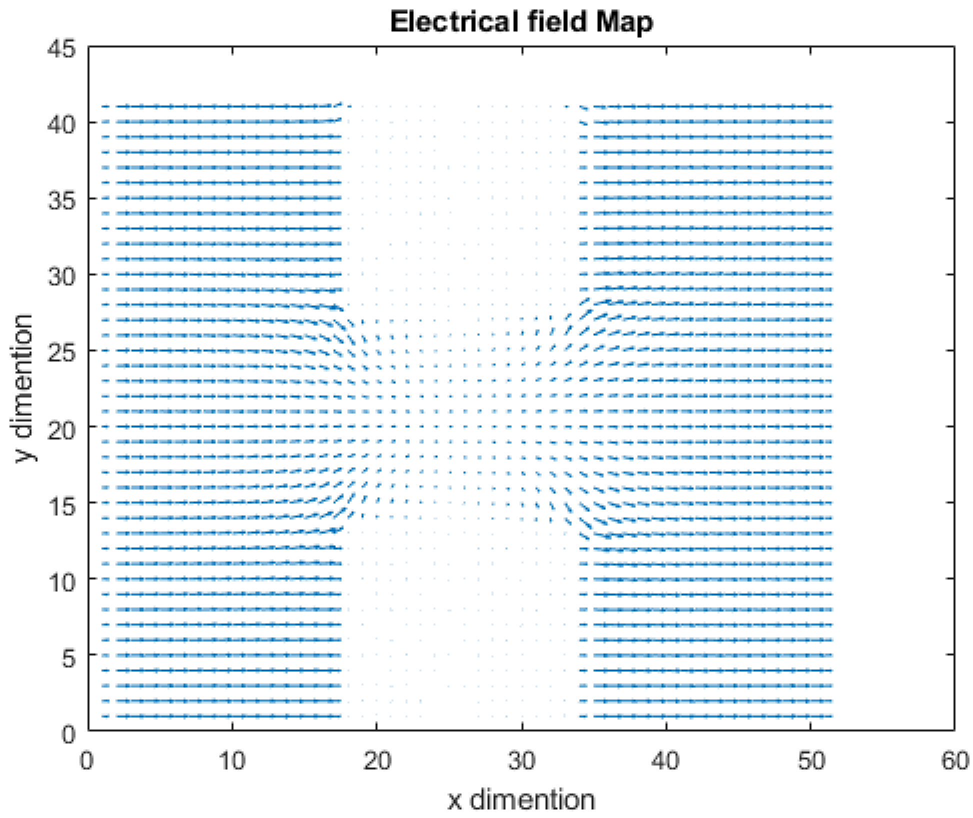
where J and E are vectors, R is scalar. Therefore the J matrix and E matrix are going to be identical in direction, different in magnitude. More specifically, the magnitude is going to be the same as well outside the box region, with a resistance equals to 1.

a2p2q1




Voltage map
z dimention
y dimention
x dimention
1
0.8
0.6
0.4
0.2
0
40
30
20
10
0
0
20
40
60


Current Density Map, notice box region
y dimention
x dimention
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
60



Part2 Question b,c,d

The current drawing in the graph is the current in the sigma = 1 region. The current is NOT constant from left to right:

$$sum(Jx(column1)! = sum(Jx(column2))! = sum(Jx(column_Others))$$

The main idea of this part is simple. The voltage drop across the whole region is constant (1V) while the box region varies. As the box region increases resistivity/size, lower voltage is dropped over the box region. Therefore more voltage has to be dropped over regions outside of the boxes, hence higher current (with constant resistivity outside the boxed region).

To be honest, this is a bit counter-intuitive compared to the resistor analogy made before. This is due to the current is not constant across x direct, so the current divider does not work!

The increment in mesh size does not highly affect the current, with a size smaller than 8. Higher mesh size (lower resolution) makes the box effectively larger and bottleneck smaller. Therefore a higher current is observed. Similar case for the increased box size, since both effectively sweep the size of the box.

Higher sigma means a higher conductivity in the box region, more voltage is dropped over the boxed region, less voltage drop over region outside the box, less current on the edge is observed!

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a2p2_b
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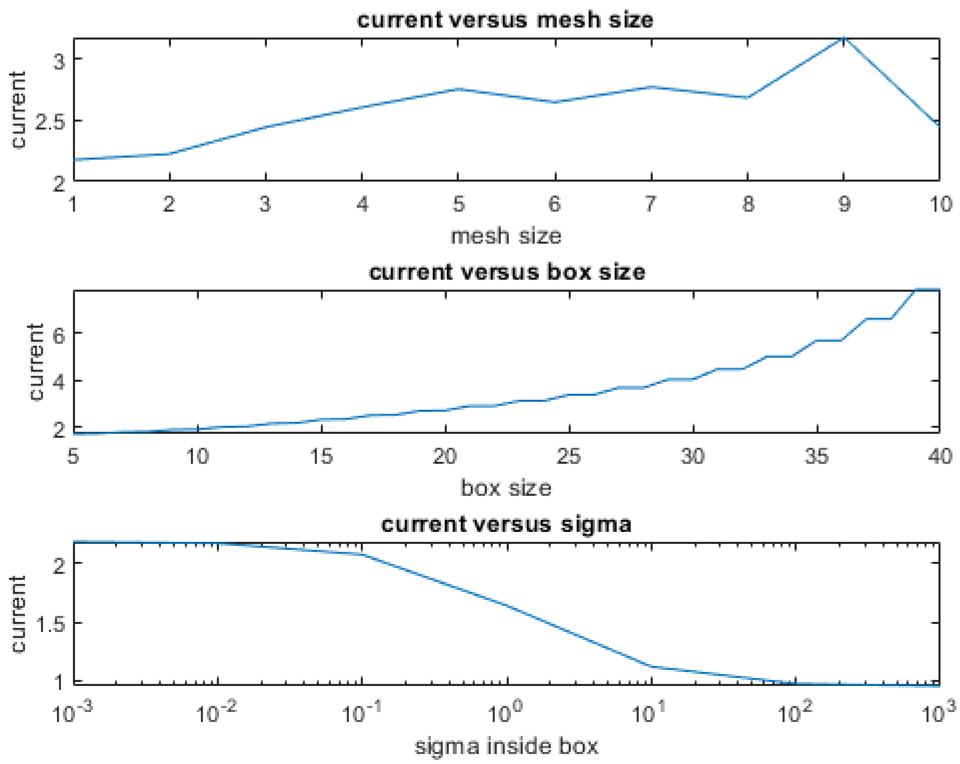

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