IE 643 ; Assignment-1 , Buay (213101002)

Q. 1 Here,

$H = (w, b)$ is hyperplane, $w \neq \vec{0} \in \mathbb{R}^d$, $b \in \mathbb{R}$.

$= \{ x \in \mathbb{R}^d : \langle w, x \rangle = b \}$

Eqn of first hyperplane:

$\langle w, x \rangle - b = 0$

Let $\tilde{w} = \alpha w$

and $\tilde{H} = (\tilde{w}, \tilde{b})$ be new hyperplane.

Such that $\|\tilde{w}\|_2 = \beta > 0$ (L_2-norm $\|\cdot\|$)

$\Rightarrow \alpha \|w\| = \beta$

$\alpha = \dfrac{\beta}{\|w\|}$

Since $\beta > 0$,

$\alpha > 0$. as $\|w\| > 0$.

Also, $\tilde{w} = \alpha w$

$\tilde{w} = \dfrac{\beta}{\|w\|} w$

Similarly, $\tilde{b} = \dfrac{\beta}{\|w\|} b$

This is the required relationship between the two different hyperplanes.

1. (b) Since $M \leq \frac{R^2 \|w^*\|^2}{\gamma^2}$

let $w^* = \frac{\beta}{\|w\|} w$ from above result with $\beta > 0$.

$\|w^*\| = \beta$

$\|w^*\|^2 = \beta^2 > 0$

$\therefore \frac{R^2 \|w^*\|^2}{\gamma^2} = \frac{R^2}{\eta^2}$

where, $\eta = \frac{\gamma}{\beta} > 0$ (because $\gamma > 0, \beta > 0$)

So, $\boxed{M \leq \frac{R^2}{\eta^2}}$

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Question (2):

Starting point $w^0 = [\theta \ \theta \dots \theta]^T \quad \theta \in [0, 1]$

(a) Here initially the hyperplane or line will always be making 45° angle since $w[0] = w[1] = \dots$ ~~w~~.

If 'n' is the number of data points in the sample then at the beginning, it can make at most n wrong predictions.

(b) The bound on the no. of mistakes obtained in (a) is tighter than ~~part~~ one shown in class. It is because of the choice of w^0 and how θ lies between 0 ~~and~~ 1.

(c) 'n' is the maximum number of mistakes that is possible. Hence, it is tight by pigeonhole principle.