Q1)

Given $y^t \in \{+1, -1\}$ $\quad H = (w^*, b)$

$$y^t(\langle w^*, x^t \rangle + b^*) \geq \gamma > 0$$

Show, $\tilde{D} = \{(\tilde{x}^t, \tilde{y}^t)\}_{t=1,2,3}$ exists

where $\tilde{x}^t = \alpha^t x^t + \beta^t$ $\quad \tilde{y}^t = y^t$

(a) Hyperplane for $\tilde{D}$ $\quad H = (v^*, 0)$

Proof

Given dataset, $x^t \in R^d$ which satisfies

$$y^t(w^* x^t + b^*) \geq \gamma$$

considering the transformation (A)

$$y^t(w^* (A x^t) + b^*) \geq \gamma$$

Here I have considered A as an universal $(d \times d)$ affine transformation Matrix:

So $y^t([A w^*]^T x^t + b^*) \geq \gamma$

$$\Rightarrow y^t((w^*)' x^t + (b^*)') \geq \gamma \longrightarrow (1)$$

Now we get a new $(w^*)'$ weighted vector.

Thus we can hence prove that the dataset is linearly separable under any kind of transformation.

Given $H(v^*, 0)$ where v^* is the transformed Matrix.

From the previous Eqn ① we can ensure that there exist some z^* where $z^* x^t = (C^{*x})'$ thus giving rise to $y^t(\underbrace{(w^{*\prime} + z^*)}_{v^*} x^t) \geq \gamma$

Hence proved there exist a $(v^*, 0)$ separating Hyperplane.

Now,

$$\tilde{y}^t(\langle v^*, u^t x^t + \beta^t \rangle + 0) \geq \gamma$$

$$\tilde{y}^t(\langle v^*, u^t x^t + \beta^t \rangle) \geq \gamma$$

$$y^t(\langle v^*, u^t x^t + \beta^t \rangle) \geq \gamma$$

$$y^t(v^* u^t x^t + v^* \beta^t) \geq \gamma$$

$$\boxed{v^* = v^* \beta^t \qquad w^* = v^* u^t}$$

→ Obtained by comparing the Equations

So $u^t = \dfrac{w^*}{v^*}$ $\qquad \beta^t = \dfrac{v^*}{v^*}$

Now, given $x^t \in R^d$.

So since $\gamma \in W$ $\qquad v^*, \beta^t, u^t \in R^d$.

Now, for α-S to hold true

$$\|\tilde{x}^t - x^t\|_2 = \|x^t\|\underbrace{(\mu^t - 1)}_{\text{Constant}} + \underbrace{\beta^t}_{\text{Constant}}$$

So if we consider $\mu_t \in$ Scalar, then.

$$\boxed{\mu_t = K \in W \ \{\text{Whole Number}\}}$$

If we are considering $\mu_t \in$ Vector then:

$$U^* \mu_t = W^*$$

However $U^* \mu_t$ = Single Value since this will be a Dot product.

(Thus μ_t cannot be vector. It is a Scalar)

For, $b^* = U^* \beta^t$

$$\boxed{b^* = \sum_{i=0}^{d} U_i^* \beta_i^t}$$

β^t is a Vector.

Relation between W^* & U^*:

$$W^* = K U^*$$

P.T.O

(6) Absence of b^* in $y^t(\langle w^*, x^t \rangle) \geq \gamma$

→ In this case, we are considering w^* to be a complete set of weights including bias.

→ $w^* = [\theta_0 \ \theta_1 \ \cdots \ \theta_d]$

Included Bias term.

→ This helps me with the ease of calculations & applying all the terms in Matrix Calculation.