Assignment 2

Table of Contents

Part1 Question a

With constant resistance across the material and boundary conditions on left and right, the voltage drop should be a straight line (dV/dx is negative constant).

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a2p1q1_1D
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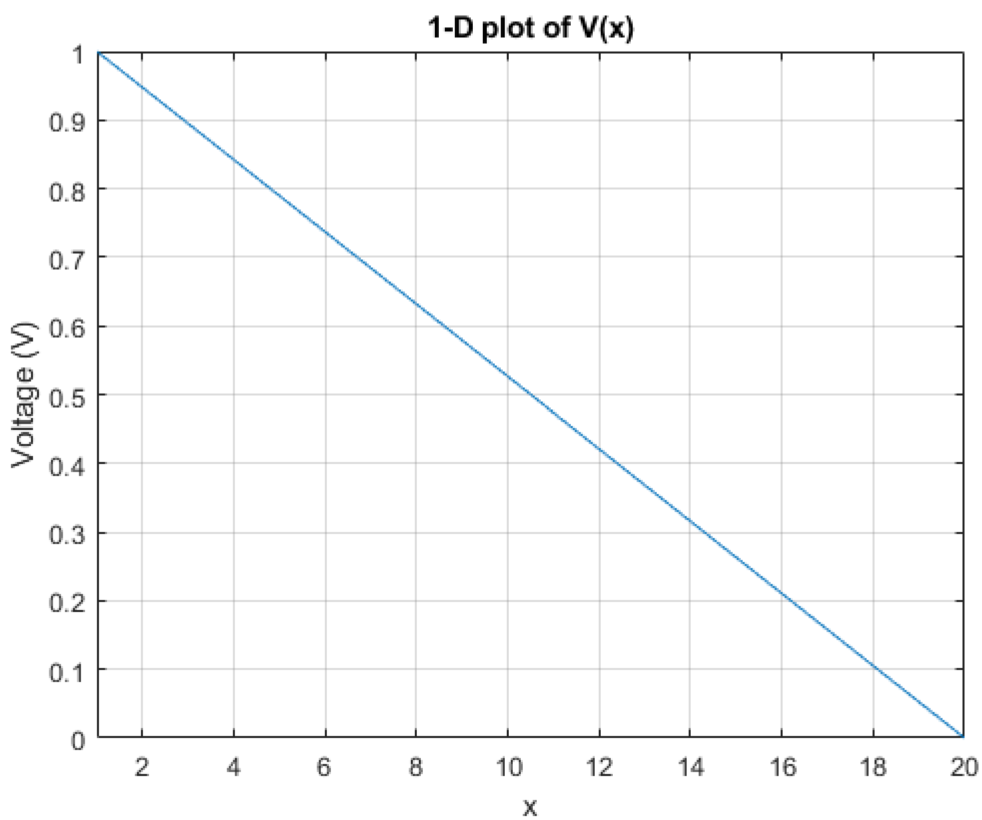

Part1 Question b analytical

The boundary conditions are two x boundaries fixed to V0(1V in this case), two other sides fixed to the ground. Due to constant conductance(sigma), the result is a smooth surface.

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a2p1q1_analytical
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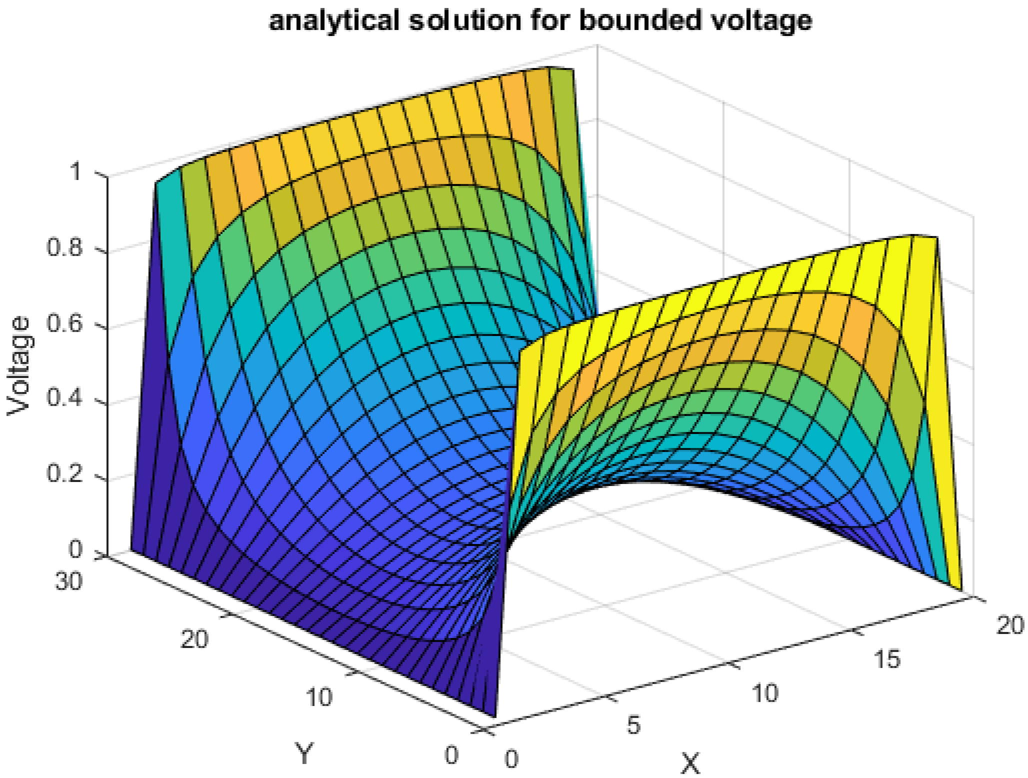


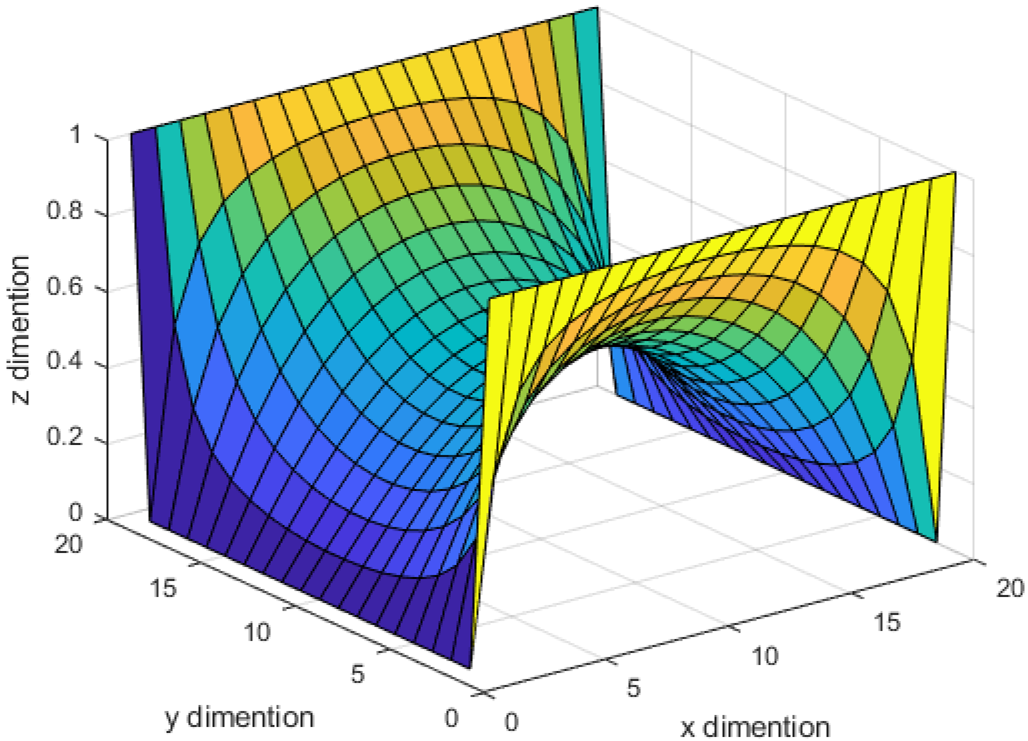
Part1 Question b numerical

The boundary conditions are two sides fixed to V0 (1V) and two sides fixed to the ground. The current flow from 1V to 0V, with a 90degree change of direction. The curve is smooth because of constant conductance.

The accuracy of the numerical method depends on the mesh size. On the other hand, the accuracy of the analytical solution depends on the number of the series (N=1,3,5...) included. The more series included, the more accurate result can be expected, but due to the infinite number of series available, the result is still an (close enough) approximation.

Notice for the numeric case the four corners are different from analytical. This is because in analytical V(0,0)=0, due to Y boundary condition; in numerical V(0,0)=1, due to the X boundary condition. All other points are similar if not the same.

```
a2p1q1_numeric
```



Part2 Question a

This part changes the material so the sigma is no longer constant. The main idea is the current flow across three parallel resistors! One resistor has a sigma of 1 (bottleneck), and the other two box resistors have a sigma of 0.01.

Sigma plot shows a lower conductivity in the box region, as expected. Therefore more current will flow across the higher conductivity region since the resistors on the sides are shorted.

The voltage map shows the box regions have sharp voltage drop, due to the high resistivity. The elecctrical field shows a higher electric field in box regions (agrees with the sharp slope of voltage) . Most importantly, the electric field shows the direction of the field is avoiding the highly resistive area.

The current density map is the most straight forward plot, where more current flows across middle highly conductive area. This is like a lower resistance shorts the high resistances on the sides.

The current density and the electrical field plots are almost identical in direction. This is due to ohms law:

$$I = V/R$$

$$dI/dx = (dV/dx)/(dR/dx)$$

where R is a constant inside box and outside the box, dR/dx is zero; dV/dx is the electrical field, dI/dx is the current density

$$J = E/R$$

where J and E are vectors, R is scalar. Therefore the J matrix and E matrix are going to be identical in direction, different in magnitude. More specifically, the magnitude is going to be the same as well outside the box region, with a resistance equals to 1.

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a2p2q1
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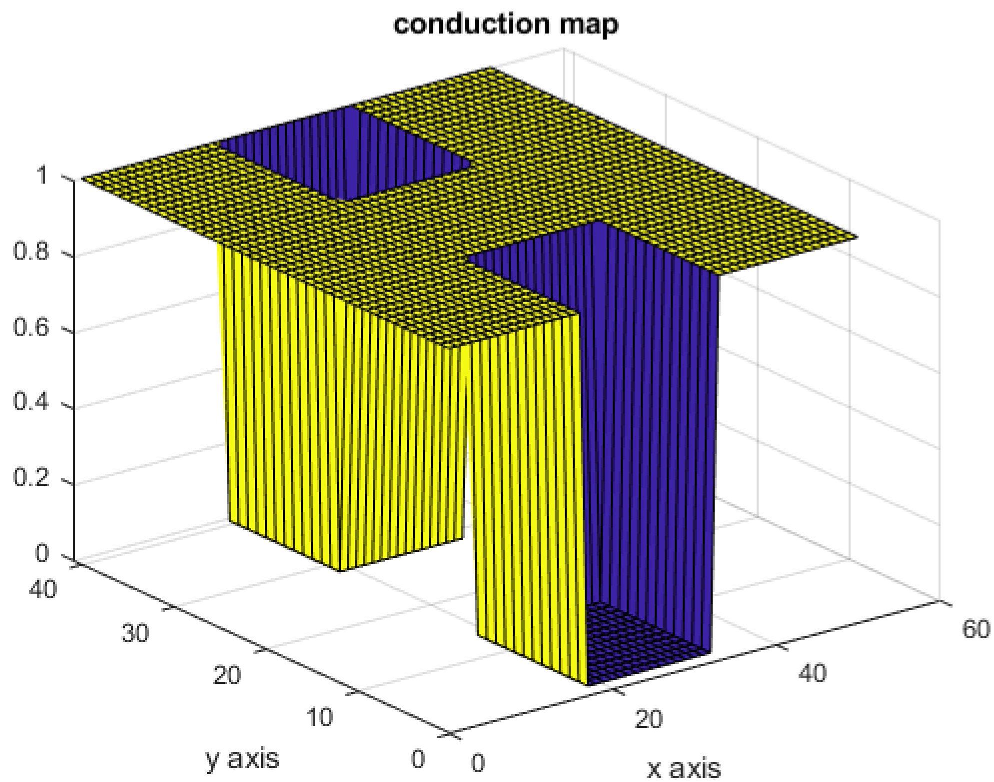
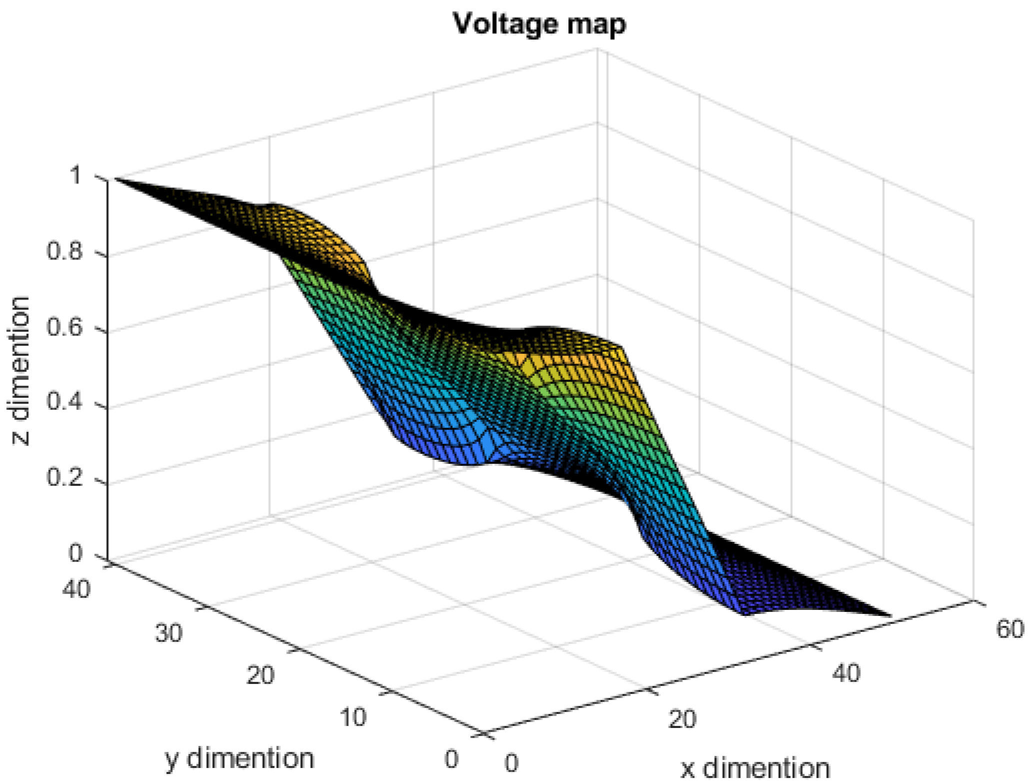
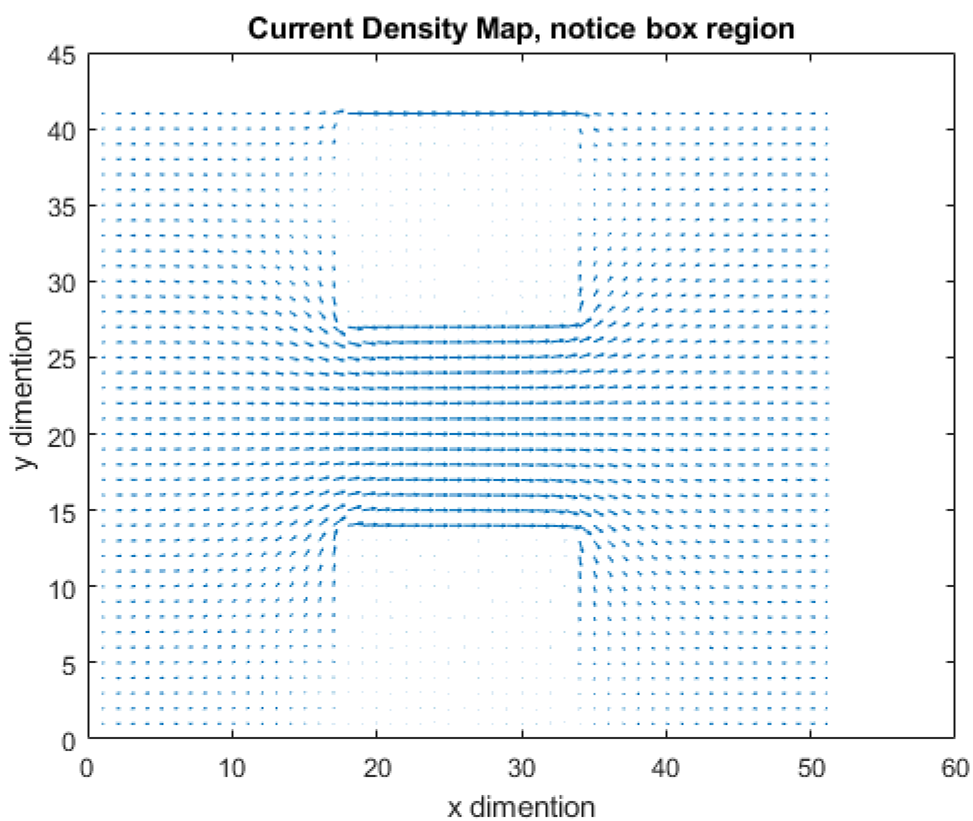
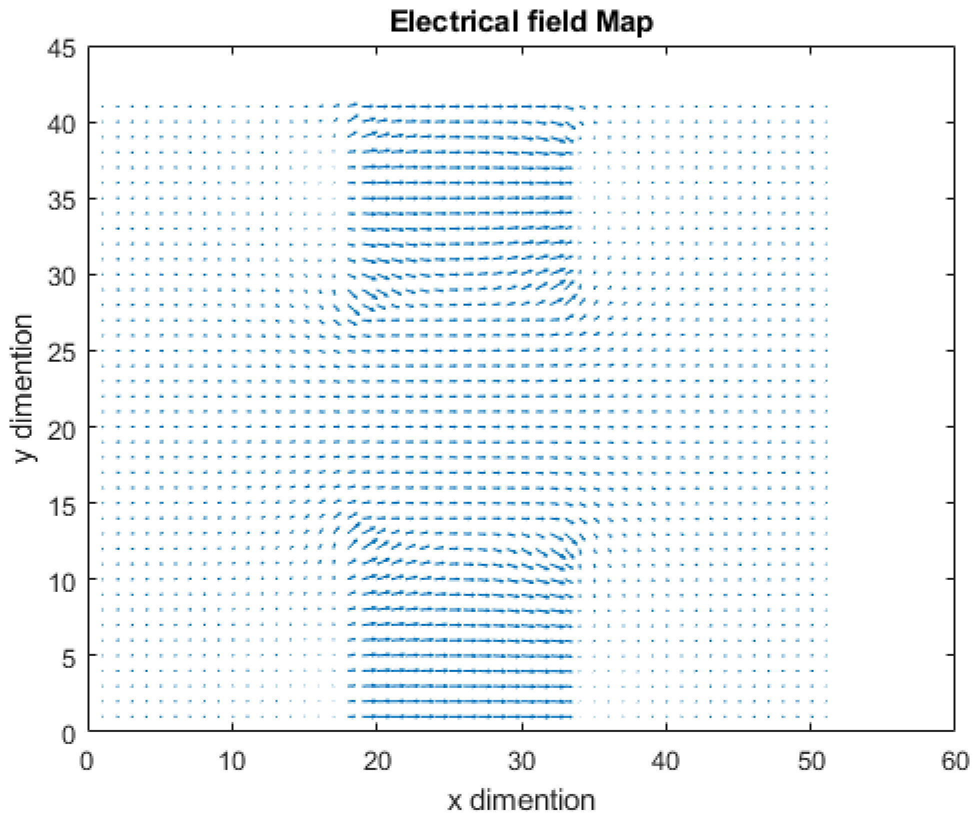



Voltage map
z dimention
1
0.8
0.6
0.4
0.2
0
40
30
20
10
0
y dimention
0
20
40
60
x dimention


Current Density Map, notice box region
45
40
35
30
25
20
15
10
5
0
y dimention
0
10
20
30
40
50
60
x dimention



Part2 Question b,c,d

The increment in mesh size does not highly affect the current. The increase in mesh size indicates a lower resolution, which means a higher error.

The total resistance across the area is: $Rtotal = Rflat + Rbox||Rchannel||Rbox$

Where the Rflat and Rchannel resistivity is constant (1 ohm/unit), the size and sigma changes the box resistance. Notice although resistivity of Rflat and Rchannel does not change, the resistance might change due to the change of dimension.

Increase in box size with constant sigma cause an increase in resistance for Rbox and Rchannel, therefore a decrease in current is expected according to ohm's law. Notice the sharp change in current after the box size>21. This is because y demension maximum is 41, so the two high resistance area merged and Rchannel term is gone. After the change, the change of current versus change of box size can be approximated as linear relationship. This is due to geometric relationship between resistivity and the resistance:

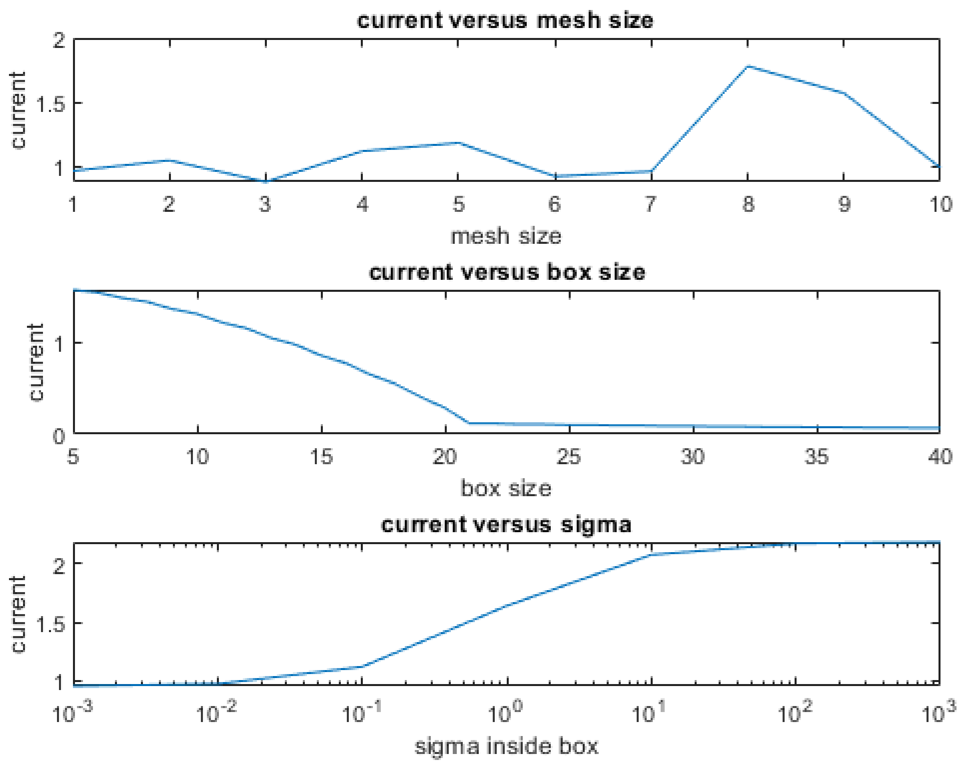
$$R = rho * l/A$$

where in this case l is the independent variable, and A is constant 41 (2D case, the area is the cross-sectional length).

Higher sigma means a higher conductivity, lower resistivity in the box region. The resistance is dominated by the smallest terms in the above resistors, since higher terms will be 'shorted'. This explains why the current on left and right extremes are quite constant.

The total resistance (Rtotal) is decreased if box resistance (Rbox) is decreased, this leads to a increasing trend in current.

`a2p2_b`



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